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ORIGINAL PAPER

Dynamic consensus of nonlinear time-delay multi-agent systems with input saturation: an impulsive control algorithm

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Abstract This paper aims to solve the dynamic consensus problem for a class of nonlinear multi-agent systems with input saturation and time delay. Due to the existing nonlinearity of the system, the low-gain feedback method widely used to handle saturation in multi-agent systems is no longer applicable. Moreover, to reduce both the communication and control energy consumption, an impulsive control algorithm is designed. Based on the stability theory of impulsive systems, as well as the property of the Laplacian matrix and convex hull, the set invariance conditions in the format of LMI are obtained. In addition, an optimization method is proposed for simultaneously designing the control parameters and assessing the attraction domain. Finally, the performance of the proposed consensus algorithms is demonstrated by two numerical experiments.

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1 Introduction

A multi-agent system (MAS) refers to a system formed by a group of autonomous natural or artificial individuals through pairwise interactions [12,16,33]. Such systems exist widely in nature, such as fish schools [7], birds flocks [3], and so on. The underlying cooperative mechanism of global cooperative behavior has attracted wide attention of scholars in many research fields [30]. In the current research on MASs, a basic problem is consensus control, which focuses on designing a distributed controller for each agent that relies only on neighboring information, so that all agents reach the consensus of designed behaviors. Consensus control is a fundamental core subject in the field of MAS collaborative control [15,17,40,43]. It may provide some insight and potential application prospects in formation control, multi-sensor information fusion, smart grid, and other related distributed systems [18,19,23,30].

Considering the limitation of energy storage of each agent device, it is crucial to reduce energy consumption for MASs. In order to reduce both communication and control energy consumption, corresponding researches have proposed sampling control [34,38,41], event-triggered control [5,22,37], impulsive control [29,31,36] and other related methods. These methods have achieved remarkable results in reducing communication energy consumption by collecting state information of neighboring agents only at discrete time instants. However, controllers designed based on the first two control methods often require to have a zero-order holder, which results in the fact that although the control commands update at discrete time instants, the control output is not interrupted. Unlike this, an impulsive control method not only collects information from neighboring agents at impulsive moments, but also outputs control signals simultaneously. Therefore, impulsive control methods can effectively save both communication and control energy. Additionally, it should be noted that time delay is often encountered in real control systems [6,32,35], which is significant to consider for the impulsive consensus of time-delay MASs. In [26], the exponential leader-following consensus problem is solved for a class of nonlinear MASs which have unknown time-varying bounded delays and partial mixed impulses. In [9], the distributed impulsive control method is proposed to solve the control problem of networked leader-following consensus of nonlinear MASs. With the consideration of time delay induced by network, a nonlinear MAS with time-delay impulses is formulated, and a general consensus criterion is proposed for several cases of network-induced time delays. In [8], the consensus tracking problem is addressed based on a memory sampled-data control method for a class of MASs with communication delay. In [14], a hybrid consensus protocol for MASs with both fixed and switching topologies is proposed, which considers continuous time communications among agents and information exchanges at delayed instants on a sequence of discrete times.

However, none of the aforementioned studies considers the effect of input saturation on MASs, and as far as the authors know, there is currently no research on the consensus control of nonlinear time-delay MASs with input saturation based on impulsive control method. It is necessary to consider the existing input saturation in the control system, since the system output cannot be increased, if it exceeds the limitations of space, energy, and actuator structure in real control systems [24]. Still, ignoring input saturation in the system may induce instability to the MAS. Recently, consensus control of linear saturated MASs has attracted considerable attention from many researches [4,21,25,27,39,42]. The leader-following consensus control protocol was designed for the first time in [25] for linear saturated MASs. Following the research line, other previous studies considered input saturation and/or external disturbances based on algebraic Riccati equation, event-triggered method, self-triggered method, and observer-based consensus tracking method [4,21,27,39,42]. Note that the low-gain feedback method is adopted in all the previous works, which requires that the system is asymptotically null controllable with bounded controls [25]. However, in most practical control systems, the cooperated agents always have nonlinear features [13,28]. It is urgent to design effective anti-saturation algorithms for nonlinear MASs, since the present method cannot be directly extended to nonlinear MASs with input saturation. Moreover, they all require the system to be asymptotically null controllable with bounded controls, which can only achieve consensus where the states eventually tend to zero (or an amplitude oscillation), but not dynamic consensus.

To fill the research gap, the objective of this paper is to propose impulsive consensus algorithms for a class of nonlinear MASs with input saturation and time delay. Accordingly, set invariance conditions in the format of LMI are derived based on the stability theory for analyzing impulsive system, as well as the property of Laplacian matrix and convex hull. In addition, by enlarging the covering area of the shape reference set, the attraction domain estimation is obtained. During estimating the attraction domain, by regarding the impulsive control gain as an index in the process of LMI optimization, the proposed distributed impulsive consensus method can ensure all saturated individuals with time delay can achieve dynamic consensus. The main contributions of the paper are summarized as follows:

- (i) It is the first time to propose an impulsive control algorithm for nonlinear MASs with time delay and input saturation.
- (ii) Compared with previous researches on consensus control of MASs with input saturation [4,21,25,27,39,42], the anti-saturation control method designed in this paper can make the nonlinear agent state reach dynamic consensus with exponential convergence.
- (iii) The proposed LMI optimization algorithm can be used to simultaneously design the parameters of the controller and assess the attraction domain, which is convenient to make the attraction domain as large as possible by using the existing MATLAB functions

The following notations are useful in facilitating the analysis. $\mathbb{R}$ is the set of real number. $\mathbb{C}([-\tau^*, 0], \mathbb{R}^n)$ means the set of continuous functions from $[-\tau^*, 0]$ to $\mathbb{R}^n$. $\mathbb{PC}$ denotes the class of piecewise right continuous function. For a given matrix with $m \times n$ elements, $\mathbb{R}^{m\times n}$ means the set of $m \times n$ real matrix. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ represent the minimum and maximum eigenvalues of A, respectively. $rank(A)$ is the rank of A. A^T and A^{-1} mean the transposition and inverse of matrix A. If A is a symmetric positive-definite matrix, then $A > 0$, the rest may be deduced by analogy and so forth. $\mathbf{1}_n$ means a n-dimensional vector with all elements 1. I_n denotes the identity matrix of order n. $\| \cdot \|$ denotes the standard Euclidean norm. $| u |$ represents the absolute value of scalar u. For a given function $g(t) \in [W, R]$, $W = [0, +\infty)$, the upper right-hand derivative is denoted as $D^+g(t) = \overline{lim}_{h\to 0^+} \frac{1}{h}(g(t+h) - g(t))$, where D is the distributional derivative.

2 Preliminaries and problem formulation

2.1 Algebraic graph theory

In the present study, the group of agents communicate under the topology modeled by an undirected graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$. Therein, $\mathscr{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ denote the set of nodes and the set of edges, respectively. Accordingly, $A = (a_{ij})$ denotes the adjacency matrix, where $a_{ii} = 0$, $a_{ij} = 1$ if $(i, j) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise, and the elements of the corresponding Laplacian matrix $\mathscr{L}$ are defined as $\mathscr{L}_{ij} = -a_{ij}$ if $i \neq j$, and $\mathscr{L}_{ii} = \sum_{j\neq i} a_{ij}$. This study considers the MAS consisting of N agents with labels being $\{1, \ldots, N\}$ under the undirected communication topology.

Assumption 1 *The undirected graph $\mathscr{G}$ which denotes the communication topology is connected.*

2.2 Problem formulation

Each agent has the following nonlinear dynamics:

$$\begin{aligned}\dot{x}_i(t) = {} & Ax_i(t) + f_1(x_i(t)) \\ & + f_2(x_i(t-\tau(t))) + U_i(t),\end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $x_i(t) \in \mathbb{R}^n$ means the state of node i, $i = 1, \ldots, N$, $\tau(t)$ is time-varying delay satisfying $0 < \tau(t) < \tau^*$, $f_1(\cdot)$, $f_2(\cdot)$ denote continuously nonlinear differentiable function, and $U_i(t)$ denotes the input for agent i.

Assumption 2 $f_1(\cdot)$, $f_2(\cdot)$ *satisfies the Lipschitz condition, i.e., a constant L exists such that* $\| f_j(x) - f_j(y) \| \leq L \| x - y \|$, $\forall x, y \in \mathbb{R}^n$, $j = 1, 2$.

The designed impulsive controller in this paper is as follows:

$$U_i(t) = \sum_{k=1}^{\infty} Sat(u_i(t))\delta(t - t_k), \quad i = 1, \ldots, N, \tag{2}$$

where $\delta(t - t_k)$ denotes the Dirac function, $\{t_k\}$ is the time sequence satisfying $0 < t_0 < t_1 < \ldots < t_k < t_{k+1} < \ldots$ and $\lim_{k\to+\infty} t_k = +\infty$, $t_{k+1} - t_k \leq \alpha$, $\alpha > 0$. At time instant t_k, jumps in the state variable $x_i(t)$ are denoted by $\Delta x_i(t_k) = x_i(t_i^+) - x_i(t_k^-)$, $x_i(t_k^+) = x_i(t_k)$ and $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$, and $Sat(u_i(t))$ represents a saturation function with $Sat(u_i(t)) = [Sat(u_{i1}(t)), \ldots, Sat(u_{in}(t))]^T$,

$$Sat(u_{im}(t)) = \begin{cases} 1, & u_{im}(t) > 1, \\ u_{im}(t), & -1 \leq u_{im}(t) \leq 1, \quad m = 1, \ldots, n. \\ -1, & u_{im}(t) < -1. \end{cases} \tag{3}$$

Design

$$u_i(t) = -K \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), \quad i = 1, \ldots, N, \tag{4}$$

where $K \in \mathbb{R}^{n\times n}$.

Remark 1 When $f_1(x_i(t)) = 0$, $f_2(x_i(t-\tau(t))) = 0$, there are mainly two approaches in the saturation system theory [10]. Firstly, $u(t) = -(\mathscr{L} \otimes I_n)x$, where $x = [x_1^T, \ldots, x_N^T]^T$. Secondly, $u(t) = -(\varpi\mathscr{L}\otimes I_n)x$, where ϖ is a scalar larger than zero. The first approach can always locally address the consensus control problem, because a positive scalar ι always exists such that all the states from $\chi = \{x \in \mathbb{R}^n : \| x \| < \iota\}$ satisfy $\| (\mathscr{L} \otimes I_n)x \|_\infty \leqslant 1$. When $\varpi \to 0$, the second approach is a semi-global one. The essential idea of avoiding saturation in the first approach is by restriction of the feasible domain, which has a drawback that it has a small interesting domain. The second one is by selection of sufficiently small connection weights, resulting in a slow convergence rate. This paper adopts the combination of real and virtual controllers to deal with input saturation, which is different from the two previous methods. The design of virtual controller is similar to the first method, whereas the output of real controller can exceed the saturation limit. Comparing with the second method that can generally obtain asymptotical consensus, the proposed method can obtain exponential consensus. Therefore, the MAS has a larger interesting domain compared to the first method, and faster convergence rate to the second method.

Therefore, system (1) can be rewritten as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + f_1(x_i(t)) \\ \qquad\quad + f_2(x_i(t-\tau(t))),\ t \neq t_k, \\ \Delta x_i(t_k) = Sat(u_i(t_k^-)). \end{cases} \tag{5}$$

Thus, the error system is written as follows

$$\begin{cases} \dot{e}(t) = (I_N \otimes A)e(t) + f_1(x(t)) - \mathbf{1}_N \otimes \overline{f}_1(x(t)) \\ + f_2(x(t-\tau(t))) - \mathbf{1}_N \otimes \overline{f}_2(x(t-\tau(t))),\ t \neq t_k, \\ \Delta e(t_k) = Sat(u(t_k^-)) - \mathbf{1}_N \otimes \overline{Sat}(u(t_k^-)). \end{cases} \tag{6}$$

where $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, $e_i(t) = x_i(t) - \bar{x}(t)$, $\bar{x}(t) = \frac{1}{N}\sum_{j=1}^{N} x_j(t)$, $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $f_1(x(\cdot)) = [f_1^T(x_1(\cdot)), \ldots, f_1^T(x_N(\cdot))]^T$, $\overline{f}_1(x(\cdot)) = \frac{1}{N}\sum_{j=1}^{N} f_1(x_j(\cdot))$, $f_2(x(\cdot))$ and $\overline{f}_2(x(\cdot))$, $Sat(u(\cdot))$ and $\overline{Sat}(u(\cdot))$ are defined analogously.

The initial conditions of the dynamical system (6) are

$$e_i(t) = \phi_i(t),\ \ -\tau^* \le t \le 0,\quad i = 1, \ldots, N, \tag{7}$$

where $\phi_i(t) \in \mathbb{C}([-\tau^*, 0], \mathbb{R}^n)$.

Define the region in the state space with no saturation occurring as

$$\mathscr{M}(K) = \bigcap_{i=1}^{N} \mathscr{M}(K)_{[i]}, \tag{8}$$

where $\mathscr{M}(K)_{[i]} := \{e(t) : \| \ K \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) \|_\infty \leqslant 1\}$.

Inspired by [1], the definition on the contractively invariant set for the impulsive system (6) can be given as follows.

Definition 1 For any given $\beta > 0$, and $V(t) = e^T(t)e(t)$, the symmetric polyhedron $F(\beta) := \{e(t) : V(t) \leqslant \beta\}$ is defined as a contractively invariant set of (6), if and $\lim_{t\to\infty} e(t) \to 0$ for all $e(t) \in F(\beta)$.

Denote the set of $n \times n$ diagonal matrix as $\mathscr{D}$, where the diagonal elements are 0 or 1. There exist 2^n elements in $\mathscr{D}$ denoted as D_i, $i = 1, \ldots, 2^n$. The following lemmas are given to support the derivation.

Lemma 1 [10] *Let $u, v \in \mathbb{R}^n$ with $u = [u_1, u_2, \ldots, u_n]^T$ and $v = [v_1, v_2, \ldots, v_n]^T$. Suppose that $|v_i| \leqslant 1$ for $i = 1, \ldots, n$, thus*

$$Sat(u) \in co\{D_i u + (I - D_i)v :\ i = 1, \ldots, 2^n\}, \tag{9}$$

where $co\{\cdot\}$ represents the convex hull, and $D_i \in \mathscr{D}$.

Lemma 2 *Let $\mathscr{L} \in \mathbb{R}^{N\times N}$ denote the Laplacian matrix of a connected undirected graph, and matrix $B \in \mathbb{R}^{N\times N}$ has the following entries, $B = \frac{1}{N}\begin{bmatrix} N-1 & -1 & \ldots & -1 \\ -1 & N-1 & \ldots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \ldots & N-1 \end{bmatrix}$. Thus, a matrix Ξ will always exist such that $\mathscr{L} = \Xi B$, and Ξ has infinitely many solutions.*

Proof Let $\mathscr{L} = \left[l_1\ l_2 \ldots l_N\right] = \left[\zeta_1\ \zeta_2 \ldots \zeta_N\right] B$, ζ_i is a proper column vector. It is equal to consider the linear nonhomogeneous equations $B\left[\zeta_1\ \zeta_2 \ldots \zeta_N\right]^T = \left[l_1\ l_2 \ldots l_N\right]^T$, since these two formulas can be obtained by a transposed operation and B is a symmetric matrix. It suffices to know $\mathscr{L}\mathbf{1} = \mathbf{0}$, which indicates that $rank(\mathscr{L}) \le n-1$. Since the graph is connected,

$rank(\mathscr{L}) \geq n-1$ [20]. Therefore, $rank(\mathscr{L}) = n-1 = rank(B)$.

It has been known that row rank of any matrices is equal to the column rank. By elementary row operations, it can be obtained that $rank(\Lambda_i) \leq n-1$, since the sum of every column of the augmented matrix $\Lambda_i = \begin{bmatrix} B \; l_i \end{bmatrix}$ is zero and Λ_i has a row with all zero entries after row operations. Obviously, $rank(\Lambda_i) \geq rank(B) = n-1$, then $rank(\Lambda_i) = rank(B) = n-1$. Therefore, $l_i = B\zeta_i$ has an infinite set of solutions. It is direct to obtain the result of $\mathscr{L} = \Xi B$, which completes the proof. □

Lemma 3 [35] *Let* $0 \leq \tau_i(t) \leq \tau$, $\overline{F}(t,\ u,\ \bar{u}_1, \ldots, \bar{u}_m) : \mathbb{R}^+ \times \overbrace{\mathbb{R} \times \ldots \times \mathbb{R}}^{m+1} \to \mathbb{R}$ *be nondecreasing in* $\bar{u}_i$ *for each fixed* $(t,\ u,\ \bar{u}_1, \ldots, \bar{u}_{i-1}, \bar{u}_{i+1}, \ldots, \bar{u}_m)$, $i = 1, \ldots, m$, *and* $I_k(u) : \mathbb{R} \to \mathbb{R}$ *be nondecreasing in* u. *Suppose that* $u(t),\ v(t) \in \mathbb{PC}$ *satisfying*

$$\begin{cases} D^+u(t) \leq \overline{F}(t,\ u(t),\ u(t-\tau_1(t)), \ldots, \\ \quad u(t-\tau_m(t))),\ t \neq t_k, \\ u(t_k) \leq I_k(u(t_k^-)),\ k \in \mathbb{N}, \end{cases} \tag{10}$$

$$\begin{cases} D^+v(t) > \overline{F}(t,\ v(t),\ v(t-\tau_1(t)), \ldots, \\ \quad v(t-\tau_m(t))),\ t \neq t_k, \\ v(t_k) \geq I_k(v(t_k^-)),\ k \in \mathbb{N}. \end{cases} \tag{11}$$

Then $u(t) \leq v(t)$, $\forall t \in [-\tau, 0]$, *implying* $u(t) \leq v(t)$, $\forall t \geq 0$.

3 Main results

The results of the condition for set invariance and estimation of the invariant set are presented as follows.

3.1 The condition for set invariance

Theorem 1 *Assume Assumption 1 and Assumption 2 hold, for given constant* θ *satisfying* $0 < \theta < min\{1,\ exp(-\alpha\lambda_3)\}$, *if there exist some constant* $\beta > 0$, *and matrices* $K,\ H$, *such that the following inequality holds:*

$$\begin{bmatrix} I_{Nn} & T_l \\ * & \theta I_{Nn} \end{bmatrix} \geq 0, \quad l = 1, \ldots, 2^n, \tag{12}$$

and $F(\beta) \subset \mathscr{M}(H)$, *where* $\lambda_3 = \lambda_{max}(2I_N \otimes A + (2L+1)I_{nN})$, $(\lambda_3 + \frac{ln\theta}{\alpha})\theta + L^2 < 0$, $T_l = I_{Nn} - (I_{Nn} + \frac{1}{N}(\boldsymbol{1}_N\boldsymbol{1}_N^T) \otimes I_n)(\Xi \otimes (D_lK + (I_n - D_l)H))$. *Then the MAS (5) with the design of impulsive controller (4) can reach exponential dynamic consensus in the following sense:*

$$\| e_i(t) \|^2 \leq \theta^{-1} sup_{-\tau^* \leq s \leq 0} \left\{ \sum_{i=1}^{N} \| \phi_i(t) \|^2 \right\} exp(-\lambda t), \tag{13}$$

where $\lambda > 0$ *denotes a unique solution of*

$$\lambda - \theta_2 + \theta^{-1} L^2 exp(\lambda\tau^*) = 0, \tag{14}$$

with

$$\theta_2 = -\left(\lambda_3 + \frac{ln\theta}{\alpha}\right), \tag{15}$$

and the contractively invariant set of the system is $F(\beta)$.

Proof When $t \neq t_k$, calculate the derivative of $V(t)$ in terms of t along (6) yields

$$\begin{aligned} D^+V(t) = & 2e^T(t)(I_N \otimes A)e(t) + 2e^T(t)(f_1(x(t)) \\ & - \mathbf{1}_N \otimes \bar{f}_1(x(t))) \\ & + 2e^T(t)(f_2(x(t-\tau(t))) \\ & - \mathbf{1}_N \otimes \bar{f}_2(x(t-\tau(t)))). \end{aligned} \tag{16}$$

Since $e(t) = x(t) - \frac{1}{N}((\mathbf{1}_N\mathbf{1}_N^T) \otimes I_n)x(t)$, then

$$\begin{aligned} & e^T(t)(\mathbf{1}_N \otimes f_1(\bar{x}(t)) - \mathbf{1}_N \otimes \bar{f}_1(x(t))) = 0, \\ & e^T(t)(\mathbf{1}_N \otimes f_2(\bar{x}(t-\tau(t))) \\ & \quad - \mathbf{1}_N \otimes \bar{f}_2(x(t-\tau(t)))) = 0. \end{aligned} \tag{17}$$

Then,

$$\begin{aligned} D^+V(t) = & 2e^T(t)(I_N \otimes A)e(t) + 2e^T(t)(f_1(x(t)) \\ & - \mathbf{1}_N \otimes f_1(\bar{x}(t))) \\ & + 2e^T(t)(f_2(x(t-\tau(t))) \\ & - \mathbf{1}_N \otimes f_2(\bar{x}(t-\tau(t)))). \end{aligned} \tag{18}$$

From Assumption 2, it is easy to show that

$$\begin{aligned}&\| f_1(x(t)) - \mathbf{1} \otimes f_1(\bar{x}(t)) \| \leq L \| e(t) \|,\\&\| f_2(x(t-\tau(t)))\\&\quad - \mathbf{1} \otimes f_2(\bar{x}(t-\tau(t))) \| \leq L \| e(t-\tau(t)) \| .\end{aligned} \tag{19}$$

Since

$$\begin{aligned}&e^T(t)(f_1(x(t)) - \mathbf{1}_N \otimes f_1(\bar{x}(t)))\\&\quad \leq \| e(t) \| \| (f_1(x(t)) - \mathbf{1}_N \otimes f_1(\bar{x}(t))) \|\\&\quad \leq L \| e(t) \| \| e(t) \| = Le^T e(t).\end{aligned} \tag{20}$$

and

$$\begin{aligned}&2e^T(t)(f_2(x(t-\tau(t))) - \mathbf{1}_N \otimes f_2(\bar{x}(t-\tau(t))))\\&\quad \leq e^T(t)e(t) + L^2 e^T(t-\tau(t))e(t-\tau(t)),\end{aligned} \tag{21}$$

then, it can be obtained that

$$\begin{aligned}D^+V(t) \leq& e^T(t)(2I_N \otimes A + (2L+1)I_{nN})e(t)\\&+ L^2 e^T(t-\tau(t))e(t-\tau(t))\\ \leq& \lambda_3 V(t) + L^2 V(t-\tau(t)).\end{aligned} \tag{22}$$

When $t = t_k$, according to Lemma 1, it is similar to [1] that there exists a set of $0 < \eta_l(t_k) < 1$, $l = 1, \ldots, 2^n$, such that

$$\begin{aligned}Sat(u_i(t_k^-)) = &-\sum_{l=1}^{2^n} \eta_l(t_k)(D_l K\\&+ (I_n - D_l)H) \sum_{j=1}^{N} a_{ij}(x_i(t_k^-)\\&- x_j(t_k^-)),\end{aligned} \tag{23}$$

Then,

$$\begin{aligned}Sat(u(t_k^-)) = &-\sum_{l=1}^{2^n} \eta_l(t_k)(\mathscr{L}\\&\otimes(D_l K + (I - D_l)H))x(t_k^-).\end{aligned} \tag{24}$$

Since $e(t_k^-) = (B \otimes I_n)x(t_k^-)$, then based on Lemma 2, there exists a matrix $\Xi \in \mathbb{R}^{N \times N}$ such that the following equation holds

$$\begin{aligned}Sat(u(t_k^-)) =& -\sum_{l=1}^{2^n} \eta_l(t_k)((\Xi B)\\&\otimes (D_l K + (I - D_l)H))x(t_k^-)\\=& -\sum_{l=1}^{2^n} \eta_l(t_k)(\Xi \otimes (D_l K + (I-D_l)H))\\&(B \otimes I_n))x(t_k^-)\\=& -\sum_{l=1}^{2^n} \eta_l(t_k)(\Xi\\&\otimes (D_l K + (I - D_l)H))e(t_k^-).\end{aligned} \tag{25}$$

Then,

$$\begin{aligned}e(t_k) =& e(t_k^-) - \sum_{l=1}^{2^n} \eta_l(t_k)(\Xi \otimes (D_l K\\&+ (I - D_l)H))e(t_k^-)\\&+ \frac{1}{N}((\mathbf{1}_N \mathbf{1}_N^T) \otimes I_n)(\Xi \otimes (D_l K\\&+ (I - D_l)H))e(t_k^-)\\=& -\sum_{l=1}^{2^n} \eta_l(t_k)(I_{Nn} - (I_{Nn}\\&+ \frac{1}{N}(\mathbf{1}_N \mathbf{1}_N^T) \otimes I_n)(\Xi \otimes (D_l K\\&+ (I - D_l)H)))e(t_k^-).\end{aligned} \tag{26}$$

Thus, from (12), it holds that

$$V(t_k) \leq \theta V(t_k^-). \tag{27}$$

For any $\epsilon > 0$, let $\upsilon(t)$ be a unique solution of the following impulsive time-delay system:

$$\begin{cases}\dot{\upsilon}(t) = \lambda_3 \upsilon(t) + L^2 \upsilon(t-\tau(t)) + \epsilon, \ t \neq t_k,\\ \upsilon(t_k) = \theta \upsilon(t_k^-),\\ \upsilon(t) = \sum_{i=1}^{N} \| \phi_i(t) \|^2, \ -\tau^* \leq t \leq 0.\end{cases} \tag{28}$$

According to Lemma 3, it has $\upsilon(t) \geq V(t) \geq 0$ for any $t \geq 0$.

By using the formula for the variation of parameters [11], the following integral equation for $\upsilon(t)$ can be obtained:

$$\upsilon(t) = W(t,0)\upsilon(0) + \int_0^t W(t,s)(L^2\upsilon(s-\tau(s))+\epsilon)\mathrm{d}s, \quad t \geq 0, \tag{29}$$

where $W(t,s)(t > s \geq 0)$ denotes the Cauchy matrix of the following linear impulsive system:

$$\begin{cases} \dot{w}(t) = \lambda_3 w(t), \quad t \geq 0, \ t \neq t_k, \\ w(t_k^+) = \theta w(t_k^-). \end{cases} \tag{30}$$

Since $0 < \theta < 1, \ \alpha \geq t_k - t_{k-1}$, it holds that

$$\begin{aligned} W(t,s) &= \exp(\lambda_3(t-s)) \prod_{s<t_k\leq t} \theta \\ &\leq \exp(\lambda_3(t-s))\theta^{\frac{t-s}{\alpha}-1} \\ &= \theta^{-1}\exp((\lambda_3 + \frac{ln\theta}{\alpha})(t-s)). \end{aligned} \tag{31}$$

Let $\theta_1 = \theta^{-1} sup_{-\tau^* \leq s \leq 0}\{\sum_{i=1}^{N} \parallel \phi_i(t) \parallel^2\}$, then

$$\begin{aligned} \upsilon(t) &\leq \theta^{-1}\sum_{i=1}^{N} \parallel \phi_i(0) \parallel^2 \exp((\lambda_3 + \frac{ln\theta}{\alpha})t) \\ &\quad + \int_0^t \theta^{-1}\exp((\lambda_3 + \frac{ln\theta}{\alpha})(t-s)) \\ &\quad (L^2\upsilon(s-\tau(s))+\epsilon)\mathrm{d}s \\ &\leq \int_0^t \exp(-\theta_2(t-s)) \\ &\quad (\theta^{-1}L^2\upsilon(s-\tau(s))+\theta^{-1}\epsilon)\mathrm{d}s \\ &\quad + \theta_1\exp(-\theta_2 t), \ t \geq 0. \end{aligned} \tag{32}$$

In the following, it will be proved that $\upsilon(t) \leq \theta_1\exp(-\lambda t) + \frac{\epsilon}{\theta_2\theta - L^2}, \ \forall t \geq 0$ by contradiction.

Firstly, for $-\tau^* \leq t \leq 0$, since $(\lambda_3 + \frac{ln\theta}{\alpha})\theta + L^2 < 0$, it hold that

$$\upsilon(t) \leq \theta^{-1}\sum_{i=1}^{N} \parallel \phi_i(t) \parallel^2 < \theta_1\exp(-\lambda t) + \frac{\epsilon}{\theta_2\theta - L^2}. \tag{33}$$

Subsequently, it shall be proved for $t \geq 0$, the following inequality holds:

$$\upsilon(t) \leq \theta_1\exp(-\lambda t) + \frac{\epsilon}{\theta_2\theta - L^2}. \tag{34}$$

If (34) is not true, then there exists a $t^* > 0$ such that

$$\upsilon(t^*) \geq \theta_1\exp(-\lambda t^*) + \frac{\epsilon}{\theta_2\theta - L^2}, \tag{35}$$

and

$$\upsilon(t) \leq \theta_1\exp(-\lambda t) + \frac{\epsilon}{\theta_2\theta - L^2}, \ for \ t < t^*. \tag{36}$$

From (32)

$$\begin{aligned} \upsilon(t^*) &\leq \theta_1\exp(-\theta_2 t^*) \\ &\quad + \int_0^{t^*} \exp(-\theta_2(t^*-s))(\theta^{-1}L^2\upsilon(s-\tau(s)) \\ &\quad + \theta^{-1}\epsilon)\mathrm{d}s \\ &< \exp(-\theta_2 t^*)(\theta_1 + \frac{\epsilon}{\theta_2\theta - L^2} \\ &\quad + \int_0^{t^*} \exp(\theta_2 s)(\theta^{-1}L^2\theta_1 \\ &\quad \exp(-\lambda(s-\tau(s))) + \theta^{-1}L^2\frac{\epsilon}{\theta_2\theta - L^2} \\ &\quad + \theta^{-1}\epsilon)\mathrm{d}s) \\ &\leq \exp(-\theta_2 t^*)(\theta_1 + \frac{\epsilon}{\theta_2\theta - L^2} \\ &\quad + \theta^{-1}L^2\theta_1\exp(\lambda\tau^*) \\ &\quad \int_0^{t^*} \exp((\theta_2 - \lambda)s)\mathrm{d}s \\ &\quad + \frac{\theta_2\epsilon}{\theta_2\theta - L^2}\int_0^{t^*} \exp(\theta_2 s)\mathrm{d}s) \\ &= \theta_1\exp(-\theta_2 t^*) + \frac{\epsilon}{\theta_2\theta - L^2}\exp(-\theta_2 t^*) \\ &\quad + \exp(-\theta_2 t^*)\theta^{-1}L^2\theta_1\exp(\lambda\tau^*) \\ &\quad \frac{\exp((\theta_2-\lambda)t^*)-1}{\theta_2-\lambda} \\ &\quad + \exp(-\theta_2 t^*)\frac{\theta_2\epsilon(\exp(\theta_2 t^*)-1)}{(\theta_2\theta - L^2)\theta_2} \\ &= \theta_1\exp(-\lambda t^*) + \frac{\epsilon}{\theta_2\theta - L^2}, \end{aligned} \tag{37}$$

where the second inequality comes from (36), the third inequality comes from the fact that $\tau(t) < \tau^*$, and the last equality comes form (14). Obviously (37) leads to a contradiction with (35). Therefore, the inequality (34) holds. Let $\epsilon \to 0$, for $t \geq 0$, it holds that

$$\upsilon(t) \leq \theta_1 \exp(-\lambda t), \tag{38}$$

which further implies that

$$V(t) \leq \upsilon(t) \leq \theta_1 \exp(-\lambda t). \tag{39}$$

This completes the proof. □

Remark 2 the existence of the solution to (14) should be discussed Let $g(\lambda) = \lambda - \theta_2 + \theta^{-1}L^2\exp(\lambda\tau^*)$. Since $\theta_2 > 0,\ 0 < \theta < 1$ and $-\theta_2 + L^2\theta^{-1} < 0$, we have $g(0) = -\theta_2 + \theta^{-1}L^2 < 0,\ g(\infty) > 0$ and $g'(\lambda) = 1 + \tau^*\theta^{-1}L^2\exp(\lambda\tau^*) > 0$. Consequently, it can be concluded that $g(\lambda) = 0$ has a unique solution $\lambda > 0$.

Remark 3 To achieve dynamic consensus control of an MAS, the error variable is designed with the following form $e_i(t) = x_i(t) - \frac{1}{N}\sum\limits_{j=1}^{N} x_j(t)$. Since the saturation of the impulsive controller exists, it is difficult to analyze the stability of the system via Lyapunov function at impulsive instants. In order to solve the problem, Lemma 2 is obtained based on the property of Laplacian matrix and nonhomogeneous linear equations, which simplifies the derivation process of the condition for set invariance. Moreover, it can be observed from (39) that the impulsive controller designed in this paper can achieve exponential dynamic consensus for the time-delay system with input saturation under the circumstance that the general low-gain method does not work [21,25,27,39,42].

Remark 4 Note from Remark 2 that the values of τ^* have no influence on the existence of the solution. From Theorem 1, the upper bound τ^* of time-varying delay in (5) will not influence the achievement of consensus of the MAS, but relates to the convergence rate λ.

Remark 5 With respect to dealing with time delay in MASs, [6,32] use the Lyapunov–Krasovskii function method to deal with time delay. Compared with our work, the advantages may lie in that they are possible to obtain smaller conservativeness by selecting appropriate inequalities and lemmas with more complicated design and analysis. However, they both have not considered the practically existing saturation problems. Still, they both require to know the upper bound of the time-varying delay, as well as the bound of the derivative of time delay which is not assumed to be known in our work.

3.2 Estimation of the invariant set

To estimate the invariant set, the following process is adopted. Define a bounded convex setx_R as a shape reference set to estimate the attraction domain, whose the typical format is a polyhedron as follows

$$x_R = co\{\xi_1, \xi_2, \ldots, \xi_\varsigma\}, \tag{40}$$

where $\xi_1, \xi_2, \ldots, \xi_\varsigma$ are pre-given vectors in $\mathbb{R}^{Nn}$.

Next, choose the maximized set γx_R from $F(\beta)$ that satisfies the condition in Theorem 1. The problem can be solved in the following optimization process:

$$\begin{aligned} &\sup_{\gamma>0,\ \beta>0,\ K,\ H} \gamma, \\ &s.t.\ (a)\ \gamma x_R \subset F(\beta), \\ &\qquad (b)\ (12), \\ &\qquad (c)\ F(\beta) \subset \mathscr{M}(H). \end{aligned} \tag{41}$$

Then, rewrite the constraint (41) in the optimization into the LMI format. The transformation is conducted as follows:

From Definition 1 based on the lemma of Schur complement, constraint (a) holds if the next relations are satisfied

$$\begin{bmatrix} g & \xi_s{}^T \\ \xi_s & \beta I_{Nn} \end{bmatrix} \geqslant 0, \tag{42}$$

where $g = \gamma^{-2},\ s = 1, \ldots, \varsigma$.

The condition $F(\beta) \subset \mathscr{M}(H)$ is equivalent to

$$(\Xi \otimes H)_i(\beta I_{Nn})(\Xi \otimes H)_i^T \leqslant 1, \tag{43}$$

where $(\cdot)_i$ denote the $i-th$ row of the corresponding matrix. According to the lemma of Schur complement, condition (c) holds if the following inequality is satisfied:

$$\begin{bmatrix} 1 & (\Xi \otimes H)_i \\ * & \beta I_{Nn} \end{bmatrix} \geqslant 0. \tag{44}$$

Then, the optimization process (41) can be formulated into the following problem with the constraints in LMI formats:

$$\begin{aligned} &\min_{g>0,\ \beta>0,\ K,\ H,} \ g, \\ &s.t.\ \ \text{(a) LMI (42)}, \\ &\qquad \text{(b) LMI (12)}, \\ &\qquad \text{(c) LMI (44)}, \quad i = 1, \ldots, N. \end{aligned} \tag{45}$$

Remark 6 From (41), it can be observed that this paper considers the controller parameters as the variables in the constraints of the optimization process for estimating the invariant set. Thus, it would be likely to choose a feedback gain to make the invariant set itself as large as possible in estimating the invariant set. Moreover, this paper transforms all the constraints in (41) into the format of LMI by variable substitution, the Schur complement lemma, and the inclusion relationship between sets, which makes the problem possible to be solved in MATLAB.

4 Numerical simulation

4.1 Example 1

In this section, the following example will illustrate the performance of Theorem 1 and the optimization method for estimating the invariant set.

The communication topology of the considered MAS is shown in Fig. 1, where the MAS consists of 4 agents labeled as 1 $\sim$ 4. The Laplacian of the network is $\mathscr{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$. Based on Lemma 2, let $\Xi = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & 3 & 1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$. The dynamics of each agent are indicated by the following nonlinear equation:

$$\begin{aligned} \dot{x}_i(t) = &\begin{bmatrix} 0.7 & 0 \\ 0.3 & 0.2 \end{bmatrix} x_i(t) + \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} \tanh(x_{i1}(t)) \\ \tanh(x_{i1}(t)) \end{bmatrix} \\ &+ \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} \tanh(x_{i1}(t-\tau(t))) \\ \tanh(x_{i1}(t-\tau(t))) \end{bmatrix} + U_i(t), \end{aligned} \tag{46}$$

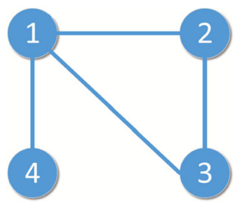

Fig. 1 Communication topology

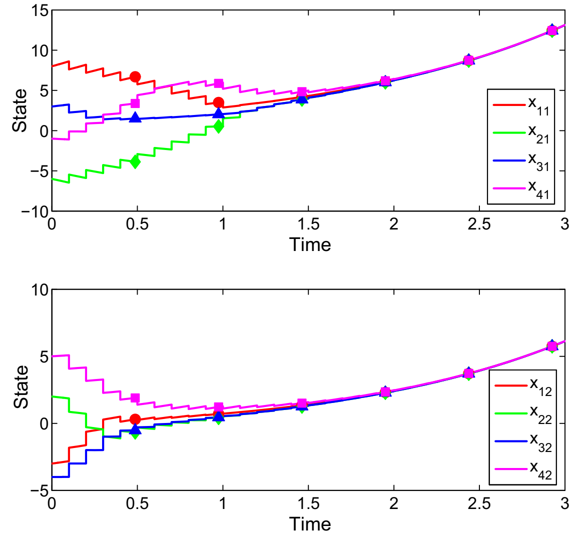

Fig. 2 State trajectories under saturated impulsive control inputs when $\tau^* = 0.0001$

where $x_i = [x_{i1}\ x_{i2}]^T$, $U_i(t) = [U_{i1}(t)\ U_{i2}(t)]^T$, $u_i(t) = [u_{i1}(t)\ u_{i2}(t)]^T$, $i = 1, \ldots, 4$. The control input u_i is designed as (4). Obviously, the nonlinear term satisfies the Lipschitz condition with $L = 0.3$.

If parameters of the maximum upper bound of the impulse interval are $\alpha = 0.1$, $\theta = 0.7$, solving (45) by using the LMI toolbox of MATLAB obtains $K = \begin{bmatrix} 0.3390 & 0 \\ 0 & 0.3390 \end{bmatrix}$.

Let the initial condition be $x_1 = [8\ -3]^T$, $x_2 = [-6\ 2]^T$, $x_3 = [3\ -4]^T$, $x_4 = [-1\ 5]^T$. When delay upper bound $\tau^* = 0.0001$, Fig. 2 shows the state trajectories variation of the four agents. The symbols with solid curves in different colors exhibit the state evolution of the four agents. It can be obtained that by solving (14), $\lambda = 0.438$ and $\lambda = 0.379$ correspond to $\tau^* = 0.0001$ and $\tau^* = 1$, respectively. Compared with the low-gain method which can only achieve zero consensus for the MASs, the impulsive consensus algorithm proposed in this study can make the MASs reach dynamical consensus. Figure 3 shows the impulsive

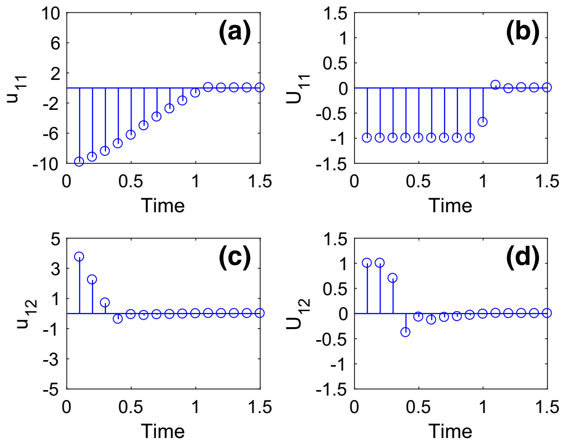

Fig. 3 The control input of the first agent

control input of the first agent of MAS (46), where the control input can reach the saturation.

4.2 Example 2

In this section, the lead-acid battery model presented in [2] is introduced to indicate the application potential of the proposed methods. Moreover, by comparison with [27], the priority of the proposed impulsive consensus algorithms for saturated MASs is clarified.

As shown in [2], the equivalent circuit of the lead-acid battery model is described by Fig. 4, and the symbols in Fig. 4 are defined in [2]. In a complete industry process, it is quite possible to consider the distributed control problem of the multiple lead-acid batteries.

The communication topology of the considered MAS is assumed the same as Fig. 1. The dynamics of each agent are indicated by the following equation:

$$\dot{x}_i(t) = Ax_i(t) + Bv_i(t), \tag{47}$$

where $A = \begin{bmatrix} -\frac{1}{R_1C_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tilde{R}\tilde{C}} \end{bmatrix}$, $B = \begin{bmatrix} \frac{1}{R_1C_1} \\ -1 \\ 1 \end{bmatrix}$, $\tilde{C}$, $\tilde{R}$

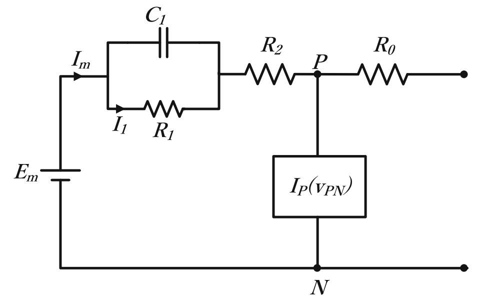

Fig. 4 The circuit of the lead-acid battery model

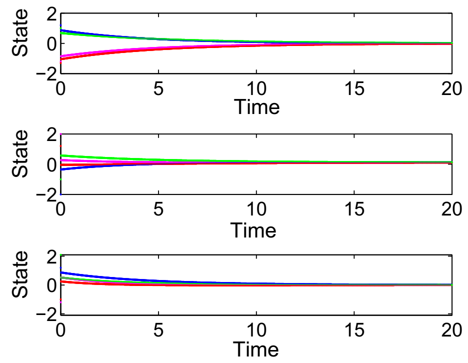

Fig. 5 State trajectories in three dimensions under the proposed anti-saturation impulsive consensus algorithms in this paper

are the battery thermal capacitance and thermal resistance between the battery and its environment, respectively. $x_i = [x_{i1}\ x_{i2}\ x_{i3}]^T$, and the control input v_i has a saturation feature with the bound of 1.

Let the initial condition be $x_1 = [-1.2\ 2\ -1.2]^T$, $x_2 = [1.2\ -2\ 1.2]^T$, $x_3 = [-1.3\ 1.2\ -1]^T$, $x_4 = [1\ -1\ 2.1]^T$. From Corollary 1, we choose $\alpha = 0.001$, $\tilde{K} = [0.0307 - 0.1533\ 0.1533]$. Figure 5 shows that the states can reach consensus under the anti-saturation impulsive consensus algorithms proposed in this paper.

For comparison, the event-triggered controller is designed based on the anti-saturation event-triggered consensus algorithms in [27]. From Theorem 1 in [27], the following control parameters are designed:

$$\varepsilon = 0.0018,\ P = \begin{bmatrix} 0.0045 & 9.8117 \times 10^{-5} & -6.5420 \times 10^{-7} \\ 9.8117 \times 10^{-5} & 0.0736 & 8.40119.8117 \times 10^{-5} \\ -6.54209.8117 \times 10^{-7} & 8.40119.8117 \times 10^{-5} & 0.0018 \end{bmatrix}, \gamma = 5, \theta = 0.25.$$

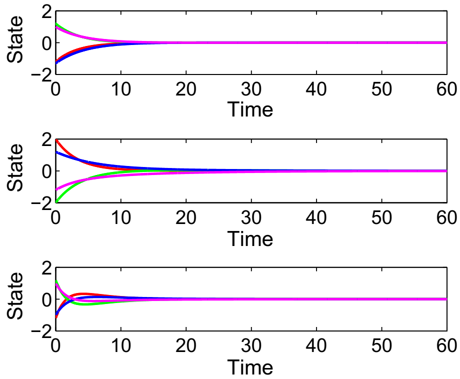

Fig. 6 State trajectories in three dimensions under the anti-saturation event-triggered consensus algorithms in [27]

Figure 6 shows the consensus process of the states under the anti-saturation event-triggered consensus algorithms in [27].

From the comparison, the systems under the two control methods can both achieve consensus. However, if matrix A does not satisfy the ANCBC condition, the method of [27] becomes invalid. The method proposed in this paper has no requirement of matrix A, and it still works for the MASs with nonlinearity and time-varying delay.

5 Conclusion

In this paper, impulsive consensus algorithms for a class of nonlinear saturated MASs with time delay are proposed, which are designed by taking advantages of the stability theory of impulsive systems, as well as the property of convex hull and the Laplacian matrix. To assess the attraction domain of the leaderless MASs, the shape reference set is also introduced. Moreover, the performance of the proposed impulsive consensus algorithms for saturated MASs with time delay is demonstrated by two numerical experiments. This study may provide some insight onto distributed control of MAS with both saturation and time delay, multi-sensor information fusion, smart grid, and other practical distributed systems. In future work, the impulsive consensus control problem of nonlinear saturated MAS with time delay will be considered under more complex communication conditions, such as switching topology and packet loss.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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