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ORIGINAL PAPER

Finite-time-estimation-based surrounding control for a class of unknown nonlinear multi-agent systems

Akbar Sharghi · Mahdi Baradarannia · Farzad Hashemzadeh

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Abstract This paper deals with the surrounding control problem for a class of multi-agent systems. The followers and leaders have nonlinear heterogeneous dynamics, and the dynamics of the leaders are time-varying. In this work, it is supposed that dynamics of each follower consists of unknown parameters. For this reason, an adaptive law is used to adjust unknown parameters and an estimator to estimate the center of the leaders is used. Also, a new estimator is presented to estimate the summation of distances of the leaders from their geometric center. This estimation is used in the surrounding control protocol, especially when a sudden change happens in the movement of any leader. It is proved that the proposed estimator is stable in finite time sense. Some numerical examples to verify the theoretical results are given.

Keywords Surrounding control problem · Multi-agent systems · Center of leaders estimator · Adaptive control

1 Introduction

Distributed cooperative control of multi-agent systems has gained much attention in the past decade [1,2]. Multiplicity of the agents in such systems is an important benefit. In some applications, performing a task is impossible with a single agent. Some examples are surveillance with the aid of multiple sensors or sensor-enabled robots [3], the release of an injured soldier by a group of autonomous robots [4], protection of vehicles on ground by armed robots [5], etc. Nowadays, the application of multi-agent systems is in broad areas such as consensus [6,7], containment control [8], formation control [9], flocking [10], and rendezvous [11].

One of the applications of cooperative control is surrounding control problem [12]. In this problem, the goal is to design an algorithm wherein some agents, which are usually called followers, are trying to construct a regular shape around some other agents, which are called leaders or targets. In recent works, the surrounding problem has been investigated for stationary or moving leaders. In [13], the surrounding problem was studied for a group of unmanned air vehicles, which surrounds one target by using decentralized nonlinear model predictive control.

The surrounding control problem and target enclosing problem have common cognate; in both issues, the aim is to achieve an encircle formation around leaders which are either stationary or moving, by a group of agents. To solve the enclosing problem, some control methods have been implemented. For example, in [14] cyclic pursuit method has been proposed, and in [15] authors consider an agent, that must move to the vicinity of the other agent with unknown location and then encircle it at a prescribed distance. A hybrid

A. Sharghi · M. Baradarannia (✉) · F. Hashemzadeh
Control Engineering Department, Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran
e-mail: mbaradaran@tabrizu.ac.ir

control based on reachability specifications was presented in [16] to accomplish the cooperative surrounding problem with multiple robots. One of the novel and interesting works in target enclosing problem has been investigated in [17], where a rotary surrounding control around a group of moving targets is presented for a second-order multi-agent system.

In the surrounding and target enclosing problem, the followers need to know the geometric center of the leaders and the radius of placement from this geometric center to construct a regular shape around the leaders. The target enclosing problem for the moving and stationery leaders is considered in [18], where authors assume that each follower is connected exactly to one leader and an estimator is used to achieve the geometric center of leaders. The connection of each follower exactly with one leader is a conservative condition. In [19], this condition has been removed and a distributed estimator for each follower is proposed. Also, in [19] it is assumed that the movement of the leaders around their center is bounded.

In this article, we consider the surrounding control of multi-agent systems with local information and moving or stationary targets. It is assumed that the dynamics of each follower is different from the others and consists of unknown parameters. In the surrounding problem, the main challenge is to create a regular shape around the leaders. For this reason, each follower needs to know the distance of its placement from the geometric center of the leaders. It should be noted that the presented method in [18, 19] can only estimate the geometric center of the leaders. So if the dynamics of leaders are in a way that the distances between leaders increase as time grows, the followers will not enclose the leaders. The proposed method in the current paper can overcome this deficiency.

In [20], the surrounding control of multi-agent systems with unknown nonlinear dynamics was considered. In this work, the authors designed an adaptive control strategy based on the estimation of geometric center of the leaders. To surround the leaders by the followers, the authors assumed that the maximum distance between the center of the leaders and each follower is known to each follower, and then, they used it in the control strategy. This assumption may be restrictive in some applications, especially when one or some of the leaders make sudden change in their movement. The current paper proposes an estimator which estimates the summation of the distances of leaders from their geometric center. This estimator is used to correct the placement of surrounding followers, i.e., if the distances between leaders change, the followers will adapt the enclosing radius.

The rest of the paper is organized as follows: Some preliminaries and assumptions are given in Sect. 2. Problem formulation is given in Sect. 3. The main results are presented in Sect. 4. An estimator is introduced for the center of leaders, and then, it is used to estimate the summation of the distances of leaders from their geometric center. Using this estimator, the surrounding control input is presented. To show the efficiency of proposed method, two simulation examples are presented in Sect. 5. Finally, the conclusion of the paper is given in Sect. 6.

2 Preliminaries

In multi-agent systems, the communication network between agents is shown by a graph. For a multi-agent system consisting of $\mathcal{P}$ agents, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represents the corresponding graph, where $\mathcal{V} = \{v_1, ..., v_{\mathcal{P}}\}$ is a nonempty set of *nodes*, $\mathcal{E} = \{e_{ij} = (v_i, v_j)\} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of *edges*. $(v_i, v_j) \in \mathcal{E}$ shows an edge. It means that agent v_j knows the information of agent v_i and the node v_j is a neighbor of node v_i. The set of neighbors of agent v_i is represented by N_i. The graph $\mathcal{G}$ is an *undirected graph* of order $\mathcal{P}$ if $(v_i, v_j) \in \mathcal{E}$ implies $(v_j, v_i) \in \mathcal{E}$. An undirected graph is *connected* if there exists a sequence of distinct edges such that consecutive edges are joint between any two vertices.

The *Adjacency* matrix of the graph $\mathcal{G}$ is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{\mathcal{P}\times\mathcal{P}}$, where $a_{ij} = 1$ if $a_{ij} \in \mathcal{E}$ and $a_{ij} = 0$ if $a_{ij} \notin \mathcal{E}$. The graph *Laplacian* matrix is defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{\mathcal{P}\times\mathcal{P}}$, where $l_{ii} = \sum_{j=1,\ j\neq i}^{\mathcal{P}} a_{ij}$ and $l_{ij} = -a_{ij},\ \ i \neq j$. The *Adjacency* and *Laplacian* matrices of an undirected graph are symmetric.

In this paper, a multi-agent system with N followers and M leaders is considered. It is supposed that the graph $\mathcal{G}$ of the multi-agent system consisting of followers and leaders is an undirected graph, and each leader communicates with at least one follower. In this paper, the set of nodes of the followers and leaders are shown with V_{F} and V_{L}, respectively. The interconnection relationship between each follower and corresponding leader is indicated with the *Explore* matrix, which is shown by $\mathcal{B} = [b_{ik}] \in \mathbb{R}^{N\times M}$. It is defined as $b_{ik} = 1$ if the ith follower communicates with the

kth leader and $b_{ik} = 0$, otherwise. Kronecker product is indicated with $\otimes$, and the 2-norm of a vector is denoted by $\|.\|$.

3 Problem formulation

Consider a multi-agent system which has N followers and M leaders. The dynamics of the followers and leaders are assumed to be nonidentical. The follower dynamics are represented by

$$\dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i, t) + \mathbf{u}_i(t), \quad i \in V_{\mathrm{F}}, \tag{1}$$

where $\mathbf{x}_i(t) = (x_{ix}, x_{iy})^{\mathrm{T}}$ is the state of the ith follower, $\mathbf{u}_i(t) \in \mathbb{R}^2$ is the control protocol and $\mathbf{f}_i(\mathbf{x}_i, t)$ is a vector of unknown nonlinear functions which is assumed to be Lipschitz in $\mathbf{x}_i$ and continuous in t. It is assumed that the leaders move in $\mathbb{R}^2$ with dynamics

$$\dot{\mathbf{l}}_k(t) = \mathbf{v}_k(t), \quad k \in V_{\mathrm{L}}, \tag{2}$$

where $\mathbf{l}_k(t) \in \mathbb{R}^2$ and $\mathbf{v}_k(t) \in \mathbb{R}^2$, respectively, represent the position and the velocity of the kth leader.

The goal is to design a adaptive control law for the followers to create a certain geometrical configuration around the leaders. For this purpose, we should have

$$\lim_{t\to\infty} (\mathbf{x}_i(t) - \bar{\mathbf{l}}(t)) = \rho_i \begin{bmatrix} \cos\left(\frac{2\pi i}{N}\right) \\ \sin\left(\frac{2\pi i}{N}\right) \end{bmatrix}, \quad i \in V_{\mathrm{F}} \tag{3}$$

here $\bar{\mathbf{l}}(t)$ denotes the center of leaders, $\rho_i \in \mathbb{R}^+$ is the radius of the ith follower with respect to the center of leaders.

Assumption 1 The velocity of each leader is bounded, namely $\|\dot{\mathbf{l}}_j(t)\| \le p_j$, $j \in V_{\mathrm{L}}$.

4 Main results

Assume that the graph topology $\mathcal{G}$ of the multi-agent system is connected and undirected. The estimator of the geometric center of leaders for the ith follower is represented by $\widehat{\mathbf{x}}_i(t)$ and is given as follows [19],

$$\widehat{\mathbf{x}}_i(t) = \phi_i(t) + \widetilde{\mathbf{l}}_i(t), \quad i \in V_{\mathrm{F}}. \tag{4}$$

Here, $\phi_i(t)$ is a dynamic variable with the following dynamics,

$$\dot{\phi}_i(t) = \begin{cases} \alpha \sum\limits_{k=1}^{N} a_{ik}\left[\frac{\widehat{\mathbf{x}}_k(t)-\widehat{\mathbf{x}}_i(t)}{\left\|\widehat{\mathbf{x}}_k(t)-\widehat{\mathbf{x}}_i(t)\right\|}\right], & \widehat{\mathbf{x}}_k(t) \ne \widehat{\mathbf{x}}_i(t) \\ 0, & \widehat{\mathbf{x}}_k(t) = \widehat{\mathbf{x}}_i(t) \end{cases} \tag{5}$$

where $\phi_i(0) = 0$ and $\alpha > 0$ is a constant, and $\widetilde{\mathbf{l}}_i(t)$ is defined as

$$\begin{bmatrix} \widetilde{\mathbf{l}}_1(t) \\ \widetilde{\mathbf{l}}_2(t) \\ \vdots \\ \widetilde{\mathbf{l}}_N(t) \end{bmatrix} = \frac{N}{M}\left(I_n \otimes \begin{bmatrix} \frac{b_{11}}{\sum_{k=1}^N b_{k1}} & \frac{b_{12}}{\sum_{k=1}^N b_{k2}} & \cdots & \frac{b_{1M}}{\sum_{k=1}^N b_{kM}} \\ \frac{b_{21}}{\sum_{k=1}^N b_{k1}} & \frac{b_{22}}{\sum_{k=1}^N b_{k2}} & \cdots & \frac{b_{2M}}{\sum_{k=1}^N b_{kM}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b_{N1}}{\sum_{k=1}^N b_{k1}} & \frac{b_{N2}}{\sum_{k=1}^N b_{k2}} & \cdots & \frac{b_{NM}}{\sum_{k=1}^N b_{kM}} \end{bmatrix}\right) \begin{bmatrix} \mathbf{l}_1(t) \\ \mathbf{l}_2(t) \\ \vdots \\ \mathbf{l}_M(t) \end{bmatrix} \tag{6}$$

in which $\mathbf{l}_j(t)$ denotes the position of the jth leader.

In [19], it has been shown that the estimator defined by (4)–(6) will converge to the leaders center in a finite time T, i.e.,

$$\widehat{\mathbf{x}}_i(t) = \frac{1}{M}\sum_{k=1}^{M} \mathbf{l}_k(t), \quad t > T = \frac{1}{\delta}\sqrt{V(0)} \tag{7}$$

where $\delta \triangleq \frac{\sqrt{2N}[c_1-(N-1)c_2]}{2(N-1)}$ with $c_1 > c_2$, wherein c_1 and c_2 are real positive constants. The initial condition of Lyapunov function is represented by $V(0)$.

The increment of the distances between leaders will cause the increment of their radius around their geometric center. Therefore, a system is needed to estimate these changes. In the following subsection, a distributed estimator for each follower is presented to estimate the summation of leaders' distances from their geometric center. This is used to obtain the radius of the ith follower around the geometric center of leaders.

4.1 Estimator for the summation of distances of the leaders from their geometric center

A estimator $\widehat{\mathbf{x}}_{ui}(t)$ is proposed to estimate the summation of the distances of leaders from their geometric center. $\widehat{\mathbf{x}}_{ui}(t)$, which is calculated for the ith follower, is as follows,

$$\widehat{\mathbf{x}}_{ui}(t) = \psi_i(t) + \widetilde{\mathbf{d}}_i(t), \quad i \in V_{\mathrm{F}} \tag{8}$$

where $\widetilde{\mathbf{d}}_i(t)$ is defined as

$$\begin{bmatrix}\widetilde{\mathbf{d}}_1(t)\\ \widetilde{\mathbf{d}}_2(t)\\ \vdots \\ \widetilde{\mathbf{d}}_N(t)\end{bmatrix} = N\left(I_n \otimes \begin{bmatrix} \frac{b_{11}}{\sum_{k=1}^N b_{k1}} & \frac{b_{12}}{\sum_{k=1}^N b_{k2}} & \cdots & \frac{b_{1M}}{\sum_{k=1}^N b_{kM}} \\ \frac{b_{21}}{\sum_{k=1}^N b_{k1}} & \frac{b_{22}}{\sum_{k=1}^N b_{k2}} & \cdots & \frac{b_{2M}}{\sum_{k=1}^N b_{kM}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b_{N1}}{\sum_{k=1}^N b_{k1}} & \frac{b_{N2}}{\sum_{k=1}^N b_{k2}} & \cdots & \frac{b_{NM}}{\sum_{k=1}^N b_{kM}} \end{bmatrix}\right) \begin{bmatrix}\mathbf{l}_1(t)\\ \mathbf{l}_2(t)\\ \vdots \\ \mathbf{l}_M(t)\end{bmatrix} - \begin{bmatrix}\widehat{\mathbf{x}}_1(t)\\ \widehat{\mathbf{x}}_2(t)\\ \vdots \\ \widehat{\mathbf{x}}_N(t)\end{bmatrix} \tag{9}$$

and $\psi_i(t)$ satisfies

$$\dot{\psi}_i(t) = \begin{cases} \gamma \sum_{k=1}^{N} a_{ik}\left[\frac{\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right], & \widehat{\mathbf{x}}_{uk}(t) \neq \widehat{\mathbf{x}}_{ui}(t) \\ 0, & \widehat{\mathbf{x}}_{uk}(t) = \widehat{\mathbf{x}}_{ui}(t) \end{cases} \tag{10}$$

where $\psi_i(0) = 0$ and $\gamma > 0$. Before introducing Theorem 1, a lemma from [21] is presented.

Lemma 1 [21] *Suppose there is a positive definite Lyapunov function $V(\mathbf{x}, t) : \mathbb{R}^n \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ and there are positive real constants $C > 0$ and $0 < \alpha < 1$, such that $\dot{V}(\mathbf{x}, t) + CV^{\alpha}(\mathbf{x}, t) \leq 0$. Then, $V(\mathbf{x}, t)$ is locally finite-time convergent with a settling time T_s where*

$$T_s < \frac{V^{1-\alpha}(\mathbf{x}_0, t)}{C(1-\alpha)} \tag{11}$$

Theorem 1 *Consider a graph topology $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ corresponding to an undirected and connected multi-agent system. If there exists a constant $\xi > 0$ for which $\|\dot{\widetilde{\mathbf{d}}}_i(t)\| < \xi$ and γ is selected in a way that $\gamma > (N-1)\xi$, then for all followers, the estimator vector defined by (8) converges to the summation of the distance vector of the leaders from their geometric center in a finite time.*

Proof The proof is organized in two steps: (1) The estimator defined by (8) converges in a finite time. (2) The estimator vector converges to the summation of distances vector of the leaders from their geometric center.

The candidate of Lyapunov function is selected as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\left(\left[\widehat{\mathbf{x}}_{ui}(t) - \frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]^{\mathrm{T}} \times \left[\widehat{\mathbf{x}}_{ui}(t) - \frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]\right). \tag{12}$$

Time derivative of $V(t)$ and using (8) and (10) result in.

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N}\left[\widehat{\mathbf{x}}_{ui}(t) - \frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]^{\mathrm{T}} \times \left[\dot{\widehat{\mathbf{x}}}_{ui}(t) - \frac{1}{N}\sum_{j=1}^{N}\dot{\widehat{\mathbf{x}}}_{uj}(t)\right] \\ &= \sum_{i=1}^{N}\left[\widehat{\mathbf{x}}_{ui}(t) - \frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]^{\mathrm{T}} \times \left[\dot{\psi}_i(t) + \dot{\widetilde{\mathbf{d}}}_i(t) - \frac{1}{N}\sum_{j=1}^{N}\dot{\widehat{\mathbf{x}}}_{uj}(t)\right].\end{aligned} \tag{13}$$

We will continue the proof in two cases, when $\dot{\psi}_i(t) \neq 0$, and when $\dot{\psi}_i(t) = 0$. When $\dot{\psi}_i(t) \neq 0$, $\dot{V}(t)$ is as follows

$$\begin{aligned}&\sum_{i=1}^{N}\left[\widehat{\mathbf{x}}_{ui}(t) - \frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]^{\mathrm{T}} \times \left[\gamma\sum_{k=1}^{N}a_{ik}\left[\frac{\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right]\right] = \gamma\sum_{i=1}^{N}\sum_{k=1}^{N}a_{ik}\widehat{\mathbf{x}}_{ui}^{\mathrm{T}}(t) \\ &\times \left[\frac{\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right] - \frac{\gamma}{N}\sum_{j=1}^{N}\mathbf{x}_{uj}^{\mathrm{T}}(t)\sum_{i=1}^{N}\sum_{k=1}^{N}a_{ik} \\ &\times \left[\frac{\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right] = \frac{\gamma}{2}\sum_{i=1}^{N}\sum_{k=1}^{N}a_{ik}[\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uk}(t)]^{\mathrm{T}} \\ &\times \left[\frac{\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right] - \frac{\gamma}{N}\sum_{j=1}^{N}\mathbf{x}_{uj}^{\mathrm{T}}(t)\left(\sum_{i=1}^{N}\sum_{k=1}^{N}a_{ik}\right. \\ &\times \left.\left[\frac{\widehat{\mathbf{x}}_{uk}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right] - \sum_{i=1}^{N}\sum_{k=1}^{N}a_{ik}\left[\frac{\widehat{\mathbf{x}}_{ui}(t)}{\left\|\widehat{\mathbf{x}}_{uk}(t)-\widehat{\mathbf{x}}_{ui}(t)\right\|}\right]\right) \\ &= -\frac{\gamma}{2}\sum_{i=1}^{N}\sum_{k=1}^{N}\left[a_{ik}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uk}(t)\right\|\right].\end{aligned} \tag{14}$$

Since $\mathcal{G}$ is connected and $\left\|\dot{\widetilde{\mathbf{d}}}_i(t)\right\| < \xi$, by using triangle inequality, one has

$$\sum_{i=1}^{N}\left\{\left[\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}\right]^{\mathrm{T}}\dot{\tilde{\mathbf{d}}}_i(t)\right\}\leq\sum_{i=1}^{N}\left\|[\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}]^{\mathrm{T}}\dot{\tilde{\mathbf{d}}}_i(t)\right\|\leq\xi\sum_{i=1}^{N}\left\|\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}\right\|=\frac{\xi}{N}\sum_{i=1}^{N}\left\|N\widehat{\mathbf{x}}_{ui}(t)-\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}\right\|\leq\frac{\xi}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\|\widehat{\mathbf{x}}_{ui}-\widehat{\mathbf{x}}_{uj}\|\leq(N-1)\xi\max_{i,j=1,\ldots,N}\|\widehat{\mathbf{x}}_{ui}-\widehat{\mathbf{x}}_{uj}\|\leq\frac{(N-1)\xi}{2}\sum_{i=1}^{N}\sum_{k=1}^{N}\left[a_{ik}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uk}(t)\right\|\right]. \tag{15}$$

Also

$$\sum_{i=1}^{N}\left(\left[\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]^{\mathrm{T}}\left[-\frac{1}{N}\sum_{j=1}^{N}\dot{\widehat{\mathbf{x}}}_{uj}(t)\right]\right)=\left[\sum_{i=1}^{N}\widehat{\mathbf{x}}_{ui}^{\mathrm{T}}-\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}^{\mathrm{T}}\right]\left[-\frac{1}{N}\sum_{j=1}^{N}\dot{\widehat{\mathbf{x}}}_{uj}(t)\right]=0. \tag{16}$$

From (14)–(16), one can write

$$\dot{V}(t)\leq\frac{(N-1)\xi-\gamma}{2}\sum_{i=1}^{N}\sum_{k=1}^{N}\left[a_{ik}\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uk}(t)\|\right]. \tag{17}$$

Since $\gamma>(N-1)\xi$, $\dot{V}(t)\leq 0$. When $\dot{\psi}_i(t)=0$, we obtain

$$\sum_{i=1}^{N}\left[\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]^{\mathrm{T}}[\dot{\psi}_i(t)]=0 \tag{18}$$

From (18), inequality (15) and equality (16) are also hold. Because $\widehat{\mathbf{x}}_{uk}(t)=\widehat{\mathbf{x}}_{ui}(t)$, the time derivative of Lyapunov function is also $\dot{V}(t)\leq 0$ in this case. In addition,

$$\left\|\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}\right\|\leq\frac{1}{N}\sum_{j=1}^{N}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|. \tag{19}$$

Thus, based on (12), one has

$$V(t)\leq\frac{1}{2}\sum_{i=1}^{N}\left(\frac{1}{N}\sum_{j=1}^{N}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|\right)^2\leq\frac{1}{2}\sum_{i=1}^{N}\left[\frac{N-1}{N}\max_{j=1,\ldots,N}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|\right]^2\leq\frac{(N-1)^2}{2N}\left(\max_{i,j=1,\ldots,N}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|\right)^2. \tag{20}$$

Then

$$\sqrt{V(t)}\leq\frac{N-1}{\sqrt{2N}}\max_{i,j=1,\ldots,N}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|\leq\frac{N-1}{2\sqrt{2N}}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[a_{ij}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|\right]. \tag{21}$$

From (17) and (21), one has

$$\dot{V}(t)+C\sqrt{V(t)}\leq\left(\frac{(N-1)\xi-\gamma}{2}+C\frac{N-1}{2\sqrt{2N}}\right)\sum_{i=1}^{N}\sum_{j=1}^{N}\left[a_{ij}\left\|\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right\|\right]. \tag{22}$$

If C is chosen to satisfy $0<C\leq\frac{-\sqrt{2N}\left((N-1)\xi-\gamma\right)}{(N-1)}$, then from Lemma 1 it is concluded that $V(t)$ is convergent in finite time with a settling time T_{s} as follows:

$$T_{\mathrm{s}}\leq\frac{-2\sqrt{V(0)}(N-1)}{\sqrt{2N}((N-1)\xi-\gamma)}. \tag{23}$$

Therefore, from (17) and (23) we have

$$\lim_{t\to T_s}\left[\widehat{\mathbf{x}}_{ui}(t)-\frac{1}{N}\sum_{j=1}^{N}\widehat{\mathbf{x}}_{uj}(t)\right]=0$$
$$\lim_{t\to T_s}\left[\widehat{\mathbf{x}}_{ui}(t)-\widehat{\mathbf{x}}_{uj}(t)\right]=0. \tag{24}$$

It is proved that the estimator defined by (8)–(10) converges in finite time with a settling time defined by (23). In the next step, it is shown that this estimator converges to the summation of distance vectors of the leaders from their geometric center. From (8) and (10), we have

$$\sum_{i=1}^{N}\widehat{\mathbf{x}}_{ui}(t)=\gamma\sum_{i=1}^{N}\int_{0}^{\tau=t}\sum_{k=1}^{N}a_{ik}\left[\frac{\widehat{\mathbf{x}}_{uk}(\tau)-\widehat{\mathbf{x}}_{ui}(\tau)}{\left\|\widehat{\mathbf{x}}_{uk}(\tau)-\widehat{\mathbf{x}}_{ui}(\tau)\right\|}\right]\mathrm{d}\tau+\sum_{i=1}^{N}\widetilde{\mathbf{d}}_{i}(t)=\gamma\int_{0}^{\tau=t}\sum_{i=1}^{N}\sum_{k=1}^{N}a_{ik}\left[\frac{\widehat{\mathbf{x}}_{uk}(\tau)-\widehat{\mathbf{x}}_{ui}(\tau)}{\left\|\widehat{\mathbf{x}}_{uk}(\tau)-\widehat{\mathbf{x}}_{ui}(\tau)\right\|}\right]\mathrm{d}\tau+\sum_{i=1}^{N}\widetilde{\mathbf{d}}_{i}(t). \tag{25}$$

Since the graph is undirected,

$$\sum_{i=1}^{N}\widehat{\mathbf{x}}_{ui}(t)=\sum_{i=1}^{N}\widetilde{\mathbf{d}}_{i}(t). \tag{26}$$

Also from (9) and (7), one has

$$\sum_{i=1}^{N}\widehat{\mathbf{x}}_{ui}(t)=N\sum_{k=1}^{M}\mathbf{l}_{k}(t)-\sum_{j=1}^{N}\widehat{\mathbf{x}}_{j}(t). \tag{27}$$

So, from (24), $\widehat{\mathbf{x}}_{ui}(t)=\sum_{k=1}^{M}\mathbf{l}_{k}(t)-\widehat{\mathbf{x}}_{i}(t)$, when $t\geq\max\{T,T_{\mathrm{s}}\}$. The values of T and T_{s} are defined in (7) and (11), respectively. □

Remark 1 Given Assumption 1 and defining a real positive constant $\delta_{kj},\ k\in V_{\mathrm{F}},\ j\in V_{\mathrm{L}}$ with $0\leq\delta_{kj}\leq 1$ when the follower k is connected to the leader j and $\delta_{kj}=0$ when the follower k is not connected to the leader j, we have

$$\left\|\dot{\widetilde{\mathbf{d}}}_{i}(t)\right\|=\left\|\left(\delta_{i1}\dot{\mathbf{l}}_{1}(t)+\delta_{i2}\dot{\mathbf{l}}_{2}(t)+\cdots+\delta_{iM}\dot{\mathbf{l}}_{M}(t)\right)-\dot{\widehat{\mathbf{x}}}_{i}(t)\right\|. \tag{28}$$

From (4) and (6), we deduce that

$$\begin{aligned}\left\|\dot{\widetilde{\mathbf{d}}}_{i}(t)\right\|&=\Big\|\left(\delta_{i1}\dot{\mathbf{l}}_{1}(t)+\delta_{i2}\dot{\mathbf{l}}_{2}(t)+\cdots+\delta_{iM}\dot{\mathbf{l}}_{M}(t)\right)\\&\quad-\frac{1}{M}\left(\delta_{i1}\dot{\mathbf{l}}_{1}(t)+\delta_{i2}\dot{\mathbf{l}}_{2}(t)+\cdots+\delta_{iM}\dot{\mathbf{l}}_{M}(t)\right)\\&\quad-\dot{\phi}_{i}(t)\Big\|\leq\Big\|\frac{M-1}{M}\left(\delta_{i1}\dot{\mathbf{l}}_{1}(t)+\delta_{i2}\dot{\mathbf{l}}_{2}(t)+\cdots\right.\\&\quad\left.+\delta_{iM}\dot{\mathbf{l}}_{M}(t)\right)\Big\|+\left\|\dot{\phi}_{i}(t)\right\|\leq\frac{M-1}{M}\\&\quad\times\left(\delta_{i1}p_{1}+\delta_{i2}p_{2}+\cdots+\delta_{iM}p_{M}\right)+\alpha\sum_{k=1}^{N}\left\|a_{ik}\right\|\\&\leq\left(\frac{M-1}{M}\right)\sum_{j=1}^{M}p_{j}+\alpha\sum_{k=1}^{N}\left|a_{ik}\right|.\end{aligned} \tag{29}$$

Then from (29), an upper bound for ξ is $\left(\frac{M-1}{M}\right)\sum_{j=1}^{M}p_{j}+\alpha\sum_{k=1}^{N}\left|a_{ik}\right|$. To use estimator (8), the parameter γ in (10) should be selected. From Theorem 1, we need ξ to determine γ. This remark proposed an upper bound for ξ.

4.2 Surrounding control protocol

Suppose that the nonlinear dynamics of the follower $\mathbf{f}_{i}(\mathbf{x}_{i},t)$ can be parameterized as

$$\mathbf{f}_{i}(\mathbf{x}_{i},t)=\vartheta(t,\mathbf{x}_{i})\chi_{i},\quad i\in V_{\mathrm{F}} \tag{30}$$

where $\vartheta(t,\mathbf{x}_{i})\in\mathbb{R}^{2\times 2}$ is a symmetric matrix of a nonlinear function with bounded elements and $\chi_{i}\in\mathbb{R}^{2}$ is an unknown constant parameter vector.

Theorem 2 presents an adaptive surrounding control protocol for the followers with dynamics (1) by using the proposed estimator (8).

Theorem 2 *Consider the multi-agent system* (1) *and* (2) *with estimators* (4) *and* (8). *The control protocol*

$$\mathbf{u}_{i}(t)=\dot{\widehat{\mathbf{x}}}_{i}(t)+v\frac{\mathrm{d}}{\mathrm{d}t}\|\widehat{\mathbf{x}}_{ui}(t)\|\left(I_{n}\otimes\begin{bmatrix}\cos\left(\frac{i.2\pi}{N}\right)&0\\0&\sin\left(\frac{i.2\pi}{N}\right)\end{bmatrix}\right)\mathbf{1}-\vartheta(t,\mathbf{x}_{i})\hat{\chi}_{i}-c\rho_{i} \tag{31}$$

causes the followers to create certain geometrical configuration with adaptive law given by

$$\dot{\hat{\chi}}_{i}(t)=\frac{c_{f}}{c}\vartheta(t,\mathbf{x}_{i})\rho_{i} \tag{32}$$

and

$$\rho_{i}=\mathbf{x}_{i}(t)-\widehat{\mathbf{x}}_{i}(t)-v\|\widehat{\mathbf{x}}_{ui}(t)\|\left(\begin{bmatrix}\cos\left(\frac{2\pi i}{N}\right)\\\sin\left(\frac{2\pi i}{N}\right)\end{bmatrix}\right) \tag{33}$$

where c, v and c_{f} are real positive constants.

Proof For $i\in V_{\mathrm{F}}$, define $\zeta_{i}=\hat{\chi}_{i}-\chi_{i}$. Consider the following Lyapunov function candidate

$$V(t)=\frac{1}{2c}\sum_{i=1}^{N}\rho_{i}^{\mathrm{T}}\rho_{i}+\frac{1}{2c_{f}}\sum_{i=1}^{N}\zeta_{i}^{\mathrm{T}}\zeta_{i}. \tag{34}$$

By evaluating time derivative of $V(t)$, we have

$$\dot{V}(t)=\frac{1}{c}\sum_{i=1}^{N}\rho_{i}^{\mathrm{T}}\dot{\rho}_{i}+\frac{1}{c_{f}}\sum_{i=1}^{N}\dot{\zeta}_{i}^{\mathrm{T}}\zeta_{i}=\frac{1}{c}\sum_{i=1}^{N}\rho_{i}^{\mathrm{T}}\big(\dot{\mathbf{x}}_{i}(t)-$$

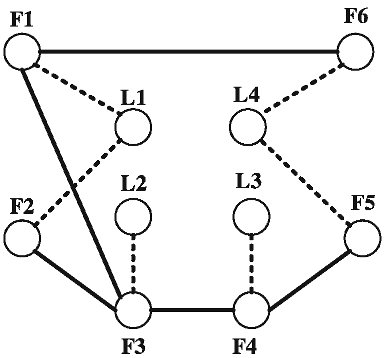

Fig. 1 Communication topology of a group of agents with 6 followers and 4 leaders (F_i the ith follower, L_j the jth leader)

$$\dot{\hat{\mathbf{x}}}_i(t) - v\frac{\mathrm{d}}{\mathrm{d}t}\|\hat{\mathbf{x}}_{ui}(t)\|\left(I_n \otimes \begin{bmatrix} \cos\left(\frac{i.2\pi}{N}\right) & 0 \\ 0 & \sin\left(\frac{i.2\pi}{N}\right) \end{bmatrix}\right)\mathbf{1}\Big) + \frac{1}{c_f}\sum_{i=1}^{N} \dot{\hat{\chi}}_i^{\mathrm{T}} \zeta_i. \tag{35}$$

Using (1) and (30)–(32), it follows that

$$\dot{V}(t) = \frac{1}{c}\sum_{i=1}^{N} \rho_i^{\mathrm{T}}\Big(\vartheta(t, \mathbf{x}_i)\chi_i + \mathbf{u}_i(t) - \dot{\hat{\mathbf{x}}}_i(t) - v\frac{\mathrm{d}}{\mathrm{d}t}\|\hat{\mathbf{x}}_{ui}(t)\|\left(I_n \otimes \begin{bmatrix} \cos\left(\frac{i.2\pi}{N}\right) & 0 \\ 0 & \sin\left(\frac{i.2\pi}{N}\right) \end{bmatrix}\right)\mathbf{1}\Big) + \frac{1}{c}\sum_{i=1}^{N} \rho_i^{\mathrm{T}}\vartheta(t, \mathbf{x}_i)\zeta_i = -\sum_{i=1}^{N} \rho_i^{\mathrm{T}}\rho_i \le 0. \tag{36}$$

By employing (1), (30), (31), (33) and Barbalat's lemma, we conclude that $\lim_{t\to\infty} \rho_i = 0$ where $i \in V_{\mathrm{F}}$. Hence, the followers create certain geometrical configurations around the geometric center of the leaders. □

5 Simulation examples

To validate the proposed methods, three examples are given in this section. Consider a group of agents with $N = 6$ followers and $M = 4$ leaders and assume $n = 2$. The communication topology given in Fig. 1 is undirected. In this figure, solid line represents the communication among followers and dash line represents the communication between the followers and the leaders.

In this example, the unknown nonlinear dynamics of the followers are chosen as

$$\mathbf{f}_i(\mathbf{x}_i, t) = \vartheta(t, \mathbf{x}_i)\chi_i = \begin{bmatrix} i\cos(2t) & 0 \\ 0 & 0.2e^{-|tx_{iy}|} \end{bmatrix}\chi_i \tag{37}$$

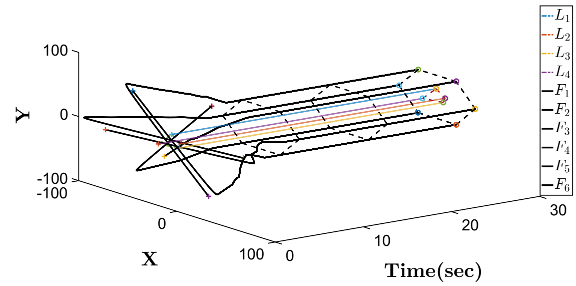

Fig. 2 Trajectories of agents in Example 1 with stationary leaders

for the followers 1–3 and

$$\mathbf{f}_i(\mathbf{x}_i, t) = \vartheta(t, \mathbf{x}_i)\chi_i = \begin{bmatrix} \cos(0.1|x_{ix}|) & 0 \\ 0 & 0.2i\sin(4t) \end{bmatrix}\chi_i. \tag{38}$$

for the followers 4–6.

The state vectors of the followers are defined as $\mathbf{x}_i = (x_{ix}, x_{iy})^{\mathrm{T}}$, and the unknown constant parameter vectors are chosen as $\chi_1 = [0.5\ 0.7]^{\mathrm{T}}$, $\chi_2 = [-0.5\ 0.5]^{\mathrm{T}}$, $\chi_3 = [1\ 0.5]^{\mathrm{T}}$, $\chi_4 = [0.6\ 0.5]^{\mathrm{T}}$, $\chi_5 = [-1\ 0.8]^{\mathrm{T}}$ and $\chi_6 = [-0.8\ -1]^{\mathrm{T}}$.

Example 1 Consider the surrounding control problem for a multi-agent system with communication topology given in Fig. 1. In this example, it is assumed that the positions of the leaders are stationary and the nonlinear dynamics of the followers are given by (37) and (38). The initial positions of the followers and the leaders are chosen as $\mathbf{X}(0) = \big[0, 20, -10, 3, -11, -10, -6, -5, -6, 6, -8, 18\big]^{\mathrm{T}}$ and $\mathbf{L}(0) = \big[-5.2, 17.5, 4, 7.1, 2.25, -0.3, -19, -4.2\big]^{\mathrm{T}}$, respectively. With the control input (31) and adaptive laws (32) and estimators (4) and (8), the simulation results are shown in Fig. 2 with $v = 2$, $c = 13$, $c_f = 0.2$, $\alpha = 2$ and $\gamma = 5$. In this figure, $+$ and o represent start and end points of the agents, respectively. Solid line denotes the trajectory of the followers, and dash line shows the trajectory of the leaders. In Fig. 3, the surrounding error vector of the system is illustrated.

To show the performance of estimator (8), the norm of the estimated vector $\hat{\mathbf{x}}_{ui}$ of each follower is illustrated in Fig. 4. It is easy to see that the estimator converges to a constant value after a finite time.

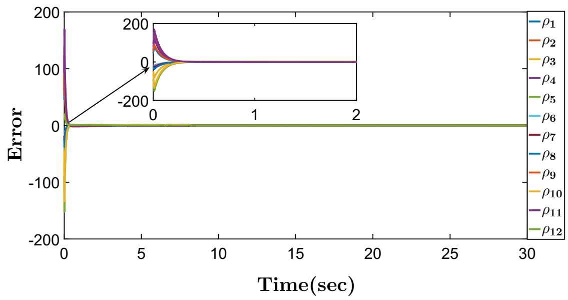

Fig. 3 Global surrounding error vector in Example 1

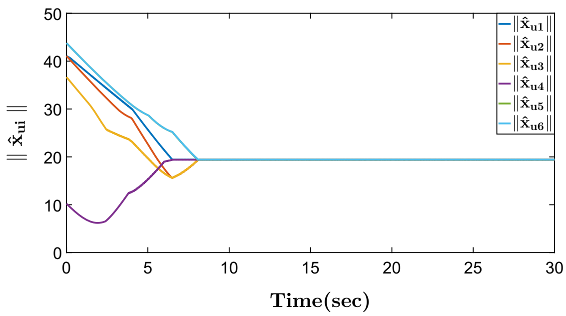

Fig. 4 Norm of the estimated vector $\hat{\mathbf{x}}_{ui}$ in Example 1

Example 2 In this example, it is assumed that the leaders are moving. The dynamics of the leaders are chosen as,

$$\mathbf{L}(t) = \begin{bmatrix} -15-0.2t \\ \sin(t) \\ 14 \\ -17+0.1t \\ 2.5 \\ -0.3t \\ -4.015t \\ -4-0.2t \end{bmatrix}. \tag{39}$$

Under the control protocol (31) and adaptive laws (32) and estimators (4) and (8), simulation results are shown in Figs. 5, 6 and 7 with $\upsilon = 2,\ c = 15,\ c_f = 0.2$, $\alpha = 5$ and $\gamma = 15$. From Fig. 5, it is easy to see that the followers are surrounding the leaders in certain geometrical configurations. Figure 6 represents surrounding error vector of the system. In this example, because the leaders are moving, the summation of the distances of leaders from their geometric center gets larger as time increases. In Fig. 7, we illustrate the norms of the summation of the distances that each follower estimates.

Example 3 In this example, we assume that the leaders have two different time-varying dynamics, in two intervals. In $t \in [0, 9)$, the dynamics of leaders is to be the same as Example 2. In $t \in [9, 20]$, the dynamics of leaders are chosen as follows

$$\mathbf{L}(t) = \begin{bmatrix} -60-0.8t \\ 28+t \\ 100 \\ 68+0.4t \\ 10+4t \\ -28-12t \\ -8.06t \\ -12.8t \end{bmatrix}. \tag{40}$$

The constant parameter of the control protocol (31) and adaptive laws (32) is $\upsilon = 7.5,\ c = 15,\ c_f = 0.2,$

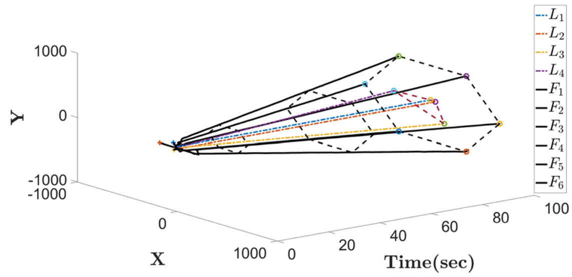

Fig. 5 Trajectories of agents in Example 2 for moving leaders

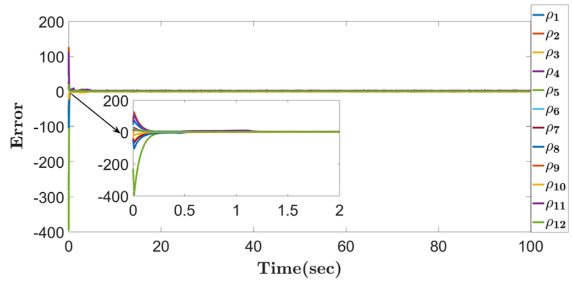

Fig. 6 Global surrounding error vector in Example 2

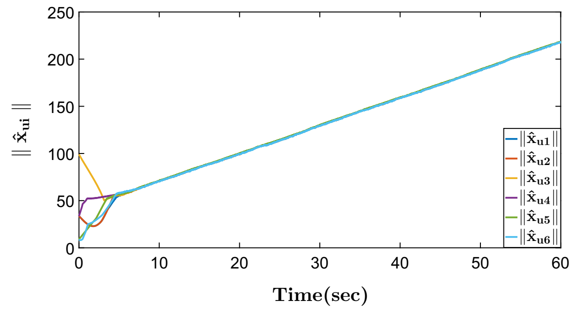

Fig. 7 Norm of the estimated vector $\hat{\mathbf{x}}_{ui}$ that each follower makes from the summation of distances of the leaders to their geometric center

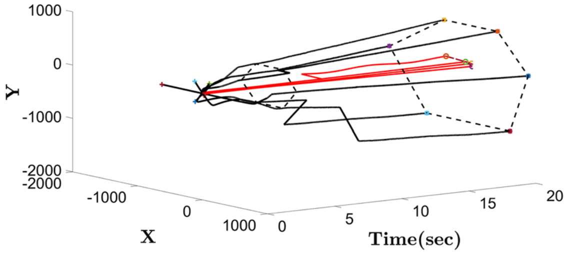

Fig. 8 Trajectories of agents in Example 3 for moving leaders

$\alpha = 6$ and $\gamma = 15$. The simulation result is shown in Fig. 8. From this figure, it can be seen that if any of the leaders makes a sudden change in its motion, the proposed method can estimate it and produce necessary forces to keep the followers enclosing the leaders.

Remark 2 In [18], the enclosing control problem is investigated for a multi-agent system with stationary and moving target. In this paper, similar to containment problem, the followers should be placed around the convex hall created by the leaders with a specified distance. In [19], the surrounding control for a multi-agent system with linear second-order dynamics is considered. Using the estimator of the geometric center of the leaders, a control protocol has been created such that the followers create a equidistant circular formation around the leaders. In [20], the surrounding problem is studied for a multi-agent system with nonlinear dynamics, where the dynamics of the agents have unknown parameters. Using the geometric center of the leaders, two adaptive control strategies are proposed. In [19,20], it is assumed that the movement of the leaders around their geometric center is bounded. Each follower must know the maximum distance between the leaders and the geometric center; then, by using this maximum distance, the control strategies have been created. To overcome these restricting assumptions, in this article an estimator to estimate the summation of distances of the leaders from their geometric center is proposed. If the distance between any of the leaders and their geometric center increases, the estimated summation also increases. So each follower gets aware of this increment, and in consequence, the control input gets adjusted. Example 3 clearly shows this case.

6 Conclusion

In this paper, we studied the surrounding control of the stationary and moving leaders, where a group of followers wants to surround a team of leaders. It was assumed that the dynamics of the followers are nonlinear and nonidentical with unknown parameters. First, an estimator for the geometric center of the leaders was introduced. Then, based on the estimation of the center of leaders, an estimator was presented to predict the summation of the distances of leaders from their geometric center. This estimation was used to calculate the placement radius of the followers around the geometric center of the leaders. This made the movement of the leaders adaptive to the position changes of the leaders. For future work, the redesign of the proposed estimators wherein the convergence time is fixed and is not dependent on the initial conditions, can be considered. On the other hand, the total number of the followers is used in the estimator of the summation of the distances of leaders from their geometric center. Another direction for future work is to take this number as an unknown parameter and try to find suitable adaptive laws for its estimation. Stochastic disturbance and time delay may be found in very dynamical system. The mathematical analysis and designing a suitable controller for the considered system in this article, when exposed to the stochastic disturbance and time delay, can be another headline for future works.

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