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Finite frequency fault detection for a class of nonhomogeneous Markov jump systems with nonlinearities and sensor failures

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Abstract In this paper, the finite frequency fault detection (FD) problem is addressed for a class of non-homogeneous Markov jump systems with nonlinearities and sensor failures. Compared with the existing sensor fault models that contain many known faulty modes, the fault model in this paper is more general since it not only covers more types of sensor failures but also does not need to know the fault information in advance. Then, by means of finite frequency stochastic performance indices, a novel FD scheme is proposed. Some new lemmas, in which the nonlinear item and nonhomogeneous Markov switching are dealt appropriately, are developed to capture the stability of the system and desired finite frequency performances. Then, by the derived lemmas, sufficient conditions with potentially less conservativeness are investigated to guarantee the existence of the FD filters. Finally, an application to HiMAT vehicle is given to illustrate the effectiveness of the derived theoretical results.

Keywords Fault detection · Finite frequency · Nonhomogeneous · Nonlinearities · Sensor failure

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1 Introduction

In recent decades, it is witnessed that Markov jump systems (MJSs) have received intensive research attention due to their strong ability to model the practical systems with sudden parameter variations, such as manufacturing systems, economic systems, electrical systems and communication systems [1–8]. MJS is a kind of system that includes two parts, finite discrete jump modes and system states which are, respectively, governed by Markov process and discrete-time differential equations [9]. As the pivotal factor of Markov processes, transition probabilities (TPs) play important roles in determining relevant system performances. For normal MJSs with affirmatory and known TPs, great efforts have been made to concentrate on the problems of stable and stability, $\mathcal{H}_\infty$ estimation and filtering, and so on [10–12]. For more information, one can see [13] for a survey. Further, for semi-MJSs and MJSs with partially known even uncertain TPs, there were also fruitful results concerned with the similar problems mentioned above, for example, [14,15] and the references therein. It should be noticed that these results have been presented in the case of the TPs are

constant, i.e., the considered Markov chain is a homogeneous one. Nevertheless, in many practical systems, such a premise may be not satisfied. Taking the vertical take-off and landing (VTOL) system, which can be seen in [16], for example, it can be modeled into normal MJSs in an ideal situation. However, the derived jump probabilities will be varied when the system experiences sudden disturbances such as wind and temperature. Thus, it is a matter of course to consider nonhomogeneous MJSs, and consequently there are a few kinds of literature focusing on such kinds of systems, such as [16] proposed an $\mathcal{H}_\infty$ estimation method in discrete-time domain for the MJSs with time-varying TPs, [17] addressed a robust $\mathcal{H}_\infty$ filtering scheme for kinds of uncertain discrete-time nonhomogeneous MJSs. However, the research on such kinds of systems is still in its infancy and deserves further considerations.

On the other hand, FD plays an important role in modern control field due to the increasing demands on reliability and low maintenance costs [18–22]. And consequently, great research efforts have been made on the FD problem for MJSs. For example, [23] presented a generalized $\mathcal{H}_2$ FD method for two-dimensional MJSs, [24] dealt with the FD and isolation problem for MJSs based on a geometric method, [25] designed the FD filter for MJSs with nonlinear stochastic time delays, and [26] developed an FD strategy for discrete-time MJSs with partial knowledge of TPs. Further, from the references therein, one can see more information about the results of FD for MJSs. Nevertheless, this method cannot be utilized for nonhomogeneous MJSs with nonlinearities, especially when the sensors experience failures.

It should be also noticed that the methods mentioned above are all developed in time domain. However, for some fault signals, they usually occur in a certain frequency domain, such as the actuator stuck fault is always in low-frequency range. In such situations, the aforementioned FD methods are imprecise due to the signal frequency constraints were not considered. Consequently, the FD strategy proposed in a finite frequency domain is considered as a more practical one nowadays. Of course, with the aid of the generalized Kalman–Yakubovič–Popov (GKYP) lemma, many FD schemes have been investigated, such as in [28–30], but all of them are proposed for certain linear systems. However, it should be declared that for systems which are not linear time-invariant ones, the GKYP lemma cannot be utilized directly. And thus, it

is valuable to explore the finite frequency FD method for such kinds of systems. Accordingly, there are a few results concerned with such a topic. For example, in [31], FD and isolation scheme have been presented in finite frequency domain for a switched stochastic system with multi-stochastic parameters, and in [18], FD strategy for fuzzy systems has been considered. Nevertheless, those developed methods cannot be utilized to detect the sensor faults that occurred in certain frequency range for nonhomogeneous MJSs, especially for such kinds of systems with nonlinearities, which cannot be ignored in practical cases when designing the system.

Motivated by the aforementioned points, it is imperative to develop a finite frequency FD strategy for nonhomogeneous MJSs with sensor failures and nonlinearities. Nevertheless, such an interesting topic is with some difficulties listed below: (1) How to model the possible sensor failures appropriately? (2) How to detect the failures with small amplitude? (3) How to deal with the nonlinearities and nonhomogeneous Markov switching when developing the solvable conditions in finite frequency domain? Specifically, in this paper, the listed questions are solved legitimately. Firstly, a novel sensor fault model is given, which is more general than existing ones, since it not only involves more kinds of failures but also does not need to know the fault information beforehand. And then, an FD strategy is presented, in which the residual is sensitive to the fault and the reference input in faulty case, meanwhile robust to the disturbance and the reference input in fault-free case. Subsequently, sufficient conditions with potentially less conservativeness are developed to capture the required performance for the system, in which the nonlinear item and nonhomogeneous Markov switching are dealt appropriately. And the FD filter gains can be characterized by derived conditions. Lastly, the effectiveness of the investigated FD scheme is verified by an application to HiMAT vehicle.

2 Modeling of the system and problem formulation

Consider a class of MJSs given in a probability space $\{\mathbf{\Omega}, \mathbf{F}, \mathbf{P}\}$ as the form of

$$
\begin{aligned}
x(k+1) &= A_{\alpha_k} x(k) + E_{\alpha_k} f\big(x(k), \alpha_k\big) \\
&\quad + B_{1\alpha_k} w(k) + B_{2\alpha_k} s(k) \\
y(k) &= C_{\alpha_k} x(k) + D_{1\alpha_k} w(k) + D_{2\alpha_k} s(k)
\end{aligned}
\tag{1}
$$

Table 1 Illustration of finite frequency range

	ϖ_{s1}	ϖ_{s2}	$\varXi_{\varpi_s}$
LF	$-\varpi_{sl}$	ϖ_{sl}	$\{|\varpi_s| \leq \varpi_{sl}|\varpi_s \in \mathbf{R}\}$
MF	ϖ_{s1}	ϖ_{s2}	$\{(\varpi_s - \varpi_{s1})(\varpi_s - \varpi_{s2}) \leq 0|\varpi_s \in \mathbf{R}\}$
HF	ϖ_{sh}	$2\pi - \varpi_{sh}$	$\{|\varpi_s| \geq \varpi_{sh}|\varpi_s \in \mathbf{R}\}$

where $x(k) \in \mathbf{R}^n$ is the system state, $y(k) \in \mathbf{R}^m$ is the measured output, $w(k) \in \mathbf{L}_2[0, \infty)$ is the disturbance, $s(k) \in \mathbf{L}_2[0, \infty)$ is the reference input with frequency $\varpi_s, \varpi_s \in \varXi_{\varpi_s} := \{\varsigma(\varpi_s - \varpi_{s1})(\varpi_s - \varpi_{s2}) \leq 0|\varpi_s \in \mathbf{R}\}$. $f(\cdot, \cdot) : \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}^n$ is a nonlinear function. Further, the parameter $\{\alpha_k, k \geq 0\}$ represents a nonhomogeneous Markov stochastic process with range set of $\mathbb{S} = \{1, 2, \ldots, N\}$ and TP matrix $\Lambda(k) = (\lambda_{ij}(k))$ given by $\lambda_{ij}(k) = \mathbf{P}\{\alpha_{k+1} = j|\alpha_k = i\}$ with $\lambda_{ij}(k) > 0, \forall i, j \in \mathbb{S}$ and $\sum_{j \in \mathbb{S}} \lambda_{ij}(k) = 1$. The system matrices $A_{\alpha_k}, E_{\alpha_k}, B_{1\alpha_k}, B_{2\alpha_k}, C_{\alpha_k}, D_{1\alpha_k}, D_{2\alpha_k}$ are known matrices with proper dimensions, which characterize the relationships between the system variables. In the sequel of the paper, the subscript α_k associated with $\alpha_k = i$ is denoted by i.

Remark 1 It is noted that $\varXi_{\varpi_s}$ is utilized to limit the frequency range. From the reference [27], it is obvious that $\varXi_{\varpi_s}$ can characterize low-frequency (LF), middle-frequency (MF) and high-frequency (HF) domain when $\varsigma = 1$ by selecting the parameters as in Table 1 while characterize the full frequency domain when $\varsigma = -1$ and $\varpi_{s1} = \varpi_{s2} = 0$.

Assumption 1 The nonlinear function $f(\cdot, \cdot)$ is assumed to be bounded as

$$
\begin{aligned}
&\mathbf{E}\{f(x(k), i)|x(k), i\} = 0; \\
&\mathbf{E}\{f(x(k), i)f^\mathrm{T}(x(k), i)|x(k), i\} \\
&= \sum_{\kappa=1}^{\mathfrak{K}} \tau_{\kappa i} \tau_{\kappa i}^\mathrm{T}(x^\mathrm{T}(k)\Theta_{\kappa i} x(k))
\end{aligned}
\tag{2}
$$

where $\tau_{\kappa i}, \Theta_{\kappa i}, (\kappa = 1, \ldots, \mathfrak{K})$ are known column vectors and positive definite matrices.

Remark 2 The description (2) in statistical sense is more general since it can involve various types of nonlinearities, such as the nonlinear stochastic sequences, whose powers are dependent on the sector-bounded nonlinear function of the system states.

In this paper, the time-varying TP matrix $\Lambda(k)$ is considered to be with a polytopic structure described by

$$
\Lambda(k) = \Lambda(o(k)) = \sum_{\iota=1}^{L} o_\iota(k)\Lambda^\iota
\tag{3}
$$

where $o_\iota(k) \geq 0, \iota \in \mathfrak{L} = \{1, 2, \ldots, L\}$ are scalar functions of k satisfying $\sum_{\iota \in \mathfrak{L}} o_\iota(k) = 1, \Lambda^\iota$ are known matrices, which denote the vertices of the polytope.

Remark 3 It is noticed that equation (3) characterized a kind of time-dependent Markov chain, and this kind of process is called as nonhomogeneous Markov process. Further, if the time-varying one is fixed, i.e., $\Lambda(k) = \Lambda$, it is the normal homogeneous one, which performs as a special case of the considered process in this manuscript. In [16], the author proposed an $\mathcal{H}_\infty$ estimation method for a class of nonhomogeneous MJSs, in which the TP matrix $\Lambda(k)$ is subject to the piecewise homogeneous as $\Lambda^{\sigma_k}, \sigma_k$ takes values in a finite set with a finite mode, which is also a specific form of the considered one in this paper.

Further, when the sensors encounter possible failures, the following model will be adopted to describe the jth measured output

$$
y_\iota^j(k) = \varrho_\iota^j(k)y^j(k) + f^j(k), \quad \iota = 0, 1
\tag{4}
$$

where $\varrho_\iota^j(k)$ denotes the unknown efficiency coefficient, and $f^j(k)$ represents the stuck or drift fault with unknown bound occurred on the jth sensor with frequency $\varpi_f, \varpi_f \in \varXi_{\varpi_f} := \{\varsigma(\varpi_f - \varpi_{f1})(\varpi_f - \varpi_{f2}) \leq 0|\varpi_f \in \mathbf{R}\}$. Moreover, $\iota = 0$ means the fault-free case in which $\varrho_\iota^j(k) = 1$ and $f^j(k) = 0$, while $\iota = 1$ means faulty case, wherein $0 \leq \varrho_\iota^j(k) \leq 1$. Thus, we have

$$
y_\iota(k) = \varrho_\iota(k)y(k) + f(k)
\tag{5}
$$

with $\varrho_\iota(k) = \mathrm{diag}\{\varrho_\iota^j(k)\}$.

Remark 4 There are a few kinds of literature considering sensor failures, such as the multi-mode model has been adopted in [18]. However, some information about the fault in each mode should be exactly known

beforehand, which restricts the application of the proposed method. In the model (5), $\varrho_\iota(k)$ is assumed to be unknown. Such an assumption is more reasonable to describe the practical systems. Moreover, except outage and stuck ones, the model (5) can express more kinds of failures such as loss of effectiveness and drift phenomena compared to [18]; thus, it is more general.

The FD filter as the following form will be adopted to generate the residuals

$$\hat{x}(k + 1) = A_{fi}\hat{x}(k) + B_{1fi}y_\iota(k) + B_{2fi}s(k)$$
$$r(k) = C_{fi}\hat{x}(k) + D_{fi}y_\iota(k) \tag{6}$$

where $\hat{x}(k) \in \mathbf{R}^n$ denotes the filter state, $r(k) \in \mathbf{R}$ is the residual signal, A_{fi}, B_{1fi}, B_{2fi}, C_{fi} and D_{fi} are filter gains. Then, by defining the augmented vector $\tilde{x}(k) = [\, x^{\mathrm{T}}(k) \; \hat{x}^{\mathrm{T}}(k) \,]^{\mathrm{T}}$, the whole FD systems together with the possible faults can be obtained as follows

$$\tilde{x}(k + 1) = \mathcal{A}_i^\iota \tilde{x}(k) + \mathcal{E}_i^\iota f\big(\mathcal{T}\tilde{x}(k), i\big)$$
$$\qquad\qquad + \mathcal{B}_{1i}^\iota w(k) + \mathcal{B}_{2i}^\iota s(k) + \mathcal{B}_{3i}^\iota f(k) \tag{7}$$
$$r(k) = \mathcal{C}_i^\iota \tilde{x}(k) + \mathcal{D}_{1i}^\iota w(k) + \mathcal{D}_{2i}^\iota s(k) + \mathcal{D}_{3i}^\iota f(k)$$

where $\mathcal{T} = [\, I \; 0\,]$, $\mathcal{E}_i^\iota = [\, Ei^{\mathrm{T}} \; 0\,]^{\mathrm{T}}$, and

$$
\begin{bmatrix}
\mathcal{A}_i^\iota & \mathcal{B}_{1i}^\iota & \mathcal{B}_{2i}^\iota & \mathcal{B}_{3i}^\iota \\
\mathcal{C}_i^\iota & \mathcal{D}_{1i}^\iota & \mathcal{D}_{2i}^\iota & \mathcal{D}_{3i}^\iota
\end{bmatrix}
$$
$$
= \begin{bmatrix}
A_i & 0 & B_{1i} & B_{2i} & 0 \\
B_{1fi}\varrho_\iota C_i & A_{fi} & B_{1fi}\varrho_\iota D_{1i} & B_{1fi}\varrho_\iota D_{2i}+B_{2fi} & B_{1fi} \\
D_{fi}\varrho_\iota C_i & C_{fi} & D_{fi}\varrho_\iota D_{1i} & D_{fi}\varrho_\iota D_{2i} & D_{fi}
\end{bmatrix} \tag{8}
$$

Denoting the operator $\| \cdot \|_2^{\mathbf{E}} = \sum_0^\infty \{\mathbf{E}| \cdot |^2\}^{1/2}$, and then the following definitions are firstly recalled.

Definition 1 The system (7) is stochastic stable (SS) if the inequality $\|\tilde{x}(k)\|_2^{\mathbf{E}} < \infty$ holds $\forall i \in \mathbb{S}$ when the system (7) is without disturbance and fault.

Definition 2 For all $i \in \mathbb{S}$, the system (7) is with a finite frequency stochastic (FFS) H_-index β to $s(k)$ if

$$\|r(k)\|_2^{\mathbf{E}} \geq \beta \|s(k)\|_2^{\mathbf{E}} \tag{9}$$

holds under zero initial conditions for all the solutions in certain frequency domain satisfying

$$e^{j\varpi_s} \sum_{k=0}^\infty \big(\tilde{x}(k + 1) - e^{j\varpi_{s1}}\tilde{x}(k)\big)$$
$$\big(\tilde{x}(k + 1) - e^{-j\varpi_{s2}}\tilde{x}(k)\big)^{\mathrm{T}} \leq 0 \tag{10}$$

where $\varpi_s = (\varpi_{s1} + \varpi_{s2})/2$.

Definition 3 For all $i \in \mathbb{S}$, the system (7) is with a FFS $\mathcal{H}_\infty$ performance γ to $s(k)$ if

$$\|r(k)\|_2^{\mathbf{E}} \leq \gamma \|s(k)\|_2^{\mathbf{E}} \tag{11}$$

holds under zero initial conditions for all the solutions in certain frequency domain satisfying (10).

Remark 5 It is obvious, from the Definitions 2–3, that β indicates the sensitivity to the reference input in finite frequency domain. On contrary, γ characterizes the robustness to it in a finite frequency range. And thus, the large β and the smaller γ they are, the better performance we can get.

Till now, the FD strategy to be addressed in this paper can be formulated as: For the considered nonhomogeneous MJSs (1) and the possible sensor failure, design a set of filters as form of (6) such that the dynamic of the system (7) is SS and the possible sensor failures can be detected effectively by satisfying the following requirement:

(a) The system (7) is SS and with an $\mathcal{H}_\infty$ performance γ to the disturbance $\omega(k)$.
(b) In faulty case, the impact of $s(k)$ on $r(k)$ is maximized in certain frequency range, i.e., the system (7) is with a FFS H_-index β to $s(k)$. Meanwhile in fault-free case, the impact is minimized in certain frequency range, i.e., with a finite frequency stochastic $\mathcal{H}_\infty$ performance γ_d.
(c) In faulty case, the impact of $f(k)$ on $r(k)$ is maximized in certain frequency range, i.e., the system (7) is with a FFS H_-index β_f to the fault.

Remark 6 In the proposed FD scheme which is presented above, it requires that the residual $r(k)$ is sensitive to $s(k)$ and $f(k)$ in faulty cases instead of to $f(k)$ only. This operation will make it possible to detect the fault with small amplitude. And the effectiveness will be shown in simulation further.

3 Main results

3.1 Performance analysis

In this subsection, the analysis conditions to capture the desired performances will be presented by the following lemmas and corollaries.

Lemma 1 *The system* (7) *is SS and with a prescribed* $\mathcal{H}_\infty$ *performance* γ, $\gamma > 0$, *to* $\omega(k)$ *if there exist matrices* $P_{i\iota}^\iota = P_{i\iota}^{\iota\mathrm{T}} > 0$ *and scalars* $\delta_{\kappa i}^\iota$ *such that the following inequalities*

$$\begin{bmatrix} -\delta_{\kappa i}^\iota & \tau_{\kappa i}^{\mathrm{T}} \mathcal{E}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \\ * & -\bar{\mathcal{P}}_i^\iota \end{bmatrix} < 0 \qquad (12)$$

$$\mathfrak{J}_i^{\iota\mathrm{T}} \Psi_i^\iota \mathfrak{J}_i^\iota + \mathfrak{V}_i^{\iota\mathrm{T}} \Pi_i^\iota \mathfrak{V}_i^\iota < \mathfrak{M}_i^{\iota\mathrm{T}} \Upsilon_i^\iota \mathfrak{M}_i^\iota \qquad (13)$$

hold $\forall i \in \mathbb{S}$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, *and* $\iota = 0, 1$, *where*

$$\mathfrak{J}_i^\iota = [\mathfrak{J}_{i1}^\iota \; \mathfrak{J}_{i2}^\iota], \; \mathfrak{V}_i^\iota = [\mathfrak{V}_{i1}^\iota \; \mathfrak{V}_{i2}^\iota], \; \mathfrak{M}_i^\iota = [\mathfrak{M}_{i1}^\iota \; \mathfrak{M}_{i2}^\iota],$$

$$\Psi_i^\iota = \mathrm{diag}\{\bar{\mathcal{P}}_i^\iota, -\mathcal{P}_i^\iota\}, \; \Pi_i^\iota = \mathrm{diag}\{\Omega'', 0\},$$

$$\Upsilon_i^\iota = \mathrm{diag}\{-I, \gamma^2 I\}$$

with

$$\mathfrak{J}_{i1}^\iota = [\mathcal{A}_i^{\iota\mathrm{T}} \; I]^{\mathrm{T}}, \; \mathfrak{J}_{i2}^\iota = [\mathcal{B}_{1i}^{\iota\mathrm{T}} \; 0]^{\mathrm{T}}, \; \mathfrak{V}_{i1}^\iota = [\Omega' \; 0]^{\mathrm{T}},$$

$$\mathfrak{V}_{i2}^\iota = [0 \; I]^{\mathrm{T}}, \; \mathfrak{M}_{i1}^\iota = [\mathcal{C}_i^{\iota\mathrm{T}} \; 0]^{\mathrm{T}}, \; \mathfrak{M}_{i2}^\iota = [\mathcal{D}_{1i}^{\iota\mathrm{T}} \; I]^{\mathrm{T}}$$

wherein $\bar{\mathcal{P}}_i^\iota = \sum_{j=1}^N \sum_{l=1}^L \sum_{J=1}^L o_l(k) \xi_J(k) \lambda_{ij}^\iota P_{jJ}^\iota$, $\mathcal{P}_i^\iota = \sum_{l=1}^L o_l(k) P_{i\iota}^\iota$, $\Omega' = \mathcal{T}^{\mathrm{T}}[\Theta_{1i}^{1/2} \; \Theta_{2i}^{1/2} \; \cdots \; \Theta_{\mathfrak{K}i}^{1/2}]$, $\Omega'' = \mathrm{diag}\{\delta_{1i} I, \delta_{2i} I, \ldots, \delta_{\mathfrak{K}i} I\}$ *with* $\xi_J(k) = o_\iota(k + 1)$.

Proof Considering the following Lyapunov mode-dependent functional when $f_\iota = 0$ and $s(k) = 0$

$$V(\tilde{x}(k), k, i) = \tilde{x}^{\mathrm{T}}(k) \left(\sum_{l=1}^L o_l(k) P_{i\iota}^\iota\right) \tilde{x}(k) \qquad (14)$$

By the expression of (7), the forward difference defined as $\Delta V_i(k) = \mathbf{E}\{V(\tilde{x}(k+1), k+1, j)\} - V(\tilde{x}(k), k, i)$ can be derived as

$$\begin{aligned} &\Delta V_i(k) \\ &= \tilde{x}^{\mathrm{T}}(k+1) \bar{\mathcal{P}}_i^\iota \tilde{x}(k+1) - \tilde{x}^{\mathrm{T}}(k) \mathcal{P}_i^\iota \tilde{x}(k) \\ &= \tilde{x}^{\mathrm{T}}(k) (\mathcal{A}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{A}_i^\iota - \mathcal{P}_i^\iota) \tilde{x}(k) + \tilde{x}^{\mathrm{T}}(k) \mathcal{A}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{B}_{1i}^\iota \omega(k) \\ &\quad + \omega^{\mathrm{T}}(k) \mathcal{B}_{1i}^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{A}_i^\iota \tilde{x}(k) + \omega^{\mathrm{T}}(k) \mathcal{B}_{1i}^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{B}_{1i}^\iota \omega(k) \end{aligned}$$

$$+ \mathbf{E}\{f(\mathcal{T}\tilde{x}(k), i)^{\mathrm{T}} \mathcal{E}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{E}_i^\iota f(\mathcal{T}\tilde{x}(k), i)\} \qquad (15)$$

Now, recalling the property

$$a^{\mathrm{T}} M b = \mathrm{tr}(b a^{\mathrm{T}} M) \qquad (16)$$

where a, b are nonzero vectors and M is an arbitrary matrix with suitable dimensions, one has

$$\begin{aligned} &\mathbf{E}\left\{f(\mathcal{T}\tilde{x}(k), i)^{\mathrm{T}} \mathcal{E}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{E}_i^\iota f(\mathcal{T}\tilde{x}(k), i)\right\} \\ &= \sum_{\kappa=1}^{\mathfrak{K}} \left[\tilde{x}^{\mathrm{T}}(k) \mathcal{T}^{\mathrm{T}} \Theta_{\kappa i} \mathcal{T} \tilde{x}(k)\right] \mathrm{tr}(\tau_{\kappa i} \tau_{\kappa i}^{\mathrm{T}} \mathcal{E}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{E}_i^\iota) \end{aligned}$$

$$(17)$$

Further, (12) implies that $\mathrm{tr}(\tau_{\kappa i} \tau_{\kappa i}^{\mathrm{T}} \mathcal{E}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{E}_i^\iota) < \delta_{\kappa i}^\iota$. Consequently, when $w(k) = 0$, we have

$$\begin{aligned} \Delta V_i(k) &\leq \tilde{x}^{\mathrm{T}}(k) \left[\mathcal{A}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_i^\iota \mathcal{A}_i^\iota - \mathcal{P}_i^\iota + \sum_{\kappa=1}^{\mathfrak{K}} \delta_{\kappa i}^\iota \mathcal{T}^{\mathrm{T}} \Theta_{\kappa i} \mathcal{T}\right] \tilde{x}(k) \\ &= \tilde{x}^{\mathrm{T}}(k) \Phi_i^{\iota'} \tilde{x}(k) \end{aligned}$$

$$(18)$$

Applying Schur's complement to (13), it has $\Phi_i^{\iota'} < 0$, then one has

$$\Delta V_i(k) \leq -\lambda_{\min}(-\Phi_i^{\iota'}) \tilde{x}^{\mathrm{T}}(k) \tilde{x}(k) \leq -\xi \tilde{x}^{\mathrm{T}}(k) \tilde{x}(k) \qquad (19)$$

where $\xi = \inf\{\lambda_{\min}(-\Phi_i^{\iota'})\}$, $i \in \mathbb{S}$, which implies that

$$\begin{aligned} &\|\tilde{x}(k)\|_2^{\mathbf{E}} \\ &\leq \sum_{k=0}^\infty \frac{1}{\xi} \left[V(\tilde{x}(k), k, i) - \mathbf{E}\{V(\tilde{x}(k+1), k+1, j)\}\right] \\ &\leq \mathbf{E}\{V(\tilde{x}(0), 0, \alpha_0)\} < \infty \end{aligned}$$

$$(20)$$

Thus, it can be deduced, from Definition 1, that the system (7) is SS.

Further, we are going to verify the $\mathcal{H}_\infty$ performance. Considering the index $\mathfrak{J}_{rw} = \sum_{k=0}^\infty [\mathbf{E}\{r^{\mathrm{T}}(k) r(k) - \gamma^2 \omega^{\mathrm{T}}(k) \omega(k)\}]$ under $\tilde{x}(0) = 0$, which can be written into

$$\mathfrak{I}_{rw} = \sum_{k=0}^{\infty} \left[\mathbf{E}\{ r^{\mathrm{T}}(k)r(k) - \gamma^2 \omega^{\mathrm{T}}(k)\omega(k) \} + \Delta V_i(k) \right]$$
$$+ \mathbf{E}\{ V(\tilde{x}(0), 0, \alpha_0) \} - \mathbf{E}\{ V(\tilde{x}(\infty), \infty, \alpha_\infty) \}$$
$$\leq \sum_{k=0}^{\infty} \left[\mathbf{E}\{ r^{\mathrm{T}}(k)r(k) - \gamma^2 \omega^{\mathrm{T}}(k)\omega(k) \} + \Delta V_i(k) \right]$$
$$\leq \sum_{k=0}^{\infty} \left([\, \tilde{x}^{\mathrm{T}}(k)\ \omega^{\mathrm{T}}(k)\,] \Phi_i^{\iota''} [\, \tilde{x}^{\mathrm{T}}(k)\ \omega^{\mathrm{T}}(k)\,]^{\mathrm{T}} \right) \quad (21)$$

where $\Phi_i^{\iota''} = \mathfrak{I}_i^{\iota\mathrm{T}} \Psi_i^\iota \mathfrak{I}_i^\iota + \mathfrak{V}_i^{\iota\mathrm{T}} \Pi_i^\iota \mathfrak{V}_i^\iota - \mathfrak{M}_i^{\iota\mathrm{T}} \Upsilon_i^\iota \mathfrak{M}_i^\iota$. It is obvious that the inequality (13) ensures that $\mathfrak{I}_{rw} < 0$, that is $\| r(k) \|_2^{\mathbf{E}} \leq \gamma \| \omega(k) \|_2^{\mathbf{E}}$, which implies the system (7) is with an $\mathcal{H}_\infty$ performance γ. The proof is completed. $\square$

Lemma 2 *The system* (7) *is with a prescribed FFS H_{-}index β, $\beta > 0$, to $s(k)$ if there exist matrices $P_{si\iota}^\iota = P_{si\iota}^{\iota\mathrm{T}} > 0$, $Q_{si\iota}^\iota = Q_{si\iota}^{\iota\mathrm{T}} > 0$ and scalars $\delta_{s\kappa i}^\iota$ such that*

$$\begin{bmatrix} -\delta_{s\kappa i}^\iota & \tau_{\kappa i}^{\mathrm{T}} \mathcal{E}_i^{\iota\mathrm{T}} \bar{\mathcal{P}}_{si}^\iota \\ * & -\bar{\mathcal{P}}_{si}^\iota \end{bmatrix} < 0 \quad (22)$$

$$\mathfrak{I}_{si}^{\iota\mathrm{T}} \Psi_{si}^\iota \mathfrak{I}_{si}^\iota - \mathfrak{V}_{si}^{\iota\mathrm{T}} \Pi_{si}^\iota \mathfrak{V}_{si}^\iota < \mathfrak{M}_{si}^{\iota\mathrm{T}} \Upsilon_{si}^\iota \mathfrak{M}_{si}^\iota \quad (23)$$

hold $\forall i \in \mathbb{S}$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, and $\iota = 1$, where

$$\mathfrak{I}_{si}^\iota = [\, \mathfrak{I}_{i1}^\iota\ \mathfrak{I}_{si2}^\iota \,],\ \mathfrak{V}_{si}^\iota = [\, \mathfrak{V}_{si1}^\iota\ \mathfrak{V}_{i2}^\iota \,],$$
$$\mathfrak{M}_{si}^\iota = [\, \mathfrak{M}_{i1}^\iota\ \mathfrak{M}_{si2}^\iota \,],$$
$$\Pi_{si}^\iota = \mathrm{diag}\{ \Omega_s^{''}, 0 \},\ \Upsilon_{si}^\iota = \mathrm{diag}\{ I, -\beta^2 I \},$$
$$\Psi_{si}^\iota = \mathcal{P}_{si}^\iota \otimes \mathrm{diag}\{ 0, I \} - \bar{\mathcal{P}}_{si}^\iota \otimes \mathrm{diag}\{ I, 0 \} + \mathcal{Q}_{si}^\iota \otimes \beth,$$

with $\mathfrak{I}_{si2}^\iota = [\, \mathcal{B}_{2i}^{\iota\mathrm{T}}\ 0 \,]^{\mathrm{T}}$, $\mathfrak{V}_{si1}^\iota = [\, \Omega'\ 0 \,]^{\mathrm{T}}$, $\mathfrak{M}_{si2}^\iota = [\, \mathcal{D}_{2i}^{\iota\mathrm{T}}\ I \,]^{\mathrm{T}}$, wherein $\bar{\mathcal{P}}_{si}^\iota = \sum_{j=1}^{N} \sum_{l=1}^{L} \sum_{J=1}^{L} o_\iota(k) \xi_J(k) \lambda_{ij}^\iota P_{sjJ}^\iota$, $\mathcal{P}_{si}^\iota = \sum_{\iota=1}^{L} o_\iota(k) P_{si\iota}^\iota$, $\Omega_s^{''} = \mathrm{diag}\{ \delta_{s1i}^\iota, \delta_{s2i}^\iota, \ldots, \delta_{s\mathfrak{K}i}^\iota \}$. Furthermore, $\beth = \begin{bmatrix} 0 & 1 \\ 1 & -2\cos\varpi_{sl} \end{bmatrix}$ in LF domain, $\beth = \begin{bmatrix} 0 & e^{j\varpi_{sa}} \\ e^{-j\varpi_{sa}} & -2\cos\varpi_{sb} \end{bmatrix}$ in MF domain, $\beth = \begin{bmatrix} 0 & -1 \\ -1 & 2\cos\varpi_{sh} \end{bmatrix}$ in HF domain, wherein $\varpi_{sa} = \frac{\varpi_{s1} + \varpi_{s2}}{2}$, and $\varpi_{sb} = \frac{\varpi_{s2} - \varpi_{s1}}{2}$.

Proof Under zero initial conditions and LF domain, pre- and post-multiplying the inequality (23) by $[\, \tilde{x}^{\mathrm{T}}(k)\ s^{\mathrm{T}}(k) \,]$ and its transpose, it is obvious that the

following inequality can be obtained

$$\mathbf{E}\Big(\tilde{x}^{\mathrm{T}}(k) \mathcal{P}_{si}^\iota \tilde{x}(k) - \tilde{x}^{\mathrm{T}}(k+1) \bar{\mathcal{P}}_{si}^\iota \tilde{x}(k+1)$$
$$- \tilde{x}^{\mathrm{T}}(k) \sum_{k=0}^{\infty} \delta_{s\kappa i}^\iota \mathcal{T}^{\mathrm{T}} \Theta_{\kappa i} \mathcal{T} \tilde{x}(k) - r^{\mathrm{T}}(k)r(k)$$
$$+ \beta^2 s^{\mathrm{T}}(k)s(k) + \tilde{x}^{\mathrm{T}}(k+1) \mathcal{Q}_{si}^\iota \tilde{x}(k)$$
$$+ \tilde{x}^{\mathrm{T}}(k) \mathcal{Q}_{si}^\iota \tilde{x}(k+1) - 2\cos\varpi_{sl} \tilde{x}^{\mathrm{T}}(k) \mathcal{Q}_{si}^\iota \tilde{x}(k) \Big) \leq 0 \quad (24)$$

On the one hand, for matrices $P_{si\iota}^\iota > 0$, define an energy functional $V_{si}(k) = \tilde{x}^{\mathrm{T}}(k)(\sum_{\iota=1}^{L} o_\iota(k) P_{si\iota}^\iota) \tilde{x}(k)$ as the same form of (14), then from the inequality (22) and the proof to Lemma 1, one can prove $\Delta V_{si}(k) \leq [\, \tilde{x}^{\mathrm{T}}(k)\ s^{\mathrm{T}}(k) \,] \Phi_{si}^{\iota'} [\, \tilde{x}^{\mathrm{T}}(k)\ s^{\mathrm{T}}(k) \,]^{\mathrm{T}} < 0$ easily, where $\Phi_{si}^{\iota'} = \mathfrak{I}_{si}^{\iota\mathrm{T}} (\bar{\mathcal{P}}_i^\iota \otimes \mathrm{diag}\{ I, 0 \} - \mathcal{P}_{si}^\iota \otimes \mathrm{diag}\{ 0, I \}) \mathfrak{I}_{si}^\iota - \mathfrak{V}_{si}^{\iota\mathrm{T}} \Pi_{si}^\iota \mathfrak{V}_{si}^\iota$, that means the sum of the first three items in the left side of inequality (24) is larger than zero, which yields that

$$\mathbf{E}\big(-r^{\mathrm{T}}(k)r(k) + \beta^2 s^{\mathrm{T}}(k)s(k) + \mathbb{I}_f \big) \leq 0 \quad (25)$$

where $\mathbb{I}_f = \tilde{x}^{\mathrm{T}}(k+1) \mathcal{Q}_{si}^\iota \tilde{x}(k) + \tilde{x}^{\mathrm{T}}(k) \mathcal{Q}_{si}^\iota \tilde{x}(k+1) - 2\cos\varpi_{sl} \tilde{x}^{\mathrm{T}}(k) \mathcal{Q}_{si}^\iota \tilde{x}(k)$.

On the other hand, recalling the property (16) again and then summing $\mathbb{I}_f$ from $k = 0$ to ∞, we have

$$\sum_{k=0}^{\infty} \mathbb{I}_f = \sum_{k=0}^{\infty} \big(\mathrm{tr}\big(\big(\tilde{x}(k)\tilde{x}^{\mathrm{T}}(k+1) + \tilde{x}(k+1)\tilde{x}^{\mathrm{T}}(k)$$
$$- 2\cos\varpi_{sl} \tilde{x}(k)\tilde{x}^{\mathrm{T}}(k) \big) \mathcal{Q}_{si}^\iota \big) \quad (26)$$

Under zero initial conditions and LF domain, performing some calculations and Euler's formula on the constraint (10), we have

$$\sum_{k=0}^{\infty} \big(\tilde{x}(k)\tilde{x}^{\mathrm{T}}(k+1) + \tilde{x}(k+1)\tilde{x}^{\mathrm{T}}(k)$$
$$- 2\cos\varpi_{sl} \tilde{x}(k)\tilde{x}^{\mathrm{T}}(k) \big) \geq 0 \quad (27)$$

Thus, for $\mathcal{Q}_{si}^\iota > 0$, we have $\sum_{k=0}^{\infty} \big(\tilde{x}(k)\tilde{x}^{\mathrm{T}}(k+1) + \tilde{x}(k+1)\tilde{x}^{\mathrm{T}}(k) - 2\cos\varpi_{sl}\tilde{x}(k)\tilde{x}^{\mathrm{T}}(k) \big) \mathcal{Q}_{si}^\iota \geq 0$, which implies $\sum_{k=0}^{\infty} \mathbb{I}_f \geq 0$. Summing the inequality (25) from $k = 0$ to ∞, one can easily obtain $\| r(k) \|_2^{\mathbf{E}} \geq \beta \| s(k) \|_2^{\mathbf{E}}$ holds, that is the system (7) is with a LF

stochastic H_-index. By the similar process, the conditions for MF and HF can be obtained naturally by choosing the suitable frequency parameters, and it is omitted here. The proof is completed. □

Remark 7 If the system considered in this paper is a linear one rather than MJS, i.e., the system is independent on i, and it is without the nonlinear items $f(x(k), \alpha_k)$, the condition in Lemma 2 can be reduced to the condition of GKYP lemma shown in [27]. Through such conditions, the finite frequency performance requirement has been transferred into time-domain conditions, which facilitated the design procedure.

Following the idea of Lemmas 1–2, the analysis conditions to capture the sensitivity performance to $f(k)$ and the robustness performance to $s(k)$ can be easily derived, which are shown in the following two corollaries.

Corollary 1 *The system* (7) *is with a prescribed FFS $\mathcal{H}_\infty$ performance γ_d, $\gamma_d > 0$, to $s(k)$ if there exist matrices $P_{di\iota}^\iota = P_{di\iota}^{\iota T} > 0$, $Q_{di\iota}^\iota = Q_{di\iota}^{\iota T} > 0$ and scalars $\delta_{d\kappa i}^\iota$ such that*

$$\begin{bmatrix} -\delta_{d\kappa i}^\iota & \tau_{\kappa i}^{T}\mathcal{E}_{i_-}^{\iota T}\bar{\mathcal{P}}_{di}^\iota \\ * & -\bar{\mathcal{P}}_{di}^\iota \end{bmatrix} < 0 \tag{28}$$

$$\mathfrak{I}_{di}^{\iota T}\Psi_{di}^\iota\mathfrak{I}_{di}^\iota - \mathfrak{V}_{di}^{\iota T}\Pi_{di}^\iota\mathfrak{V}_{di}^\iota < \mathfrak{M}_{di}^{\iota T}\Upsilon_{di}^\iota\mathfrak{M}_{di}^\iota \tag{29}$$

hold $\forall i \in \$$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, and $\iota = 0$, where

$$\mathfrak{I}_{di}^\iota = [\mathfrak{I}_{i1}^\iota \ \mathfrak{I}_{si2}^\iota], \mathfrak{V}_{di}^\iota = [\mathfrak{V}_{di1}^\iota \ \mathfrak{V}_{i2}^\iota],$$
$$\mathfrak{M}_{di}^\iota = [\mathfrak{M}_{i1}^\iota \ \mathfrak{M}_{si2}^\iota],$$
$$\Pi_{di}^\iota = \text{diag}\{\Omega_d^{''}, 0\}, \Upsilon_{di}^\iota = \text{diag}\{-I, \gamma_d^2 I\},$$
$$\Psi_{di}^\iota = \mathcal{P}_{di}^\iota \otimes \text{diag}\{0, I\} - \bar{\mathcal{P}}_{di}^\iota \otimes \text{diag}\{I, 0\}$$
$$+ \mathcal{Q}_{di}^\iota \otimes \mathsf{J}$$

with $\mathfrak{V}_{di1}^\iota = [\Omega^{'} \ 0]^T$, $\Omega_d^{''} = \text{diag}\{\delta_{d1i}I, \delta_{d2i}I, \ldots, \delta_{d\mathfrak{K}i}I\}$, $\bar{\mathcal{P}}_{di}^\iota = \sum_{j=1}^{N}\sum_{l=1}^{L}\sum_{J=1}^{L} o_\iota(k)\xi_J(k)\lambda_{ij}^\iota P_{djJ}^\iota$, and $\mathcal{P}_{di}^\iota = \sum_{l=1}^{L} o_\iota(k)P_{di\iota}^\iota$.

Corollary 2 *For the system* (7) *in faulty case, it is with a prescribed FFS H_-index β_f, $\beta_f > 0$, to $f(k)$ if there exist matrices $P_{fi\iota}^\iota = P_{fi\iota}^{\iota T} > 0$, $Q_{fi\iota}^\iota = Q_{fi\iota}^{\iota T} > 0$ and scalars $\delta_{f\kappa i}^\iota$ such that*

$$\begin{bmatrix} -\delta_{f\kappa i}^\iota & \tau_{\kappa i}^{T}\mathcal{E}_i^{\iota T}\bar{\mathcal{P}}_{fi}^\iota \\ * & -\bar{\mathcal{P}}_{fi}^\iota \end{bmatrix} < 0 \tag{30}$$

$$\mathfrak{I}_{fi}^{\iota T}\Psi_{fi}^\iota\mathfrak{I}_{fi}^\iota - \mathfrak{V}_{fi}^{\iota T}\Pi_{fi}^\iota\mathfrak{V}_{fi}^\iota < \mathfrak{M}_{fi}^{\iota T}\Upsilon_{fi}^\iota\mathfrak{M}_{fi}^\iota \tag{31}$$

hold $\forall i \in \$$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, and $\iota = 1$, where

$$\mathfrak{I}_{fi}^\iota = [\mathfrak{I}_{i1}^\iota \ \mathfrak{I}_{fi2}^\iota], \mathfrak{V}_{fi}^\iota = [\mathfrak{V}_{fi1}^\iota \ \mathfrak{V}_{i2}^\iota],$$
$$\mathfrak{M}_{fi}^\iota = [\mathfrak{M}_{i1}^\iota \ \mathfrak{M}_{fi2}^\iota],$$
$$\Pi_{fi}^\iota = \text{diag}\{\Omega_f^{''}, 0\}, \Upsilon_{fi}^\iota = \text{diag}\{I, -\beta_f^2 I\},$$
$$\Psi_{fi}^\iota = \mathcal{P}_{fi}^\iota \otimes \text{diag}\{0, I\} - \bar{\mathcal{P}}_{fi}^\iota \otimes \text{diag}\{I, 0\}$$
$$+ \mathcal{Q}_{fi}^\iota \otimes \mathsf{J}_f$$

with $\mathfrak{I}_{fi2}^\iota = [\mathcal{B}_{3i}^{\iota T} \ 0]^T$, $\mathfrak{V}_{fi1}^\iota = [\Omega^{'} \ 0]^T$, $\mathfrak{M}_{fi2}^\iota = [\mathcal{D}_{3i}^{\iota T} \ I]^T$, wherein $\bar{\mathcal{P}}_{fi}^\iota = \sum_{j=1}^{N}\sum_{l=1}^{L}\sum_{J=1}^{L} o_\iota(k)\xi_J(k)\lambda_{ij}^\iota P_{fjJ}^\iota$, $\mathcal{P}_{fi}^\iota = \sum_{l=1}^{L} o_\iota(k)P_{fi\iota}^\iota$, $\Omega_f^{''} = \text{diag}\{\delta_{f1i}I, \delta_{f2i}I, \ldots, \delta_{f\mathfrak{K}i}I\}$. Furthermore, $\mathsf{J}_f = \begin{bmatrix} 0 & 1 \\ 1 & -2\cos\varpi_{fl} \end{bmatrix}$ in LF domain, $\mathsf{J}_f = \begin{bmatrix} 0 & e^{j\varpi_{fa}} \\ e^{-j\varpi_{fa}} & -2\cos\varpi_{fb} \end{bmatrix}$ in MF domain, $\mathsf{J}_f = \begin{bmatrix} 0 & -1 \\ -1 & 2\cos\varpi_{fh} \end{bmatrix}$ in HF domain, wherein $\varpi_{fa} = \frac{\varpi_{f1}+\varpi_{f2}}{2}$, and $\varpi_{fb} = \frac{\varpi_{f2}-\varpi_{f1}}{2}$.

3.2 Fault detection filters synthesis

Based on the conclusions in above subsection, the linear conditions to design the FD filters will be developed thoroughly in this subsection.

Lemma 3 *For the known scalar ϵ and the real matrices Y, M, F, and E with appropriate dimensions subjecting $F^T F \leq \epsilon^2 I$, the inequality $Y + \boldsymbol{He}(MFE) < 0$ holds if and only if there exists a scalar $\sigma > 0$ such that $Y + \sigma MM^T + \frac{1}{\sigma}\epsilon^2 E^T E < 0$.*

Theorem 1 *The system* (7) *is with a prescribed LF stochastic H_- index β, $\beta > 0$, to $s(k)$ if there exist matrices $P_{si\iota}^{\iota T} = P_{si\iota}^\iota = \begin{bmatrix} P_{si1\iota}^\iota & * \\ P_{si2\iota}^\iota & P_{si3\iota}^\iota \end{bmatrix} > 0$, $Q_{si\iota}^{\iota T} = Q_{si\iota}^\iota = \begin{bmatrix} Q_{si1\iota}^\iota & * \\ Q_{si2\iota}^\iota & Q_{si3\iota}^\iota \end{bmatrix} > 0$, $\mathcal{Y}_i$, $\mathcal{M}_i$, $\mathcal{N}_i$, $\mathcal{G}_i$, $\mathcal{X}_{3i}$, $\mathcal{A}_{fi}$, $\mathcal{B}_{1fi}$, $\mathcal{B}_{2fi}$, $\mathcal{C}_{fi}$, $\mathcal{D}_{fi}$ and scalars $\delta_{s\kappa i}^\iota$ and ε_s such that*

$$\begin{bmatrix} -\frac{1}{\mathfrak{K}}\delta_{s\kappa i}^\iota & * & * \\ \bar{\mathcal{P}}_{si1}^\iota E_i^\iota\tau_{\kappa i} & -\bar{\mathcal{P}}_{si1}^\iota & \\ \bar{\mathcal{P}}_{si2}^\iota E_i^\iota\tau_{\kappa i} & -\bar{\mathcal{P}}_{si2}^\iota & -\bar{\mathcal{P}}_{si3}^\iota \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} \Gamma_{s11} & * & * & * & * & * \\ \Gamma_{s21} & \Gamma_{s22} & * & * & * & * \\ 0 & \Gamma_{s32} & -I & * & * & * \\ \Gamma_{s41} & \Gamma_{s42} & -\bar{v}_2\mathcal{G}_i & \Gamma_{s44} & * & * \\ 0 & \Gamma_{s52} & 0 & 0 & \Gamma_{s55} & * \\ 0 & \Gamma_{s62} & 0 & \Gamma_{s64} & 0 & -\varepsilon_s I \end{bmatrix} < 0 \qquad (33)$$

hold $\forall i \in \mathbb{S}$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, *and* $\iota = 1$, *where*

$$\Gamma_{s11} = \begin{bmatrix} -\bar{\mathcal{P}}_{si1}^\iota & * \\ -\bar{\mathcal{P}}_{si2}^\iota & -\bar{\mathcal{P}}_{si3}^\iota \end{bmatrix},$$

$$\Gamma_{s21} = \begin{bmatrix} Q_{si1}^\iota - \mathcal{Y}_i & Q_{si1}^{\iota\mathrm{T}} - \mathcal{N}_i \\ Q_{si2}^\iota - \mathcal{M}_i & Q_{si3}^\iota - \mathcal{N}_i \end{bmatrix},$$

$$\Gamma_{s22} = \begin{bmatrix} \Gamma_{s22}^1 & * \\ \Gamma_{s22}^2 & \Gamma_{s22}^3 \end{bmatrix}, \ \Gamma_{s32} = \begin{bmatrix} -\mathcal{G}_i^\mathrm{T} & -\mathcal{G}_i^\mathrm{T} \end{bmatrix},$$

$$\Gamma_{s41} = \begin{bmatrix} -\bar{v}_1\mathcal{Y}_i & -\bar{v}_1\mathcal{N}_i \end{bmatrix}, \ \Gamma_{s42} = \begin{bmatrix} \Gamma_{s42}^1 & \Gamma_{s42}^2 \end{bmatrix},$$

$$\Gamma_{s44} = \beta^2 I + \mathbf{He}(\bar{v}_1\mathcal{Y}_i B_{2i} + \bar{v}_1\mathcal{B}_{2fi}) + \varepsilon_s D_{2i}^\mathrm{T} D_{2i},$$

$$\Gamma_{s52} = \mathcal{X}_{3i} \otimes \Omega'^\mathrm{T}, \ \Gamma_{s62} = \begin{bmatrix} \mathcal{B}_{1fi}^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T} & \mathcal{B}_{1fi}^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T} \end{bmatrix}$$

$$\Gamma_{s55} = -\Omega_s'' - \mathbf{He}(\mathcal{X}_{3i} \otimes \mathrm{diag}\{I, I, \ldots, I\}_\mathfrak{K}),$$

$$\Gamma_{s64} = \mathcal{B}_{1fi}^\mathrm{T}\bar{v}_1^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T}\bar{v}_2^\mathrm{T}$$

with $\Gamma_{s22}^1 = \mathcal{P}_{si1}^\iota - 2\cos\varpi_{sl}Q_{si1}^\iota + \mathbf{He}(\mathcal{Y}_i A_i) + \varepsilon_s C_i^\mathrm{T} C_i$, $\Gamma_{s22}^2 = \mathcal{P}_{si2}^\iota - 2\cos\varpi_{sl}Q_{si2}^\iota + \mathcal{M}_i A_i + \mathcal{A}_{fi}^\mathrm{T} + \mathcal{C}_{fi}^\mathrm{T}$, $\Gamma_{s22}^3 = \mathcal{P}_{si3}^\iota - 2\cos\varpi_{sl}Q_{si3}^\iota + \mathbf{He}(\mathcal{A}_{fi} + \mathcal{C}_{fi})$, $\Gamma_{s42}^1 = B_{2i}^\mathrm{T}\mathcal{Y}_i^\mathrm{T} + \mathcal{B}_{2fi}^\mathrm{T} + \bar{v}_1\mathcal{Y}_i A_i + \varepsilon_s D_{2i}^\mathrm{T} C_i$, *and* $\Gamma_{s42}^2 = B_{2i}^\mathrm{T}\mathcal{Y}_i^\mathrm{T} + \mathcal{B}_{2fi}^\mathrm{T} + \bar{v}_1\mathcal{A}_{fi} + \bar{v}_2\mathcal{C}_{fi}$.

Proof It is clear, from Lemma 2, that the conditions (22)–(23) guarantee the system (7) can be with a FFS H_- index β. Expanding the inequality (22), the condition (32) can be easily derived. Further, we will focus on linearizing the inequality (23), which can be transformed into

$$\mathbb{J}_{si}^{\iota\mathrm{T}}\left(\begin{bmatrix} \Psi_{si}^\iota & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \mathfrak{V}_{si}^{\iota\mathrm{T}}\Pi_{si}^\iota\mathfrak{V}_{si}^\iota + \mathfrak{M}_{si}^{\iota\mathrm{T}}\Upsilon_{si}^\iota\mathfrak{M}_{si}^\iota \end{bmatrix}\right)\mathbb{J}_{si}^\iota \leq 0 \qquad (34)$$

where $\mathbb{J}_{si}^\iota = \begin{bmatrix} \mathfrak{I}_{i1}^\iota & \mathfrak{I}_{si2}^\iota \\ 0 & I \end{bmatrix}$, and then it can be equivalent to

$$\left(\begin{bmatrix} \Psi_{si}^\iota & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \mathfrak{V}_{si}^{\iota\mathrm{T}}\Pi_{si}^\iota\mathfrak{V}_{si}^\iota + \mathfrak{M}_{si}^{\iota\mathrm{T}}\Upsilon_{si}^\iota\mathfrak{M}_{si}^\iota \end{bmatrix}\right) + \mathbf{He}\left(\begin{bmatrix} -I \\ \mathcal{A}_i^{\iota\mathrm{T}} \\ \mathcal{B}_{2i}^{\iota\mathrm{T}} \end{bmatrix}\begin{bmatrix} 0 \\ \mathcal{X}_{1i} \\ v_1\mathcal{X}_{1i} \end{bmatrix}^\mathrm{T}\right) \leq 0 \qquad (35)$$

by applying Finsler's Lemma [18], where $\mathcal{X}_{1i}$ is the introduced slack matrix and v_1 is a given scalar. And then, the above inequality can be transformed into

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \mathcal{C}_i^\iota & \mathcal{D}_{2i}^\iota \\ 0 & 0 & I \end{bmatrix}^\mathrm{T} \Gamma_{ss}^1 \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \mathcal{C}_i^\iota & \mathcal{D}_{2i}^\iota \\ 0 & 0 & I \end{bmatrix} \leq 0 \qquad (36)$$

where

$$\Gamma_{ss}^1 = \begin{bmatrix} \Psi_{si}^\iota & 0 \\ 0 & -\Upsilon_{si}^\iota \end{bmatrix} - \mathrm{diag}\{0, \ \mathfrak{V}_{si}^{\iota\mathrm{T}}\Pi_{si}^\iota\mathfrak{V}_{si}^\iota, \ 0\}$$
$$+ \mathbf{He}([-I \ \mathcal{A}_i^\iota \ 0 \ \mathcal{B}_{2i}^\iota]^\mathrm{T}[0 \ \mathcal{X}_{1i}^\mathrm{T} \ 0 \ \mathcal{X}_{1i}^\mathrm{T}v_1^\mathrm{T}])$$

Exploiting Finsler's Lemma again, (36) is equivalent to

$$\Gamma_{ss}^1 + \mathbf{He}\left(\begin{bmatrix} 0 & \mathcal{C}_i^\iota & -I & \mathcal{D}_{2i}^\iota \end{bmatrix}^\mathrm{T}\begin{bmatrix} 0 & \mathcal{X}_{2i}^\mathrm{T} & 0 & \mathcal{X}_{2i}^\mathrm{T}v_2^\mathrm{T} \end{bmatrix}\right) \leq 0 \qquad (37)$$

where $\mathcal{X}_{2i}$ is the additional slack matrix. Further, the above inequality can be rewritten into

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & \Omega_s'^\mathrm{T} & 0 & 0 \end{bmatrix}^\mathrm{T} \Gamma_{ss}^2 \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & \Omega_s'^\mathrm{T} & 0 & 0 \end{bmatrix} \leq 0 \qquad (38)$$

where $\Gamma_{ss}^2 = \mathrm{diag}\left\{\Gamma_{ss}^1 + \mathrm{diag}\{0, \ \mathfrak{V}_{si}^{\iota\mathrm{T}}\Pi_{si}^\iota\mathfrak{V}_{si}^\iota, \ 0\}, -\Omega_s''\right\}$. Utilizing the same preceding method adopted above, the inequality (38) is equivalent to

$$\Gamma_{ss}^2 + \mathbf{He}\left(\begin{bmatrix} 0 & \Omega_s'^\mathrm{T} & 0 & 0 & -I \end{bmatrix}^\mathrm{T}\begin{bmatrix} 0 & 0 & 0 & 0 & \mathcal{X}_{3i} \end{bmatrix}\right) \leq 0 \qquad (39)$$

where $\mathcal{X}_{3i}$ is the slack matrix introduced by Finsler's lemma.

Without loss of generality, decomposing the matrices $\mathcal{X}_{1i}$, $\mathcal{X}_{2i}$ and the vectors v_1, v_2 into the form of $\mathcal{X}_{1i} = \begin{bmatrix} \mathcal{Y}_i & \mathcal{N}_i \\ \mathcal{M}_i & \mathcal{N}_i \end{bmatrix}$, $\mathcal{X}_{2i} = \begin{bmatrix} \mathcal{G}_i \\ \mathcal{G}_i \end{bmatrix}$, and $v_1 = \begin{bmatrix} \bar{v}_1 & 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} \bar{v}_2 & 0 \end{bmatrix}$. We expand the left part of equation (37) and rewrite it into the sum form of two parts. One part

is the items with ϱ_ι, and the other is the left items without ϱ_ι. Recalling the fact of $\varrho_\iota^{\mathrm{T}}\varrho_\iota \leq I$, applying the Lemma 3, and defining the new variables

$$\mathcal{A}_{fi} = \mathcal{N}_i A_{fi}, \ \mathcal{B}_{1fi} = \mathcal{N}_i B_{1fi}, \ \mathcal{B}_{2fi} = \mathcal{N}_i B_{2fi},$$
$$\mathcal{C}_{fi} = \mathcal{G}_i C_{fi}, \ \mathcal{D}_{fi} = \mathcal{G}_i D_{fi} \tag{40}$$

the condition (33) can be derived. The proof is completed. □

Remark 8 Theorem 1 has presented the conditions to capture the LF stochastic H_- index. From Lemma 2, the conditions ensuring the desired MF and HF stochastic H_- indices can be easily derived by utilizing different $\mathbb{J}$ and following the same proof process of Theorem 1.

Theorem 2 *The system* (7) *is SS and with a prescribed* $\mathcal{H}_\infty$ *performance* γ, $\gamma > 0$, *to* $\omega(k)$ *if there exist matrices* $P_{i\iota}^{\iota\mathrm{T}} = P_{i\iota}^\iota = \begin{bmatrix} P_{i1\iota}^\iota & * \\ P_{i2\iota}^\iota & P_{i3\iota}^\iota \end{bmatrix} > 0$, $\mathcal{Y}_i$, $\mathcal{M}_i$, $\mathcal{N}_i$, $\mathcal{G}_i$, $\mathcal{X}_{3i}$, $\mathcal{A}_{fi}$, $\mathcal{B}_{1fi}$, $\mathcal{B}_{2fi}$, $\mathcal{C}_{fi}$, $\mathcal{D}_{fi}$ *and scalars* $\delta_{\kappa i}^\iota$ *and* ε *such that*

$$\begin{bmatrix} -\frac{1}{\bar{\mathfrak{K}}}\delta_{\kappa i}^\iota & * & * \\ \bar{\mathcal{P}}_{i1}^\iota E_i^\iota \tau_{\kappa i} & -\bar{\mathcal{P}}_{i1}^\iota & * \\ \bar{\mathcal{P}}_{i2}^\iota E_i^\iota \tau_{\kappa i} & -\bar{\mathcal{P}}_{i2}^\iota & -\bar{\mathcal{P}}_{i3}^\iota \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} \Gamma_{11} & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & * & * & * \\ 0 & \Gamma_{42} & 0 & \Gamma_{44} & * & * \\ \Gamma_{51} & 0 & 0 & \mathcal{D}_{fi}^{\mathrm{T}}\bar{v}_2^{\mathrm{T}} & -\varepsilon I & * \\ 0 & \Gamma_{62} & 0 & 0 & 0 & \Gamma_{66} \end{bmatrix} < 0 \tag{42}$$

hold $\forall i \in \mathbb{S}$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, *and* $\iota = 0, 1$, *where*

$$\Gamma_{11} = \begin{bmatrix} \bar{\mathcal{P}}_{i1}^\iota - \boldsymbol{He}(\mathcal{Y}_i) & * \\ \bar{\mathcal{P}}_{i2}^\iota - \mathcal{M}_i - \mathcal{N}_i^{\mathrm{T}} & \bar{\mathcal{P}}_{i3}^\iota - \boldsymbol{He}(\mathcal{N}_i) \end{bmatrix},$$
$$\Gamma_{21} = \begin{bmatrix} A_i^{\mathrm{T}}\mathcal{Y}_i^{\mathrm{T}} & A_i^{\mathrm{T}}\mathcal{M}_i^{\mathrm{T}} \\ \mathcal{A}_{fi}^{\mathrm{T}} & \mathcal{A}_{fi}^{\mathrm{T}} \end{bmatrix},$$
$$\Gamma_{22} = \begin{bmatrix} -\mathcal{P}_{i1}^\iota + \varepsilon C_i^{\mathrm{T}}C_i & * \\ -\mathcal{P}_{i2}^\iota & -\mathcal{P}_{i3}^\iota \end{bmatrix},$$
$$\Gamma_{31} = \begin{bmatrix} B_{1i}^{\mathrm{T}}\mathcal{Y}_i^{\mathrm{T}} & B_{1i}^{\mathrm{T}}\mathcal{M}_i^{\mathrm{T}} \end{bmatrix}, \quad \Gamma_{32} = \begin{bmatrix} \varepsilon D_{1i}^{\mathrm{T}}C_i & 0 \end{bmatrix},$$
$$\Gamma_{33} = -\gamma^2 I + \varepsilon D_{1i}^{\mathrm{T}}D_{1i}, \quad \Gamma_{42} = \begin{bmatrix} 0 & \bar{v}_2 \mathcal{C}_{fi} \end{bmatrix},$$
$$\Gamma_{44} = I - \boldsymbol{He}(\bar{v}_2 \mathcal{G}_i),$$
$$\Gamma_{51} = \begin{bmatrix} \mathcal{B}_{1fi}^{\mathrm{T}} & \mathcal{B}_{1fi}^{\mathrm{T}} \end{bmatrix}, \Gamma_{62} = \mathcal{X}_{3i} \otimes \Omega'^{\mathrm{T}}$$
$$\Gamma_{66} = \Omega'' - \boldsymbol{He}(\mathcal{X}_{3i} \otimes \mathrm{diag}\{I, I, \ldots, I\}_{\bar{\mathfrak{K}}}).$$

Proof Lemma 1 has presented the conditions to guarantee the SS and desired $\mathcal{H}_\infty$ performance to $\omega(k)$. Equation (41) can be derived directly from (12). Further, we rewrite the inequality (13) into

$$\mathbb{J}_i^{\iota\mathrm{T}}\left(\begin{bmatrix} \Psi_i^\iota & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathfrak{V}_i^{\iota\mathrm{T}}\Pi_i^\iota\mathfrak{V}_i^\iota - \mathfrak{M}_i^{\iota\mathrm{T}}\Upsilon_i^\iota\mathfrak{M}_i^\iota \end{bmatrix}\right)\mathbb{J}_i^\iota \leq 0 \tag{43}$$

where $\mathbb{J}_i^\iota = \begin{bmatrix} \mathfrak{I}_{i1}^\iota & \mathfrak{I}_{i2}^\iota \\ 0 & I \end{bmatrix}$, and then it can be equivalent to

$$\left(\begin{bmatrix} \Psi_i^\iota & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathfrak{V}_i^{\iota\mathrm{T}}\Pi_i^\iota\mathfrak{V}_i^\iota - \mathfrak{M}_i^{\iota\mathrm{T}}\Upsilon_i^\iota\mathfrak{M}_i^\iota \end{bmatrix}\right)$$
$$+ \boldsymbol{He}\left(\begin{bmatrix} -I \\ \mathcal{A}_i^{\iota\mathrm{T}} \\ \mathcal{B}_{1i}^{\iota\mathrm{T}} \end{bmatrix}\begin{bmatrix} \mathcal{X}_{1i} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}\right) \leq 0 \tag{44}$$

from Finsler's Lemma. We go on to split the above inequality into the form of

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \mathcal{C}_i^\iota & \mathcal{D}_{1i}^\iota \\ 0 & 0 & I \end{bmatrix}^{\mathrm{T}} \Gamma' \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \mathcal{C}_i^\iota & \mathcal{D}_{1i}^\iota \\ 0 & 0 & I \end{bmatrix} \leq 0 \tag{45}$$

where

$$\Gamma' = \begin{bmatrix} \Psi_i^\iota & 0 \\ 0 & -\Upsilon_i^\iota \end{bmatrix} + \mathrm{diag}\{0, \ \mathfrak{V}_i^{\iota\mathrm{T}}\Pi_i^\iota\mathfrak{V}_i^\iota, \ 0\}$$
$$+ \boldsymbol{He}(\begin{bmatrix} -I & \mathcal{A}_i^\iota & 0 & \mathcal{B}_{1i}^\iota \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \mathcal{X}_{1i}^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix})$$

Then, utilizing Finsler's Lemma again, one has (45) is equivalent to

$$\Gamma' + \boldsymbol{He}\left(\begin{bmatrix} 0 & \mathcal{C}_i^\iota & -I & \mathcal{D}_{1i}^\iota \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} 0 & 0 & \mathcal{X}_{2i}^{\mathrm{T}}v_2^{\mathrm{T}} & 0 \end{bmatrix}\right) \leq 0 \tag{46}$$

Subsequently, for the item of $\mathfrak{V}_i^{\iota\mathrm{T}}\Pi_i^\iota\mathfrak{V}_i^\iota$, we adopt the same dealing method with the proof for Theorem 1, the condition (42) can be easily achieved. The proof is completed. □

Further from Corollaries 1–2, following the similar route to prove Theorem 1 and choosing relative system matrices, the linear and solvable conditions can be derived, which are given in the following two theorems.

Theorem 3 *For the system* (7) *in fault-free case, i.e.,* $\varrho_\iota = I$ *and* $\epsilon = 0$, *it is with a prescribed LF stochastic* $\mathcal{H}_\infty$ *performance* γ_d, $\gamma_d > 0$, *to* $s(k)$ *if there exist matrices* $P_{di\iota}^{\iota\mathrm{T}} = P_{di\iota}^\iota = \begin{bmatrix} P_{di1\iota}^\iota & * \\ P_{di2\iota}^\iota & P_{di3\iota}^\iota \end{bmatrix} > 0$, $Q_{di\iota}^{\iota\mathrm{T}} = Q_{di\iota}^\iota = \begin{bmatrix} Q_{di1\iota}^\iota & * \\ Q_{di2\iota}^\iota & Q_{di3\iota}^\iota \end{bmatrix} > 0$, $\mathcal{Y}_i$, $\mathcal{M}_i$, $\mathcal{N}_i$, $\mathcal{G}_i$, $\mathcal{X}_{3i}$, $\mathcal{A}_{fi}$, $\mathcal{B}_{1fi}$, $\mathcal{B}_{2fi}$, $\mathcal{C}_{fi}$, $\mathcal{D}_{fi}$ *and scalars* $\delta_{d\kappa i}^\iota$ *such that the following inequalities*

$$\begin{bmatrix} -\frac{1}{\bar{\mathfrak{K}}}\delta_{d\kappa i}^\iota & * & * \\ \bar{\mathcal{P}}_{di1}^\iota E_i^\iota \tau_{\kappa i} & -\bar{\mathcal{P}}_{di1}^\iota & \\ \bar{\mathcal{P}}_{di2}^\iota E_i^\iota \tau_{\kappa i} & -\bar{\mathcal{P}}_{di2}^\iota & -\bar{\mathcal{P}}_{di3}^\iota \end{bmatrix} < 0 \qquad (47)$$

$$\begin{bmatrix} \Gamma_{d11} & * & * & * & * \\ \Gamma_{d21} & \Gamma_{d22} & * & * & * \\ 0 & \Gamma_{d32} & -\gamma_d^2 I & * & * \\ 0 & \Gamma_{d42} & \bar{v}_2 \mathcal{D}_{fi} D_{2i} & \Gamma_{d44} & * \\ 0 & \Gamma_{d52} & 0 & 0 & \Gamma_{d55} \end{bmatrix} < 0 \qquad (48)$$

hold for all $i \in \mathbb{S}$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, *and* $\iota = 0$, *where*

$$\Gamma_{d11} = \begin{bmatrix} -\bar{\mathcal{P}}_{di1}^\iota & * \\ -\bar{\mathcal{P}}_{di2}^\iota & -\bar{\mathcal{P}}_{di3}^\iota \end{bmatrix},$$

$$\Gamma_{d21} = \begin{bmatrix} Q_{di1}^\iota - \mathcal{Y}_i & Q_{di2}^{\iota\mathrm{T}} - \mathcal{N}_i \\ Q_{di2}^\iota - \mathcal{M}_i & Q_{di3}^\iota - \mathcal{N}_i \end{bmatrix},$$

$$\Gamma_{d22} = \begin{bmatrix} \Gamma_{d22}^1 & * \\ \Gamma_{d22}^2 & \Gamma_{d22}^3 \end{bmatrix},$$

$$\Gamma_{d32} = \begin{bmatrix} \Gamma_{d32}^1 & B_{2i}^\mathrm{T}\mathcal{M}_i^\mathrm{T} + D_{2i}^\mathrm{T}\mathcal{B}_{1fi}^\mathrm{T} + \mathcal{B}_{2fi}^\mathrm{T} + D_{2i}^\mathrm{T}\mathcal{D}_{fi}^\mathrm{T} \end{bmatrix},$$

$$\Gamma_{d42} = \begin{bmatrix} -\mathcal{G}_i^\mathrm{T} + \bar{v}_2 \mathcal{D}_{fi} C_i & -\mathcal{G}_i^\mathrm{T} + \bar{v}_2 \mathcal{C}_{fi} \end{bmatrix},$$

$$\Gamma_{d44} = I - \boldsymbol{He}(\bar{v}_2 \mathcal{G}_i), \quad \Gamma_{d52} = \mathcal{X}_{3i} \otimes \Omega^{'T},$$

$$\Gamma_{d55} = -\Omega_d^{''} - \boldsymbol{He}(\mathcal{X}_{3i} \otimes \mathrm{diag}\{I, I, \ldots, I\}_{\mathfrak{K}})$$

with $\Gamma_{d22}^1 = \mathcal{P}_{di1}^\iota - 2\cos\varpi_{sl} Q_{di1}^\iota + \boldsymbol{He}(\mathcal{Y}_i A_i + \mathcal{B}_{1fi} C_i)$, $\Gamma_{d22}^2 = \mathcal{P}_{di2}^\iota - 2\cos\varpi_{sl} Q_{di2}^\iota + \mathcal{M}_i A_i + \mathcal{B}_{1fi} C_i + \mathcal{A}_{fi}^\mathrm{T} + \mathcal{C}_{fi}^\mathrm{T} + \mathcal{D}_{fi} C_i$, $\Gamma_{d22}^3 = \mathcal{P}_{di3}^\iota - 2\cos\varpi_{sl} Q_{di3}^\iota + \boldsymbol{He}(\mathcal{A}_{fi} + \mathcal{C}_{fi})$, *and* $\Gamma_{d32}^1 = B_{2i}^\mathrm{T}\mathcal{Y}_i^\mathrm{T} + D_{2i}^\mathrm{T}\mathcal{B}_{1fi}^\mathrm{T} + \mathcal{B}_{2fi}^\mathrm{T} + D_{2i}^\mathrm{T}\mathcal{D}_{fi}^\mathrm{T}$.

Theorem 4 *For the system* (7) *in faulty case, it is with a prescribed LF stochastic* H_-*index* β_f, $\beta_f > 0$, *to* $f(k)$ *if there exist matrices* $P_{fi\iota}^{\iota\mathrm{T}} = P_{fi\iota}^\iota = \begin{bmatrix} P_{fi1\iota}^\iota & * \\ P_{fi2\iota}^\iota & P_{fi3\iota}^\iota \end{bmatrix} > 0$, $Q_{fi\iota}^{\iota\mathrm{T}} = Q_{fi\iota}^\iota = \begin{bmatrix} Q_{fi1\iota}^\iota & * \\ Q_{fi2\iota}^\iota & Q_{fi3\iota}^\iota \end{bmatrix} > 0$, $\mathcal{Y}_i$, $\mathcal{M}_i$, $\mathcal{N}_i$, $\mathcal{G}_i$, $\mathcal{X}_{3i}$, $\mathcal{A}_{fi}$, $\mathcal{B}_{1fi}$,

$\mathcal{B}_{2fi}$, $\mathcal{C}_{fi}$, $\mathcal{D}_{fi}$ *and scalars* $\delta_{f\kappa i}^\iota$ *and* ε_f *such that*

$$\begin{bmatrix} -\frac{1}{\bar{\mathfrak{K}}}\delta_{f\kappa i}^\iota & * & * \\ \bar{\mathcal{P}}_{fi1}^\iota E_i^\iota \tau_{\kappa i} & -\bar{\mathcal{P}}_{fi1}^\iota & \\ \bar{\mathcal{P}}_{fi2}^\iota E_i^\iota \tau_{\kappa i} & -\bar{\mathcal{P}}_{fi2}^\iota & -\bar{\mathcal{P}}_{fi3}^\iota \end{bmatrix} < 0 \qquad (49)$$

$$\begin{bmatrix} \Gamma_{f11} & * & * & * & * & * \\ \Gamma_{f21} & \Gamma_{f22} & * & * & * & * \\ 0 & \Gamma_{f32} & -I & * & * & * \\ \Gamma_{f41} & \Gamma_{f42} & -\bar{v}_2 \mathcal{G}_i & \Gamma_{f44} & * & * \\ 0 & \Gamma_{f52} & 0 & 0 & \Gamma_{f55} & * \\ 0 & \Gamma_{f62} & 0 & \Gamma_{f64} & 0 & -\varepsilon_f I \end{bmatrix} < 0 \qquad (50)$$

hold $\forall i \in \mathbb{S}$, $\kappa = 1, 2, \ldots, \mathfrak{K}$, *and* $\iota = 1$, *where*

$$\Gamma_{f11} = \begin{bmatrix} -\bar{\mathcal{P}}_{fi1}^\iota & * \\ -\bar{\mathcal{P}}_{fi2}^\iota & -\bar{\mathcal{P}}_{fi3}^\iota \end{bmatrix},$$

$$\Gamma_{f21} = \begin{bmatrix} Q_{fi1}^\iota - \mathcal{Y}_i & Q_{fi2}^{\iota\mathrm{T}} - \mathcal{N}_i \\ Q_{fi2}^\iota - \mathcal{M}_i & Q_{fi3}^\iota - \mathcal{N}_i \end{bmatrix},$$

$$\Gamma_{f22} = \begin{bmatrix} \Gamma_{f22}^1 & * \\ \Gamma_{f22}^2 & \Gamma_{f22}^3 \end{bmatrix},$$

$$\Gamma_{f32} = \begin{bmatrix} -\mathcal{G}_i^\mathrm{T} & -\mathcal{G}_i^\mathrm{T} \end{bmatrix}, \quad \Gamma_{f41} = \begin{bmatrix} -\bar{v}_1\mathcal{Y}_i & -\bar{v}_1\mathcal{N}_i \end{bmatrix},$$

$$\Gamma_{f42} = \begin{bmatrix} \mathcal{B}_{1fi}^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T} + \bar{v}_1\mathcal{Y}_i A_i & \mathcal{B}_{1fi}^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T} + \bar{v}_1\mathcal{A}_{fi} + \bar{v}_2\mathcal{C}_{fi} \end{bmatrix},$$

$$\Gamma_{f44} = \beta_f^2 I + \boldsymbol{He}(\bar{v}_1\mathcal{B}_{1fi} + \bar{v}_2\mathcal{D}_{fi}),$$

$$\Gamma_{f52} = \mathcal{X}_{3i} \otimes \Omega_f^{'T},$$

$$\Gamma_{f55} = -\Omega_f^{''} - \boldsymbol{He}(\mathcal{X}_{3i} \otimes \mathrm{diag}\{I, I, \ldots, I\}_{\mathfrak{K}}),$$

$$\Gamma_{f62} = \begin{bmatrix} \mathcal{B}_{1fi}^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T} & \mathcal{B}_{1fi}^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T} \end{bmatrix},$$

$$\Gamma_{f64} = \mathcal{B}_{1fi}^\mathrm{T}\bar{v}_1^\mathrm{T} + \mathcal{D}_{fi}^\mathrm{T}\bar{v}_2^\mathrm{T}$$

with $\Gamma_{f22}^1 = \mathcal{P}_{fi1}^\iota - 2\cos\varpi_{fl} Q_{fi1}^\iota + \boldsymbol{He}(\mathcal{Y}_i A_i) + \varepsilon_f C_i^\mathrm{T} C_i$, $\Gamma_{f22}^2 = \mathcal{P}_{fi2}^\iota - 2\cos\varpi_{fl} Q_{fi2}^\iota + \mathcal{M}_i A_i + \mathcal{A}_{fi}^\mathrm{T} + \mathcal{C}_{fi}^\mathrm{T}$, *and* $\Gamma_{f22}^3 = \mathcal{P}_{fi3}^\iota - 2\cos\varpi_{fl} Q_{fi3}^\iota + \boldsymbol{He}(\mathcal{A}_{fi} + \mathcal{C}_{fi})$.

Remark 9 It is noticed from the Theorems 1–4 that some given vectors $\bar{v}_1$ and $\bar{v}_2$ are needed to derive the linear solvable conditions, and of course, different values will result in different performance levels. Particularly, but without loss of generality, it can be chosen as a unit vector.

Now, with the aid of the derived conditions in Theorems 1–4, a set of feasible solutions to the proposed design scheme can be obtained by the optimization problem stated as follows for acceptable γ and γ_d:

$$\max \quad a_{\mathrm{s}}\beta + a_{\mathrm{f}}\beta_f$$
$$s.t. \ (32), (33), i \in \mathbb{S}, \kappa = 1, 2, \ldots, \mathfrak{K}, \iota = 1$$
$$\quad (41), (42), i \in \mathbb{S}, \kappa = 1, 2, \ldots, \mathfrak{K}, \iota = 0, 1 \quad (51)$$
$$\quad (47), (48), i \in \mathbb{S}, \kappa = 1, 2, \ldots, \mathfrak{K}, \iota = 0$$
$$\quad (49), (50), i \in \mathbb{S}, \kappa = 1, 2, \ldots, \mathfrak{K}, \iota = 1$$

where a_{s} and a_{f} are given weighting scalars subjecting to $a_{\mathrm{s}} + a_{\mathrm{f}} = 1$. After getting the solutions, the FD filter gains can be calculated according to (40).

The following algorithm presents the process to solve the above optimization problem.

Algorithm 1

Step 1. Solving the following optimization problems to obtain the optimal values of the robustness performance indices γ and γ_d,

$$\min \quad \gamma$$
$$s.t. \ (41), (42), i \in \mathbb{S}, \kappa = 1, 2, \ldots, \mathfrak{K}, \iota = 0, 1$$
$$\min \quad \gamma_d$$
$$s.t. \ (47), (48), i \in \mathbb{S}, \kappa = 1, 2, \ldots, \mathfrak{K}, \iota = 0$$

Step 2. For the γ and γ_d derived in Step 1, solving (51). If it is solvable, break. Otherwise go to *Step 3*.

Step 3. Increasing the values of γ and γ_d steply and separately, then solving (51). If it is solvable, break. Otherwise cycling in this step.

Remark 10 Normally, the scalars a_{s} and a_{f} are determined empirically according to the fault type that frequently occurred. For example, if the faults with large amplitude always occur, they can be chosen as $a_{\mathrm{s}} = 0.5$ and $a_{\mathrm{f}} = 0.5$. And if the faults with small amplitude always occur, we should increase the value of a_{s} to strengthen the FD ability.

3.3 Detection threshold

After generating the residual signals, the last step is to determine the threshold and construct the detection logic. In this paper, the threshold J_{th} will be chosen as

$$J_{\mathrm{th}} = \sup_{f(k)=0, s(k)=0} J_{\mathrm{r}}(\tau) \quad (52)$$

where $J_{\mathrm{r}}(\tau)$ represents the average energy of the residual over the time window $[k_0, k_\tau]$, which is denoted by

$$J_{\mathrm{r}}(\tau) = \sqrt{\frac{1}{\tau} \sum_{k=k_0}^{k_\tau} r^{\mathrm{T}}(k) r(k)}. \quad (53)$$

And then the detection logic can be made through

$$J_{\mathrm{r}}(\tau) \geq J_{\mathrm{th}} \Rightarrow \text{Alarm}$$

Remark 11 It should be emphasized that J_{th} is just an ideally choice. In practice, it should be adjusted by $\bar{J}_{\mathrm{th}} = J_{\mathrm{th}} + \Delta J$, where $\bar{J}_{\mathrm{th}}$ means the adopted value, and ΔJ represents the worst-case value of systems related to dynamic inversion, such as parasitism in power systems. Furthermore, it is different for diverse systems, which do not cause $J_{\mathrm{r}}(\tau)$ to exceed $\bar{J}_{\mathrm{th}}$.

4 Simulation examples

Here, a simulation example will be presented to validate the effectiveness of the proposed FD method. Considering the highly maneuverable aircraft technology (HiMAT) vehicle given in [32], choosing two operating points within the flight envelopes, whose trim conditions are shown in Table 2, as the switching modes, and the sampling period $T = 1s$, a switched system model under the reference input $s(k)$ can be derived as the form of

$$x(k+1) = A_i x(k) + E_i f(x(k), i) + B_{1i} w(k) + B_{2i} s(k)$$
$$y(k) = C_i x(k) + D_{1i} w(k) + D_{2i} s(k) \quad (54)$$

where $x(k) = [x_1(k) \ x_2(k)]$, which represents the attack angle (deg) and the pitch rate (deg/s), respectively. The corresponding system matrices are

$$\left[\begin{array}{c|c|c|c} A_1 & B_{11} & B_{21} & E_1 \\ \hline C_1 & D_{11} & D_{21} & \end{array}\right] =$$
$$\left[\begin{array}{cc|cc|cc} -0.0213 & -0.0035 & 0.0170 & -0.0881 & 0.0036 & -0.0076 \\ 0.0832 & -0.0047 & -0.0775 & 0.3817 & -0.0151 & 0.0296 \\ \hline 0.4000 & 0.2000 & 0.1010 & 0.5001 & & \end{array}\right]$$

$$\left[\begin{array}{c|c|c|c} A_2 & B_{12} & B_{22} & E_2 \\ \hline C_2 & D_{12} & D_{22} & \end{array}\right] =$$
$$\left[\begin{array}{cc|cc|cc} -0.0200 & -0.0036 & -0.0177 & 0.0500 & 0.0006 & 0.0059 \\ 0.0752 & -0.0061 & 0.0579 & -0.0552 & -0.0092 & -0.0193 \\ \hline 0.3400 & 0.1500 & -0.1010 & 0.5001 & & \end{array}\right]$$

In addition, it is assumed that the derived system is switched under the direction of nonhomogeneous Markov chain with $\mathbb{S} = \{1, 2\}$ and TP been in a polytope, whose vertices are

Table 2 Trim condition for two operator points of HiMAT

Operation points	1	2
Mach number	0.6	0.9
Altitude (ft)	20000	25000
Attack angle (∘)	1.48	4.27

Table 3 Comparison of β and β_f for different ϖ_{sl} and ϖ_{fl}

$\varpi_{sl} = \varpi_{fl} = 0$	$\varpi_{sl} = \varpi_{fl} = 0.005$	$\varpi_{sl} = \varpi_{fl} = 1/(2\pi)$
$\beta = 0.3072$	$\beta = 0.3073$	$\beta = 0.2894$
$\beta_f = 0.3329$	$\beta_f = 0.3330$	$\beta_f = 0.3140$

Table 4 Comparison of β and β_f under different γ and γ_d

$\gamma = \gamma_d = 0.81$	$\gamma = \gamma_d = 0.9$	$\gamma = \gamma_d = 1$	$\gamma = \gamma_d = 2$
$\beta = 0.2425$	$\beta = 0.2950$	$\beta = 0.3072$	$\beta = 0.3879$
$\beta_f = 0.2594$	$\beta_f = 0.3215$	$\beta_f = 0.3329$	$\beta_f = 0.3911$

$$\Lambda^1 = \begin{bmatrix} 0.45 & 0.55 \\ 0.68 & 0.32 \end{bmatrix}, \ \Lambda^2 = \begin{bmatrix} 0.2 & 0.8 \\ 0.14 & 0.86 \end{bmatrix},$$

$$\Lambda^3 = \begin{bmatrix} 0.47 & 0.53 \\ 0.73 & 0.27 \end{bmatrix}$$

Further, $o_\iota(k)$, $\iota = 1, 2, 3$ are assumed to be as $o_1(k) = 0.7$, $o_2(k) = 0.3\cos^2(k)$, $o_3(k) = 0.3\sin^2(k)$. And thus, one has $\Lambda(k) = 0.1\Lambda^1 + 0.3\cos^2(k)\Lambda^2 + 0.3\sin^2(k)\Lambda^3$.

Next, for the sake of illustrating the system performances can be influenced by signal frequency, solving the optimization problem (51) with $\gamma = \gamma_d = 1$ under $a_1 = 0.8$ and $a_2 = 0.2$, the derived indices β and β_f for different ϖ_{sl} and ϖ_{fl} are shown in Table 3. From which, it is not difficult to see that the signal frequency affects the system performance definitely. And the signals in high frequency results in worse performance. Further, we are going to show the mutual restrictions between each performance by solving the optimization problem (51) under different situations with different γ and γ_d when $a_1 = 0.8$ and $a_2 = 0.2$ for $\varpi_{sl} = \varpi_{fl} = 0$. The results are shown in Table 4.

One can see, from Table 4, that the larger γ and γ_d are, the larger β and β_f can be obtained. That means it is contrary between the sensitivity performance β, β_f and the attenuation performance γ, γ_d, and they cannot achieve the optimal identification simultaneously.
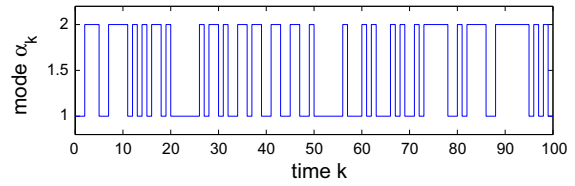
Fig. 1 The values of α_k at each time instant k

Thus, one should find an equilibrium point between them when designing the practical FD systems.

It should be stated here that this is the first attempt to develop the FD method in a finite frequency domain for nonhomogeneous MJSs with nonlinearities, especially with sensor failures. Though there are some results concerned on the finite frequency FD method, they are proposed for linear systems. Thus in this section, no comparison can be done meaningfully. In the following, to show the effectiveness of the proposed method, the system will be simulated for acceptable robustness level, i.e., $\gamma = \gamma_d = 1$ when $a_1 = 0.8$ and $a_2 = 0.2$. In such situation, from Table 4, it is shown that $\beta = 0.3072$, $\beta_f = 0.3329$. The filter gains are not listed here. Further, for the $\Lambda(k)$ given above, a series of sequence α_k can be produced. Without loss of generality, the one shown in Fig. 1 will be adopted in this simulation.

Furthermore, the model given in [33] will be utilized to generate the wind gust signal as the disturbance $\omega(k)$, i.e., $\omega(s) = \mathbf{W}(s)d$, where d is Gaussian white noise and $\mathbf{W}(s)$ represents a random vertical gust process with the following form

$$\mathbf{W}(s) = \sqrt{\frac{3U_0\sigma_w^2}{\pi L_w}} \frac{\frac{U_0}{\sqrt{3}L_w} + s}{\left[\frac{U_0}{L_w} + s\right]^2}$$

where U_0 means the trim velocity (fps) of the vehicle, σ_w denotes the rms vertical gust velocity (fps), L_w represents the characteristic length of turbulence (ft). One usual thunderstorm is with the value of $\sigma_w = 23$ fps, and $L_w = 1900$ ft. Within the above sequence α_k and wind disturbance $\omega(k)$, the system will be simulated under the following four fault cases, which are, respectively, corresponded to the fault types that (5) covered.

Case 1. The outage fault with parameters $\varrho = 0$ and $f = 0$ occurred when $30 < k < 70$. The residual signal and its evaluation function are shown in Fig. 2. From the figure, one can see $J_r(\tau) > J_{th}$ when $k = 32$. That is to say, the occurred fault can be detected two
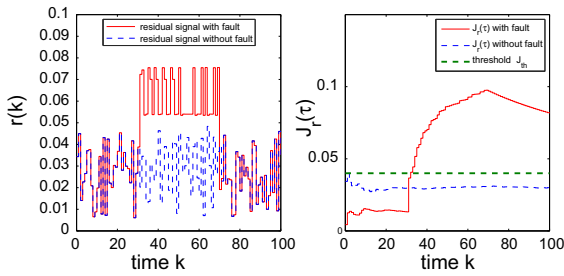
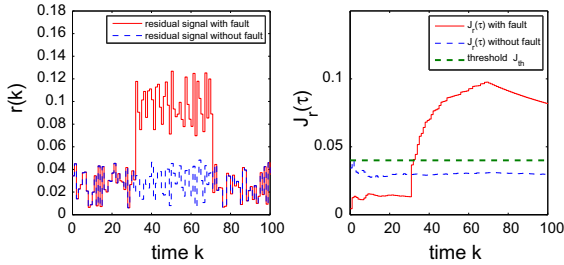**Fig. 2** Residual and its evaluation function at fault case 1

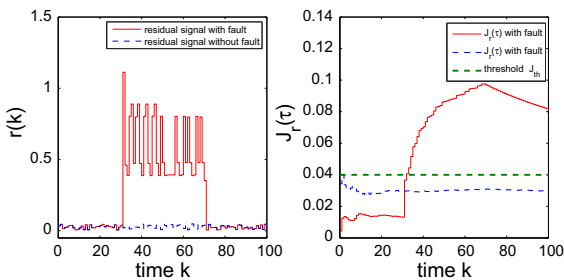**Fig. 3** Residual and its evaluation function at fault case 2

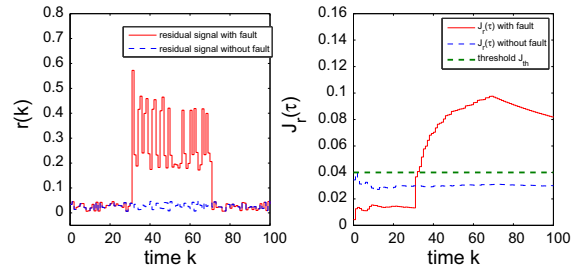**Fig. 4** Residual and its evaluation function at fault case 3

Fig. 5 Residual and its evaluation function at fault case 4

Case 4. The drift fault under the case of $\varrho = 1$ and $f = 0.5$ occurred on the sensor when $30 < k < 70$. The residual signal and its evaluation function are given in Fig. 5. It is shown that $J_r(\tau)$ exceeds the threshold J_{th} at $k = 32$, which indicates the occurred fault can be detected successfully by the investigated FD approach.

In conclusion, from the simulation results for the above four faulty cases, we can see that even though there is a small time delay of detection after the fault occurred, the proposed FD method works well for every kind of fault that the model (5) covers. Such results exhibit the effectiveness of the proposed FD strategy.

5 Conclusion

In this study, the finite frequency FD problem has been presented for a class of nonhomogeneous MJSs with nonlinearities and sensor failures. Firstly, a novel model was presented to describe the sensor failures, and the FD scheme has been proposed for such kinds of sensor faults with the aid of finite frequency stochastic performance indices. Then, sufficient conditions to capture the desired performances were developed in terms of linear matrix inequalities, in which the nonlinearities and nonhomogeneous Markov switching were dealt into solvable items. Subsequently, the filter gains were characterized by solving an optimization problem. Finally, an application to the HiMAT vehicle has been presented to show the effectiveness of the proposed method. In addition, when the systems experience time delays as the types in [34–36] when transferring the information, the research of the FD method in finite frequency domain for such class of systems will be our future work.

steps after it occurred, which shows the effectiveness of the developed FD method.

Case 2. The loss of effectiveness fault under the situation of $\varrho = 0.8$ and $f = 0$ occurred when $30 < k < 70$. The residual signal and its evaluation function are given in Fig. 3. It is shown, from which, that the $J_r(\tau)$ exceeded the threshold J_{th} when $k = 32$. It means the fault can be detected effectively and timely by the proposed FD method.

Case 3. The stuck fault with parameters $\varrho = 0$ and $f = 1$ occurred on the sensor when $30 < k < 70$. Figure 4 presents the residual and its evaluation function. It is clear that the occurred fault could be detected timely since $J_r(\tau) > J_{th}$ when $k = 32$.

Compliance with ethical standards

Conflict of interest All the authors declare that there are no potential conflicts of interest and approval of the submission.

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