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ORIGINAL PAPER

Prescribed performance-barrier Lyapunov function for the adaptive control of unknown pure-feedback systems with full-state constraints

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Abstract In this paper, an adaptive state-feedback control technique is proposed for a class of unknown pure-feedback systems. A remarkable feature is that not only the problem of full-state constraints and prescribed performance tracking is solved together, but also the design is an approximation-free control scheme for pure-feedback systems with completely unknown nonlinearities. These properties will lead to a difficult task for designing a stable controller. To this end, a novel prescribed performance-barrier Lyapunov function is developed to guarantee that all the state constraints are not violated and the tracking error is preserved within a specified prescribed performance bound at all times, simultaneously. Then, by utilizing the mean value theorem, Nussbaum gain technique, a low-pass filter and a novel bounded estimation approach at each step of back-stepping procedure, a novel adaptive dynamics surface control scheme is developed to remove the difficulties of pure-feedback characteristic, unknown nonlinearities, unknown control direction and "explosion of complexity", which can guarantee that the proposed design is universal and low-complexity. Moreover, it is proved that all the signals in the closed-loop system are global uniformly bounded. Two simulation studies are worked out to illustrate the performance of the proposed approach.

Keywords Unknown pure-feedback systems · Adaptive dynamic surface control · Prescribed performance · Barrier Lyapunov function · Full-state constraints

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1 Introduction

Robustness and tracking performance are vital indexes of controller design. Tracking performance is the system properties shown in the steady-state and transient processes, for example, convergence rate, maximum overshoot and steady-state tracking error. Robustness is the maintenance ability of tracking performance in the presence of uncertainties, such as external disturbances, system parameter variations, un-modeled dynamics, and so on. In particular, for the unknown nonlinear systems, how to guarantee these performances comprehensively is still an open and significant problem [1]. During the past several decades, back-stepping technique has been recognized as a powerful tool to design controller for a larger class of uncertain nonlinear system, and the robustness and tracking problem for unknown nonlinear systems has been intensively solved based on back-stepping technique [2–7]. A main limitation in these works is that the progress can only be applied to systems in the affine form. Compared with this progress, relatively fewer results are available for control of pure-feedback systems, which represents a more general class of triangular systems. The difficulty associated with the control design of pure feedback systems arises from the lack of the appropriate variables to be used as virtual and/or actual control in the recursive design procedure. Therefore, control synthesis and stability analysis of pure-feedback systems are challenging issues and have attracted considerable research efforts [8–19]. To enhance the robustness of uncertain nonlinearities or completely unknown nonlinearities, most of these back-stepping-based schemes were developed based on neural networks (NNs) or fuzzy logical systems (FLSs). By utilizing the mean value theorem [8–15], the auxiliary integral method [16] and the contraction mapping method [17–19], the original pure-feedback system was transformed into an equivalent model with quasistrict-feedback form. Subsequently, the back-stepping-based control scheme was developed by employing the NN [8–13,16–18] and FLS [14,15,19] to approximate the unknown nonlinearities of the equivalent strict-feedback form models. However, the repeated differentiation calculations of virtual controller in back-stepping may lead to the problem of "explosion of complexity", which results in tremendous calculation burden and undesirable numerical noise in practice. To eliminate this problem, the dynamic surface control (DSC) was developed in [20] and extended to unknown pure-feedback systems by utilizing a low-pass filter for the synthetic input at each step of back-stepping procedure. By borrowing the features that universal approximator (i.e., NN and FLS) can approximate arbitrary nonlinear continuous function to a given accuracy, an adaptive NN-DSC approach was formulated for a class of unknown pure-feedback systems, where the mean value theorem [21–24] and contraction mapping method [25,26] were employed to transformed the original systems into strict-feedback form, respectively. In [27,28], the pure-feedback system was transformed into pseudostrict-feedback form by adopting the contraction mapping approach, then, an adaptive NN output feedback DSC design [27] and an adaptive fuzzy output feedback DSC design [28] were presented for unknown multi-input and multi-output (MIMO) and single-input and single-output (SISO) pure-feedback systems, respectively. Unfortunately, all aforementioned works guarantee convergence of the steady-state tracking error to a residual set, whose size depends on explicit design parameters and some unknown bounded terms, which makes a priori selection of the design parameters satisfying certain steady-state behavior practically impossible. Furthermore, the analytical relationship between transient behavior (i.e., convergence rate and maximum overshoot) and design parameters is difficult to analyze by using mathematical tool. To formulate the relationship between performance indexes and design parameters as specific analytical functions, an alternative approach, named prescribed performance control (PPC) to guarantee the transient and steady-state performance was first proposed in [29], where prescribed performance bound (PPB) can characterize the convergence rate, maximum overshoot and maximum steady-state error of the tracking errors. With the appropriate performance function and error transformation, the tracking errors can converge to a predefined small residual set with a convergence rate no less than a predefined value and maximum overshoot less than a sufficiently small specified constant. Subsequently, PPC methodology was employed to design adaptive controller for various classes of unknown nonlinear systems by using NN [30,31] and FLS [32]. Only a few results are available in the literature for the PPC of unknown pure-feedback systems. Two low-complexity global approximation-free control scheme with prescribed performance for unknown pure-feedback systems were proposed in [33,34]. An adaptive robust control with prescribed performance for a class of unknown pure-feedback system was studied in [35]. However, a major obstacle in the application scope is that the output and state constraints are not considered in the above-mentioned results about unknown pure-feedback systems.

In recent years, adaptive control of nonlinear systems with output or state constraints has received much attention. Many significant control schemes have been developed by utilizing the back-stepping and DSC approach [36–45]. Since the barrier Lyapunov function (BLF) candidate was originally proposed in [46], the BLF-based control scheme has been widely used for the nonlinear system with state and output constraints. By employing the BLF, an adaptive control scheme was developed to tackle the problem of control output constraints in [36,37] and time-varying output constraints in [38,39]. An adaptive fuzzy control approach for a category of uncertain nonlinear systems with output constraint was developed in [40], and the problem of output constraint was handled by utilizing a BLF. A BLF-based adaptive control was developed for a class of strict-feedback nonlinear systems with full-state constraints in [41,42] and partial-state constraints in [43]. But in the aforementioned works, the research results are obtained under the condition that the considered systems are strict-feedback nonlinear systems, there are very few existing research results for unknown pure-feedback systems with state and output constraints. In [44], for a class of pure-feedback systems with output constraints, a dynamic surface design approach based on an appropriate integral BLF was presented to design an adaptive controller to ensure both the constraint satisfaction and the desired tracking ability. In [45], for a class of uncertain pure-feedback parametric systems, an adaptive recursive design procedure was constructed to remove the difficulties for avoiding nonaffine terms and guarantee that the full-state constraints are not violated by introducing BLF with the error variables.

Despite the efforts made to unknown pure-feedback systems recently, certain issues still remain open. Firstly, all aforementioned works about unknown pure-feedback systems have resorted to universal approximation theorem to deal with the unknown nonlinear dynamics of the system. Unfortunately, this universal approximation-based (i.e., NN and FLS) approach inherently introduces certain issues affecting controller complexity, closed-loop stability and robustness [33]. Secondly, the assumption is always very stringent, for example, the signs of control gain are compelled to be known [8–14,16–19,21–28,33,34,45] and the unknown nonlinear functions satisfy linear-in-the-parameters (LIP) condition [45]. Finally, there are very few results about the constrained or prescribed performance control problem of pure-feedback system. In particular, to the authors' best knowledge, in the literature, there are no results reported on the integrated control design for prescribed performance and state/output constraints, and the previous works for all the state/output constraints are very conservative due to the need for a priori knowledge of control direction.

Based on the above discussions, an adaptive DSC scheme is proposed for a class of unknown pure-feedback systems with prescribe performance and full-state constraints. The main contributions of the proposed approach are that:

(1) Reduced design complexity. Different from the results in [8–14,16–19,21–28] which focus on approximation-based techniques to tackle the unknown dynamics, this study frames a novel adaptive bounded estimation approach to deal with the unknown dynamics by combining the mean value theorem, the supremum norm theory and DSC, which is approximation-free and can avoid unnecessary repeated differentiation calculations. Namely, the proposed control scheme is low-complexity.

(2) Reduced system prior knowledge and conservatism. Compared with results in [8–14,16–19,21–28,33,34,45], the knowledge of the sign of control gain and the LIP condition of unknown nonlinear functions are not required by employing Nussbaum gain technique and supremum norm theory. Meanwhile, the conservatism of traditional BLF can be removed in this study.

(3) Prescribed performance-barrier Lyapunov function (PP-BLF). The proposed control scheme constitutes a first approach to solve the problem of state constraints and prescribed performance tracking integratedly, which can guarantee that all the state constraints are not violated, and the tracking error is preserved within a specified prescribed performance bound at all times, simultaneously. Moreover, the proposed control scheme also constitutes a first approach toward the solution of the prescribed performance tracking problem and state-constrained problem for pure feedback systems.

2 Problem formulation and preliminaries

Consider a class of unknown pure-feedback systems [8] with full-state constraints as follows

$$\begin{cases} \dot{x}_i = f_i(x_1, \ldots, x_{i+1}), \quad i = 1, \ldots, n-1 \\ \dot{x}_n = f_n(x, u) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$, $y \in R$ and $u \in R$ are the states, the output, and the input of system, respectively; $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in R^i$, $f_i(\bar{x}_{i+1}), i = 1, \ldots, n-1$ and $f_n(x, u)$ are unknown nonaffine nonlinear smooth functions. In this study, all the states are constrained in the compact sets, i.e., $|x_i| < k_{c_i}$ with k_{c_i} being a known positive constant. To facilitate design, let $x_{n+1} = u$.

The control objective in this study is to design an adaptive state feedback controller u such that: (1) All the signals in the closed-loop system are bounded; (2) the tracking error $z_1 = y - y_{\mathrm{r}}$ achieves prescribed transient and steady-state performance; (3) the full-state constraints are not violated, i.e., $|x_i| < k_{c_i}$, where $y_{\mathrm{r}} \in R$ is the reference signal.

To achieve the control goal, the following assumptions, lemmas, and definitions are required in the design.

Assumption 1 For the unknown pure-feedback system (1), the function f_i is continuously differentiable and there exists unknown positive constant $\bar{\beta}_{i,j}, i = 1, \ldots, n,\ j = 1, \ldots, i+1$ such that

$$\left|\frac{\partial f_i(x_1, \ldots, x_i, x_{i+1})}{\partial x_j}\right| \le \bar{\beta}_{i,j},$$
$$i = 1, \ldots, n, \quad j = 1, \ldots, i$$
$$0 < \left|\frac{\partial f_i(x_1, \ldots, x_i, x_{i+1})}{\partial x_{i+1}}\right| \le \bar{\beta}_{i,i+1},$$
$$i = 1, \ldots, n$$

for all $(x_1, \ldots, x_n, u) \in R^{n+1}$.

Assumption 2 For the unknown pure-feedback system (1), the $f_i(\bar{x}^0_{i+1})$ is always bounded; that is, there exists unknown positive constant Δ_i such that $\left|f_i(\bar{x}^0_{i+1})\right| \le \Delta_i$, where $x^0_j = x_j(0), j = 1, \ldots, i+1$ and $\bar{x}^0_{i+1} = [x^0_1, \ldots, x^0_{i+1}]^{\mathrm{T}}$ denote the system initial conditions.

Assumption 3 [27] For the unknown pure-feedback system (1), there exists positive constants A_0 and B_0 such that the desired trajectory y_{r}, its first-order derivative $\dot{y}_{\mathrm{r}}$ and its second-order derivative $\ddot{y}_{\mathrm{r}}$ satisfy $|y_{\mathrm{r}}| \le A_0 < k_{c_1}$ and $\Omega_0 := \{[y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}, \ddot{y}_{\mathrm{r}}] : |y_{\mathrm{r}}|^2 + |\dot{y}_{\mathrm{r}}|^2 + |\ddot{y}_{\mathrm{r}}|^2 \le B_0\} \in R^3,\ \forall t \ge 0$.

Remark 1 ⓐ: Without loss of generality, the stability results will be valid as long as the states remain within some compact sets, so the global Lipschitz condition on f_i can be relaxed to a local one in the Assumption 1. Such as $f_i = 0.1x_i^2 - x_{i+1} + 0.2\sin(x_i x_{i+1})$ also satisfies Assumption 1. ⓑ: From a practical view, the energy and change rate of nonlinear dynamic are limited. Thus, Assumption 1–3 is reasonable. ⓒ: In [8–14,16–19,21–28,33,34], the adaptive control design is required to confirm the signs of control gain, and the unknown nonlinear function $f_i(\bar{x}_{i+1})$ satisfies $f_i(\bar{x}_{i+1}) = \theta_i^{\mathrm{T}}\xi_i(\bar{x}_{i+1})$ in [45], These are conservative for practical application and the problem of state constraints. In this study, the signs of control gain and the function f_i do not require to be known.

Definition 1 [47] Any continuous even function $N(\zeta)$ is called Nussbaum-type function when there are the following properties:

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\zeta)\mathrm{d}\zeta = +\infty \tag{2}$$

$$\lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\zeta)\mathrm{d}\zeta = -\infty \tag{3}$$

There are many functions to be viewed as a Nussbaum-type function such as $\mathrm{e}^{\zeta^2}\cos((\pi/2)\zeta)$ and $\zeta^2\cos(\zeta)$. In this study, $N(\zeta) = \zeta^2\cos(\zeta)$ is used.

Lemma 1 [48] *Let $V(t)$ and $\zeta(t)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0$, $\forall t \in [0, t_f)$, and $N(\zeta)$ is a smooth Nussbaum-type function. If the following inequality holds*

$$V(t) \le c + \mathrm{e}^{-\mu t}\int_0^t (\Gamma(x(\tau))N(\zeta) + 1)\dot{\zeta}\mathrm{e}^{\mu\tau}d\tau \tag{4}$$

where $\mu > 0$, $\Gamma(x(\tau))$ is a time-varying parameter which takes values in the unknown closed intervals $I := [l^-, l^+]$ with $0 \notin I$, and c represents some suitable constant, then $V(t), \zeta(\tau)$ and $\int_0^t \Gamma(x(\tau))N(\zeta)\dot{\zeta}d\tau$ must be bounded on $[0, t_f)$.

Lemma 2 [49] *Suppose $0 \le t_f \le \infty$ and that $x : [0, t_f] \to R^N$ is a solution of the closed-loop system. If x is a bounded solution, then $t_f = \infty$.*

Lemma 3 (Youngs inequality [15]) *For $\forall(m, n) \in R^2$, the following inequality holds:*

$$mn \le \frac{\varsigma^p}{p}|m|^p + \frac{1}{q\varsigma^q}|n|^q \tag{5}$$

where $\varsigma > 0,\ p > 1,\ q > 1$ and $(p-1)(q-1) = 1$.

Lemma 4 *For the unknown nonlinear function $f_i(\bar{x}_{i+1}), i = 1, \ldots, n$ of system (1), there exists unknown constant $\xi_{i,j} \in [x^0_j, x_j]$, $i = 1, \ldots, n$, $j = 1, \ldots, i+1$, such that*

$$f_i(\bar{x}_{i+1}) = \sum_{j=1}^{i}\beta_{i,j}(x_j - x^0_j) - \beta_{i,i+1}x^0_{i+1} + \beta_{i,i+1}x_{i+1} + f_i(\bar{x}^0_{i+1}) \tag{6}$$

where $\beta_{i,j} = (\partial f_i/\partial x_j)|_{(0,\ldots,\xi_{i,j},\ldots,x_i,x_{i+1})}$.

Proof By adding and subtracting the term $f_i(\bar{x}^0_{i+1})$, then the $f_i(\bar{x}_{i+1})$ can be described as:

$$f_i(\bar{x}_{i+1}) = f_i(x_1, \ldots, x_{i+1}) - f_i(x^0_1, x_2, \ldots, x_{i+1}) + f_i(x^0_1, x_2, \ldots, x_{i+1}) - f_i(x^0_1, x^0_2, x_3, \ldots, x_{i+1})$$

$$+\cdots+f_i(x_1^0,\ldots x_i^0,x_{i+1}) - f_i(\bar{x}_{i+1}^0)+f_i(\bar{x}_{i+1}^0) \tag{7}$$

Subsequently, since the $f_i(\bar{x}_{i+1})$ is continuous and differentiable for all $(x_1,\ldots,x_i,x_{i+1}) \in R^{i+1}$ and the initial value $f_i(\bar{x}_{i+1}^0)$ are bounded, the $f_i(\bar{x}_{i+1})$ can be expressed as the following form with $\xi_{i,j} \in [x_j^0, x_j]$, $i=1,\ldots,n,\ j=1,\ldots,i+1$ ultimately by using mean value theorem.

$$\begin{aligned}f_i(\bar{x}_{i+1}) =& \left(\frac{\partial f_i}{\partial x_1}\bigg|_{(\xi_{i,1},x_2\ldots,x_i,x_{i+1})}\right)*(x_1-x_1^0)\\ &+\left(\frac{\partial f_i}{\partial x_2}\bigg|_{(x_1^0,\xi_{i,2},x_3\ldots,x_i,x_{i+1})}\right)\\ &*(x_2-x_2^0)+\cdots\\ &+\left(\frac{\partial f_i}{\partial x_{i+1}}\bigg|_{(x_1^0,\ldots,x_i^0,\xi_{i,i+1})}\right)\\ &*(x_{i+1}-x_{i+1}^0)+f_i(\bar{x}_{i+1}^0)\end{aligned} \tag{8}$$

which together with the $\beta_{i,j} = (\partial f_i/\partial x_j)|_{(0,\ldots,\xi_{i,j},\ldots,x_i,x_{i+1})}$, $i=1,\ldots,n$, $j=1,\ldots,i+1$ gives the Eq. (6).

This concludes the proof. □

3 Prescribed performance-Barrier Lyapunov function

3.1 Prescribed performance function

PPC was first proposed in [29]. PPC is achieved if the tracking error $z_1 = y - y_r$ evolves strictly within a predefined region that is bounded by a decaying function of time, i.e., prescribed performance function (PPF). The definition of PPF is given as follows.

Definition 2 [29]. A continuous function $\rho(t)$ is called PPF if

(i) $\rho(t)$ is positive and strictly decreasing;
(ii) $\lim_{t\to\infty}\rho(t)=\rho_\infty$. In this paper, the following exponential function is chosen as PPF

$$\rho(t)=(k_{b_1}-\rho_\infty)e^{-\ell t}+\rho_\infty \tag{9}$$

where $k_{b_1}=k_{c_1}-A_0>\rho_\infty$ is the boundedness of tracking error z_1, $\rho_\infty>0$ and $\ell>0$ are appropriately prescribed scalars. Then, the aforementioned second objective is obviously equivalent to the following mathematical inequality:

$$-\rho(t)<z_1<\rho(t),\quad \forall t>0 \tag{10}$$

The constant ρ_∞ represents the maximum allowable size of the tracking error z_1 at the steady state, which may even be set arbitrarily small, thus accomplishing practical convergence of z_1 to zero. Moreover, the decreasing rate of $\rho(t)$, which is determined by the constant ℓ, imposes a lower bound on the required speed of convergence of z_1. Therefore, the appropriate selection of the PPF $\rho(t)$ imposes performance characteristics on the tracking error z_1.

Remark 2 To integrate PPF with BLF, the mathematical expression of prescribed performance is given as (9) and (10) which introduce parameter k_{b_1}. Different from the results in [29–32] which employ an error transformation technique to transform the constrained tracking error (10) into an unconstrained one, this study integrates the constrained tracking error (10) into BLF (i.e., PP-BLF) to achieve PPC and state-constrained control, simultaneously.

3.2 BLF and PP-BLF

To avoid the violation of output constraints, we employ a BLF with the following definitions.

Definition 3 [36] $V(x)$ as a scalar function defined with respect to the system $\dot{x}=f(x)$ on an open region $\mathcal{D}$ including the origin, is a BLF if it has the following properties.

(1) $V(x)$ is continuous, positively definite, and has continuous first-order partial derivatives at every point of $\mathcal{D}$.
(2) $V(x)\to\infty$ as x approaches the boundary of $\mathcal{D}$.
(3) $V(x)\le d$, $\forall t\ge 0$ along the solution of $\dot{x}=f(x)$ with $x(0)\in\mathcal{D}$, where d is some positive constant.

To prevent the states or outputs of system from violating their constraints, one kind of traditional BLF is always defined in the following form with a compact set $\Omega_z=\{z_i : |z_i|<k_{b_i}, i=1,\ldots,n\}$:

$$V_{b_i}=\frac{1}{2}\ln\frac{k_{b_i}^2}{k_{b_i}^2-z_i^2},\quad i=1,\ldots,n \tag{11}$$

where $k_{b_1} > 0$ is the bounded of z_1, $k_{b_i} > 0, i = 2, \ldots, n$ is a design constant, and the k_{b_i} and z_i will be determined later on. However, the traditional BLF is incapable of the requirement of prescribed performance. In order to guarantee that the tracking error z_1 satisfies predefined inequality (10), the PP-BLF is developed in this study, which can be defined as follows with a compact set $\Omega_{z_1} = \{z_1 : |z_1/\rho(t)| < 1\}$.

$$V_{b_1} = \frac{1}{2}\ln\frac{1}{1-(z_1/\rho(t))^2} \tag{12}$$

The following lemma formalizes the result for (11) and (12) barrier functions and is used in the control design and analysis for pure-feedback system (1) to guarantee that state constraints and prescribed performance scalars are not violated.

Remark 3 In fact, the time-varying BLF in [38] can solve the problem of prescribed performance, but the relationship between BLF and PPB cannot be expressed clearly. Moreover, only strict-feedback system is considered with LIP condition in [38].

Lemma 5 [36,38] *For any χ_1, any positive constant k_{b_i} and any $z_i \in R$ satisfying $|\chi_1| < 1$ and $|z_i| < k_{b_i}$, $i = 1, 2, \ldots, n$, we have*

$$\ln\frac{k_{b_i}^2}{k_{b_i}^2 - z_i^2} \le \frac{z_i^2}{k_{b_i}^2 - z_i^2}, \quad \ln\frac{1}{1-\chi_1^2} \le \frac{\chi_1^2}{1-\chi_1^2} \tag{13}$$

Proof The inequality (13) can be verified easily based on the Lemma 2 in [36,38]. The proof is omitted here due to the limited space. □

Lemma 6 [37] *For any positive constant λ, let $\boldsymbol{S} = \{s \in \boldsymbol{R} : |s| < \lambda\} \subset \boldsymbol{R}$ and $\boldsymbol{N} = \boldsymbol{R}^l \times \boldsymbol{S} \subset \boldsymbol{R}^{l+1}$ be open sets. Consider the system*

$$\dot{\eta} = h(t, \eta) \tag{14}$$

where $\eta = [\omega, s]^{\mathrm{T}} \in \boldsymbol{N}$ and $h : \boldsymbol{R}^+ \times \boldsymbol{N} \to \boldsymbol{R}^{l+1}$ is piecewise continuous in t and locally Lipschitz in η, uniformly in t, on $R^+ \times N$. Suppose that there exists functions $U : \boldsymbol{R}^l \times \boldsymbol{R}^+ \to R^+$ and $V_b : \boldsymbol{S} \to \boldsymbol{R}^+$, continuously differentiable and positive definite in their respective domains, such that

$$V_b \to \infty \quad as \quad |s| \to \lambda \tag{15}$$

$$\Upsilon_1(\|\omega\|) \le U(\omega, t) \le \Upsilon_2(\|\omega\|) \tag{16}$$

where Υ_1 and Υ_2 are class K_∞ functions. Let $V(\eta) = V_b(s) + U(\omega, t)$ and $s(0) \in \boldsymbol{S}$. If the inequality holds:

$$\dot{V} = \frac{\partial V}{\partial \eta}h \le -\mu V + \sigma \tag{17}$$

where μ and σ are positive constants, then $s(t) \in \boldsymbol{S}, \forall t \in [0, \infty)$.

Proof The proof is omitted here due to the limited space. Interested readers can follow the similar procedures of the proof of Lemma 1 in [36]. □

4 Main results

4.1 Control scheme

In this section, the approximation-free PP-BLF-based control scheme for systems (1) is designed step-by-step in the presence of unknown dynamics and full-state constraints.

Step 1 Define the tracking error as $z_1 = x_1 - y_r$ and invoking (6), it has

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 - \dot{y}_r = \beta_{1,1}(x_1 - x_1^0) \\ &\quad -\beta_{1,2}x_2^0 + \beta_{1,2}x_2 + f_1(\bar{x}_2^0) - \dot{y}_r \\ &= \varphi_1^{\mathrm{T}}\psi_1 + \beta_{1,2}x_2 - \dot{y}_r\end{aligned} \tag{18}$$

where $\varphi_1 = [\beta_{1,1}, \beta_{1,2}, f_1(\bar{x}_2^0)]^{\mathrm{T}}$ and $\psi_1 = [x_1 - x_1^0, -x_2^0, 1]^{\mathrm{T}}$. Since the $\beta_{1,1}, \beta_{1,2}$ and $f_1(\bar{x}_2^0)$ are bounded by Assumption 1 and 2, the $\|\varphi_1\|$ is also bounded. With the supremum norm theory in mild, let

$$\vartheta_1 = \sup_{t\ge 0}\|\varphi_1\|^2 \tag{19}$$

Design the virtual control α_1 as

$$\alpha_1 = N(\zeta_1)\varpi_1 \tag{20}$$

$$\begin{aligned}\varpi_1 &= \kappa_1\chi_1 - \dot{y}_r - \rho^{-1}\dot{\rho}z_1 + \frac{\rho^{-1}\chi_1}{1-\chi_1^2} \\ &\quad + \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\hat{\vartheta}_1\|\psi_1\|^2\right)\Big/(2\delta_1^2)\end{aligned} \tag{21}$$

where $\chi_1 = z_1/\rho(t)$. The parameter updating laws are designed as

$$\dot{\zeta}_1 = \rho^{-1}\chi_1\varpi_1/(1-\chi_1^2) \tag{22}$$

$$\dot{\hat{\vartheta}}_1 = \gamma_1\left[-\hat{\vartheta}_1 + \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2\|\psi_1\|^2/(2\delta_1^2)\right] \tag{23}$$

where κ_1, δ_1 and γ_1 are designed positive constants, $\hat{\vartheta}_1$ denotes the estimation of ϑ_1.

To avoid repeatedly differentiating α_1, which leads to the so-called "explosion of complexity", the DSC technique in [20] is employed here. Namely, introduce a new variable $\overline{\alpha}_1$ and let α_1 pass through a first-order filter with time constant g_1 to gain $\overline{\alpha}_1$

$$g_1\dot{\overline{\alpha}}_1 + \overline{\alpha}_1 = \alpha_1, \qquad \overline{\alpha}_1(0) = \alpha_1(0) \tag{24}$$

By defining the output error ϕ_1 of this filter and tracking error z_2 as

$$\phi_1 = \overline{\alpha}_1 - \alpha_1 \tag{25}$$

$$z_2 = x_2 - \overline{\alpha}_1 \tag{26}$$

Differentiating ϕ_1 yields

$$\begin{aligned}\dot{\phi}_1 &= \dot{\overline{\alpha}}_1 - \dot{\alpha}_1 = -\frac{\phi_1}{g_1} - \dot{\alpha}_1 \\ &= -\frac{\phi_1}{g_1} + \left[-\frac{\partial N(\zeta_1)}{\partial \zeta_1}\dot{\zeta}_1\varpi_1 - N(\zeta_1)\left(\frac{\partial \varpi_1}{\partial x_1}\dot{x}_1 \right.\right. \\ &\quad \left.\left. + \frac{\partial \varpi_1}{\partial z_1}\dot{z}_1 + \frac{\partial \varpi_1}{\partial \rho_1}\dot{\rho}_1 - \ddot{y}_r + \frac{\partial \varpi_1}{\partial \dot{\rho}_1}\ddot{\rho}_1 + \frac{\partial \varpi_1}{\partial \hat{\vartheta}_1}\dot{\hat{\vartheta}}_1\right)\right] \\ &= -\frac{\phi_1}{g_1} + T_1(z_1, z_2, \phi_1, \rho_1, \hat{\vartheta}_1, y_r, \dot{y}_r, \ddot{y}_r)\end{aligned} \tag{27}$$

All of variables of the function T_1 are from compact sets and T_1 is a smooth function, so the $|T_1|$ has its maximum value $\overline{T}_1$, i.e., $|T_1| \le \overline{T}_1$ with $\overline{T}_1$ being an unknown constant *(Please refer to* [20] *f* or details).

Then, consider the PP-BLF in (12) and choose the following Lyapunov function candidate

$$V_1 = V_{b_1} + \frac{1}{2\gamma_1}\tilde{\vartheta}_1^2 + \frac{1}{2}\phi_1^2 \tag{28}$$

where γ_1 is a designed positive constant, $\tilde{\vartheta}_1 = \hat{\vartheta}_1 - \vartheta_1$ is the estimation error. Define a set $\Omega_{\chi_1} = \{\chi_1 : |\chi_1| < 1\}$. In the set $\Omega_{\chi_1} = \{\chi_1 : |\chi_1| < 1\}$, V_1 is continuous. Then, in the view of (12), (18), (20), (21), (22), (25), and (26), differentiating (28) yields

$$\begin{aligned}\dot{V}_1 &= \frac{\rho^{-1}\chi_1}{1-\chi_1^2}\left[\varphi_1^{\mathrm{T}}\psi_1 + \beta_{1,2}(z_2+\phi_1) + \beta_{1,2}N(\zeta_1)\varpi_1 \right. \\ &\quad \left. + \varpi_1 - \varpi_1 - \dot{y}_r - \rho^{-1}\dot{\rho}z_1\right] \\ &\quad + \frac{\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1}{\gamma_1} + \phi_1\dot{\phi}_1 \\ &= -\kappa_1\frac{\rho^{-1}\chi_1^2}{1-\chi_1^2} - \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2 \\ &\quad + \frac{\beta_{1,2}\rho^{-1}\chi_1}{1-\chi_1^2}(z_2+\phi_1) + \left[\beta_{1,2}N(\zeta_1)+1\right]\dot{\zeta}_1 \\ &\quad - \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2\hat{\vartheta}_1\|\psi_1\|^2/(2\delta_1^2) + \frac{\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1}{\gamma_1} \\ &\quad + \frac{\rho^{-1}\chi_1}{1-\chi_1^2}\varphi_1^{\mathrm{T}}\psi_1 + \phi_1\dot{\phi}_1\end{aligned} \tag{29}$$

where ρ is the abbreviation for $\rho(t)$. Consider (19), (27) and Assumption 1, using the Youngs inequality, we have

$$\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\varphi_1^{\mathrm{T}}\psi_1 \le \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2\vartheta_1\|\psi_1\|^2/(2\delta_1^2)+\frac{\delta_1^2}{2} \tag{30}$$

$$\frac{\beta_{1,2}\rho^{-1}\chi_1 z_2}{1-\chi_1^2} \le \frac{1}{2}\left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2 + \frac{1}{2}\bar{\beta}_{1,2}^2 z_2^2 \tag{31}$$

$$\frac{\beta_{1,2}\rho^{-1}\chi_1\phi_1}{1-\chi_1^2} \le \frac{1}{2}\left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2 + \frac{1}{2}\bar{\beta}_{1,2}^2\phi_1^2 \tag{32}$$

$$\begin{aligned}\phi_1\dot{\phi}_1 &= \phi_1\left(-\frac{\phi_1}{g_1} - \dot{\alpha}_1\right) \\ &\le -\frac{\phi_1^2}{g_1} + \frac{\phi_1^2}{2} + \frac{T_1^2}{2} \\ &\le -\frac{\phi_1^2}{g_1} + \frac{\phi_1^2}{2} + \frac{\overline{T}_1^2}{2}\end{aligned} \tag{33}$$

Substituting (30), (31), (32) and (33) into (29), we have

$$\begin{aligned}\dot{V}_1 &= -\kappa_1\frac{\rho^{-1}\chi_1^2}{1-\chi_1^2} - \left(\frac{1}{g_1} - \frac{1}{2} - \frac{\bar{\beta}_{1,2}^2}{2}\right)\phi_1^2 \\ &\quad + \frac{1}{2}\bar{\beta}_{1,2}^2 z_2^2 + \left[\beta_{1,2}N(\zeta_1)+1\right]\dot{\zeta}_1 \\ &\quad - \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2\tilde{\vartheta}_1\|\psi_1\|^2/(2\delta_1^2) \\ &\quad + \frac{\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1}{\gamma_1} + \frac{\delta_1^2}{2} + \frac{\overline{T}_1^2}{2}\end{aligned} \tag{34}$$

According to (23), it is easy to obtain

$$\begin{aligned}&-\left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\right)^2\tilde{\vartheta}_1\|\psi_1\|^2/(2\delta_1^2) + \frac{\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1}{\gamma_1} = -\tilde{\vartheta}_1\hat{\vartheta}_1 \\ &= -\tilde{\vartheta}_1^2 - \tilde{\vartheta}_1\vartheta_1 \le -\frac{1}{2}\tilde{\vartheta}_1^2 + \frac{1}{2}\vartheta_1^2\end{aligned} \tag{35}$$

Substituting (35) into (34), we have

$$\begin{aligned}\dot{V}_1 &\le -\kappa_1\frac{\rho^{-1}\chi_1^2}{1-\chi_1^2} - \frac{1}{2}\tilde{\vartheta}_1^2 - \left(\frac{1}{g_1} - \frac{1}{2} - \frac{\bar{\beta}_{1,2}^2}{2}\right)\phi_1^2 \\ &\quad + \left[\beta_{1,2}N(\zeta_1)+1\right]\dot{\zeta}_1 + \frac{1}{2}\bar{\beta}_{1,2}^2 z_2^2 \\ &\quad + \frac{1}{2}\vartheta_1^2 + \frac{\delta_1^2}{2} + \frac{\overline{T}_1^2}{2}\end{aligned} \tag{36}$$

From Lemma 5, it is the fact that $\ln\frac{1}{1-\chi_1^2} \le \frac{\chi_1^2}{1-\chi_1^2}$ in the interval $|\chi_1| < 1$, and the fact $\rho^{-1} \in [k_{b_1}^{-1}, \rho_\infty^{-1})$, we have

$$\dot{V}_1 \leq -\mu_1 V_1 + \varepsilon_1 + \left[\beta_{1,2} N(\zeta_1) + 1\right]\dot{\zeta}_1 + 0.5\bar{\beta}_{1,2}^2 z_2^2 \quad (37)$$

where $\mu_1 = \min\{2\kappa_1 k_{b_1}^{-1}, \gamma_1, 2g_1^{-1} - 1 - \bar{\beta}_{1,2}^2\} > 0$ and $\varepsilon_1 = 0.5\vartheta_1^2 + 0.5\delta_1^2 + 0.5\overline{T}_1^2$.

Step i, $i = 2, \ldots, n-1$. Define the variable $\varphi_i = [\beta_{i,1}, \ldots, \beta_{i,i}, \beta_{i,i+1}, f_i(\bar{x}_{i+1}^0)]^{\mathrm{T}}$, $\psi_i = [x_1 - x_1^0, \ldots, x_i - x_i^0, -x_{i+1}^0, 1]^{\mathrm{T}}$, $z_i = x_i - \overline{\alpha}_{i-1}$, and invoking (6), it also has

$$\dot{z}_i = \dot{x}_i - \dot{\overline{\alpha}}_{i-1} = \varphi_i^{\mathrm{T}}\psi_i + \beta_{i,i+1}x_{i+1} - \dot{\overline{\alpha}}_{i-1} \quad (38)$$

Similarly, the $\beta_{i,j}$, $j = 1, \ldots, i+1$ and $f_i(\bar{x}_{i+1}^0)$ are bounded, so the $\|\varphi_i\|$ is also bounded. With the supremum norm theory in mild again, let

$$\vartheta_i = \sup_{t\geq 0} \|\varphi_i\|^2 \quad (39)$$

Design the virtual control α_i as

$$\alpha_i = N(\zeta_i)\varpi_i \quad (40)$$

$$\varpi_i = \kappa_i z_i - \dot{\overline{\alpha}}_{i-1} + \frac{z_i}{k_{b_i}^2 - z_i^2} + \left(\frac{z_i}{k_{b_i}^2 - z_i^2}\hat{\vartheta}_i\|\psi_i\|^2\right)\Big/(2\delta_i^2) \quad (41)$$

where κ_i and δ_i are designed positive constants. The parameter updating laws are designed as

$$\dot{\zeta}_i = z_i\varpi_i \big/ (k_{b_i}^2 - z_i^2) \quad (42)$$

$$\dot{\hat{\vartheta}}_i = \gamma_i\left[-\hat{\vartheta}_i + \left(\frac{z_i}{k_{b_1}^2 - z_i^2}\right)^2 \|\psi_i\|^2\big/(2\delta_i^2)\right] \quad (43)$$

Similar to Step 1, to avoid repeatedly differentiating α_i, we introduce a new variable $\overline{\alpha}_i$ and let α_i pass through a first-order filter with time constant g_i to gain $\overline{\alpha}_i$.

$$g_i\dot{\overline{\alpha}}_i + \overline{\alpha}_i = \alpha_i, \qquad \overline{\alpha}_i(0) = \alpha_i(0) \quad (44)$$

By defining the output error of this filter and tracking error z_{i+1} as

$$\phi_i = \bar{\alpha}_i - \alpha_i \quad (45)$$

$$z_{i+1} = x_{i+1} - \bar{\alpha}_i \quad (46)$$

Similar to Step 1, we can obtain that

$$\dot{\phi}_i = -\frac{\phi_i}{g_i} + T_i(z_1, \ldots, z_i, \phi_1, \ldots, \phi_i, \rho_1, \hat{\vartheta}_1, \ldots, \hat{\vartheta}_i, y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}, \ddot{y}_{\mathrm{r}}) \quad (47)$$

and $|T_i|$ has its maximum value $\overline{T}_i$, i.e., $|T_i| \leq \overline{T}_i$ with $\overline{T}_i$ being an unknown constant.

Then, consider the BLF in (11) and choose the following Lyapunov function candidate

$$V_i = V_{b_i} + \frac{1}{2\gamma_i}\tilde{\vartheta}_i^2 + \frac{1}{2}\phi_i^2 \quad (48)$$

where γ_i is a designed positive constant, $\tilde{\vartheta}_i = \hat{\vartheta}_i - \vartheta_i$ and k_{b_i} is specified later on. In the set $\Omega_{z_i} = \{z_i : |z_i| < k_{b_i}\}$, V_i is continuous. Then, we obtain $\dot{V}_i$ from (11), (38), (40), (41), (42), (45), and (46) as

$$\begin{aligned}\dot{V}_i =& -\kappa_i\frac{z_i^2}{k_{b_i}^2 - z_i^2} - \left(\frac{z_i}{k_{b_i}^2 - z_i^2}\right)^2 \\ &+ \frac{\beta_{i,i+1}z_i}{k_{b_i}^2 - z_i^2}(z_{i+1} + \phi_i) \\ &+ \left[\beta_{i,i+1}N(\zeta_i) + 1\right]\dot{\zeta}_i \\ &- \left(\frac{z_i}{k_{b_i}^2 - z_i^2}\right)^2 \hat{\vartheta}_i\|\psi_i\|^2\big/(2\delta_i^2) \\ &+ \frac{\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i}{\gamma_i} + \frac{z_i}{k_{b_i}^2 - z_i^2}\varphi_i^{\mathrm{T}}\psi_i + \phi_i\dot{\phi}_i\end{aligned} \quad (49)$$

Consider (39), (47) and Assumption 1, using the Youngs inequality, we have

$$\frac{z_i}{k_{b_i}^2 - z_i^2}\varphi_i^{\mathrm{T}}\psi_i \leq \left(\frac{z_i}{k_{b_i}^2 - z_i^2}\right)^2 \vartheta_i\|\psi_i\|^2\big/(2\delta_i^2) + \frac{\delta_i^2}{2} \quad (50)$$

$$\frac{\beta_{i,i+1}z_i z_{i+1}}{k_{b_i}^2 - z_i^2} \leq \frac{1}{2}\left(\frac{z_i}{k_{b_i}^2 - z_i^2}\right)^2 + \frac{1}{2}\bar{\beta}_{i,i+1}^2 z_{i+1}^2 \quad (51)$$

$$\frac{\beta_{i,i+1}z_i\phi_i}{k_{b_i}^2 - z_i^2} \leq \frac{1}{2}\left(\frac{z_i}{k_{b_i}^2 - z_i^2}\right)^2 + \frac{1}{2}\bar{\beta}_{i,i+1}^2\phi_i^2 \quad (52)$$

$$\phi_i\dot{\phi}_i = \phi_i\left(-\frac{\phi_i}{g_i} - \dot{\alpha}_i\right) \leq -\frac{\phi_i^2}{g_i} + \frac{\phi_i^2}{2} + \frac{\overline{T}_i^2}{2} \quad (53)$$

Substituting (50), (51), (52) and (53) into (49), we have

$$\begin{aligned}\dot{V}_i \leq& -\kappa_i\frac{z_i^2}{k_{b_i}^2 - z_i^2} - \left(\frac{1}{g_i} - \frac{1}{2} - \frac{\bar{\beta}_{i,i+1}^2}{2}\right)\phi_i^2 \\ &+ \frac{1}{2}\bar{\beta}_{i,i+1}^2 z_{i+1}^2 + \left[\beta_{i,i+1}N(\zeta_i) + 1\right]\dot{\zeta}_i \\ &- \left(\frac{z_i}{k_{b_i}^2 - z_i^2}\right)^2 \tilde{\vartheta}_i\|\psi_i\|^2\big/(2\delta_i^2) \\ &+ \frac{\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i}{\gamma_i} + \frac{\delta_i^2}{2} + \frac{\overline{T}_i^2}{2}\end{aligned} \quad (54)$$

According to (43), it is easy to obtain

$$-\left(\frac{z_i}{k_{b_i}^2-z_i^2}\right)^2\tilde{\vartheta}_i\|\psi_i\|^2/(2\delta_i^2)+\frac{\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i}{\gamma_i}$$
$$\le -\frac{1}{2}\tilde{\vartheta}_i^2+\frac{1}{2}\vartheta_i^2 \tag{55}$$

Substituting (55) into (54), we have

$$\dot{V}_i \le -\kappa_i\frac{z_i^2}{k_{b_i}^2-z_i^2}-\left(\frac{1}{g_i}-\frac{1}{2}-\frac{\bar{\beta}_{i,i+1}^2}{2}\right)\phi_i^2-\frac{1}{2}\tilde{\vartheta}_i^2$$
$$+\left[\beta_{i,i+1}N(\zeta_i)+1\right]\dot{\zeta}_i+\frac{1}{2}\bar{\beta}_{i,i+1}^2z_{i+1}^2$$
$$+\frac{1}{2}\vartheta_i^2+\frac{\delta_i^2}{2}+\frac{\overline{T}_i^2}{2} \tag{56}$$

From Lemma 5, it is the fact that $\ln\frac{k_{b_i}^2}{k_{b_i}^2-z_i^2}\le\frac{z_i^2}{k_{b_i}^2-z_i^2}$ in the interval $|z_i|<k_{b_i}$, we have

$$\dot{V}_i \le -\mu_i V_i+\varepsilon_i$$
$$+\left[\beta_{i,i+1}N(\zeta_i)+1\right]\dot{\zeta}_i+0.5\bar{\beta}_{i,i+1}^2z_{i+1}^2 \tag{57}$$

where $\mu_i=\min\{2\kappa_i,\gamma_i,2g_i^{-1}-1-\bar{\beta}_{i,i+1}^2\}>0$ and $\varepsilon_i=0.5\vartheta_i^2+0.5\delta_i^2+0.5\overline{T}_i^2$.

Step n Define the variable $z_n=x_n-\overline{\alpha}_{n-1}$, $\varphi_n=[\beta_{n,1},\ldots,\beta_{n,n},\beta_{n,n+1},f_n(\bar{x}_{n+1}^0)]^{\mathrm{T}}$, $\psi_n=[x_1-x_1^0,\ldots,x_n-x_n^0,-u^0,1]^{\mathrm{T}}$ with $u^0=u(0)$, and invoking (6) with $u=x_{n+1}$, it has

$$\dot{z}_n=\dot{x}_n-\dot{\overline{\alpha}}_{n-1}=\varphi_n^{\mathrm{T}}\psi_n+\beta_{n,n+1}u-\dot{\overline{\alpha}}_{n-1} \tag{58}$$

Similarly, the $\|\varphi_n\|$ is also bounded. With the supremum norm theory in mild again, let

$$\vartheta_n=\sup_{t\ge 0}\|\varphi_n\|^2 \tag{59}$$

Then, consider the BLF in (11) again and choose the following Lyapunov function candidate

$$V_n=V_{b_n}+\frac{1}{2\gamma_n}\tilde{\vartheta}_n^2 \tag{60}$$

where γ_n is a designed positive constant, $\tilde{\vartheta}_n=\hat{\vartheta}_n-\vartheta_n$, $\hat{\vartheta}_n$ denotes the estimation of ϑ_n and k_{b_n} is specified later on. In the set $\Omega_{z_n}=\{z_n:|z_n|<k_{b_n}\}$, V_n is continuous. Then, we obtain $\dot{V}_n$ from (11) and (58) as

$$\dot{V}_n=\frac{z_n}{k_{b_n}^2-z_n^2}\left(\varphi_n^{\mathrm{T}}\psi_n+\beta_{n,n+1}u-\dot{\overline{\alpha}}_{n-1}\right)+\frac{\tilde{\vartheta}_n\dot{\hat{\vartheta}}_n}{\gamma_n} \tag{61}$$

Design the actual control u as

$$u=N(\zeta_n)\varpi_n \tag{62}$$

$$\varpi_n=\kappa_nz_n-\dot{\overline{\alpha}}_{n-1}+\frac{1}{2}\frac{z_n}{k_{b_n}^2-z_n^2}$$
$$+\left(\frac{z_n}{k_{b_n}^2-z_n^2}\hat{\vartheta}_n\|\psi_n\|^2\right)/(2\delta_n^2) \tag{63}$$

where κ_n and δ_n are designed positive constants. The parameter updating laws are designed as

$$\dot{\zeta}_n=z_n\varpi_n/(k_{b_n}^2-z_n^2) \tag{64}$$

$$\dot{\hat{\vartheta}}_n=\gamma_n\left[-\hat{\vartheta}_n+\left(\frac{z_n}{k_{b_n}^2-z_n^2}\right)^2\|\psi_n\|^2/(2\delta_n^2)\right] \tag{65}$$

Using (62), (63) and (64), the (61) can be rewritten as

$$\dot{V}_n=-\kappa_n\frac{z_n^2}{k_{b_n}^2-z_n^2}-\frac{1}{2}\left(\frac{z_n}{k_{b_n}^2-z_n^2}\right)^2$$
$$-\left(\frac{z_n}{k_{b_n}^2-z_n^2}\right)^2\hat{\vartheta}_n\|\psi_n\|^2/(2\delta_n^2)$$
$$+\frac{\tilde{\vartheta}_n\dot{\hat{\vartheta}}_n}{\gamma_n}+\frac{z_n}{k_{b_n}^2-z_n^2}\varphi_n^{\mathrm{T}}\psi_n$$
$$+\left[\beta_{n,n+1}N(\zeta_n)+1\right]\dot{\zeta}_n \tag{66}$$

Consider (59) and Assumption 1, using the Youngs inequality, we have

$$\frac{z_n}{k_{b_n}^2-z_n^2}\varphi_n^{\mathrm{T}}\psi_n\le\left(\frac{z_n}{k_{b_n}^2-z_n^2}\right)^2\vartheta_n\|\psi_n\|^2/(2\delta_n^2)+\frac{\delta_n^2}{2} \tag{67}$$

Substituting (67) into (66), we have

$$\dot{V}_n\le-\kappa_n\frac{z_n^2}{k_{b_i}^2-z_n^2}+\frac{\delta_n^2}{2}$$
$$-\left(\frac{z_n}{k_{b_n}^2-z_n^2}\right)^2\tilde{\vartheta}_n\|\psi_n\|^2/(2\delta_n^2)$$
$$+\frac{\tilde{\vartheta}_n\dot{\hat{\vartheta}}_n}{\gamma_n}+\left[\beta_{n,n+1}N(\zeta_n)+1\right]\dot{\zeta}_n \tag{68}$$

According to (65), it is easy to obtain

$$-\left(\frac{z_n}{k_{b_n}^2-z_n^2}\right)^2\tilde{\vartheta}_n\|\psi_n\|^2/(2\delta_n^2)+\frac{\tilde{\vartheta}_n\dot{\hat{\vartheta}}_n}{\gamma_n}$$
$$\le-\frac{1}{2}\tilde{\vartheta}_n^2+\frac{1}{2}\vartheta_n^2 \tag{69}$$

Substituting (69) into (68), we have

$$\dot{V}_n \le -\kappa_n \frac{z_n^2}{k_{\mathrm{b}_n}^2 - z_n^2} - \frac{1}{2}\tilde{\vartheta}_n^2 + \left[\beta_{n,n+1} N(\zeta_n) + 1\right] \dot{\zeta}_n + \frac{1}{2}\vartheta_n^2 + \frac{\delta_n^2}{2} \quad (70)$$

From Lemma 5, it is the fact that $\ln \frac{k_{\mathrm{b}_n}^2}{k_{\mathrm{b}_n}^2 - z_n^2} \le \frac{z_n^2}{k_{\mathrm{b}_n}^2 - z_n^2}$ in the interval $|z_n| < k_{\mathrm{b}_n}$, we have

$$\dot{V}_n \le -\mu_n V_n + \varepsilon_n + \left[\beta_{n,n+1} N(\zeta_n) + 1\right] \dot{\zeta}_n \quad (71)$$

where $\mu_n = \min\{2\kappa_n, \gamma_n\}$ and $\varepsilon_n = 0.5\vartheta_n^2 + 0.5\delta_n^2$.

4.2 Performance analysis of the closed-loop system

The main results of this study are summarized as the following theorem where it is proved that the aforementioned control scheme solves the control problem of the system (1).

Theorem 1 *Consider the closed-loop system consisting of the system* (1) *obeying Assumptions* 1–3*, the virtual controllers* $\alpha_i, i = 1, 2, \ldots, n-1$ *in* (20)*,* (21)*,* (40) *and* (41)*, the actual controller u in* (62) *and* (63)*, and the update laws in* (22)*,* (23)*,* (42)*,* (43)*,* (64) *and* (65) *are constructed. If the following conditions hold:*

(C1) Choosing appropriate design parameters $\kappa_i, \delta_i, \gamma_i, i = 1, \ldots, n$ and $g_i, i = 1, \ldots, n-1$ such that

$$k_{\mathrm{c}_1} > k_{\mathrm{b}_1} + \bar{y}_{\mathrm{r}},\ k_{\mathrm{c}_{i+1}} > k_{\mathrm{b}_{i+1}} + \overline{\alpha}_i^m,\ i = 1, \ldots, n-1 \quad (72)$$

where $\bar{y}_{\mathrm{r}} = \max(|y_{\mathrm{r}}|)$ and $\overline{\alpha}_i^m = \max(|\overline{\alpha}_i|)$.

(C2) the initial state $x_i^0, i = 1, \ldots, n-1$ satisfies:

$$|z_1(0)| < k_{\mathrm{b}_1},\ |z_i(0)| < k_{\mathrm{b}_i},\ i = 2, \ldots, n \quad (73)$$

where $z_1(0) = x_1^0 - y_{\mathrm{r}}(0)$ and $z_i(0) = x_i^0 - \alpha_i(0)$.

Then, the following properties hold:

(i) All the signals in the closed-loop system are bounded;

(ii) The tracking error $z_1 = y - y_{\mathrm{r}}$ is preserved within a specified prescribed performance bound at all times, i.e., $-\rho(t) < z_1 < \rho(t),\ \forall t > 0$.

(iii) the full-state constraints are not violated, i.e., $|x_i| < k_{\mathrm{c}_i}, i = 1, \ldots, n$.

Remark 4 According to (20), (21), (24), (40), (41), (44) and the properties of low-pass filter, we known that $\overline{\alpha}_i, i = 1, \ldots, n-1$ is a continuous function of $\hat{\vartheta}_i, \bar{x}_i, \rho, \dot{\rho}, y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}$ and $\ddot{y}_{\mathrm{r}}$. Because the boundedness of $\hat{\vartheta}_i, \bar{x}_i, \rho, \dot{\rho}, y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}$ and $\ddot{y}_{\mathrm{r}}$, the $\overline{\alpha}_i$ is bounded due to the continuous function properties and moreover assumed to be $|\overline{\alpha}_i| \le \alpha_i^m$ with a positive constant α_i^m. We also can prove the boundedness of $\overline{\alpha}_i$ by employing Lyapunov stability theory later on.

Remark 5 The given constrained constants $k_{\mathrm{c}_i} i = 1, \ldots, n$ need to satisfy feasibility condition (72), we can choose $k_{\mathrm{b}_1} = k_{\mathrm{c}_1} - A_0$, then it is obvious that $k_{\mathrm{c}_1} > k_{\mathrm{b}_1} + \bar{y}_{\mathrm{r}}$ according to the Assumption 3. From the (20), (21), (40) and (41), we known that $\alpha_i, i = 1, \ldots, n-1$ can be derived step-by-step in back-stepping design, and then we also can gain $\overline{\alpha}_i$ step-by-step. Subsequently, similar to [43], using the Matlab routine function $\langle fmincon.m\rangle$, we can compute the maximum solution $k_{\mathrm{b}_i} > 0, i = 2, \ldots, n$ which satisfy inequality $k_{\mathrm{c}_{i+1}} > k_{\mathrm{b}_{i+1}} + \overline{\alpha}_i^m$, $i = 1, \ldots, n-1$ in (72). The computational process is similar to that in [43] and is omitted here due to the limited space.

Proof Multiplying (71) by $\mathrm{e}^{\mu_n t}$ and integrating over $[0, t]$, we have

$$V_n \le \mathrm{c}_n + \mathrm{e}^{-\mu_n t} \int_0^t \left[\beta_{n,n+1} N(\zeta_n) + 1\right] \dot{\zeta}_n \mathrm{e}^{-\mu_n \tau} \mathrm{d}\iota \quad (74)$$

where $\mathrm{c}_n = V_n(0) + \varepsilon_n/\mu_n$. With the aid of Lemma 1, we can known that V_n and ζ_n are bounded, and z_n, $\hat{\vartheta}_n$ and $\int_0^t \left[\beta_{n,n+1} N(\zeta_n) + 1\right] \dot{\zeta}_n \mathrm{d}\iota$ are bounded.

Suppose that there exists unknown positive constant $\bar{z}_n$ such that $|z_n| \le \bar{z}_n$, and defining $\bar{\varepsilon}_{n-1} = \varepsilon_{n-1} + 0.5\bar{\beta}_{n-1,n}^2 \bar{z}_n^2$. Consider step $n-1$ and let in $i = n-1$ in (57), multiplying both sides by $\mathrm{e}^{\mu_{n-1} t}$, we have

$$V_{n-1} \le \mathrm{c}_{n-1} + \mathrm{e}^{-\mu_{n-1} t} \int_0^t \left[\beta_{n-1,n} N(\zeta_{n-1}) + 1\right] \dot{\zeta}_{n-1} \mathrm{e}^{-\mu_{n-1}\tau} \mathrm{d}\iota \quad (75)$$

where $\mathrm{c}_{n-1} = V_{n-1}(0) + \bar{\varepsilon}_{n-1}/\mu_{n-1}$. Thus, we can conclude from (75) that V_{n-1}, ζ_{n-1}, z_{n-1}, $\hat{\vartheta}_{n-1}$ and $\int_0^t \left[\beta_{n-1,n} N(\zeta_{n-1}) + 1\right] \dot{\zeta}_{n-1} \mathrm{d}\iota$ are bounded by using Lemma 1 again. Similarly, repeating $n-1$ times backwardly, we can prove in turn that V_i, ζ_i, z_i, $\hat{\vartheta}_i$ and $\int_0^t \left[\beta_{i,i+1} N(\zeta_i) + 1\right] \dot{\zeta}_i \mathrm{d}\iota$, $i = 1, \ldots, n-2$ are

Table 1 Comparison of proposed controller of existing controller

Method	Controller	Parameters
PP-BLF-based	$\alpha_1 = N(\zeta_1)\varpi_1$ $\varpi_1 = \kappa_1\chi_1 - \dot{y}_r - \rho^{-1}\dot{\rho}z_1 + \frac{\rho^{-1}\chi_1}{1-\chi_1^2}$ $+ \left(\frac{\rho^{-1}\chi_1}{1-\chi_1^2}\hat{\vartheta}_1\|\psi_1\|^2\right)/(2\delta_1^2)$ $u = N(\zeta_2)\varpi_2$ $\varpi_2 = \kappa_2 z_2 - \dot{\bar{\alpha}}_1 + \frac{1}{2}\frac{z_2}{k_{b_2}^2 - z_2^2}$ $+ \left(\frac{z_2}{k_{b_2}^2 - z_2^2}\hat{\vartheta}_2\|\psi_2\|^2\right)/(2\delta_2^2)$ The parameter updating laws $\dot{\hat{\vartheta}}_i$, $\dot{\zeta}_i$ are in (22), (23), (64) and (65) with $n = 2$.	$\kappa_1 = 2.5$ $\kappa_2 = 2$ $g_1 = 0.001$ $\gamma_1 = \gamma_2 = 2$ $\delta_1 = \delta_2 = 1$ $k_{b_2} = 1.047$
Non-BLF/PPC-based	$\alpha_1 = N(\zeta_1)\varpi_1$ $\varpi_1 = \kappa_1 z_1 - \dot{y}_r + \left(\hat{\vartheta}_1\|\psi_1\|^2\right)z_1/(2\delta_1^2)$ $u = N(\zeta_2)\varpi_2$ $\varpi_2 = \kappa_2 z_2 - \dot{\bar{\alpha}}_1 + \left(\hat{\vartheta}_2\|\psi_2\|^2\right)z_2/(2\delta_2^2)$ Similarly to PP-BLF-based method, the parameter updating laws $\dot{\hat{\vartheta}}_i$, $\dot{\zeta}_i$ can be obtained easily.	$\kappa_1 = 35$ $\kappa_2 = 0.5$ $g_1 = 0.001$ $\gamma_1 = 10$ $\gamma_2 = 5$ $\delta_1 = \delta_2 = 1$
BLF-based in Ref. [45]	$\alpha_1 = -\kappa_1 z_1 + \frac{1}{2}\frac{z_1\Phi_1}{k_{b_1}^2 - z_1^2}$ $+ \left(\frac{z_1}{k_{b_1}^2 - z_1^2}\hat{\vartheta}_1\psi_1(x_1)\right)/(2\delta_1^2)$ $u = -\kappa_2 z_2 + \frac{1}{2}\frac{z_2\Phi_2}{k_{b_2}^2 - z_2^2}$ $+ \left(\frac{z_2}{k_{b_2}^2 - z_2^2}\hat{\vartheta}_2\psi_2(\bar{x}_2)\right)/(2\delta_2^2)$ Please refer to [45] for the details of Φ_i, $\psi_i(\bar{x}_i)$, $\dot{\hat{\vartheta}}_i$	$\kappa_1 = 1.3$ $\kappa_2 = 2$ $g_1 = 0.001$ $\gamma_1 = \gamma_2 = 2$ $\delta_1 = 6$ $\delta_2 = 3$ $k_{b_2} = 1.252$

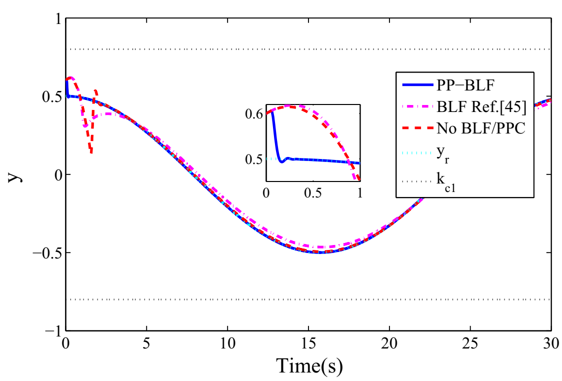


Fig. 1 The output y and reference signal y_r

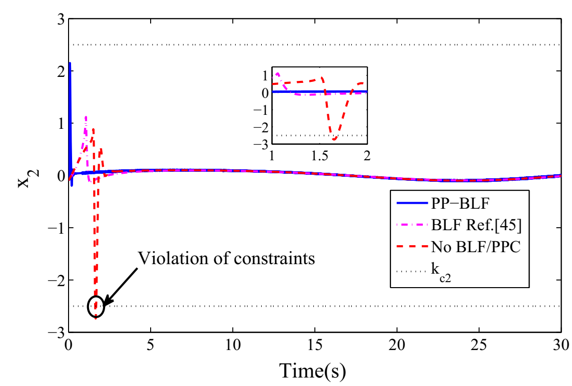


Fig. 2 The state x_2

bounded. From (20), (21), (25), (40), (41), (45), (62) and (63), the virtual controllers $\alpha_i, \overline{\alpha}_i,\ i = 1, \ldots, n-1$ and actual controller u are bounded, namely, we have proved property (i) All the signals in the closed-loop system are bounded.

Let υ_n be the upper bound of $\left|\left[\beta_{n,n+1}N(\zeta_n)+1\right]\dot{\zeta}_n\right|$, and from (71), we have

$$\dot{V}_n \leq -\mu_n V_n + \sigma_n \tag{76}$$

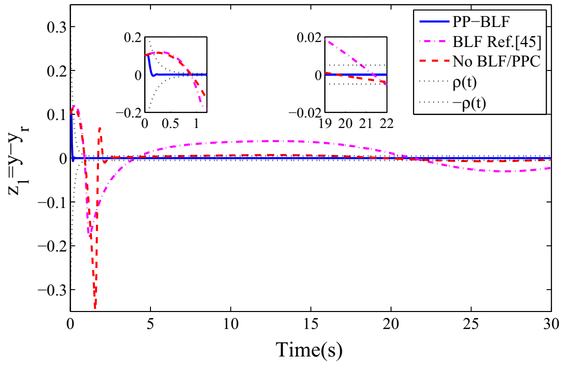


Fig. 3 The tracking error z_1 and PPB

where $\sigma_n = \varepsilon_n + \upsilon_n$. According to Lemma 6, we can obtain that $|z_n| < k_{b_n}$. Similarly, Let υ_i be the upper bound of $\left|\left[\beta_{i,i+1}N(\zeta_i)+1\right]\dot{\zeta}_i\right|$, and form (37) and (57), we have

$$\dot{V}_i \le -\mu_i V_i + \sigma_i, \ i = 1, \ldots, n-1 \tag{77}$$

where $\sigma_i = \varepsilon_i + \upsilon_i + 0.5\bar{\beta}^2_{i,i+1}\bar{z}^2_{i+1}$ with $\bar{z}_{i+1}$ being the upper bound of $|z_{i+1}|$, $i = 1, \ldots, n-1$. According to Lemma 6 again and again, we can obtain that $|z_1/\rho(t)| < 1$ and $|z_i| < k_{b_i}$, $i = 2, \ldots, n$ from (12) and (11), i.e., (ii) The tracking error $z_1 = y - y_r$ is preserved within $-\rho(t) < z_1 < \rho(t)$, $\forall t > 0$. □

Remark 6 The cyclic argument may arise from the Nussbaum-type function for unknown control direction and BLF for constraints, so the Lemmas 1, 2 and 6 are employed to handle the problem of cyclic argument and extend the solution of the closed-loop system to infinity.

Based on the fact that $\rho(t) = (k_{b_1} - \rho_\infty)e^{-\ell t} + \rho_\infty$ is positive and strictly decreasing and $-\rho(t) < z_1 < \rho(t)$, we have $-k_{b_1} < z_1 < k_{b_1}$. Then from $x_1 = z_1 + y_r$ and $|y_r| < A_0$, we have $-k_{b_1} - A_0 < x_1 < k_{b_1} + A_0$. Thus, we can obtain that $-k_{c_1} < x_1 < k_{c_1}$ according to the fact $k_{b_1} = k_{c_1} - A_0$. From the definition of z_i, $i = 2, \ldots, n$, we known that $x_i = z_i + \overline{\alpha}_{i-1}$, then based on $|z_i| < k_{b_i}$, $|\overline{\alpha}_{i-1}| \le \overline{\alpha}^m_{i-1}$ and condition (72): $k_{c_i} > k_{b_i} + \overline{\alpha}^m_{i-1}$, we can obtain $|x_i| < k_{c_i}$, i.e., we have proved property (iii).

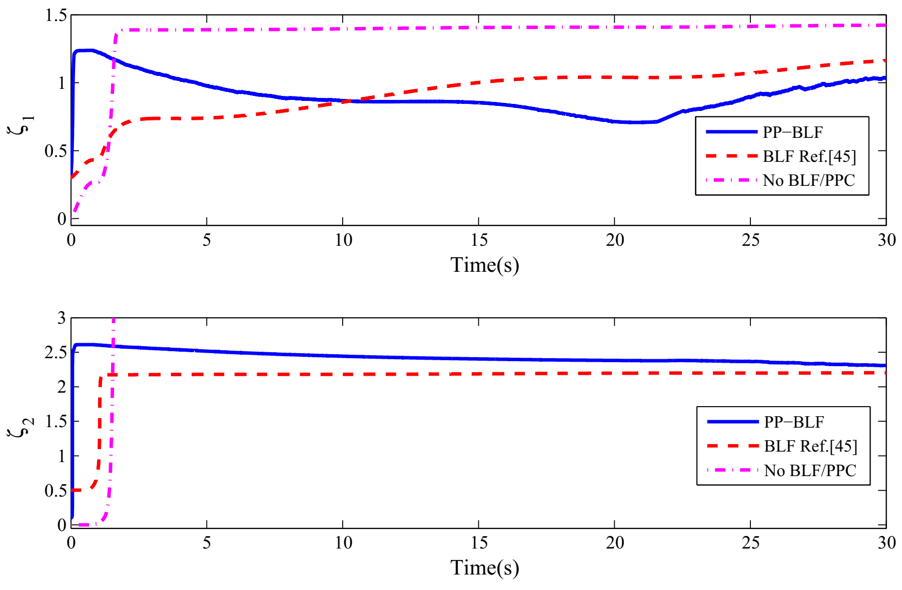
5 Simulation result

To clarify and verify the performance of the proposed approximation-free PP-BLF-based adaptive control scheme, we present some simulation studies in this section. In the simulation, the unknown systems are assumed that (i) **Numerical example** and (ii) **Single-link robot**.

5.1 Numerical example

Consider the following pure-feedback nonlinear systems

Fig. 4 The adaptive parameters ζ_1 and ζ_2



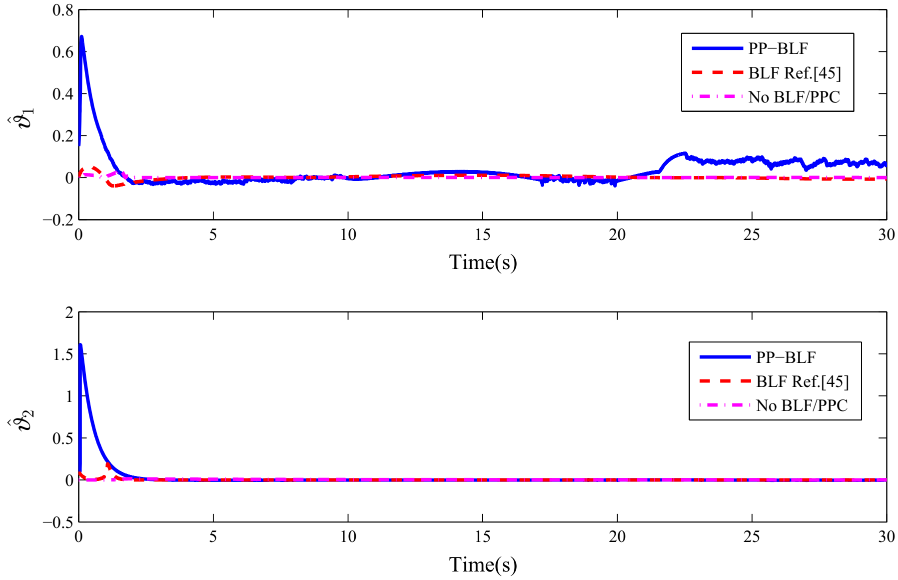


Fig. 5 The adaptive parameters ϑ_1 and ϑ_2

$$\begin{cases} \dot{x}_1 = 0.1x_1^2 - x_2 + 0.2\sin(x_1 x_2) \\ \dot{x}_2 = 0.2x_1x_2 + x_1 + 2u + 0.01u^3 \\ y = x_1 \end{cases} \quad (78)$$

where the states are constrained in $|x_1| < k_{c_1} = 0.8$ and $|x_2| < k_{c_2} = 2.5$. The initial values are $x_1^0 = 0.6, x_2^0 = -0.1, u(0) = 0, \zeta_1(0) = 0.3, \zeta_2(0) = 0.1, \vartheta_1(0) = 0.1$ and $\vartheta_2(0) = 0.1$, the reference signal is $y_r = 0.5\cos(t/5)$ with $A_0 = 0.5$ and $k_{b_1} = k_{c_1} - A_0 = 0.3$, it is easy to know that the $z_1(0)$ satisfies $|z_1(0)| < k_{b_1}$. For the control performance requirements, we allow steady-state error no more than 0.005, minimum speed of convergence as obtained by the exponential e^{-6t}. Then, the performance function can be given as $\rho(t) = (k_{b_1} - 0.005)\mathrm{e}^{-6t} + 0.005$.

In the simulation, to show the superiority, the investigated PP-BLF-based controller is compared with a non-BLF/PPC-based control scheme and a BLF-based control scheme proposed in [45]. The concrete controller and design parameters are shown in Table 1. Similar to [43], the parameter k_{b_2} is computed by utilizing the Matlab function $\langle fmincon.m \rangle$ to satisfy feasibility condition (72) for the proposed PP-BLF-based control scheme and satisfy feasibility condition in [45] *(Please refer to Theorem* 1 *in* [45] *for details)* for BLF-based control scheme.

Remark 7 Compared with results in [45], the proposed PP-BLF-based control scheme can solve the problem of state constraints, prescribed performance tracking and "explosion of complexity" integratedly, and the knowledge of the sign of control gain and the LIP condition of unknown nonlinear functions are not required from the Table 1.

Figures 1, 2, 3, 4, 5 and 6 show the simulation results. Figures 1 and 2 show that the output can accurately track the desired signals. Simultaneously, the PP-BLF-based and BLF-based adaptive control scheme can guarantee that the full-state constraints are not violated. However, the non-BLF/PPC-based control scheme cannot guarantee that full-state constraints are not violated *(State x_2 breaks the constraint in Fig.* 2). Figure 3 shows the system output tracking error and it can be seen from the figure that the tracking error with prescribed performance is achieved all the time by using the proposed PP-BLF-based controller, and the non-BLF/PPC-based and BLF-based in [45] control scheme cannot achieve the prescribed performance all the time. The estimation value of ζ_1, ζ_2, ϑ_1 and ϑ_2 is shown in Figs. 4 and 5, which are easily asymptotic with respect to the zero point. The control signals are show in Fig. 6.

5.2 Single-link robot

The single-link robot dynamic equations are

$$\begin{cases} M\ddot{q} + \frac{1}{2}mgl\sin q = u \\ y = q \end{cases} \quad (79)$$

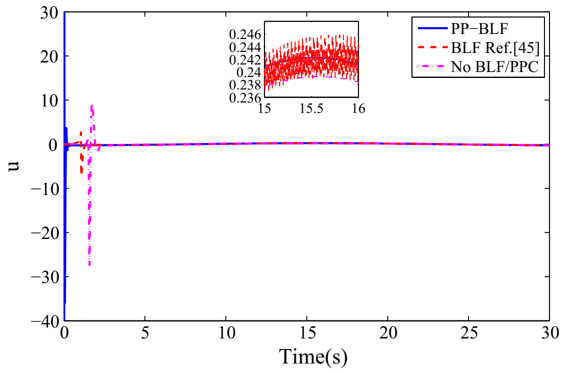


Fig. 6 The control signal u

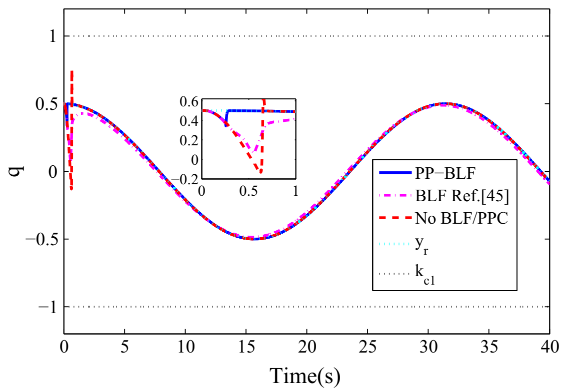


Fig. 7 The output q and reference signal y_{r}

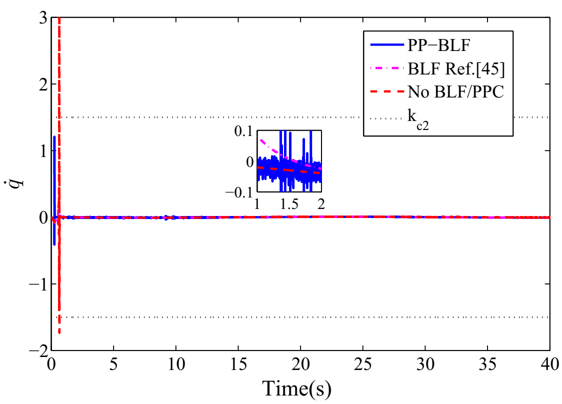


Fig. 8 The state $\dot{q}$

where M is the moment of inertia, q is the angle, u is the input torque, g is the gravity acceleration, m and l are the mass and the length of the link. The robot parameters are $m = l = 1, M = 0.5$ and $g = 0.8$. Let $x_1 = q, x_2 = \dot{q}$, we can gain that $f(x_1, x_2) = x_2,\ f(x_1, x_2, u) = (u - \frac{1}{2}mgl\sin x_1)/M$

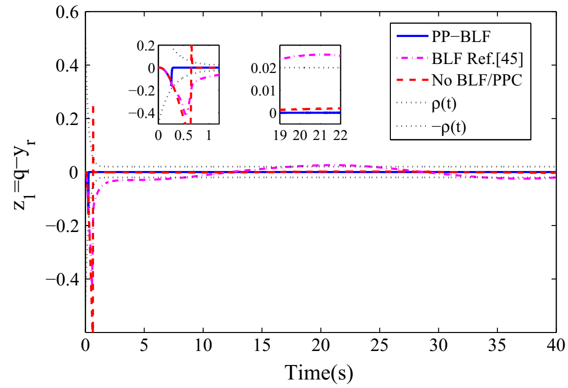


Fig. 9 The tracking error z_1 and PPB

and $y = x_1$. The states are constrained in $|x_1| < k_{c_1} = 1$ and $|x_2| < k_{c_2} = 1.5$. The initial values are $x_1^0 = 0.5, x_2^0 = 0.1, u(0) = 0, \zeta_1(0) = 0.3, \zeta_2(0) = 0.1, \vartheta_1(0) = 0.1$ and $\vartheta_2(0) = 0.1$, the reference signal is $y_{\mathrm{r}} = 0.5\cos(t/5)$ with $A_0 = 0.5$ and $k_{\mathrm{b}_1} = k_{\mathrm{c}_1} - A_0 = 0.5$, it is easy to know that the $z_1(0)$ satisfies $|z_1(0)| < k_{\mathrm{b}_1}$. Then, the PPF is selected as $\rho(t) = (k_{\mathrm{b}_1} - 0.02)\mathrm{e}^{-4t} + 0.02$.

The design parameters are chosen as $\kappa_1 = 2.05$, $\kappa_2 = 5, \gamma_1 = 2, \gamma_1 = 2, \sigma_1 = 1, \sigma_2 = 1, g_1 = 0.001$. Similar to [43], we can obtain $k_{\mathrm{b}_2} = 1.103$ by using Matlab routine. The simulation results are obtained in Figs. 7, 8, 9, 10 and 11. From these figures, we can see that the prescribed performance and state constraints requirements can be guaranteed with the investigated PP-BLF-based adaptive controller. However, the non-BLF/PPC-based and BLF-based controller cannot achieve these two indexes together.

6 Conclusions

In this paper, a novel approximation-free PP-BLF-based adaptive control technique has been proposed for unknown pure-feedback nonlinear systems with full-state constraints. By employing the mean value theorem, the systems are transformed into linear structure form. Then, a novel back-stepping design is developed with the aid of use BLF, PP-BLF, Nussbaum-type function and supremum norm theory to eliminate the difficult problems of unknown dynamics, full-state constraints and prescribed performance require-

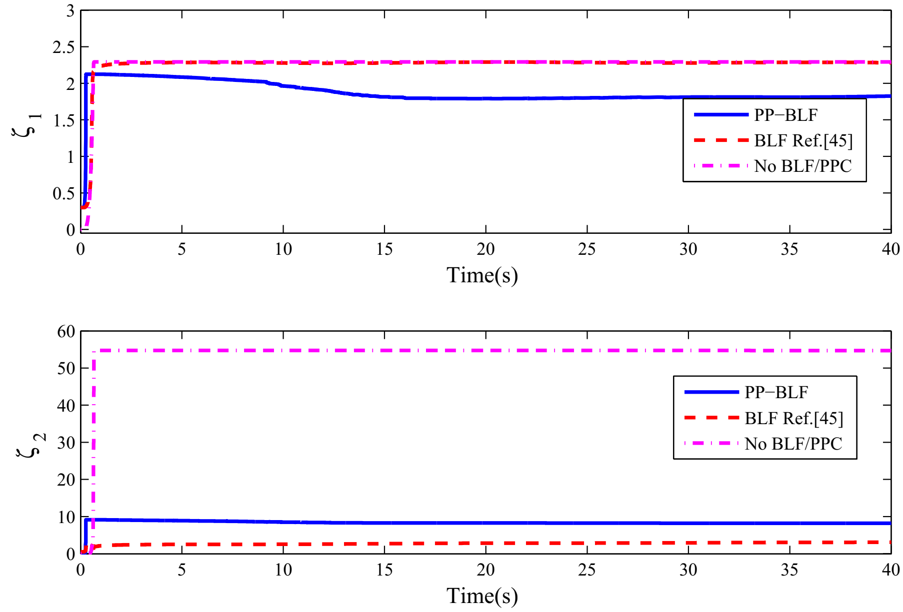


Fig. 10 The adaptive parameters ζ_1 and ζ_2

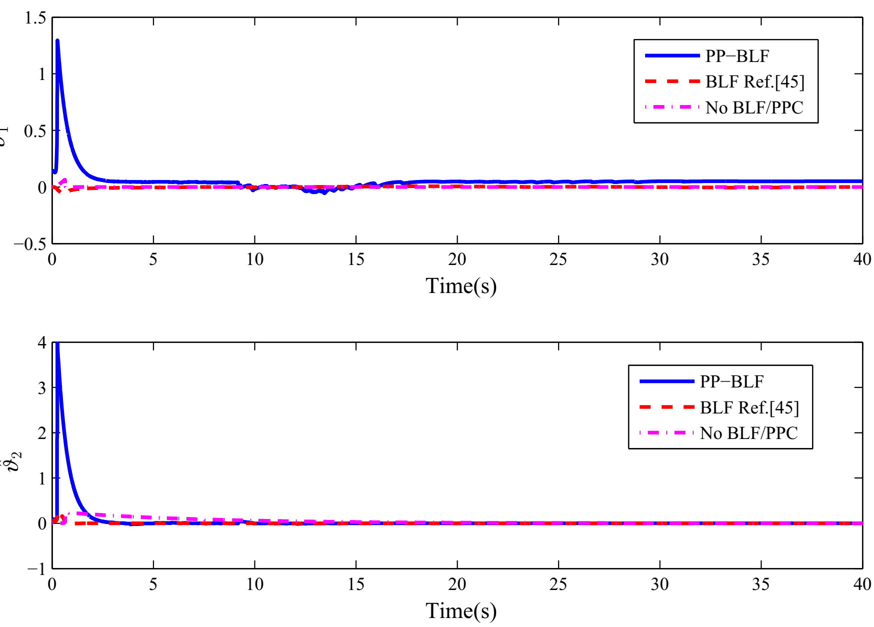


Fig. 11 The adaptive parameters ϑ_1 and ϑ_2

ments. We have proved that all the signals in the closed-loop system are uniformly bounded and the tracking error is preserved within a specified prescribed performance bound without violating the constraints. Finally, simulation results have demonstrated the feasibility and validity of the proposed control scheme.

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