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ORIGINAL PAPER

Rational solutions and lump solutions to a non-isospectral and generalized variable-coefficient Kadomtsev–Petviashvili equation

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Abstract In this work, a non-isospectral and variable-coefficient Kadomtsev–Petviashvili equation is considered using Hirota's bilinear form and a direct assumption with arbitrary functions. Analytical rational solutions in light of positive quadratic functions and lump solutions of the variable-coefficient Kadomtsev–Petviashvili equation are obtained. These lump solutions describe two types of characters by some three-dimensional graphs and contour plots, which contain bright lump wave and bright–dark lump wave. Meanwhile, periodic structure of the lump wave is also shown.

Keywords Rational solutions · Lump solutions · Variable-coefficient Kadomtsev–Petviashvili equation · Hirota's bilinear form

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1 Introduction

Nonlinear integrable equations are often used to describe problems in the fields of mechanics, fluid mechanics, plasma, fiber optic communication and Bose–Einstein condensation [1–5]. Finding exact solutions of nonlinear integrable equations has become an important research topic [6–13], and a series of special methods have been proposed, such as inverse scattering method [14], Darboux transformation method [15], truncated Painlevé expansion method [16], Hirota's bilinear method [17], (G'/G)-expansion method [18], multi-exponential function method [19] and so on.

In recent years, lump-type solutions and lump–stripe mixed solutions have attracted many scholars' attention. They can reveal very interesting dynamical properties [20–32]. The lump solution, also known as the rational solution, is an elastic collision in which the shape, amplitude and velocity remain unchanged after collision with the soliton solution. The mixed solution of lump–stripe mainly considers the interaction between lump solution and other soliton solutions. The research in this field is mainly based on Hirota's bilinear method and symbolic computation. Many nonlinear integrable equations have lump formal solutions and lump–stripe mixed solutions. Some important results have been established. Gilson and Nimmo discussed the lump solution and properties of the B-type Kadomtsev–Petviashvili (KP) equation [33]. Li [34] and his collaborators considered the interaction between lump solution and cosh function. Sun et al. [35] considered the interaction between lump solutions and exponential functions. Ma et al. [36] further considered the interaction between the lump solution and the trigonometric function and the bi-exponential function, which have more abundant dynamical properties and structures. With the development of lump formal solutions and lump–stripe mixed solutions, the dynamical properties of the solutions are discussed, which will help us to understand the physical background behind the nonlinear integrable equations.

The KP equation was first proposed in 1970 by Soviet physicists Kadomtsev and Petviashvili [37], which has been used to describe water waves of long wavelength with weakly nonlinear restoring forces and frequency dispersion, and waves in ferromagnetic media, as well as two-dimensional matter-wave pulses in Bose–Einstein condensates. The KP model has the wave dispersion changing significantly the nonlinear dynamics [38]. Nonlinear stability with infinite space and periodic boundary conditions and dynamics of solitary waves of KP-type equation have been discussed in many literatures [39–41].

In this paper, a $(3+1)$-dimensional generalized KP equation with variable coefficients is presented as follows [42]:

$$\vartheta_3(t)[\vartheta_5(t)u_y + u_{yt}] + u_x[3\vartheta_2(t)u_{xy} + \vartheta_5(t)] \\ +3\vartheta_2(t)u_y u_{xx} + \vartheta_1(t)u_{xxxy} + u_{xt} - \vartheta_4(t)u_{zz} = 0, \tag{1}$$

where $u = u(x, y, z, t)$ is the wave amplitude function, which describes the long water waves and small-amplitude surface waves with weak nonlinearity, weak dispersion and weak perturbation in fluid mechanics. $\vartheta_i(t)(i = 1, 2, 3, 4, 5)$ are arbitrary differentiable functions. $\vartheta_1(t)$ and $\vartheta_2(t)$ represent the dispersion and nonlinearity, respectively. $\vartheta_3(t)$ and $\vartheta_5(t)$ are the functions representing the perturbed effects. $\vartheta_4(t)$ represents the disturbed wave velocities along the z direction, and the subscripts represent the corresponding derivatives. When $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta_4 = 1$ and $\vartheta_5 = 0$, Eq. (1) becomes the $(3+1)$-dimensional generalized KP equation [43]. For Eq. (1), Wronskian and Grammian solutions were obtained with a constraint condition on the variable coefficients [44]. A couple of solutions have been studied by the extended transformed rational function method [45]. Pfaffian solutions were presented by the Pfaffianization procedure of Ohta and Hirota [42].

For the KP-type equations with constant coefficients, rational solutions and lump-type solutions have been obtained in many research works. However, as far as we know, rational solutions and lump-type solutions to the KP equation with the variable coefficients have not been found yet, which will become the primary work of our paper. Section 2 inquires the rational solutions and lump solutions by the Hirota's bilinear form and a direct assumption with arbitrary functions; Sect. 3 describes the spatial structures of the lump waves in figures by choosing some suitable parameters; Sect. 4 gives the conclusion.

2 Rational solutions and lump solutions for Eq. (1)

Through the transformation $u = \frac{2\vartheta_1(t)}{\vartheta_2(t)}[ln\xi(x, y, z, t)]_x$ and the constraint $\vartheta_1(t) = \vartheta_0\vartheta_2(t)e^{-\int \vartheta_5(t)\,\mathrm{d}t}$, Hirota's bilinear form of Eq. (1) can be presented as

$$[D_t D_x + \vartheta_1(t)D_x^3 D_y + \vartheta_3(t)D_t D_y \\ -\vartheta_4(t)D_z^2]\xi \cdot \xi = 0. \tag{2}$$

This is equivalent to:

$$-\xi_t\xi_x + \vartheta_4(t)\xi_z^2 - \vartheta_3(t)\xi_t\xi_y + 3\vartheta_1(t)\xi_{xy}\xi_{xx} \\ -3\vartheta_1(t)\xi_x\xi_{xxy} - \vartheta_1(t)\xi_y\xi_{xxx} + \xi[\xi_{xt} \\ -\vartheta_4(t)\xi_{zz} + \vartheta_3(t)\xi_{yt} + \vartheta_1(t)\xi_{xxxy}] = 0. \tag{3}$$

Considering the rational solutions and lump solutions for Eq. (1), we make the following assumption

$$\zeta = x\alpha_1 + y\alpha_2 + z\alpha_3 + \alpha_4(t), \\ \varsigma = x\alpha_5 + y\alpha_6 + z\alpha_7 + \alpha_8(t), \\ \xi = \zeta^2 + \varsigma^2 + \alpha_9(t), \tag{4}$$

where α_1, α_2, α_3, α_5, α_6 and α_7 are undefined constants. $\alpha_4(t)$, $\alpha_8(t)$, $\alpha_9(t)$ are unknown differentiable function. Compared with those methods in Refs. [25–32], our assumption can be used for solving variable-coefficient nonlinear integrable equation. Substituting Eq. (4) into Eq. (3) with the Mathematica software, we have following results

$$
\begin{aligned}
(I):\alpha_2 &= \alpha_6 = 0, \alpha_8(t) = \eta_1 \\
&+ \int_1^t \frac{\alpha_3^2\alpha_1^2\vartheta_4(t) + \alpha_3^2\alpha_5^2\vartheta_4(t) - \alpha_1^3\alpha_4'(t)}{\alpha_1^2\alpha_5}\,\mathrm{d}t, \\
\alpha_7 &= \frac{\alpha_3\alpha_5}{\alpha_1}, \alpha_9(t) = \eta_2 \\
&+ \int_1^t 2[-\alpha_3^2\alpha_1\eta_1\vartheta_4(t) + \alpha_1^2\eta_1\alpha_4'(t) \\
&+ [\alpha_1^2\alpha_4'(t) - \alpha_3^2\alpha_1\vartheta_4(t)] \\
&\times \int_1^t \frac{\alpha_3^2\alpha_1^2\vartheta_4(t) + \alpha_3^2\alpha_5^2\vartheta_4(t) - \alpha_1^3\alpha_4'(t)}{\alpha_1^2\alpha_5}\,\mathrm{d}t \\
&+ \alpha_3^2\alpha_5\alpha_4(t)\vartheta_4(t) - \alpha_5\alpha_1\alpha_4(t)\alpha_4'(t)]/(\alpha_1\alpha_5)\,\mathrm{d}t
\end{aligned} \tag{5}
$$

where $\alpha_1\alpha_5 \neq 0$, η_1 and η_2 are integral constants. Substituting Eqs. (4), (5) and the constraint $\vartheta_1(t) = \vartheta_0\vartheta_2(t)e^{-\int\vartheta_5(t)\,\mathrm{d}t}$ into the transformation $u = \frac{2\,\vartheta_1(t)}{\vartheta_2(t)}\,[ln\xi(x, y, z, t)]_x$, the rational solutions for Eq. (1) can be expressed as follows:

$$
\begin{aligned}
u^{(I)} = \Bigg[&2\vartheta_0 e^{-\int\vartheta_5(t)\,\mathrm{d}t} \\
&\Bigg[2\alpha_5\Bigg[\eta_1 + \int_1^t \frac{\frac{\alpha_3^2(\alpha_1^2+\alpha_5^2)\vartheta_4(t)}{\alpha_1^2} - \alpha_1\alpha_4'(t)}{\alpha_5}\,\mathrm{d}t \\
&+\alpha_5\left(x + \frac{\alpha_3 z}{\alpha_1}\right)\Bigg] + 2\alpha_1(\alpha_4(t) + \alpha_1 x + \alpha_3 z)\Bigg]\Bigg]_x \\
&/[\eta_2 + \int_1^t \Bigg[2\left(\alpha_1\alpha_4'(t) - \alpha_3^2\vartheta_4(t)\right)[\alpha_1[\eta_1 \\
&+ \int_1^t \frac{\frac{\alpha_3^2(\alpha_1^2+\alpha_5^2)\vartheta_4(t)}{\alpha_1^2} - \alpha_1\alpha_4'(t)}{\alpha_5}\,\mathrm{d}t\Bigg] \\
&-\alpha_5\alpha_4(t)]]/(\alpha_1\alpha_5)\,\mathrm{d}t + \Bigg[\eta_1 \\
&+ \int_1^t \frac{\frac{\alpha_3^2(\alpha_1^2+\alpha_5^2)\vartheta_4(t)}{\alpha_1^2} - \alpha_1\alpha_4'(t)}{\alpha_5}\,\mathrm{d}t \\
&+\alpha_5\left(x + \frac{\alpha_3 z}{\alpha_1}\right)\Bigg]^2 + (\alpha_4(t) + \alpha_1 x + \alpha_3 z)^2\Bigg],
\end{aligned} \tag{6}
$$

where ϑ_0 is an arbitrary nonzero constant. In addition to the constraint (*I*), other parameters are arbitrary.

To describe the resulting rational solutions in Eq. (6), we select the two particular values for the parameters:

$$
\begin{aligned}
&\alpha_4(t) = \sin t, \quad \vartheta_3(t) = 1, \quad \vartheta_4(t) = t, \quad \vartheta_5(t) = 0, \\
&\alpha_1 = 1, \alpha_3 = 2, \quad \alpha_5 = -3, \quad \eta_1 = \eta_2 = \vartheta_0 = 1,
\end{aligned} \tag{7}
$$

and

$$
\begin{aligned}
&\alpha_4(t) = \sin t, \quad \vartheta_3(t) = 1, \quad \vartheta_4(t) = t, \\
&\vartheta_5(t) = 0, \quad \alpha_1 = -5, \\
&\alpha_3 = 2, \quad \alpha_5 = 3, \quad \eta_1 = \eta_2 = \vartheta_0 = 1.
\end{aligned} \tag{8}
$$

Substituting Eq. (7) and Eq. (8) into solution $u^{(I)}$, three-dimensional graphs and contour graphs at $z = -5$ and $x = -5$ are shown in Figs. 1 and 2, respectively.

Next, we will present the lump solutions for Eq. (1) and discuss their spatial structures as follows

$$
\begin{aligned}
(II):\vartheta_4(t) = {}&[3(\alpha_1^2 + \alpha_5^2)(\alpha_1\alpha_2 + \alpha_5\alpha_6)\vartheta_1(t) \\
&\Big[\alpha_1^2 + \alpha_5^2 + \left(\alpha_2^2 + \alpha_6^2\right)\vartheta_3(t)^2 \\
&+ 2(\alpha_1\alpha_2 + \alpha_5\alpha_6)\vartheta_3(t)\Big]\Big] \\
&/[\alpha_9[\alpha_7(\alpha_1 + \alpha_2\vartheta_3(t)) - \alpha_3[\alpha_5 \\
&+ \alpha_6\vartheta_3(t)]]^2], \alpha_9(t) = \alpha_9, \\
\alpha_4(t) = {}&\eta_3 + \int_1^t [3\left(\alpha_1^2 + \alpha_5^2\right)(\alpha_1\alpha_2 + \alpha_5\alpha_6) \\
&\vartheta_1(t)\Big[2\alpha_3\alpha_5\alpha_7 \\
&+ \alpha_1\left(\alpha_3^2 - \alpha_7^2\right) + \Big[2\alpha_3\alpha_6\alpha_7 \\
&+ \alpha_2\left(\alpha_3^2 - \alpha_7^2\right)\Big]\vartheta_3(t)\Big]\Big] \\
&/[\alpha_9[\alpha_7(\alpha_1 + \alpha_2\vartheta_3(t)) \\
&- \alpha_3(\alpha_5 + \alpha_6\vartheta_3(t))]^2]\,\mathrm{d}t, \\
\alpha_8(t) = {}&\eta_4 + \int_1^t [3\left(\alpha_1^2 + \alpha_5^2\right)(\alpha_1\alpha_2 + \alpha_5\alpha_6) \\
&\vartheta_1(t)[\alpha_3^2[-[\alpha_5 \\
&+ \alpha_6\vartheta_3(t)]] + 2\alpha_7\alpha_3(\alpha_1 + \alpha_2\vartheta_3(t)) \\
&+ \alpha_7^2(\alpha_5 + \alpha_6\vartheta_3(t))]] \\
&/[\alpha_9[\alpha_7(\alpha_1 + \alpha_2\vartheta_3(t)) \\
&- \alpha_3(\alpha_5 + \alpha_6\vartheta_3(t))]^2]\,\mathrm{d}t
\end{aligned} \tag{9}
$$

where $\alpha_9[\alpha_7(\alpha_1 + \alpha_2\vartheta_3(t)) - \alpha_3(\alpha_5 + \alpha_6\vartheta_3(t))] \neq 0$, η_3 and η_4 are integral constants. Substituting Eq. (4), Eq. (7) and the constraint $\vartheta_1(t) = \vartheta_0\vartheta_2(t)e^{-\int\vartheta_5(t)\,\mathrm{d}t}$ into the transformation $u = \frac{2\,\vartheta_1(t)}{\vartheta_2(t)}\,[ln\xi(x, y, z, t)]_x$, the lump solutions for Eq. (1) can be expressed as follows:

$$
\begin{aligned}
u^{(II)} = \Big[&2\vartheta_0 e^{-\int\vartheta_5(t)\,\mathrm{d}t}[2\alpha_1(\alpha_4(t) + \alpha_1 x \\
&+ \alpha_2 y + \alpha_3 z) + 2\alpha_5(\alpha_8(t) + \alpha_5 x \\
&+ \alpha_6 y + \alpha_7 z)]]/[\alpha_9 + (\alpha_4(t) + \alpha_1 x \\
&+ \alpha_2 y + \alpha_3 z)^2 \\
&+ (\alpha_8(t) + \alpha_5 x + \alpha_6 y + \alpha_7 z)^2\Big],
\end{aligned} \tag{10}
$$

where ϑ_0 is an arbitrary nonzero constant. In addition to the constraint (II), other parameters are arbitrary.

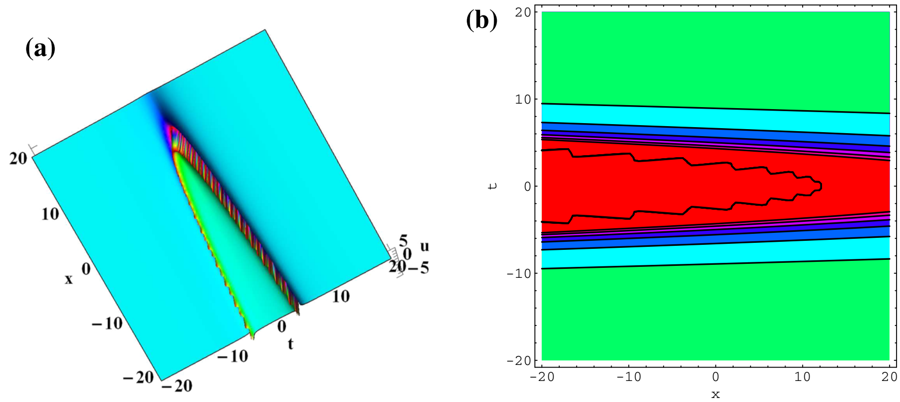

Fig. 1 Rational solution $u^{(I)}$ when $z = -5$ **a** three-dimensional graph, **b** contour graph

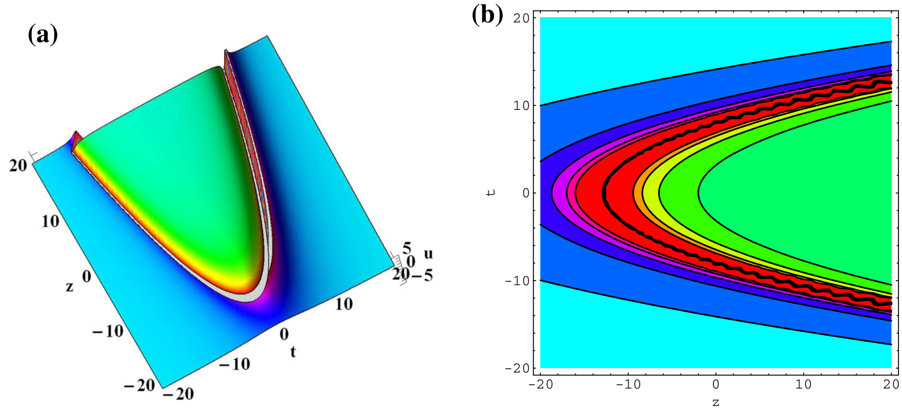

Fig. 2 Rational solution $u^{(I)}$ when $x = -5$ **a** three-dimensional graph, **b** contour graph

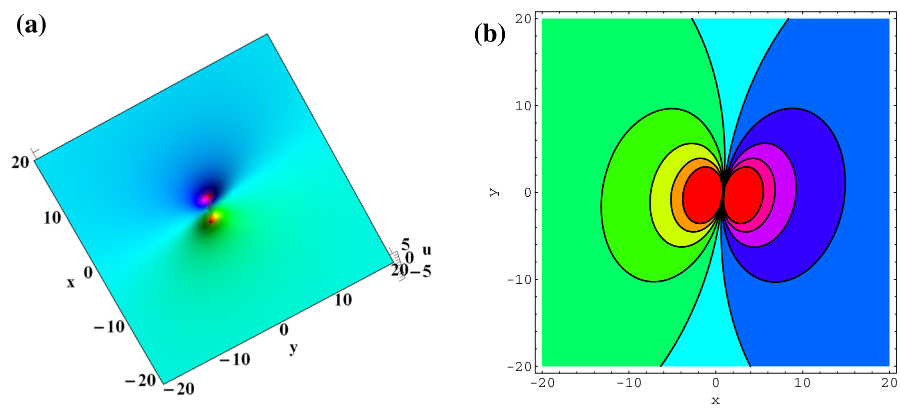

Fig. 3 Spatial structure of the bright–dark lump solution $u^{(II)}$ when $z = 0$ **a** three-dimensional graph, **b** contour plot

3 Spatial structures of lump solutions in Eq. (10)

To demonstrate the spatial structures of lump solutions in Eq. (10), we select the three particular values for the parameters:

$$\vartheta_2(t) = t, \quad \vartheta_3(t) = 1, \quad \vartheta_5(t) = t = 0, \quad \alpha_2 = \alpha_9 = 2, \\ \alpha_1 = \alpha_3 = \alpha_6 = \alpha_7 = \eta_3 = \eta_4 = \vartheta_0 = 1, \quad \alpha_5 = -3, \tag{11}$$

$$\vartheta_2(t) = t, \quad \vartheta_3(t) = 1, \quad \vartheta_5(t) = x = 0, \quad \alpha_2 = \alpha_9 = 2, \\ \alpha_1 = \alpha_3 = \alpha_6 = \alpha_7 = \eta_3 = \eta_4 = \vartheta_0 = 1, \quad \alpha_5 = -3, \tag{12}$$

and

$$\vartheta_2(t) = \sin t, \quad \vartheta_3(t) = 1, \quad \vartheta_5(t) = y = 0, \\ \alpha_2 = \alpha_9 = 2, \alpha_1 = \alpha_3 = \alpha_6 = \alpha_7 = \eta_3 \\ = \eta_4 = \vartheta_0 = 1, \alpha_5 = -3. \tag{13}$$

Substituting Eq. (11) into solution $u^{(II)}$, three-dimensional graphs and contour plots at $z = 0$ and $x = 0$ are shown in Figs. 3 and 4, respectively. Substituting Eq. (12) and Eq. (13) into solution $u^{(II)}$, three-dimensional graphs and contour plots at $y = -5; 0; 5$

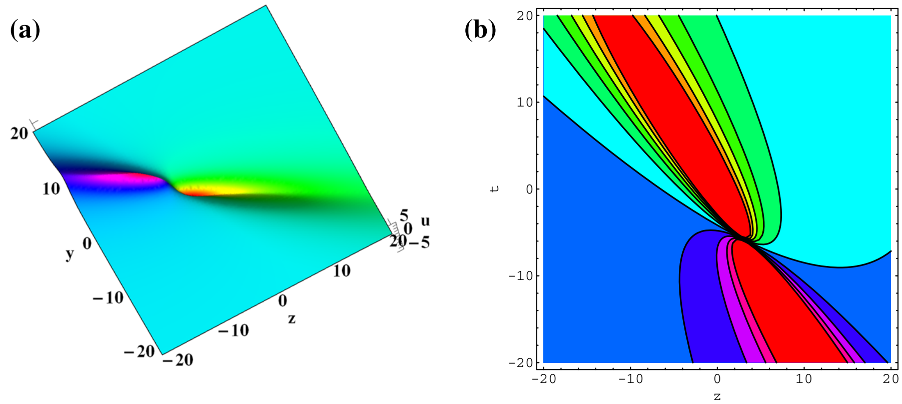

Fig. 4 Spatial structure of the bright–dark lump solution $u^{(II)}$ when $x = 0$ **a** three-dimensional graph, **b** contour plot

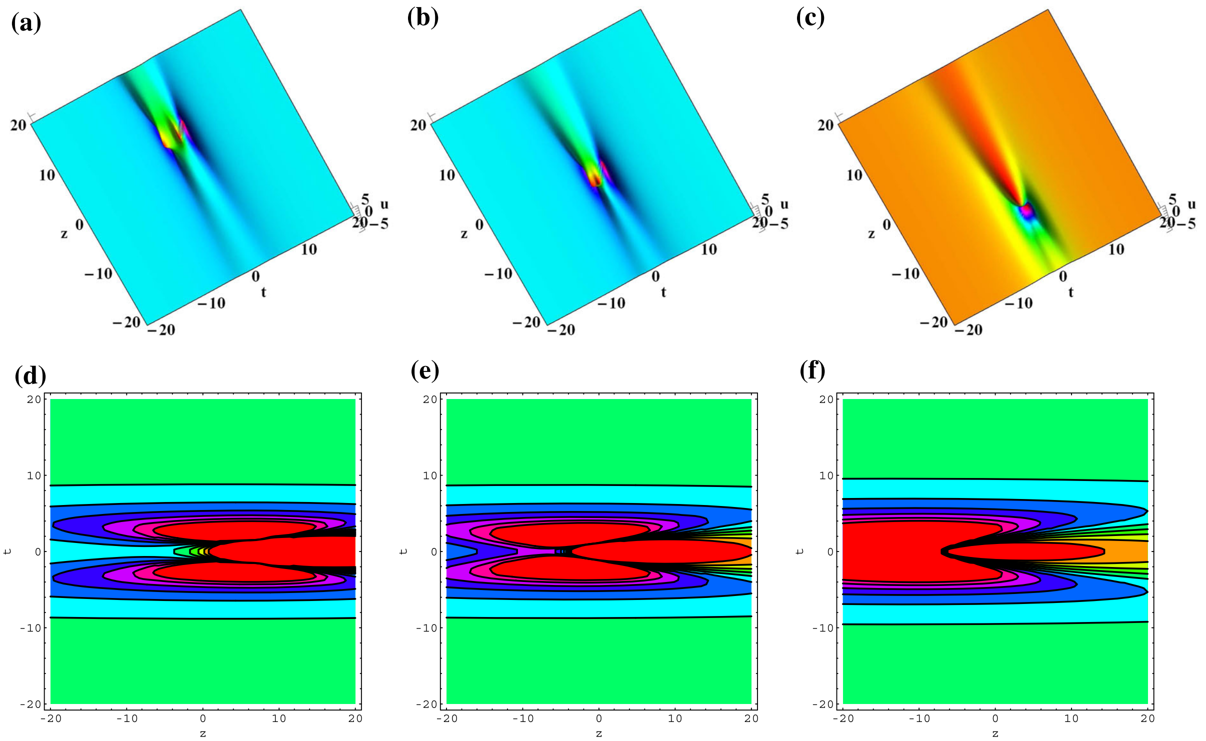

Fig. 5 Spatial structure of the bright lump solution $u^{(II)}$ when $y = -5$ (**a**, **d**), $y = 0$ (**b**, **e**) and $y = 5$ (**c**, **f**)

and $x = -5; 0; 5$ are presented in Figs. 5 and 6, respectively.

Figures 3 and 4 demonstrate the spatial structure of the bright–dark lump solution because the height of the peak is competent for the depth of the valley bottom, which contains one peak and one valley. Their peak and valley are meristic. Figure 5 shows the spatial structure of the bright lump solution, which contains one peak and two valleys. Figure 6 shows the periodic structure of the lump solution.

4 Conclusion

In this work, by applying Hirota's bilinear form and a direct assumption with arbitrary functions, we have obtained rational solutions and lump solutions to the

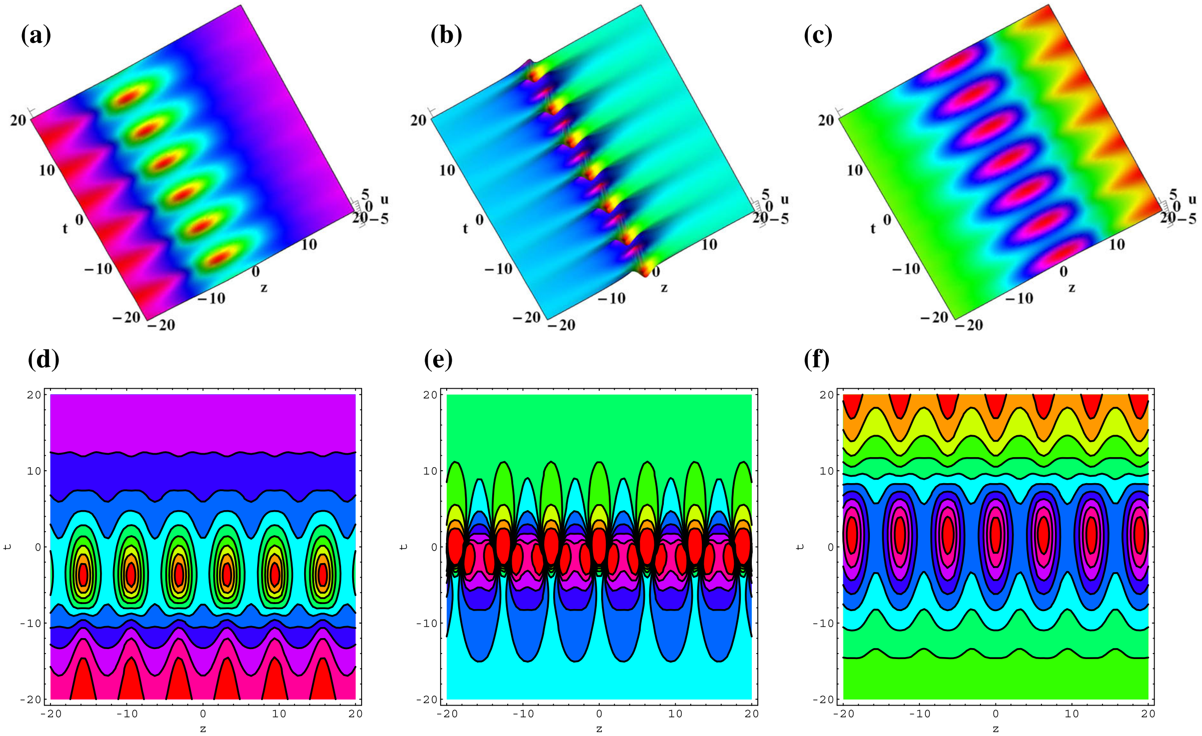

Fig. 6 Periodic structure of the lump solution $u^{(11)}$ when $x = -5$ (**a**, **d**), $x = 0$ (**b**, **e**) and $x = 5$ (**c**, **f**)

(3+1)-dimensional generalized KP equation with variable coefficients. Spatial structures of the bright lump solution and the bright–dark lump solution are shown by some three-dimensional graphs and contour plots.

Compliance with ethical standards

Conflict of interests The authors declare that there are no conflict of interests regarding the publication of this article.

Ethical standards The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

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