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Event-triggered finite-time resilient control for switched systems: an observer-based approach and its applications to a boost converter circuit system model

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Abstract Under an event-triggered communication scheme (ETCS), this note focuses on the observer-based finite-time resilient control problem for a class of switched systems. Different from the existing finite-time problems, not only the problem of finite-time boundedness (FTBs) but also the problem of input-output finite-time stability (IO-FTSy) are considered in this paper. To effectively use the network resources, an ETCS is formulated for switched systems. Considering that not all the states could be measured, thus an event-triggered observer is constructed, and then, an observer-based resilient controller is devised, which robustly stabilizes the given systems in the meaning of finite-time control. Based on time-delay method and Lyapunov functional approach, interesting results are derived to verify the properties of the FTBs and the IO-FTSy of the event-triggered (ET) closed-loop error switched systems. All the matrix inequalities can be converted to linear matrix inequalities (LMIs) so as to simultaneously obtain the controller gain and observer gain. Finally, the applicability of the proposed control scheme is verified via a boost converter circuit system.

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1 Introduction

Switched systems belong to the category of hybrid systems, which consists of a limited number of subsystems and a switching law orchestrating the switching among the subsystems. Over the past years, switched systems have already obtained many attentions because of their powerful potentials in process control, electricity system , aircraft control systems, automobile controlling systems, etc. From a theoretical point of view, different properties of switched systems are characterized [1–5]. Unfortunately, the research for switched systems under communication networks is insufficient. With the rapid development of computer and networked technologies, data transmission via communication networks has received considerable research attentions [6–8]. Though communication network scheme allows for reduced wiring as well as for lower installation cost, it often be accompanied by some problems, such as network congestion and communication packet losses, which are quite challenging issues.

In order to deal with those challenging issues in network environment, the simplest approach is periodic sampling, which is termed as time-triggered scheme, and many results have been established for switched systems [9–11]. However, time-triggered strategy may lead to resource waste since some redundant sampled data are sent even when those data do not significantly change the performance of the research object. To overcome this obstacle in general time-triggered communication, one can prefer event-triggered technique, which allows sample points satisfying the predefined event-triggered condition (ETC) to be transmitted. Since the event-triggered control scheme can save precious communication resources, many researchers have paid attention to it [12–14]. It is well known that time delay is the unavoidable feature in practice, and the results of time-delay systems have been widely reported in the past several years [15–18]. With that in mind, time-delay method is generally used to mode an ETC system as a time-delay system.

It is noted that the controlled systems in mentioned above papers require that the total states of systems are always available. In fact, this requirement is not realistic for many real-world systems; thus, control based on state feedback cannot be realized. Instead, observer-based control can be used to implement a good system performance when some states of systems are not usually measurable. Therefore, the observer-based control plays an active role in the field of control. Especially, the observer-based event-triggered control technique is widely adopted to estimate all the states which cannot always be measured in practice. For various dynamic systems, some results on observer-based event-triggered control are presented in [19–21]. Compared with the existing analysis on a single system, the analysis of switched system's observer-based event-triggered control problem is more complicated due to its hybrid nature. In [22], assume that the observer is designed in advance, and then, the event-triggered control issue is studied for switched linear systems. For the switched system in [23], the problem of event-triggered finite-time stabilization is studied by using an observer-based control approach, where the controller gain and the observer gain cannot be obtained at the same time due to the nonlinear terms. Moreover, the authors in [22,23] do not consider the uncertainty in the actuator. It is worth mentioning that the designed controller under ETCS may be sensitive in practice due to the imprecision inherent in networked communication. Therefore, how to design an appropriate ETCS to save the network resources and an observer-based robust controller to avoid the sensitivity issue are one of the most interesting and serious challenges for switched systems, which is one motivation of this study.

As we know, Lyapunov asymptotic stability has been widely studied in the majority of the existing studies. However, under some circumstances such as chemical reaction process, missile systems, and certain aircraft maneuvers, it is required that the system state x_t should not exceed some bounds in the predefined finite-time span. Noticing this, the definition of FTBs comes out [24], which requires that the response of $x_t^T Rx_t$ falls into the predefined bounded domain during a specified time span, where R is a positive definite matrix. FTB has been proved to be a meaningful research topic and found wide applications in practice [25–29]. As sometimes the output y_t is also required to be restrained within a bound, the concept of IO-FTSy is provided in [30]. By IO-FTSy, we mean that for given input signals and a positive definite matrix Γ, the response of $y_t^T \Gamma y_t$ lies in an assigned threshold during a finite-time span. Afterward, some results on IO-FTS are presented for various systems [31–34]. It is noted that in the above studies, the results are obtained in nonevent-triggered context. Nevertheless, the redundant signals in nonevent-triggered context will increase the load on the communication, so the research on IO-FTS needs to be done based on an ETCS. This also motivates us to perform the present study.

In view of the above-mentioned facts, we aim at investigating to solve the observer-based finite-time resilient control problem for switched systems with ETCS. Compared to the existing works, there are some main contributions in this paper as follows:

- **(i)** Although the research on switched systems is very rich, most studies do not consider the waste of communication resources, such as high energy consumption and equipment abrading. Here, an appropriate ETCS is proposed for switched systems to save the cost of communications.
- **(ii)** Different from the finite-time control in [25–32] that only makes state/output bounded in a specified time interval, a new finite-time control approach is developed, which requires that during a finite-time span, the response of $x_t^T Rx_t$ falls into the predefined bounded domain, i.e., FTBs; the response of $y_t^T \Gamma y_t$ under zero initial condition also lies in a limited area, i.e., IO-FTSy.
- **(iii)** The ET closed-loop error switched system is constructed by using the time-delay system

method. Further, when considering the unmeasured states and the uncertainty in controller, the observer-based resilient control approach is applied to resolve the FTB and IO-FTS problems for the ET closed-loop error switched system.

- (**iv**) In the field of observer-based controller design, when seeking the controller gain and observer gain, the following situation often occurs: the controller gain or observer gain should be given beforehand [22,23,35]. Different from the above-mentioned calculation methods, the controller gain and the observer gain here can be acquired simultaneously by solving strict LMIs.
- (**v**) Under the ETCS, the observer-based control technique is used to solve the FTB and IO-FTS problems of the boost converter circuit systems, which has been shown to be more effective.

1.1 Notations

Throughout this paper, $S - Q < 0 (\leq 0)$ means that $S - Q$ is a negative definite (negative semidefinite) matrix. S^T indicates the transpose of matrix S. We use $*$, $\lambda_{\min}(\cdot)$, and $\lambda_{\max}(\cdot)$ to, respectively, denote the symmetric terms of matrices, the minimum and the maximum eigenvalues of a matrix $(\cdot)$. diag$\{X, Y\}$ stands the block-diagonal matrix of X and Y. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The symbol $\mathcal{L}_{p,[t_0,t_0+T]}$ represents the space of vector-valued signals $\varepsilon_{(\cdot)}$, and the norm is defined by $\varepsilon_{(\cdot)} \in \mathcal{L}_{p,[t_0,t_0+T]} \iff (\int_{t_0}^{t_0+T} |\varepsilon_s|^p \mathrm{d}s)^{\frac{1}{p}} < \infty$.

2 Problem formulation and preliminaries

Consider the following switched system:

$$\begin{cases} \dot{x}_t = A_{\sigma_t} x_t + B_{\sigma_t} u_t + D_{\sigma_t} \omega_t, \\ y_t = E x_t, \end{cases} \tag{1}$$

where $x_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^q$, $\omega_t \in \mathbb{R}^m$, and $y_t \in \mathbb{R}^l$ are the state vector, controlled input, disturbance input, and measured output, respectively. $\sigma_t : [0, \infty) \mapsto \mathcal{N} = \{1, 2, \ldots, N\}$ is a piecewise constant function which represents the switching signal. Further, the switching sequence is represented by

$$\{(0, \sigma_0), (t_1, \sigma_{t_1}), \ldots, (t_k, \sigma_{t_k}), \ldots | \sigma_{t_k} \in \mathcal{N},\ k \in \mathbb{N}\},$$

where the switching points satisfy $0 < t_1 < \cdots < t_k < \ldots$. The σ_{t_k}th subsystem works when $t \in [t_k, t_{k+1})$.

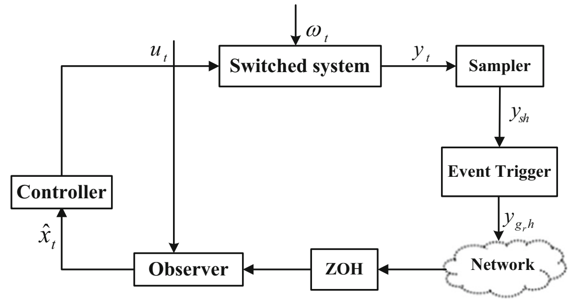

Fig. 1 The switched control system with ETCS

For $\sigma_t = i$, A_i, B_i, D_i, and E are real known constant matrices.

In order to enhance the utilization ratio of the precious communication resources, we will design a suitable ETCS to decide whether the sampling point is transmitted or not. The diagram of the observer-based controller with an ETCS for switched systems is shown in the following figure.

In Fig. 1, at a constant period h, the measured outputs are sampled and the set of sampled instants is given by $\mathcal{T} = \{sh | s \in \mathbb{N}\}$. The set of the event generator release sample instants of the measured output is represented by $\mathcal{T}_1 = \{g_r h | g_r \in \mathbb{N}\} \subseteq \mathcal{T}$. Here, the event trigger is a very critical component in the control loop, since it determines whether a newly sampled signal y_{sh} is transmitted or not. The ETC is given as follows:

$$g_{r+1}h = g_r h + \min_{\nu \in \mathbb{N}} \Big\{ \nu h \mid e^T_{(g_r+\nu)h} \Phi_i e_{(g_r+\nu)h} \geq \delta_i y^T_{(g_r+\nu)h} \Phi_i y_{(g_r+\nu)h} \Big\}, \tag{2}$$

where $e_{(g_r+\nu)h} = y_{(g_r+\nu)h} - y_{g_r h}$, $\delta_i > 0$ is the coefficient of the threshold. $\{(g_r + \nu)h\}$ is the set of real-time sampling instants between two successive event release instants, and $\Phi_i > 0$ is a weighting matrix to be determined.

Remark 1 It should be pointed that the sampling process of the measured output y_t can be described as follows:

- Firstly, the output signal y_t is sampled at a constant period h. The set of sampled output signals is given by $\mathcal{T} = \{y_{sh} | s \in \mathbb{N}\}$.
- Secondly, the event trigger is applied to determine whether the sampled data y_{sh} should be transmitted or not. The set of transmitted instants can be represented by $\mathcal{T}_1 = \{g_r h | g_r \in \mathbb{N}\} \subseteq \mathcal{T}$.

Since the event trigger is based on the periodic sampling at a constant rate, it is clear that the minimum event-triggered interval is the constant sampling interval h. Therefore, there is no Zeno behavior.

Remark 2 The ZOH is used to hold the input signal of the observer when there is no latest transmitted data arrived at the observer side. However, once a transmitted datum arrives at the ZOH, the ZOH immediately updates its store and actuates the observer.

Remark 3 As we know, the ETC plays an important role, which can judge whether the newly sampled signal y_{sh} is transmitted or not. From ETC in (2), we say that the number of sampling points will be affected by the triggered thresholds $\delta_i > 0$, which will be demonstrated via a boost converter circuit system in Sect. 4.

For the sake of simplicity, let $s_{r,\nu} \triangleq g_r + \nu$, we have $[g_r h, g_{r+1}h) = \cup_{\nu=0}^{l_r}[s_{r,\nu}h, s_{r,\nu+1}h)$, where $l_r = g_{r+1} - g_r - 1$. Define a piecewise function $d_t = t - s_{r,\nu}h$, $t \in [s_{r,\nu}h, s_{r,\nu+1}h)$, $0 \leq d_t \leq h$. Then, it is not difficult to find

$$\begin{aligned} \bar{y}_t &\triangleq y_{g_r h} = y_{s_{r,\nu}h} - e_{s_{r,\nu}h} = y_{t-d_t} - e_{s_{r,\nu}h}, \\ &\forall t \in [s_{r,\nu}h, s_{r,\nu+1}h). \end{aligned} \tag{3}$$

Next, we focus on solving the problems of the waste of communication resource and the unmeasured state x_t. Based on the ETCS and the measurable output y_t, we shall consider the following Luenberger observer for system (1):

$$\begin{cases} \dot{\hat{x}}_t = A_i \hat{x}_t + B_i u_t + L_i(\bar{y}_t - \hat{y}_t), \\ \hat{y}_t = E\hat{x}_t, \end{cases} \tag{4}$$

where $\sigma(t) = i \in \mathcal{N}$, $\hat{x}_t \in \mathbb{R}^n$ is the estimated state and $\hat{y}_t \in \mathbb{R}^p$ is the estimated output. $\bar{y}_t = y_{g_r h} \in \mathbb{R}^p$ is the system output measurement that meets ETC. L_{σ_t} is the observer gain to be designed.

In order to overcome the uncertainty of the actuator, we shall design a resilient controller using $\hat{x}_t$ to guarantee that the closed-loop control systems are both FTB and IO-FTS as follows:

$$u_t = \bar{K}_i(t)\hat{x}_t = (K_i + \Delta K_i(t))\hat{x}_t, \tag{5}$$

where $\triangle K_i(t)$ represents controller gain variation satisfying

$$\triangle K_i(t) = M_i F_i(t) N_i, \tag{6}$$

M_i and N_i are given real matrices. $F_i(t)$ is an unknown time-varying matrix function which meets

$$F_i^T(t)F_i(t) \leq I.$$

The estimation error $\bar{x}_t = x_t - \hat{x}_t$ satisfies

$$\dot{\bar{x}}_t = A_i\bar{x}_t - L_i E x_{t-d_t} + L_i E\hat{x}_t + L_i e_{s_{r,\nu}h} + D_i\omega_t. \tag{7}$$

From (4), (5), and (7), one can obtain the ET closed-loop error switched system

$$\begin{cases} \dot{\tilde{x}}_t = \tilde{A}_i\tilde{x}_t + \tilde{A}_{di}H\tilde{x}_{t-d_t} + \tilde{L}_i e_{s_{r,\nu}h} + \tilde{D}_i\omega_t, \\ y_t = \tilde{E}\tilde{x}_t, \\ \tilde{x}_\theta = \varphi_\theta, \quad \theta \in [-h, 0], \end{cases} \tag{8}$$

where φ_θ is a continuously differentiable initial function in $[-h,\ 0]$,

$$\begin{aligned} &\tilde{x}_t \triangleq [\hat{x}_t^T \ \ \bar{x}_t^T]^T, \quad \tilde{x}_{t-d_t} \triangleq [\hat{x}_{t-d_t}^T \ \ \bar{x}_{t-d_t}^T]^T, \\ &\tilde{A}_i \triangleq \begin{bmatrix} A_i + B_i\bar{K}_i(t) - L_i E & 0 \\ L_i E & A_i \end{bmatrix}, \quad \tilde{A}_{di} \triangleq \begin{bmatrix} L_i E \\ -L_i E \end{bmatrix}, \\ &\tilde{L}_i \triangleq \begin{bmatrix} -L_i \\ L_i \end{bmatrix}, \quad \tilde{D}_i \triangleq \begin{bmatrix} 0 \\ D_i \end{bmatrix}, \\ &\tilde{E} = \begin{bmatrix} E & E \end{bmatrix}, H = \begin{bmatrix} I & I \end{bmatrix}. \end{aligned}$$

Now we state the following definitions and Lemmas before the later development in this paper.

Definition 1 (FTBs) [36]. Given positive constants a_1, a_2, and T_f with $a_1 < a_2$, a positive definite matrix $\hat{R}$, ET closed-loop error switched system (8) is said to be finite-time bounded (FTB) wrt $(a_1, a_2, \hat{R}, T_f)$, if $t \in [0, T_f]$

$$\max_{\theta\in[-h,0]}\left\{\tilde{x}_\theta^T\hat{R}\tilde{x}_\theta, \dot{\tilde{x}}_\theta^T\hat{R}\dot{\tilde{x}}_\theta\right\} \leq a_1 \Rightarrow \tilde{x}_t^T\hat{R}\tilde{x}_t < a_2.$$

Remark 4 Noting the state $\tilde{x}_t = [\hat{x}_t^T \ \ \bar{x}_t^T]^T$, we shall choose $\hat{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$, where R represents a positive definite matrix.

Definition 2 (IO-FTSy) [31]. Given positive constant T_f, a positive definite matrix Γ, a class of exogenous disturbance signals $\mathcal{W}$ in $[0, T_f]$, under zero initial condition, switched system (8) shall be input-output finite-time stable (IO-FTS) wrt $(\mathcal{W}, \Gamma, T_f)$, if $t \in [0, T_f]$

$$\omega_t \in \mathcal{W} \Rightarrow y_t^T\Gamma y_t < 1.$$

Remark 5 In general, for the IO-FTSy problem, the following exogenous disturbances are considered by most of scholars:

$$\mathcal{W} \triangleq \left\{\omega_t \in \mathcal{L}_{2,[0,T_f]} : \int_0^{T_f} \omega_s^T G \omega_s \mathrm{d}s \leq 1\right\}, \tag{9}$$

$$\mathcal{W} \triangleq \left\{\omega_t \in \mathcal{L}_{\infty,[0,T_f]} : \max_{t\in[0,T_f]} \omega_t^T G \omega_t \leq 1\right\}, \tag{10}$$

where G represents a positive definite matrix. Throughout this paper, we only consider the exogenous disturbances $\omega_t \in \mathcal{L}_{2,[0,T_f]}$. Following the similar manipulations, one can deal with the disturbance in (10). Here, we just omit it.

Remark 6 It is known that a system is IO $\mathcal{L}_p$-stable, if its output belongs to the same class with the corresponding input, where $\mathcal{L}_p$ ($1 \leq p < \infty$) is defined to be a set of all piecewise continuous functions $\varepsilon_{(\cdot)}$ such that $\|\varepsilon_t\|_{\mathcal{L}_p} = \left(\int_0^\infty \|\varepsilon_t\|^p \mathrm{d}t\right)^{\frac{1}{p}} < \infty$. One can see that IO-FTSy and IO $\mathcal{L}_p$-stability are rather different concepts. IO-FTSy characterizes the short-time performances of the system's output, which not only requires the existence of the bounds but also pays more attention to the quantitative relations of the input and output signals. However, the concept of IO $\mathcal{L}_p$-stability is generally concerned with the output's property in an infinite time interval, which only demands the existence of the bounds.

Lemma 1 [37]. *For any constant matrix $M > 0$ and all continuously differentiable function $\phi_{(\cdot)}$ in $[a, b] \to \mathbb{R}^n$, the following inequality holds*

$$\begin{aligned} &-(b-a)\int_a^b \dot{\phi}_s^T M \dot{\phi}_s ds \\ &\quad \leq -(\phi_b - \phi_a)^T M (\phi_b - \phi_a) - 3\vartheta^T M \vartheta, \end{aligned}$$

where $\vartheta = \phi_b + \phi_a - \frac{2}{b-a}\int_a^b \phi_s ds$.

Lemma 2 [38]. *For full rank matrix $E \in \mathbb{R}^{m\times n}$, $\mathrm{rank}(E) = m$, the singular value decomposition (SVD) for E can be described as $E = O[S \ \ 0]V^T$, where $O \cdot O^T = I$ and $V \cdot V^T = I$. Let matrices $X > 0$, $M \in \mathbb{R}^{m\times m}$ and $N \in \mathbb{R}^{n\times n}$. Then, there exists $\bar{X}$ such that $EX = \bar{X}E$ if and only if the following condition holds:*

$$X = V\begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix} V^T.$$

Problem. Consider system (1). Given positive scalars a_1, a_2, T_f, positive definite matrices $\hat{R}$, G, Γ, and disturbance inputs $\mathcal{W}$ during the finite-time interval $[0, T_f]$, **our main tasks** are

(i) constructing an output-based event-triggered scheme to solve the problem of communication resources abused in network environment;
(ii) designing a resilient controller based on observer to guarantee that ET closed-loop error switched system (8) is both FTBs and IO-FTSy wrt (a_1, a_2, $\hat{R}$, $\mathcal{W}$, Γ, T_f).

3 Main results

3.1 FTB and IO-FTS analysis of the closed-loop error switched system

Theorem 1 *For switched system (1), given positive constants a_1, a_2, T_f, α, δ_i, μ, h, with $a_2 > a_1$, $\mu > 1$, and positive define matrix $\hat{R}$, there is a controller in the form of (5) making ET closed-loop error switched system (8) FTB wrt (a_1, a_2, T_f, $\hat{R}$), if there exist positive definite matrices P_i, Q_i, and W_i such that*

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \tag{11}$$

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad W_i \leq \mu W_j, \tag{12}$$

$$\tau_a > \tau_{a,1}^* = \frac{T_f \ln \mu}{\ln(\lambda_2 a_2) - \ln(\gamma + 1) - \alpha T_f - N_0 \ln \mu}, \tag{13}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega(1,1) & \Omega(1,2) & \Omega(1,3) \\ * & \Omega(2,2) & \Omega(2,3) \\ * & * & \Omega(3,3) \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0 & -P_1^i L_i & 6Q_i & \Omega(1,7) \\ P_2^i D_i & P_2^i L_i & 6Q_i & A_i^T \\ 0 & 0 & 6Q_i & 0 \end{bmatrix},$$

$$\Omega_3 = \begin{bmatrix} -G & 0 & 0 & D_i^T \\ * & -\Phi_i & 0 & 0 \\ * & * & -12Q_i & 0 \\ * & * & * & -h^{-2}Q_i^{-1} \end{bmatrix},$$

$$\Omega(3,3) = -4Q_i + \delta_i E^T \Phi_i E,$$

$$\begin{aligned} \Omega(1,1) &= \left(A_i^T + \bar{K}_i^T(t)B_i^T - E^T L_i^T\right) P_1^i - \alpha P_1^i \\ &\quad + W_i + P_1^i(A_i + B_i\bar{K}_i(t) - L_i E) - 4Q_i, \end{aligned}$$

$$\Omega(1,2) = E^T L_i^T P_2^i - 4Q_i,\ \Omega(1,3) = P_1^i L_i E - 2Q_i,\quad \Omega(2,3) = -P_2^i L_i E - 2Q_i,$$
$$\Omega(2,2) = A_i^T P_2^i + P_2^i A_i - \alpha P_2^i - 4Q_i,\quad \Omega(1,7) = A_i^T + \bar{K}_i^T(t) B_i^T,\ P_i = \hat{R}^{\frac{1}{2}} \bar{P}_i \hat{R}^{\frac{1}{2}},$$
$$\lambda_2 = \lambda_{\min}(\bar{P}_i),$$
$$\gamma = \lambda_1 a_1 \Big[1 - \frac{1}{\alpha}\Big((1-e^{\alpha h})\Big(1+\frac{h}{\alpha}\Big) + h^2\Big)\Big],$$
$$H^T W_i H = \hat{R}^{\frac{1}{2}} \bar{W}_i \hat{R}^{\frac{1}{2}},$$
$$\lambda_1 = \max_{i\in\mathcal{N}} \left\{\lambda_{\max}(\bar{P}_i), \lambda_{\max}(\bar{Q}_i), \lambda_{\max}(\bar{W}_i)\right\},$$
$$H^T Q_i H = \hat{R}^{\frac{1}{2}} \bar{Q}_i \hat{R}^{\frac{1}{2}}.$$

Proof Construct the following Lyapunov–Krasovskii functional for system (8)

$$\begin{aligned} V_i(\tilde{x}_t, t) &= \tilde{x}_t^T P_i \tilde{x}_t + \int_{t-d_t}^{t} e^{\alpha(t-s)} \tilde{x}_s^T H^T W_i H \tilde{x}_s \mathrm{d}s \\ &\quad + h \int_{-h}^{0}\int_{t+\theta}^{t} e^{\alpha(t-s)} \dot{\tilde{x}}_s^T H^T Q_i H \dot{\tilde{x}}_s \mathrm{d}s\mathrm{d}\theta \\ &= \tilde{x}_t^T P_i \tilde{x}_t + \int_{t-d_t}^{t} e^{\alpha(t-s)} x_s^T W_i x_s \mathrm{d}s \\ &\quad + h \int_{-h}^{0}\int_{t+\theta}^{t} e^{\alpha(t-s)} \dot{x}_s^T Q_i \dot{x}_s \mathrm{d}s\mathrm{d}\theta. \end{aligned} \tag{14}$$

The derivative of $V_i(\tilde{x}_t, t)$ along the trajectory of system (8) is described by

$$\begin{aligned} \dot{V}_i(\tilde{x}_t, t) &= \tilde{x}_t^T (\tilde{A}_i^T P_i + P_i \tilde{A}_i - \alpha P_i)\tilde{x}_t \\ &\quad + 2\tilde{x}_t^T P_i \tilde{A}_{di} x_{t-d_t} + x_t^T W_i x_t \\ &\quad + 2\tilde{x}_t^T P_i \tilde{L}_i e_{s_{r,\nu}h} + \alpha V_i(\tilde{x}_t, t) \\ &\quad + h^2 \dot{x}_t^T Q_i \dot{x}_t + 2\tilde{x}_t^T P_i \tilde{D}_i \omega_t \\ &\quad - h\int_{t-h}^{t} e^{\alpha(t-s)} \dot{x}_s^T Q_i \dot{x}_s \mathrm{d}s. \end{aligned} \tag{15}$$

From Lemma 1, we derive

$$\begin{aligned} &- h\int_{t-h}^{t} e^{\alpha(t-s)} \dot{x}_s^T Q_i \dot{x}_s \mathrm{d}s \le -h \int_{t-d_t}^{t} \dot{x}_s^T Q_i \dot{x}_s \mathrm{d}s \\ &\le - \begin{bmatrix} x_t - x_{t-d_t} \\ x_t + x_{t-d_t} - \frac{2}{d_t}\int_{t-d_t}^{t} x_s \mathrm{d}s \end{bmatrix}^T \begin{bmatrix} Q_i & 0 \\ 0 & 3Q_i \end{bmatrix} \begin{bmatrix} x_t - x_{t-d_t} \\ x_t + x_{t-d_t} - \frac{2}{d_t}\int_{t-d_t}^{t} x_s \mathrm{d}s \end{bmatrix} \\ &= \begin{bmatrix} x_t \\ x_{t-d_t} \\ \frac{1}{d_t}\int_{t-d_t}^{t} x_s \mathrm{d}s \end{bmatrix}^T \begin{bmatrix} -4Q_i & -2Q_i & 6Q_i \\ -2Q_i & -4Q_i & 6Q_i \\ 6Q_i & 6Q_i & -12Q_i \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-d_t} \\ \frac{1}{d_t}\int_{t-d_t}^{t} x_s \mathrm{d}s \end{bmatrix} \\ &= \begin{bmatrix} \hat{x}_t \\ \bar{x}_t \\ x_{t-d_t} \\ \frac{1}{d_t}\int_{t-d_t}^{t} x_s \mathrm{d}s \end{bmatrix}^T \begin{bmatrix} -4Q_i & -4Q_i & -2Q_i & 6Q_i \\ -4Q_i & -4Q_i & -2Q_i & 6Q_i \\ -2Q_i & -2Q_i & -4Q_i & 6Q_i \\ 6Q_i & 6Q_i & 6Q_i & -12Q_i \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \bar{x}_t \\ x_{t-d_t} \\ \frac{1}{d_t}\int_{t-d_t}^{t} x_s \mathrm{d}s \end{bmatrix}. \end{aligned} \tag{16}$$

Due to $\bar{x}_t = x_t - \hat{x}_t$, we have

$$h^2 \dot{x}_t^T Q_i \dot{x}_t = h^2 \begin{bmatrix} \hat{x}_t \\ \bar{x}_t \\ \omega_t \end{bmatrix}^T \begin{bmatrix} \Omega(1,7) \\ A_i^T \\ D_i^T \end{bmatrix} Q_i \begin{bmatrix} \Omega(1,7)^T & A_i & D_i \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \bar{x}_t \\ \omega_t \end{bmatrix}. \tag{17}$$

Define $P_i = \begin{bmatrix} P_1^i & 0 \\ 0 & P_2^i \end{bmatrix}$. From (2), (8), and (15)–(17), we get

$$\begin{aligned} \dot{V}_i(\tilde{x}_t, t) &\le \xi_t^T \begin{bmatrix} \Omega(1,1) & \Omega(1,2) & \Omega(1,3) & 0 & -P_1^i L_i & 6Q_i \\ * & \Omega(2,2) & \Omega(2,3) & P_2^i D_i & P_2^i L_i & 6Q_i \\ * & * & \Omega(3,3) & 0 & 0 & 6Q_i \\ * & * & * & -G & 0 & 0 \\ * & * & * & * & -\Phi_i & 0 \\ * & * & * & * & * & -12Q_i \end{bmatrix} \xi_t \\ &\quad + h^2 \begin{bmatrix} \hat{x}_t \\ \bar{x}_t \\ \omega_t \end{bmatrix}^T \begin{bmatrix} \Omega(1,7) \\ A_i^T \\ D_i^T \end{bmatrix} Q_i \begin{bmatrix} \Omega(1,7)^T & A_i & D_i \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \bar{x}_t \\ \omega_t \end{bmatrix} \\ &\quad + \omega_t^T G \omega_t + \alpha V_i(\tilde{x}_t, t), \end{aligned} \tag{18}$$

where

$$\xi_t = \mathrm{col}\left\{\hat{x}_t,\ \bar{x}_t,\ x_{t-d_t}, \omega_t,\ e_{s_{r,\nu}}, \frac{1}{d_t}\int_{t-d_t}^{t} x_s^T \mathrm{d}s\right\}.$$

Then, applying Schur complement and taking (11) into account, we derive

$$\dot{V}_i(\tilde{x}_t, t) \leq \omega_t^T G\omega_t + \alpha V_i(\tilde{x}_t, t). \tag{19}$$

This together with (12) implies that when $t \in [t_k, t_{k+1})$

$$\begin{aligned} V_{\sigma_t}(\tilde{x}_t, t) &\leq e^{\alpha(t-t_k)} V_{\sigma_{t_k}}(\tilde{x}_{t_k}, t_k) \\ &\quad + \int_{t_k}^{t} e^{\alpha(t-s)}\omega_s^T G\omega_s \mathrm{d}s \\ &\leq \mu e^{\alpha(t-t_k)} V_{\sigma_{t_k^-}}(\tilde{x}_{t_k^-}, t_k^-) \\ &\quad + \int_{t_k}^{t} e^{\alpha(t-s)}\omega_s^T G\omega_s \mathrm{d}s \\ &\leq \mu e^{\alpha(t-t_{k-1})} V_{\sigma_{t_{k-1}}}(\tilde{x}_{t_{k-1}}, t_{k-1}) \\ &\quad + \mu\int_{t_{k-1}}^{t} e^{\alpha(t-s)}\omega_s^T G\omega_s \mathrm{d}s \\ &\quad \vdots \\ &\leq \mu^{N_\sigma(0,t)} e^{\alpha t} V_{\sigma_0}(\tilde{x}_0, 0) \\ &\quad + \mu^{N_\sigma(0,t)} \int_{0}^{t} e^{\alpha(t-s)}\omega_s^T G\omega_s \mathrm{d}s. \end{aligned} \tag{20}$$

By Definition 1, (9) and (14), we have

$$\begin{aligned} \tilde{x}_t^T \hat{R}\tilde{x}_t &\leq \frac{1}{\lambda_2} V_{\sigma_t}(\tilde{x}_t, t) \\ &\leq \frac{1}{\lambda_2}\mu^{N_\sigma(0,T_f)} e^{\alpha T_f}(V_{\sigma_0}(\tilde{x}_0, 0) + 1), \end{aligned} \tag{21}$$

and

$$V_{\sigma_0}(\tilde{x}_0, 0) \leq \gamma. \tag{22}$$

Using the average dwell time (ADT) technique in [25], we get $N_\sigma(0, T_f) \leq N_0 + \frac{T_f}{\tau_a}$. Further, from (13), (21), and (22), one can see $\tilde{x}_t^T \hat{R}\tilde{x}_t < a_2$. As a result, ET closed-loop error switched system (8) is FTBs wrt $(a_1, a_2, \hat{R}, T_f)$. □

Remark 7 For the ADT technique, the parameters $\tau_a > 0$, $N_0 \geq 0$ are, respectively, called the ADT and the chatter bound. We say a switching signal $\sigma(t) \in \mathcal{N}$ satisfying the ADT condition if the total switching numbers meet with $N_\sigma(t_1, t_2) \leq N_0 + \frac{t_2-t_1}{\tau_a}$.

Theorem 2 *For switched system* (1)*, given positive constants* a_1, a_2, T_f, α, δ_i, μ, h, *with* $a_2 \geq a_1$, $\mu > 1$, *and positive define matrices* $\hat{R}$, G, Γ, *there is a controller in the form of* (5) *making the corresponding ET closed-loop error switched system* (8) *IO-FTS wrt* $(\mathcal{W}, \Gamma, T_f)$*, if there exist positive definite matrices* P_i, Q_i, *and* W_i, *such that* (11)–(13) *and the following inequalities hold:*

$$\begin{bmatrix} -e^{-2\alpha T_f} P_i & \tilde{E}^T \\ * & -\Gamma^{-1} \end{bmatrix} < 0, \tag{23}$$

$$\tau_a > \tau_{a,2}^* = \frac{T_f \ln\mu}{\alpha T_f - N_0 \ln\mu}. \tag{24}$$

Proof Under zero initial condition, from (20) and (24), we know

$$\begin{aligned} \tilde{x}_t^T P_i\tilde{x}_t \leq V_i(\tilde{x}_t, t) &\leq \mu^{N_\sigma(0,t)} \int_0^t e^{\alpha(t-s)}\omega_s^T G\omega_s \mathrm{d}s \\ &\leq e^{2\alpha T_f}\int_0^{T_f} \omega_s^T G\omega_s \mathrm{d}s. \end{aligned} \tag{25}$$

Thus, the following inequality holds:

$$\begin{aligned} &y_t^T\Gamma y_t - \int_0^{T_f} \omega_s^T G\omega_s \mathrm{d}s \\ &\quad \leq \tilde{x}_t^T(\tilde{E}^T\Gamma\tilde{E} - e^{-2\alpha T_f} P_i)\tilde{x}_t. \end{aligned} \tag{26}$$

From (23) and Schur complement formula, we get

$$y_t^T\Gamma y_t < \int_0^{T_f} \omega_s^T G\omega_s \mathrm{d}s < 1. \tag{27}$$

Therefore, ET closed-loop error switched system (8) is IO-FTSy wrt $(\mathcal{W}, \Gamma, T_f)$. □

From Theorems 1 and 2, we can get the following result:

Theorem 3 *For switched system* (1)*, given positive constants* a_1, a_2, T_f, α, δ_i, μ, h, *with* $a_2 > a_1$, $\mu > 1$, *and positive definite matrices* $\hat{R}$, G, Γ, *there is a controller in the form of* (5) *marking the corresponding ET closed-loop error switched system* (8) *both FTB and IO-FTS wrt* $(a_1, a_2, \hat{R}, \mathcal{W}, \Gamma, T_f)$*, if there exist positive definite matrices* P_i, Q_i, W_i *such that* (11)–(13) *and* (23)–(24) *hold.*

Remark 8 It should be pointed out that Theorem 1 is concerned with the FTBs of the state of system (8). However, sometimes it is not the state, but the output required to be restrained within a bound. In such cases, Theorem 2 ensures the IO-FTSy of system (8). Here, Theorem 3 proposes conditions about FTBs and IO-FTSy for system (8).

3.2 Both FTB and IO-FTS resilient controller design of the ET closed-loop error switched system

Based on the result in Theorem 3, a resilient controller is designed in this section to guarantee that ET closed-loop error switched system (8) is both FTB and IO-FTS.

Theorem 4 *For switched system* (1)*, given positive constants* $a_1, a_2, T_f, \alpha, \delta_i, \mu, h, \varrho$ *with* $a_2 > a_1$, $\mu > 1$, *and positive definite matrices* $\hat{R}$, G, Γ, *there is a controller in the form of* (5) *marking the corresponding ET closed-loop error switched system* (8) *both FTB and IO-FTS wrt* $(a_1, a_2, \hat{R}, \mathcal{W}, \Gamma, T_f)$*, if there exist positive definite matrices* X_i, $\bar{\Phi}_i$, $\hat{P}_1^i$, $\hat{Q}_i$, $\hat{W}_i$, *and real matrices* Y_i, $\bar{Y}_i$, *such that the following inequalities hold:*

$$\begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} -\mu \hat{P}_1^j & 0 & X_j^T & 0 \\ * & -\mu X_j & 0 & X_j^T \\ * & * & \hat{P}_1^i - 2X_i & 0 \\ * & * & * & -X_i \end{bmatrix} \le 0, \tag{29}$$

$$\begin{bmatrix} -\mu \hat{Q}_j & X_j^T \\ * & \hat{Q}_i - 2X_i \end{bmatrix} \le 0, \quad \begin{bmatrix} -\mu \hat{W}_j & X_j^T \\ * & \hat{W}_i - 2X_i \end{bmatrix} \le 0, \tag{30}$$

$$\begin{bmatrix} -e^{-2\alpha T_f} \hat{P}_1^i & 0 & X_i^T E^T \\ * & -e^{-2\alpha T_f} X_i & X_i^T E^T \\ * & * & -\Gamma^{-1} \end{bmatrix} < 0, \tag{31}$$

$$\tau_a > \tau_a^* = \max\{\tau_{a,1}^*, \tau_{a,2}^*\}, \tag{32}$$

where

$$\Xi_1 = \begin{bmatrix} -X_i^T - X_i & \Xi(1,2) & 0 & \bar{Y}_i E \\ * & \Xi(2,2) & E^T \bar{Y}_i^T - 4\hat{Q}_i & -2\hat{Q}_i \\ * & * & \Xi(3,3) & -\bar{Y}_i E - 2\hat{Q}_i \\ * & * & * & -4\hat{Q}_i \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} 0 & -\bar{Y}_i & 0 & 0 & B_i M_i & 0 & 0 \\ 0 & 0 & 6\hat{Q}_i & \Xi(2,8) & 0 & X_i^T N_i^T & 0 \\ D_i & \bar{Y}_i & 6\hat{Q}_i & X_i^T A_i^T & 0 & 0 & 0 \\ 0 & 0 & 6\hat{Q}_i & 0 & 0 & 0 & E^T \end{bmatrix},$$

$$\Xi_3 = \begin{bmatrix} -G & 0 & 0 & D_i^T & 0 & 0 & 0 \\ * & -\bar{\Phi}_i & 0 & 0 & 0 & 0 & 0 \\ * & * & -12\hat{Q}_i & 0 & 0 & 0 & 0 \\ * & * & * & \Xi(8,8) & B_i M_i & 0 & 0 \\ * & * & * & * & -\varrho I & 0 & 0 \\ * & * & * & * & * & -\varrho^{-1} I & 0 \\ * & * & * & * & * & * & \Xi(11,11) \end{bmatrix},$$

$\Xi(1,2) = \hat{P}_1^i + A_i X_i + B_i Y_i - \bar{Y}_i E$,

$\Xi(2,2) = -\alpha \hat{P}_1^i + \hat{W}_i - 4\hat{Q}_i$,

$\Xi(2,8) = X_i^T A_i^T + Y_i^T B_i^T$,

$\Xi(3,3) = X_i^T A_i^T + A_i X_i - \alpha X_i - 4\hat{Q}_i$,

$\Xi(8,8) = h^{-2}(\hat{Q}_i - 2X_i)$,

$\Xi(11,11) = \delta_i^{-1}(\bar{\Phi}_i - 2I)$.

Moreover, the admissible gains of controller and observer can be given by

$$K_i = Y_i X_i^{-1}, \quad L_i = \bar{Y}_i \bar{X}_i^{-1}.$$

Proof Firstly, define $P_2^{i^{-1}} = X_i$, $X_i^T P_1^i X_i = \hat{P}_1^i$, $X_i^T W_i X_i = \hat{W}_i$, $X_i^T Q_i X_i = \hat{Q}_i$, and $Y_i = K_i X_i$. The SVD for E can be described as $E = J_1[Z \ \ 0]J_2^T$, where $J_1 \cdot J_1^T = I$ and $J_2 \cdot J_2^T = I$. Thus, for $X_i = J_2 \begin{bmatrix} X_1^i & 0 \\ 0 & X_2^i \end{bmatrix} J_2^T$, there exists $\bar{X}_i = J_1^T Z X_1^i Z^{-1} J_1^{-1}$ such that $E X_i = \bar{X}_i E$.
Then, letting

$$\bar{\Phi}_i = \bar{X}_i^T \Phi_i \bar{X}_i, \ \bar{Y}_i = L_i \bar{X}_i.$$

Next, using Schur complement and applying the inequalities $-\bar{\Phi}_i^{-1} \le \bar{\Phi}_i - 2I$, $-Q_i^{-1} \le \hat{Q}_i - 2X_i$, (28) can be transformed to

$$\begin{bmatrix} \tilde{\Xi}_1 & \tilde{\Xi}_2 \\ * & \tilde{\Xi}_3 \end{bmatrix} < 0, \tag{33}$$

where

$$\tilde{\Xi}_1 = \begin{bmatrix} -X_i^T - X_i & \Xi(1,2) & 0 & \bar{Y}_i E \\ * & \Xi(2,2) & E^T \bar{Y}_i^T - 4\hat{Q}_i & -2\hat{Q}_i \\ * & * & \Xi(3,3) & -\bar{Y}_i E - 2\hat{Q}_i \\ * & * & * & -4\hat{Q}_i + \delta_i E^T \bar{\Phi}_i E \end{bmatrix},$$

$$\tilde{\Xi}_2 = \begin{bmatrix} 0 & -\bar{Y}_i & 0 & 0 & B_i M_i & 0 \\ 0 & 0 & 6\hat{Q}_i & X_i^T A_i^T + Y_i^T B_i^T & 0 & X_i^T N_i^T \\ D_i & \bar{Y}_i & 6\hat{Q}_i & X_i^T A_i^T & 0 & 0 \\ 0 & 0 & 6\hat{Q}_i & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Xi}_3 = \begin{bmatrix} -G & 0 & 0 & D_i^T & 0 & 0 \\ * & -\bar{\Phi}_i & 0 & 0 & 0 & 0 \\ * & * & -12\hat{Q}_i & 0 & 0 & 0 \\ * & * & * & -h^{-2} Q_i^{-1} & B_i M_i & 0 \\ * & * & * & * & -\varrho I & 0 \\ * & * & * & * & * & -\varrho^{-1} I \end{bmatrix}.$$

Pre- and post-multiplying (33) by

$$\mathrm{diag}\left\{X_i^{-1}, X_i^{-1}, X_i^{-1}, X_i^{-1}, I, \bar{X}_i^{-1}, X_i^{-1}, I, \ I, I\right\}$$

and its transpose, respectively, one can see that inequalities (28) imply

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix} < 0, \tag{34}$$

where

$$\Pi_1 = \begin{bmatrix} -P_2^{i^T} - P_2^i & \Pi(1,2) & 0 & P_2^i L_i E \\ * & \Pi(2,2) & \Omega(1,2) & -2Q_i \\ * & * & \Omega(2,2) & \Omega(2,3) \\ * & * & * & \Omega(3,3) \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} 0 & -P_2^i L_i & 0 & 0 & P_2^i B_i M_i & 0 \\ 0 & 0 & 6Q_i & A_i^T + K_i^T B_i^T & 0 & N_i^T \\ P_2^i D_i & P_2^i L_i & 6Q_i & A_i^T & 0 & 0 \\ 0 & 0 & 6Q_i & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_3 = \begin{bmatrix} -G & 0 & 0 & D_i^T & 0 & 0 \\ * & -\Phi_i & 0 & 0 & 0 & 0 \\ * & * & -12Q_i & 0 & 0 & 0 \\ * & * & * & -h^{-2}Q_i^{-1} & B_i M_i & 0 \\ * & * & * & * & -\varrho I & 0 \\ * & * & * & * & * & -\varrho^{-1} I \end{bmatrix},$$

$\Pi(1,2) = P_1^i + P_2^i A_i + P_2^i B_i K_i - P_2^i L_i E,$

$\Pi(2,2) = -\alpha P_1^i + W_i - 4Q_i.$

From (34), we know that

$$\begin{bmatrix} \hat{\Pi}_1 & \hat{\Pi}_2 \\ * & \hat{\Pi}_3 \end{bmatrix} < 0, \tag{35}$$

where

$$\hat{\Pi}_1 = \begin{bmatrix} -P_2^{i^T} - P_2^i & P_1^i + P_2^i \hat{A}_i & 0 & P_2^i L_i E \\ * & \Pi(2,2) & \Omega(1,2) & -2Q_i \\ * & * & \Omega(2,2) & \Omega(2,3) \\ * & * & * & \Omega(3,3) \end{bmatrix},$$

$$\hat{\Pi}_2 = \begin{bmatrix} 0 & -P_2^i L_i & 0 & 0 \\ 0 & 0 & 6Q_i & \Omega(1,7) \\ P_2^i D_i & P_2^i L_i & 6Q_i & A_i^T \\ 0 & 0 & 6Q_i & 0 \end{bmatrix},$$

$$\hat{\Pi}_3 = \begin{bmatrix} -G & 0 & 0 & D_i^T \\ * & -\Phi_i & 0 & 0 \\ * & * & -12Q_i & 0 \\ * & * & * & -h^{-2}Q_i^{-1} \end{bmatrix},$$

where $\hat{A}_i(t) = A_i + B_i \bar{K}_i(t) - L_i E$. Moreover, (35) can be also rewritten as

$$\Upsilon + \Lambda^T P_2^{i^T} \Delta + \Delta^T P_2^i \Lambda < 0, \tag{36}$$

where

$\Lambda = \begin{bmatrix} -I & \hat{A}_i(t) & 0 & L_i E & 0 & -L_i & 0 & 0 \end{bmatrix},$

$\Delta = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$

$$\Upsilon = \begin{bmatrix} 0 & P_1^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \Pi(2,2) & \Omega(1,2) & -2Q_i & 0 & 0 & 6Q_i & \Omega(1,7) \\ * & * & \Omega(2,2) & \Omega(2,3) & P_2^i D_i & P_2^i L_i & 6Q_i & A_i^T \\ * & * & * & \Omega(3,3) & 0 & 0 & 6Q_i & 0 \\ * & * & * & * & -G & 0 & 0 & D_i^T \\ * & * & * & * & * & -\Phi_i & 0 & 0 \\ * & * & * & * & * & * & -12Q_i & 0 \\ * & * & * & * & * & * & * & -h^{-2}Q_i^{-1} \end{bmatrix}.$$

Then, pre- and post-multiplying (36) by

$$\Theta = \begin{bmatrix} \hat{A}_i^T(t) & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ E^T L_i^T & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ -L_i^T & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

and its transpose, respectively, we can obtain $\Theta \Upsilon \Theta^T < 0$, i.e., inequality (11) holds. Therefore, inequality (28) implies (11).

Pre- and post-multiplying (29) by diag$\left\{X_j^{-1}, X_j^{-1}, I, I\right\}$, using Schur complement and applying the inequalities $-P_1^{i^{-1}} \leq \hat{P}_1^i - 2X_i$, we get $P_i \leq \mu P_j$.

Pre- and post-multiplying (30) by diag$\left\{X_j^{-1}, I\right\}$ and its transpose, respectively, using Schur complement and applying the inequalities $-Q_i^{-1} \leq \hat{Q}_i - 2X_i$, $-W_i^{-1} \leq \hat{W}_i - 2X_i$, we get $Q_i \leq \mu Q_j$ and $W_i \leq \mu W_j$.

In addition, pre- and post-multiplying (31) by diag$\left\{X_i^{-1}, X_i^{-1}, I\right\}$ and combining with $P_i = \begin{bmatrix} P_1^i & 0 \\ 0 & P_2^i \end{bmatrix}$, $\tilde{E} = \begin{bmatrix} E & E \end{bmatrix}$, we can have (23).

Finally, from (28) to (32), one can see that ET closed-loop error switched system (8) is both FTB and IO-FTS wrt $(a_1, a_2, \hat{R}, \mathcal{W}, \Gamma, T_f)$. □

Remark 9 Noting that the nonlinear terms $P_1^i L_i E - 2Q_i$ and $-P_2^i L_i E - 2Q_i$ exist in (11), the MATLAB LMI Toolbox cannot be directly used to solve the matrix inequalities. Different from $P = \text{diag}\{\bar{P}, \bar{P}\}$ in [39], we let $P_i = \text{diag}\{P_1^i, P_2^i\}$ (with $P_1^i \neq P_2^i$) and adopt SVD method to cast them into LMI forms.

Remark 10 It is worth mention that the feasibility problem of (11) cannot be directly expressed in terms of LMIs-based feasibility problem, since the presence of the nonlinear terms in (11). The usual calculation methods in [22,23,35] to deal with this problem may increase the conservativeness. Here, the feasibility

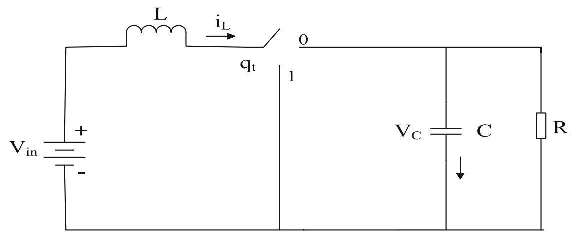

Fig. 2 A boost converter circuit system

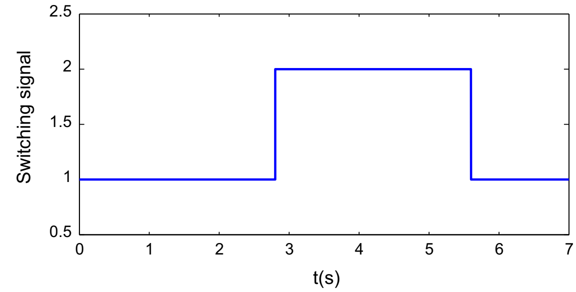

Fig. 3 The switching signal

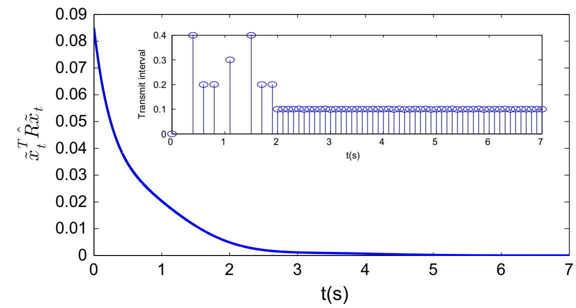

Fig. 4 Response of $\tilde{x}_t^T \hat{R} \tilde{x}_t$

problem of (11) has been transformed into the feasibility problem of (35), which can be recast in terms of LMIs (28). Then, the gains of controller and observer in this paper can be acquired simultaneously by solving strict LMIs.

4 Simulation example

In this part, we borrow the boost converter circuit from [40] to verify the availability of our event-triggered finite-time control technique.

As shown in Fig. 2, under the different modes ($\sigma_t = q_t + 1 \in \{1, 2\}$), the system matrices of the boost converter circuit are given by

$$q_t = 0: \ A_1 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, \ B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \tag{37}$$

$$q_t = 1: \ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}, \ B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}. \tag{38}$$

Let the state variable $x_t = [i_L, \ V_C]^T$ and the control input $u_t = V_{in}$. Suppose that the other system matrices are

$$E = \begin{bmatrix} 0.8 \ 0.2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -0.15 \\ 0.2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -0.13 \\ 0.2 \end{bmatrix}.$$

Now, we choose the circuit parameters as $L = 1H$, $C = 1F$, and $R = 1\Omega$. Take the parameters $\alpha = 0.28$, $a_1 = 0.12$, $a_2 = 19$, $\mu = 2.1$, $T_f = 7$, $h = 0.1$, $R = I_{2\times2}$, $\varrho = 0.1$, $\Gamma = 0.1$, and $G = 2$. And the external disturbance input is taken as $\omega_t = \frac{0.1}{1+t}$.

To investigate the effect of the triggered parameters on the release interval, we will divide them into two cases.

Case 1 Choosing the same triggered thresholds as $\delta_1 = 5.0 \times 10^{-8}$, $\delta_2 = 5.0 \times 10^{-8}$ and solving LMIs in Theorem 4, we can get the controller gains:

$$K_1 = \begin{bmatrix} -1.5932 \ 1.0993 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.7311 \ -0.0391 \end{bmatrix},$$

the observer gains:

$$L_1 = \begin{bmatrix} 0.4176 \\ -0.0830 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.3121 \\ -0.0097 \end{bmatrix},$$

and the event-triggered matrices $\Phi_1 = 23.3314$, $\Phi_2 = 8.1242$.

From (32), we have $\tau_a^* = 2.7641$. Choose $\tau_a = 2.8$ and the initial state as $[-0.2 -0.01 \ 0.15 -0.15]^T$. The switching signal is depicted in Fig. 3. The responses of $\tilde{x}_t^T \hat{R} \tilde{x}_t$ and the corresponding event-triggered release intervals (ETRIs) for the closed-loop error switched system are depicted in Fig. 4. The responses of the estimation error and the corresponding ETRIs are shown in Fig. 5. From Fig. 5, we can know that the observer state $\hat{x}_t$ can track the real state x_t smoothly, where $\bar{x}_t = x_t - \hat{x}_t$ indicates the estimation error.

Next, letting the initial values be zero, the responses of $y_t^T \Gamma y_t$ versus the corresponding ETRIs of the ET closed-loop error switched system are provided in Fig. 6.

Case 2 Choosing the different triggered thresholds as $\delta_1 = 1.0 \times 10^{-6}$, $\delta_2 = 5.0 \times 10^{-8}$ and solving LMIs

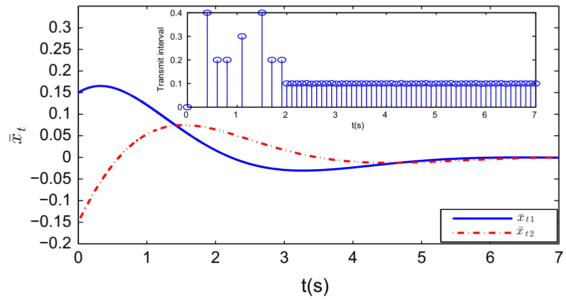

Fig. 5 Response of the estimation error

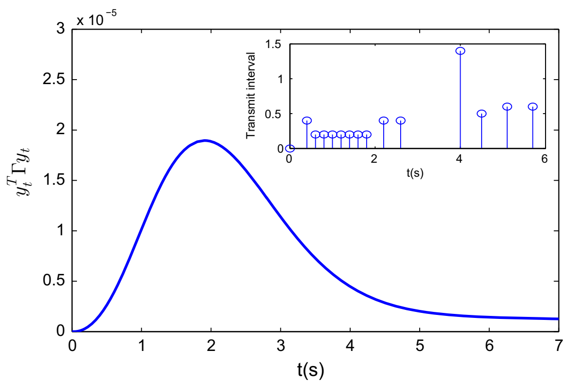

Fig. 6 Response of $y_t^T \Gamma y_t$ under zero initial condition

in Theorem 4, we can get the controller gains:

$$K_1 = \big[-1.5935\ 1.0952 \big],$$
$$K_2 = \big[-1.7369\ -0.0453 \big],$$

the observer gains:

$$L_1 = \begin{bmatrix} 0.4216 \\ -0.0980 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.3084 \\ -0.0144 \end{bmatrix},$$

and the event-triggered matrices $\Phi_1 = 24.1247$, $\Phi_2 = 8.0772$.

From (32), we have $\tau_a^* = 2.7336$. In order to compare with the results in Case 1, we choose the same switching rule in Fig. 3 and the same initial state. Correspondingly, the response of $\tilde{x}_t^T \hat{R} \tilde{x}_t$, the estimation error $\bar{x}_t = x_t - \hat{x}_t$, and its ETRIs are plotted in Figs. 7 and 8. Under the zero initial values, the responses of $y_t^T \Gamma y_t$ versus the corresponding ETRIs are provided in Fig. 9.

Remark 11 From the comparisons of the two cases and the simulation results in Figs. 4, 5, 6, 7, 8 and 9, one can clearly see that the triggered thresholds δ_i can affect the number of sampling points transmitted to the observer.

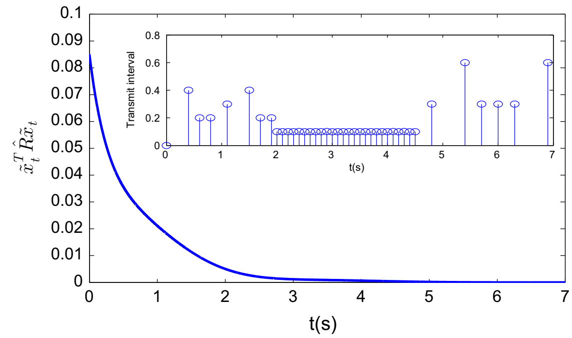

Fig. 7 Response of $\tilde{x}_t^T \hat{R} \tilde{x}_t$

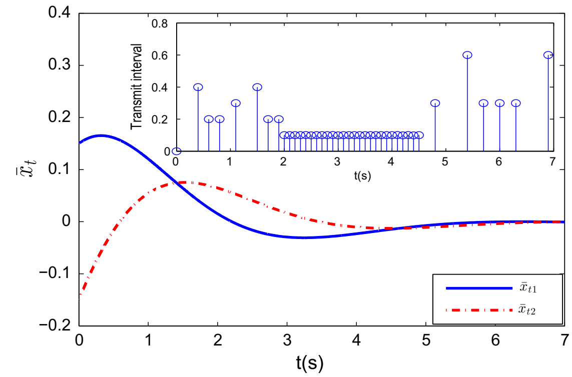

Fig. 8 Response of the state error

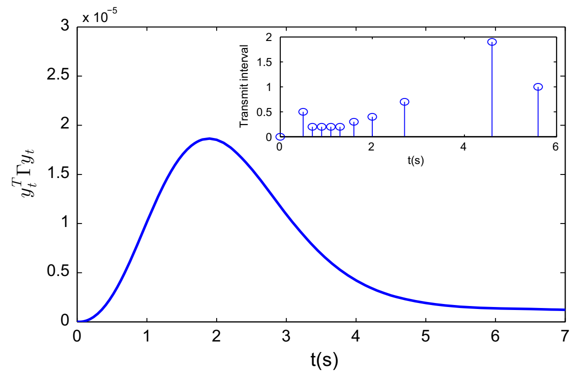

Fig. 9 Response of $y_t^T \Gamma y_t$ under zero initial condition

The smaller triggered parameters indicate the less sampled data can be transmitted to the observer.

Remark 12 From Figs. 4, 5 and 6, in the finite-time interval $[0, 7]$, under the observer-based event-triggered resilient controller, the response of $x_t^T \hat{R} x_t$ keeps below a_2 when the initial condition satisfies $x_0^T \hat{R} x_0 \leq a_1$ and $y_t^T \Gamma y_t < 1$ when the initial condition is zero. Therefore, from Definitions 1 and 2, we conclude that ET closed-loop error switched system (8) is both FTB and IO-FTS. The same conclusion can be obtained from Figs. 7, 8 and 9.

5 Conclusion

In this paper, we have investigated the FTBs and IO-FTSy problems for switched system under ETCS by applying observer-based resilient control approach. First, we have concentrated on the design of output-based event-triggered scheme to improve the utilization of network resources. Then, an event-triggered observer was constructed to estimate the unmeasurable states. Furthermore, the observer-based resilient controller was designed for the ET closed-loop error switched system. By constructing the mode-dependent Lyapunov–Krasovskii functional and resorting to the ADT approach, sufficient conditions of FTBs and IO-FTSy have been established for ET closed-loop error switched system. Finally, as an application, the boost converter circuit model has been provided to verify its FTB and IO-FTS characteristics through simulations. In future research, we shall extend the proposed results to the semi-Markovian jump systems with event-triggered scheme.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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