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ORIGINAL PAPER

A fuzzy wavelet neural network-based approach to hypersonic flight vehicle direct nonaffine hybrid control

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Abstract A direct nonaffine hybrid control methodology is proposed for a generic hypersonic flight models based on fuzzy wavelet neural networks (FWNNs). The addressed strategy extends the previous indirect nonaffine control approaches stemming from simplified models of affine formulations. To cope with nonaffine effects on control design, analytically invertible models are constructed and then novel hybrid controllers are developed directly using nonaffine models. Furthermore, by employing FWNNs to devise adaptive terms, inversion errors are canceled via fuzzy neural approximations. In addition, robust terms are designed to achieve larger stable region in comparison with earlier work using Lyapunov synthesis. Finally, numerical simulation results from a hypersonic flight vehicle model are given to clarify the efficiency of the proposed direct nonaffine control scheme in the presence of parametric uncertainties.

Keywords Hypersonic flight vehicle · Direct nonaffine control · Fuzzy wavelet neural networks · Analytically invertible models · Inversion errors

List of symbols

m	Vehicle mass
ρ	Density of air
$\bar{q}$	Dynamic pressure
S	Reference area
h	Altitude
V	Velocity
γ	Flight-path angle
θ	Pitch angle
α	Angle of attack ($\alpha = \theta - \gamma$)
Q	Pitch rate
T	Thrust
D	Drag
L	Lift
M	Pitching moment
I_{yy}	Moment of inertia
$\bar{c}$	Aerodynamic chord
z_{T}	Thrust moment arm
Φ	Fuel equivalence ratio
δ_{e}	Elevator angular deflection
N_i	ith generalized force
$N_i^{\alpha_j}$	jth order contribution of α to N_i
N_i^0	Constant term in N_i
$N_2^{\delta_e}$	Contribution of δ_{e} to N_2
$\beta_i\,(h, \bar{q})$	ith trust fit parameter
η_i	ith generalized elastic coordinate
ζ_i	Damping ratio for elastic mode η_i
ω_i	Natural frequency for elastic mode η_i
$C_{\mathrm{D}}^{\alpha^i}$	ith order coefficient of α in D
$C_{\mathrm{D}}^{\delta_e^i}$	ith order coefficient of δ_{e} in D
C_{D}^{0}	Constant coefficient in D
$C_{\mathrm{L}}^{\alpha^i}$	ith order coefficient of α in L

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$C_{\mathrm{L}}^{\delta_e}$	Coefficient of δ_{e} contribution in L
C_{L}^{0}	Constant coefficient in L
$C_{\mathrm{M},\alpha}^{\alpha^i}$	ith order coefficient of α in M
$C_{\mathrm{M},\alpha}^{0}$	Constant coefficient in M
$C_{\mathrm{T}}^{\alpha^i}$	ith order coefficient of α in T
C_{T}^{0}	Constant coefficient in T
h_0	Nominal altitude for air density approximation
ρ_0	Air density at the altitude h_0
$\tilde{\psi}_i$	Constrained beam coupling constant for η_i
c_e	Coefficient of δ_e in M
$1/h_s$	Air density decay rate
$\Re^n$	n-dimensional Euclidean space
$\Re$	The set of all real numbers
$\Re^+$	The set of all positive real numbers
$\|\|\bullet\|\|$	The 2-norm of a vector
$\|\bullet\|$	The absolute value of a scalar

1 Introduction

Hypersonic flight vehicles (HFVs) have spurred considerable attention owing to its promising prospect for a reliable and cost-efficient access to near-space for both civilian and military applications [1,2]. To capture all of the potential effects on HFVs controllability, we must take into consideration aerodynamics, aerodynamic heating, and flexible airframe, as well as the interactions among these disciplines. For this reason, the vehicle model we developed is highly coupled and nonlinear. This makes hypersonic flight control a challenging research area. Moreover, owing to the inconstant of the vehicle characteristics including aerodynamics and thrust level with varying flight conditions, significant uncertainties affect the vehicle model [3–7]. Besides, HFVs usually experience extreme aerothermal loads that cause dynamic forces and moments to rapidly change, which further increase the model uncertainties.

The nonlinear and coupled motion model for HFVs implies that the vehicle model is completely nonaffine in the control inputs (For example, "$\dot{x} = f(x, u)$" is nonaffine and "$\dot{x} = g(x) + F(x)u$" is affine, where x is the state, u is the control input, $f()$, $g()$ and $F()$ are given functions.). Noting that it is extreme difficult to directly design controls using such a highly coupled nonaffine system, lots of efforts are made to simplify HFV's nonaffine models as affine ones, followed by affine control designs for HFVs [8–13]. In [14,15], simplified affine models are firstly constructed applying the input/output linearization technique based on Lie derivative notations, and then affine sliding mode controllers are presented for HFVs to provide robust tracking of reference trajectories. Moreover, under rigorous assumptions/restrictions, HFVs' motion models are directly simplified as affine ones of the formulations $\dot{y}_i = f_i(y_i) + g_i(y_i)u_i$ ($f_i()$ and $g_i() \neq 0$ are given functions, u_i and y_i are the control input and state, respectively.) [16–19], on this basis, various affine control methodologies are addressed. However, model simplifications mean that we neglect certain key dynamic characteristics of HFVs, probably resulting in invalidation of control systems [20]. Besides, additional techniques have to be attached to improve these controllers' robust performance against system uncertainties especially the ones caused by model simplifications.

Noticing the fact that HFVs' motion models are nonlinear in the control inputs, nonaffine control designs for them also spur considerable interests. In [21], an indirect nonaffine control method is investigated, that is, the HFV's nonaffine model is firstly equivalently converted into an affine one by adding and subtracting the same term $k_i u_i$ (u_i is the control input, and k_i is a positive constant to be designed.) in each subsystem of HFV's model, and then an affine controller is devised. Another indirect nonaffine control strategy [22,23] is exploited by transforming the nonaffine motion model into an affine one based on Mean Value Theorem, which yields an unknown control direction problem that is handed by introducing a Nussbaum-type function.

Despite excellent tracking performance obtained by the previous studies, it is worth pointing out that there is still a considerable lack of effort in direct nonaffine control designs. Therefore, in this paper, we propose a novel direct nonaffine control methodology for HFVs. The special contributions are as summarized follows.

(1) A direct nonaffine hybrid control is presented based on the invertible theory, which avoids model simplifications and exhibits fine practicability and reliability.
(2) To guarantee the addressed hybrid controllers with satisfactory robustness against pragmatic uncertainties, fuzzy wavelet neural networks (FWNNs) are applied to estimate unknown flight dynamics. Furthermore, the computational costs for online

learning parameters are reduced by developing modified regulation laws.

(3) In contrast to the existing methods, larger stable region of the closed-loop control system is achieved owing to the developed robust terms.

The rest of this paper is structured as follows. Problem formulations are presented in Sect. 2. Section 3 shows the design procedure of hybrid controller. Simulation results are given in Sect. 4, and the conclusions are shown in Sect. 5.

2 Problem formulation

2.1 Vehicle model

In this study, we use a longitudinal motion model developed by Parker, formulated as [7]

$$\dot{V} = T\cos(\theta-\gamma)/m - D/m - g\sin\gamma \tag{1}$$

$$\dot{h} = V\sin\gamma \tag{2}$$

$$\dot{\gamma} = L/(mV) + T\sin(\theta-\gamma)/(mV) - (g/V)\cos\gamma \tag{3}$$

$$\dot{\theta} = Q \tag{4}$$

$$\dot{Q} = (M + \tilde{\psi}_1\ddot{\eta}_1 + \tilde{\psi}_2\ddot{\eta}_2)/I_{\mathrm{yy}} \tag{5}$$

$$k_1\ddot{\eta}_1 = -2\zeta_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1 + N_1 - \tilde{\psi}_1 M/I_{\mathrm{yy}} - \tilde{\psi}_1\tilde{\psi}_2\ddot{\eta}_2/I_{\mathrm{yy}} \tag{6}$$

$$k_2\ddot{\eta}_2 = -2\zeta_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2 + N_2 - \tilde{\psi}_2 M/I_{\mathrm{yy}} - \tilde{\psi}_2\tilde{\psi}_1\ddot{\eta}_1/I_{\mathrm{yy}} \tag{7}$$

In the above model, velocity V, altitude h, flight-path angle γ, pitch angle θ and pitch rate Q are the rigid body states, η_1 and η_2 are the flexible states. The attack angle $\alpha = \theta - \gamma$. The thrust force T, drag force D, lift force L, pitching moment M and the generalized force N_1 and N_2 are defined as [7]

$$\begin{aligned}
T &\approx \beta_1(h,\bar{q})\Phi\alpha^3 + \beta_2(h,\bar{q})\alpha^3 + \beta_3(h,\bar{q})\Phi\alpha^2 + \beta_4(h,\bar{q})\alpha^2 \\
&\quad + \beta_5(h,\bar{q})\Phi\alpha + \beta_6(h,\bar{q})\alpha + \beta_7(h,\bar{q})\Phi + \beta_8(h,\bar{q}), \\
D &\approx \bar{q}SC_{\mathrm{D}}^{\alpha^2}\alpha^2 + \bar{q}SC_{\mathrm{D}}^{\alpha}\alpha + \bar{q}SC_{\mathrm{D}}^{\delta_\mathrm{e}^2}\delta_\mathrm{e}^2 + \bar{q}SC_{\mathrm{D}}^{\delta_\mathrm{e}}\delta_\mathrm{e} + \bar{q}SC_{\mathrm{D}}^{0}, \\
M &\approx z_\mathrm{T}T + \bar{q}S\bar{c}C_{M,\alpha}^{\alpha^2}\alpha^2 + \bar{q}S\bar{c}C_{M,\alpha}^{\alpha}\alpha + \bar{q}S\bar{c}C_{M,\alpha}^{0} + \bar{q}S\bar{c}c_\mathrm{e}\delta_\mathrm{e}, \\
L &\approx \bar{q}SC_{\mathrm{L}}^{\alpha}\alpha + \bar{q}SC_{\mathrm{L}}^{\delta_\mathrm{e}}\delta_\mathrm{e} + \bar{q}SC_{\mathrm{L}}^{0}, \\
N_1 &= N_1^{\alpha^2}\alpha^2 + N_1^{\alpha}\alpha + N_1^0, \\
N_2 &= N_2^{\alpha^2}\alpha^2 + N_2^{\alpha}\alpha + N_2^{\delta_\mathrm{e}}\delta_\mathrm{e} + N_2^0, \quad \bar{q} = \bar{\rho}V^2/2, \\
\bar{\rho} &= \bar{\rho}_0\exp\left(-(h-h_0)/h_\mathrm{s}\right),
\end{aligned}$$

where the control inputs Φ and δ_e denote fuel equivalence ratio and elevator angular deflection, respectively. For more detailed definitions of other parameters and coefficients, the reader could refer to [7] or Nomenclature

Remark 1 Some sample plots [7] of aerodynamic data derived from the above vehicle model are shown in Fig. 1. Obviously, the HFVs' model is completely nonaffine, which is due to that the above motion model contains these nonlinear terms "δ_e^2," "α^3" and "α^2." (For back-stepping designs, α is treated as a virtual control input.) For this reason, in what follows, we directly employ the vehicle nonaffine model to devise controllers. Furthermore, to represent the entire model's features, the pole-zero map [7] is depicted in Fig. 2.

2.2 Control objective

To facilitate the subsequent control designs, the motion model of HFVs is formally decomposed into the velocity subsystem (i.e., Eq. (1)) and the altitude subsystem (i.e., Eqs. (2)–(5)).

The velocity subsystem (1) is rewritten as a succinct nonaffine formulation:

$$\begin{cases} \dot{V} = f_V(V,\Phi) \\ y_V = V \\ u_V = \Phi \end{cases} \tag{8}$$

where $f_V(V,\Phi)$ is an unknown differentiable function; u_V and y_V are the control input and output of velocity subsystem, respectively.

Building upon the previous studies [22,23], the altitude subsystem (3)–(5) can be transformed as the following nonaffine form:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = F_h(\boldsymbol{x},\delta_\mathrm{e}) \\ y_h = z_1 = \gamma \\ u_h = \delta_\mathrm{e} \end{cases} \tag{9}$$

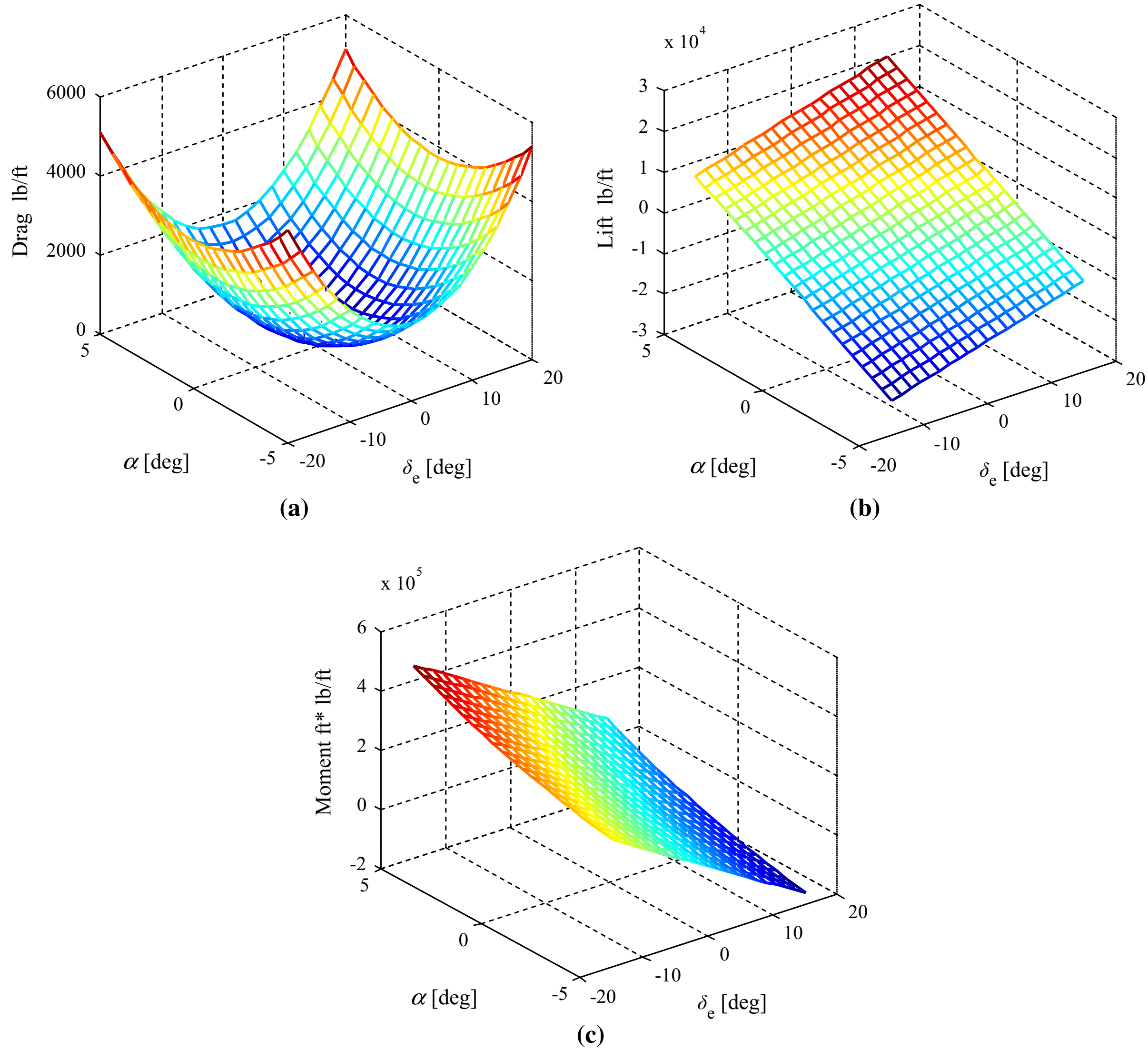

Fig. 1 The responses of drag, lift and moment along with the varying of α and δ_e. **a** The response of drag. **b**, the response of lift, **c** the response of moment

where $F_h(\boldsymbol{x}, \delta_{\mathrm{e}})$ is an unknown differentiable function and $\boldsymbol{x} = [\gamma, \theta, Q]^{\mathrm{T}} \in \Re^3$; u_h and y_h mean the control input and output of altitude subsystem, respectively.

The pursued control objective is that velocity V and altitude h follow their reference trajectories V_{ref} and h_{ref} in the presence of parametric uncertainties by developing direct nonaffine hybrid controllers Φ and δ_{e}.

2.3 FWNN approximate

FWNN is combined wavelet theory with fuzzy logic and neural networks (NNs). Because fuzzy logics can improve wavelet neural approximation performance, FWNN exhibits excellent performance and global approximation [24,25]. Thus, we employ it as an accurate function approximator in this study.

For the simplify of formulation, by employing a singleton fuzzifier, product inference and weighted average defuzzifier, the output of FWNN can be described as

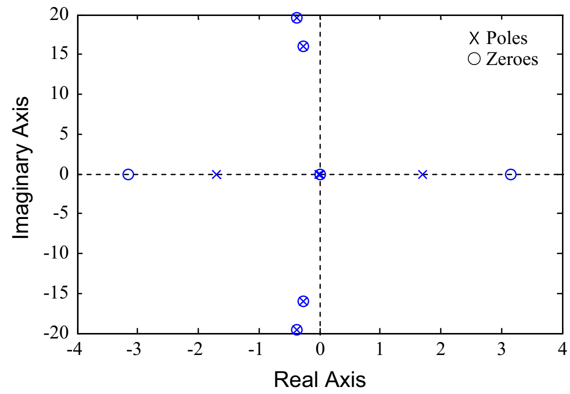

Fig. 2 Pole-zero maps of the Jacobian linearization of the adopted HFV model. Inputs $u = [\Phi, \delta_e]$, outputs $y = [V, \gamma]$

$$y = \boldsymbol{W}^{\mathrm{T}}\boldsymbol{\psi}(\boldsymbol{\ell}) \tag{10}$$

where $\boldsymbol{\ell} = [\ell_1, \ell_2, \cdots, \ell_n]^{\mathrm{T}} \in \Re^n$ is the input vector and $\boldsymbol{W}^{\mathrm{T}} = [w_1, w_1, \cdots, w_N]^{\mathrm{T}} \in \Re^N$ is the weight matrix; $\boldsymbol{\psi}(\boldsymbol{\ell}) = [\psi_1(\boldsymbol{\ell}), \psi_2(\boldsymbol{\ell}), \cdots, \psi_N(\boldsymbol{\ell})]^{\mathrm{T}}$ is the fuzzy wavelet basis function vector and $\psi_j()$ has the following formulation:

$$\psi_j = \prod_{i=1}^{n} g_{ji}(\ell_i)\phi_j \Big/ \sum_{j=1}^{N} \phi_j \tag{11}$$

where $g_{ji}(\ell_i) = 1 - b_{ji}^2(\ell_i - c_{ji})^2$ and $\phi_j = \prod_{i=1}^{n} \mu_{A_{ji}}(\ell_i)$ is the firing strength with the membership function $\mu_{A_{ji}}(\ell_i)$ given by $\mu_{A_{ji}}(\ell_i) = \exp(-b_{ji}^2(\ell_i - c_{ji})^2)$; b_{ji} and c_{ji} are dilation and translation parameters, respectively.

Traditionally, Taylor expansion linearization technique is usually applied to convert the nonlinear fuzzy wavelet basis function into a partially linear function of ψ_j, b_{ji} and c_{ji} [24]. Then, parameters ψ_j, b_{ji} and c_{ji} are all online regulated for the sake of achieving desired approximation performance. This leads to high computational burden and reduces the real-time performance of control system. Therefore, an advanced learning scheme is adopted to directly turn $\varphi = \|\boldsymbol{\psi}(\boldsymbol{\ell})\|^2$ by setting appropriate values for b_{ji} and c_{ji}. In this way, there is only one learning parameter φ. Thus, the online computational load is low.

3 Hybrid control design

3.1 Velocity control design

Assumption 1 [21] $\partial f_V(V, \Phi)/\partial \Phi$ is continuous and positive.

Define velocity tracking error $\tilde{V} = V - V_{\mathrm{ref}}$. Then using (8), $\dot{\tilde{V}}$ is derived as

$$\dot{\tilde{V}} = \dot{V} - \dot{V}_{\mathrm{ref}} = f_V(V, \Phi) - \dot{V}_{\mathrm{ref}} \tag{12}$$

From (12), a hybrid pseudocontrol υ_V is developed as

$$\upsilon_V = \hat{f}_V(V, \Phi) \tag{13}$$

where $\hat{f}_V(V, \Phi)$ denotes the approximate of $f_V(V, \Phi)$. Define the inversion error $\delta_V = f_V(V, \Phi) - \hat{f}_V(V, \Phi)$ and then Eq. (12) becomes

$$\dot{\tilde{V}} = \delta_V + \hat{f}_V(V, \Phi) - \dot{V}_{\mathrm{ref}} \tag{14}$$

Assumption 2 It is concluded from Assumption 1 that $\partial \hat{f}_V(V, \Phi)/\partial \Phi$ also is continuous and positive.

The hybrid controller υ_V is designed as

$$\upsilon_V = \upsilon_{V1} + \upsilon_{V2} - \upsilon_{V3} + \upsilon_{V4} \tag{15}$$

where $\upsilon_{V1} = \dot{V}_{\mathrm{ref}}$ and $\upsilon_{V2} = -K_{V1}\tilde{V} - K_{V2}\int_0^t \tilde{V}(\tau)\,\mathrm{d}\tau$; $K_{V1} \in \Re^+$ and $K_{V2} \in \Re^+$ are constants to be chosen; υ_{V4} is a robust term and υ_{V3} will be devised to cancel δ_V.

The control input Φ is derived from (13) as follows

$$\Phi = \hat{f}_V^{-1}(V, \upsilon_V) \tag{16}$$

Substituting (13) and (15) into (14), we have

$$\begin{aligned}
\dot{\tilde{V}} &= \delta_V + \upsilon_V - \dot{V}_{\mathrm{ref}} \\
&= \delta_V + \upsilon_{V1} + \upsilon_{V2} - \upsilon_{V3} + \upsilon_{V4} - \dot{V}_{\mathrm{ref}} \\
&= -K_{V1}\tilde{V} - K_{V2}\int_0^t \tilde{V}(\tau)\mathrm{d}\tau + \delta_V - \upsilon_{V3} + \upsilon_{V4}
\end{aligned} \tag{17}$$

with

$$\begin{aligned}
\delta_V - \upsilon_{V3} = {}& f_V(V, \hat{f}_V^{-1}(V, \upsilon_{V1}+\upsilon_{V2}-\upsilon_{V3}+\upsilon_{V4})) \\
& - \hat{f}(V, \hat{f}_V^{-1}(V, \upsilon_{V1}+\upsilon_{V2}-\upsilon_{V3}+\upsilon_{V4})) - \upsilon_{V3}
\end{aligned}$$

$$= f_V(V, \hat{f}_V^{-1}(V, \upsilon_{V1} + \upsilon_{V2} - \upsilon_{V3} + \upsilon_{V4})) \\ -\upsilon_{V1} - \upsilon_{V2} - \upsilon_{V4} \tag{18}$$

By defining $\upsilon_{Vl} \triangleq \upsilon_{V1} + \upsilon_{V2}$ and $\upsilon_V^* \triangleq \hat{f}_V(V, f_V^{-1}(V, \upsilon_{Vl}))$, we obtain $f_V^{-1}(V, \upsilon_{Vl}) = \hat{f}_V^{-1}(V, \upsilon_V^*) \Rightarrow \upsilon_{Vl} = f_V(V, \hat{f}_V^{-1}(V, \upsilon_V^*))$. Then, Eq. (18) becomes

$$\begin{aligned}\delta_V - \upsilon_{V3} &= f_V(V, \hat{f}_V^{-1}(V, \upsilon_V)) - \upsilon_{Vl} - \upsilon_{V4} \\ &= f_V(V, \hat{f}_V^{-1}(V, \upsilon_V)) \\ &\quad - f_V(V, \hat{f}_V^{-1}(V, \upsilon_V^*)) - \upsilon_{V4}\end{aligned} \tag{19}$$

By Mean Value Theorem, Eq. (19) is further rewritten as

$$\begin{aligned}\delta_V - \upsilon_{V3} &= f_V(V, \hat{f}_V^{-1}(V, \upsilon_V)) \\ &\quad - f_V(V, \hat{f}_V^{-1}(V, \upsilon_V^*)) - \upsilon_{V4} \\ &= f_V(\bar{\upsilon}_V)(\upsilon_V - \upsilon_V^*) - \upsilon_{V4} \\ &= f_V(\bar{\upsilon}_V)(\upsilon_{Vl} - \upsilon_{V3} + \upsilon_{V4} \\ &\quad - \hat{f}_V(V, f_V^{-1}(V, \upsilon_{Vl}))) - \upsilon_{V4} \\ &= f_V(\bar{\upsilon}_V)(\bar{\upsilon}_{Vl} - \upsilon_{V3} + \upsilon_{V4}) - \upsilon_{V4}\end{aligned} \tag{20}$$

where $\bar{\upsilon}_{Vl} = \upsilon_{Vl} - \hat{f}_V(V, f_V^{-1}(V, \upsilon_{Vl})) = f_V(V, \hat{f}_V^{-1}(V, \upsilon_V^*)) - \upsilon_V^*$ is an unknown term; $f_V(\bar{\upsilon}_V) = \frac{\partial f_V}{\partial \Phi}\frac{\partial \Phi}{\partial \upsilon_V}\Big|_{\upsilon_V = \bar{\upsilon}_V} = \frac{\partial f_V}{\partial \Phi}\frac{\partial \hat{f}_V}{\partial \Phi}\Big|_{\Phi = \hat{f}_V^{-1}(V, \bar{\upsilon}_V)} > 0$ and $\bar{\upsilon}_V = \theta_V \upsilon_V + (1 - \theta_V)\upsilon_V^*$ with $\theta_V \in [0, 1]$.

Substituting (20) into (17) yields

$$\dot{\tilde{V}} = -K_{V1}\tilde{V} - K_{V2}\int_0^t \tilde{V}(\tau)\mathrm{d}\tau + f_V(\bar{\upsilon}_V)(\bar{\upsilon}_{Vl} \\ -\upsilon_{V3}) + f_V(\bar{\upsilon}_V)\upsilon_{V4} \tag{21}$$

If $\bar{\upsilon}_{Vl}$ is known, we can set $\upsilon_{V3} = \bar{\upsilon}_{Vl}$ to stabilize (8). However, $\bar{\upsilon}_{Vl}$ is usually unknown and hereon we employ one FWNN to approximate it. Based on the universal approximation theorem [24,25], there must exist an ideal weight vector $\boldsymbol{W}_V^* \in \Re^N$ such that

$$\bar{\upsilon}_{Vl} = \boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) + \varepsilon_V \tag{22}$$

where $\boldsymbol{\ell}_V = V$ is the input of FWNN and $\boldsymbol{\psi}_V(\boldsymbol{\ell}_V)$ has the same formulation as (11); ε_V is an approximation error and there is a constant $\varepsilon_{VM} \in \Re^+$ such that $|\varepsilon_V| \le \varepsilon_{VM}$.

Define $\varphi_V = \left\|\boldsymbol{W}_V^*\right\|^2 \in \Re$ and υ_{V3} is devised as

$$\upsilon_{V3} = \frac{1}{2}\tilde{V}\hat{\varphi}_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \tag{23}$$

where $\hat{\varphi}_V$ denotes the estimate of φ_V.

For the single online parameter $\hat{\varphi}_V$, we design the following adaptive law:

$$\dot{\hat{\varphi}}_V = \frac{\eta_V}{2}\tilde{V}^2\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) - 2\hat{\varphi}_V \tag{24}$$

with $\eta_V \in \Re^+$.

Employing (22) and (23), $\bar{\upsilon}_{Vl} - \upsilon_{V3}$ equals to

$$\bar{\upsilon}_{Vl} - \upsilon_{V3} = \boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) + \varepsilon_V \\ -\frac{1}{2}\tilde{V}\hat{\varphi}_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \tag{25}$$

Considering (25), Eq. (21) becomes

$$\begin{aligned}\dot{\tilde{V}} = &- K_{V1}\tilde{V} - K_{V2}\int_0^t \tilde{V}(\tau)\mathrm{d}\tau \\ &+ f_V(\bar{\upsilon}_V)\boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) + f_V(\bar{\upsilon}_V)\varepsilon_V \\ &-\frac{1}{2}f_V(\bar{\upsilon}_V)\tilde{V}\hat{\varphi}_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &+ f_V(\bar{\upsilon}_V)\upsilon_{V4}\end{aligned} \tag{26}$$

The robust term υ_{V4} is chosen as

$$\upsilon_{V4} = -\frac{\tilde{V}}{2\rho_V^2} \tag{27}$$

where $\rho_V \in \Re^+$ is a constant to be selected.

Taking into consideration (27), it is derived from (26) that

$$\begin{aligned}\dot{\tilde{V}} = &-K_{V1}\tilde{V} - K_{V2}\int_0^t \tilde{V}(\tau)\mathrm{d}\tau \\ &+ f_V(\bar{\upsilon}_V)\boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) + f_V(\bar{\upsilon}_V)\varepsilon_V \\ &-\frac{1}{2}f_V(\bar{\upsilon}_V)\tilde{V}\hat{\varphi}_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &- f_V(\bar{\upsilon}_V)\frac{\tilde{V}}{2\rho_V^2}\end{aligned} \tag{28}$$

Theorem 1 *Consider the closed-loop system consisting of plant (8) under Assumptions 1 and 2 with controller (15) and adaptive law (24). Then, all the signals involved are semi-globally uniformly ultimately bounded.*

Proof Define estimate error $\tilde{\varphi}_V = \hat{\varphi}_V - \varphi_V$.

Define the following Lyapunov function:

$$V_V = \frac{\tilde{V}^2}{2} + \frac{K_{V2}}{2}\left(\int_0^t \tilde{V}(\tau)\mathrm{d}\tau\right)^2 + \frac{\bar{f}_V\tilde{\varphi}_V^2}{2\eta_V} \tag{29}$$

where $\bar{f}_V \in \Re^+$ is a constant to be ingeniously selected such that $\bar{f}_V \le f_V(\bar{\upsilon}_V)$when $\tilde{\varphi}_V\dot{\hat{\varphi}}_V \ge 0$ or else $\bar{f}_V > f_V(\bar{\upsilon}_V)$. We further have $\bar{f}_V\tilde{\varphi}_V\dot{\hat{\varphi}}_V/\eta_V \leq f_V(\bar{\upsilon}_V)\tilde{\varphi}_V\dot{\hat{\varphi}}_V/\eta_V$.

The time derivative of V_V is given by

$$\begin{aligned}\dot{V}_V &= \tilde{V}\dot{\tilde{V}} + K_{V2}\tilde{V}\int_0^t \tilde{V}(\tau)\mathrm{d}\tau + \frac{\bar{f}_V}{\eta_V}\tilde{\varphi}_V\dot{\hat{\varphi}}_V \\ &\leq \tilde{V}\dot{\tilde{V}} + K_{V2}\tilde{V}\int_0^t \tilde{V}(\tau)\mathrm{d}\tau \\ &\quad + \frac{f_V(\bar{\upsilon}_V)}{\eta_V}\tilde{\varphi}_V\dot{\hat{\varphi}}_V \end{aligned} \tag{30}$$

Substituting (24) and (28) into (30) leads to

$$\begin{aligned}\dot{V}_V &\leq -K_{V1}\tilde{V}^2 + f_V(\bar{\upsilon}_V)\tilde{V}\boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &\quad + f_V(\bar{\upsilon}_V)\tilde{V}\varepsilon_V \\ &\quad - \frac{1}{2}f_V(\bar{\upsilon}_V)\tilde{V}^2\hat{\varphi}_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &\quad - f_V(\bar{\upsilon}_V)\frac{\tilde{V}^2}{2\rho_V^2} \\ &\quad + \frac{1}{2}f_V(\bar{\upsilon}_V)\tilde{V}^2\tilde{\varphi}_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &\quad - \frac{2f_V(\bar{\upsilon}_V)\tilde{\varphi}_V\hat{\varphi}_V}{\eta_V} \\ &= -K_{V1}\tilde{V}^2 + f_V(\bar{\upsilon}_V)\tilde{V}\boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &\quad + f_V(\bar{\upsilon}_V)\tilde{V}\varepsilon_V \\ &\quad - \frac{1}{2}f_V(\bar{\upsilon}_V)\tilde{V}^2\varphi_V\boldsymbol{\psi}_V^{\mathrm{T}}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \\ &\quad - f_V(\bar{\upsilon}_V)\frac{\tilde{V}^2}{2\rho_V^2} \\ &\quad - \frac{2f_V(\bar{\upsilon}_V)\tilde{\varphi}_V\hat{\varphi}_V}{\eta_V} \end{aligned} \tag{31}$$

Notice that $\tilde{V}\boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) \leq \frac{\tilde{V}^2}{2}\left\|\boldsymbol{W}_V^{*\mathrm{T}}\boldsymbol{\psi}_V(\boldsymbol{\ell}_V)\right\|^2 + \frac{1}{2} = \frac{\tilde{V}^2}{2}\left\|\boldsymbol{W}_V^{*}\right\|^2\left\|\boldsymbol{\psi}_V(\boldsymbol{\ell}_V)\right\|^2 + \frac{1}{2} = \frac{\tilde{V}^2}{2}\varphi_V\boldsymbol{\psi}_V^{T}(\boldsymbol{\ell}_V)\boldsymbol{\psi}_V(\boldsymbol{\ell}_V) + \frac{1}{2}$, $2\tilde{\varphi}_V\hat{\varphi}_V \geq \tilde{\varphi}_V^2 - \varphi_V^2$ and $\tilde{V}\varepsilon_V \leq \left|\tilde{V}\right|\varepsilon_{VM} = 2\left|\frac{\tilde{V}}{2}\right|\varepsilon_{VM} \leq \frac{\tilde{V}^2}{4} + \varepsilon_{VM}^2$. Therefore, inequality (31) becomes

$$\begin{aligned}\dot{V}_V &\leq -K_{V1}\tilde{V}^2 - \frac{f_V(\bar{\upsilon}_V)\tilde{\varphi}_V^2}{\eta_V} + \frac{f_V(\bar{\upsilon}_V)\tilde{V}^2}{4} \\ &\quad - f_V(\bar{\upsilon}_V)\frac{\tilde{V}^2}{2\rho_V^2} + \frac{f_V(\bar{\upsilon}_V)\varphi_V^2}{\eta_V} + \frac{1}{2}f_V(\bar{\upsilon}_V) \\ &\quad + f_V(\bar{\upsilon}_V)\varepsilon_{VM}^2 \end{aligned} \tag{32}$$

Setting $\rho_V = \sqrt{2}$, we have

$$\begin{aligned}\dot{V}_V &\leq -K_{V1}\tilde{V}^2 - \frac{f_V(\bar{\upsilon}_V)\tilde{\varphi}_V^2}{\eta_V} + \frac{f_V(\bar{\upsilon}_V)\varphi_V^2}{\eta_V} \\ &\quad + \frac{1}{2}f_V(\bar{\upsilon}_V) + f_V(\bar{\upsilon}_V)\varepsilon_{VM}^2 \end{aligned} \tag{33}$$

Define the following compact sets:

$$\begin{aligned}\Omega_{\tilde{V}} &= \left\{\tilde{V}\,\middle|\,\left|\tilde{V}\right| \leq \sqrt{\left(\frac{f_V(\bar{\upsilon}_V)\varphi_V^2}{\eta_V} + \frac{1}{2}f_V(\bar{\upsilon}_V) + f_V(\bar{\upsilon}_V)\varepsilon_{VM}^2\right)/K_{V1}}\right\} \\ \Omega_{\tilde{\varphi}_V} &= \left\{\tilde{\varphi}_V\,\middle|\,|\tilde{\varphi}_V| \leq \sqrt{\left(\frac{\varphi_V^2}{\eta_V} + \frac{1}{2} + \varepsilon_{VM}^2\right)/\left(\frac{1}{\eta_V}\right)}\right\}\end{aligned} \tag{34}$$

If $\tilde{V} \notin \Omega_{\tilde{V}}$ or $\tilde{\varphi}_V \notin \Omega_{\tilde{\varphi}_V}$, we have $\dot{V}_V < 0$. Hence, $\tilde{V}$ and $\tilde{\varphi}_V$ are semi-globally uniformly ultimately bounded. Moreover, by choosing adequately large K_{V1}, the velocity tracking error $\tilde{V}$ can be arbitrarily small. This completes the proof. □

3.2 Altitude control design

Assumption 3 [21] $\partial F_h(\boldsymbol{x}, \delta_\mathrm{e})/\partial\delta_\mathrm{e}$ is continuous and positive.

Define altitude tracking error $\tilde{h} = h - h_\mathrm{ref}$ and the reference command $\gamma_\mathrm{d} = \arcsin\left(-\kappa_h\tilde{h}/V + \dot{h}_\mathrm{ref}/V\right)$ with $\kappa_h \in \Re^+$. When $\gamma \to \gamma_\mathrm{d}$, we have $\kappa_h\dot{\tilde{h}} = -\tilde{h}$, that is, $\dot{\tilde{h}}\tilde{h} = -\tilde{h}^2/\kappa_h \leq 0$ and $\tilde{h}$ can converge to zero when $t \to \infty$. In what follows, the design objective becomes to let $\gamma \to \gamma_\mathrm{d}$ by devising a hybrid control δ_e.

Define flight-path angle tracking error e_h and error function E_h as

$$\begin{cases} e_h = \gamma - \gamma_\mathrm{d} = z_1 - \gamma_\mathrm{d} \\ E_h = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \mu_h\right)^3 \int_0^t e_h(\tau)\mathrm{d}\tau \end{cases} \tag{35}$$

where $\mu_h \in \Re^+$ and the polynomial $(s + \mu_h)^3$ is Hurwitz.

Define

$$\begin{cases} \xi_1 = \dot{e}_h \\ \xi_2 = \dot{\xi}_1 \\ \xi_3 = \dot{\xi}_2 \end{cases} \tag{36}$$

Invoking (35) and (36), $\dot{E}_h$ is given by

$$\begin{aligned}\dot{E}_h &= \xi_3 + 3\mu_h\xi_2 + 3\mu_h^2\xi_1 + \mu_h^3 e_h \\ &= F_h(\boldsymbol{x}, \delta_\mathrm{e}) - \dddot{\gamma}_\mathrm{d} + 3\mu_h\xi_2 + 3\mu_h^2\xi_1 + \mu_h^3 e_h \end{aligned} \tag{37}$$

We design the following hybrid pseudocontrol υ_h:

$$\upsilon_h = \hat{F}_h(\boldsymbol{x}, \delta_{\mathrm{e}}) \tag{38}$$

where $\hat{F}_h(\boldsymbol{x}, \delta_{\mathrm{e}})$ represents the estimate of $F_h(\boldsymbol{x}, \delta_{\mathrm{e}})$ and the estimation error is formulated as $\delta_h = F_h(\boldsymbol{x}, \delta_{\mathrm{e}}) - \hat{F}_h(\boldsymbol{x}, \delta_{\mathrm{e}})$. Then Eq. (37) is rewritten as

$$\dot{E}_h = \delta_h + \hat{F}_h(\boldsymbol{x}, \delta_{\mathrm{e}}) - \dddot{\gamma}_{\mathrm{d}} + 3\mu_h\xi_2 + 3\mu_h^2\xi_1 + \mu_h^3 e_h \tag{39}$$

Assumption 4 Based on Assumption 3, we have that $\partial\hat{F}_h(\boldsymbol{x}, \delta_{\mathrm{e}})/\partial\delta_{\mathrm{e}}$ is continuous and positive.

We design the hybrid controller υ_h as

$$\upsilon_h = \upsilon_{h1} + \upsilon_{h2} - \upsilon_{h3} + \upsilon_{h4} \tag{40}$$

where $\upsilon_{h1} = \dddot{\gamma}_{\mathrm{d}}$ and $\upsilon_{h2} = -K_h E_h - (3\mu_h\xi_2 + 3\mu_h^2\xi_1 + \mu_h^3 e_h)$; $K_h \in \Re^+$ is a chosen parameter and υ_{h3} will be developed to handle δ_h; υ_{h4} is a robust term to be designed.

From (38), we can directly achieve the following altitude control effort:

$$\delta_{\mathrm{e}} = \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h) \tag{41}$$

Using (38) and (40), it is derived from (39) that

$$\begin{aligned}\dot{E}_h &= \delta_h + \upsilon_{h1} + \upsilon_{h2} - \upsilon_{h3} + \upsilon_{h4} - \dddot{\gamma}_{\mathrm{d}} \\ &\quad +3\mu_h\xi_2 + 3\mu_h^2\xi_1 + \mu_h^3 e_h \\ &= -K_h E_h + \delta_h - \upsilon_{h3} + \upsilon_{h4}\end{aligned} \tag{42}$$

with

$$\begin{aligned}\delta_h - \upsilon_{h3} &= F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_{h1} + \upsilon_{h2} - \upsilon_{h3} + \upsilon_{h4})) \\ &\quad -\hat{F}_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_{h1} + \upsilon_{h2} - \upsilon_{h3} + \upsilon_{h4})) - \upsilon_{h3} \\ &= F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_{h1} + \upsilon_{h2} - \upsilon_{h3} + \upsilon_{h4})) \\ &\quad -\upsilon_{h1} - \upsilon_{h2} - \upsilon_{h4}\end{aligned} \tag{43}$$

Define $\upsilon_{hl} \triangleq \upsilon_{h1} + \upsilon_{h2}$ and $\upsilon_h^* \triangleq \hat{F}_h(\boldsymbol{x}, F_h^{-1}(\boldsymbol{x}, \upsilon_{hl}))$. Then, we obtain $\upsilon_{hl} = F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h^*))$. Thus, Eq. (43) becomes

$$\begin{aligned}\delta_h - \upsilon_{h3} &= F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h)) - \upsilon_{hl} - \upsilon_{h4} \\ &= F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h)) - F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h^*)) - \upsilon_{h4}\end{aligned} \tag{44}$$

Employing Mean Value Theorem, we further get

$$\begin{aligned}\delta_h - \upsilon_{h3} &= F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h)) \\ &\quad -F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h^*)) - \upsilon_{h4} \\ &= F_h(\bar{\upsilon}_h)(\upsilon_h - \upsilon_h^*) - \upsilon_{h4} \\ &= F_h(\bar{\upsilon}_h)(\upsilon_{hl} - \upsilon_{h3} + \upsilon_{h4} \\ &\quad -\hat{F}_h(\boldsymbol{x}, F_h^{-1}(\boldsymbol{x}, \upsilon_{hl}))) - \upsilon_{h4} \\ &= F_h(\bar{\upsilon}_h)(\bar{\upsilon}_{hl} - \upsilon_{h3} + \upsilon_{h4}) - \upsilon_{h4}\end{aligned} \tag{45}$$

where $\bar{\upsilon}_{hl} = \upsilon_{hl} - \hat{F}_h(\boldsymbol{x}, F_h^{-1}(\boldsymbol{x}, \upsilon_{hl})) = F_h(\boldsymbol{x}, \hat{F}_h^{-1}(\boldsymbol{x}, \upsilon_h^*)) - \upsilon_h^*$ is an unknown term; $F_h(\bar{\upsilon}_h) = \left.\frac{\partial F_h}{\partial\delta_{\mathrm{e}}}\frac{\partial\delta_{\mathrm{e}}}{\partial\upsilon_h}\right|_{\upsilon_h=\bar{\upsilon}_h} = \left.\frac{\partial F_h}{\partial\delta_{\mathrm{e}}}\frac{\partial\hat{F}_h}{\partial\delta_{\mathrm{e}}}\right|_{\delta_{\mathrm{e}}=\hat{F}_h^{-1}(\boldsymbol{x},\upsilon_h)} > 0$ and $\bar{\upsilon}_h = \theta_h\upsilon_h + (1-\theta_h)\upsilon_h^*$ with $\theta_h \in [0, 1]$.

Substituting (45) into (42) yields

$$\dot{E}_h = -K_h E_h + F_h(\bar{\upsilon}_h)(\bar{\upsilon}_{hl} - \upsilon_{h3}) + F_h(\bar{\upsilon}_h)\upsilon_{h4} \tag{46}$$

To cancel $\bar{\upsilon}_{hl}$, we define $\upsilon_{h3} = \bar{\upsilon}_{hl}$. Owing to the fact that $\bar{\upsilon}_{hl}$ cannot be calculated since it is unknown, we also introduce one FWNN to approximate it.

$$\bar{\upsilon}_{hl} = \boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + \varepsilon_h \tag{47}$$

where $\boldsymbol{\ell}_h = \boldsymbol{x}$ is the input vector of FWNN and $\boldsymbol{W}_h^* \in \Re^N$ is an ideal weight vector; the formulation of $\boldsymbol{\psi}_h(\boldsymbol{\ell}_h)$ is the same as (11); ε_h is the approximation error and there exists a constant $\varepsilon_{h\mathrm{M}} \in \Re^+$ such that $|\varepsilon_h| \le \varepsilon_{h\mathrm{M}}$.

We select the following υ_{h3}:

$$\upsilon_{h3} = \frac{1}{2}E_h\hat{\varphi}_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \tag{48}$$

where $\hat{\varphi}_h$ denotes the estimation of $\varphi_h = \left\|\boldsymbol{W}_h^*\right\|^2 \in \Re$ and its adaptive law is designed as

$$\dot{\hat{\varphi}}_h = \frac{\eta_h}{2}E_h^2\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) - 2\hat{\varphi}_h \tag{49}$$

with $\eta_h \in \Re^+$.

Based on (47) and (48), $\delta_h - \upsilon_{h3}$ is described as

$$\bar{\upsilon}_{hl} - \upsilon_{h3} = \boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + \varepsilon_h - \frac{1}{2}E_h\hat{\varphi}_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \tag{50}$$

Then Eq. (46) leads to

$$\begin{aligned}\dot{E}_h &= -K_h E_h + F_h(\bar{\upsilon}_h)\boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + F_h(\bar{\upsilon}_h)\varepsilon_h \\ &\quad -\frac{1}{2}F_h(\bar{\upsilon}_h)E_h\hat{\varphi}_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + F_h(\bar{\upsilon}_h)\upsilon_{h4}\end{aligned} \tag{51}$$

The robust term υ_{h4} is chosen as

$$\upsilon_{h4} = -\frac{E_h}{2\rho_h^2} \tag{52}$$

where $\rho_h \in \Re^+$ is a constant to be designed.

Substituting (52) into (51), we obtain

$$\begin{aligned}\dot{E}_h &= -K_h E_h + F_h(\bar{\upsilon}_h)\boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + F_h(\bar{\upsilon}_h)\varepsilon_h \\ &\quad -\frac{1}{2}F_h(\bar{\upsilon}_h)E_h\hat{\varphi}_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) - F_h(\bar{\upsilon}_h)\frac{E_h}{2\rho_h^2}\end{aligned} \tag{53}$$

Theorem 2 *Consider the closed-loop system consisting of plant (3) under Assumptions 3 and 4 with controller (40) and adaptive law (49). Then, all the signals involved are semi-globally uniformly ultimately bounded.*

Proof Define estimate error $\tilde{\varphi}_h = \hat{\varphi}_h - \varphi_h$.

Define the following Lyapunov function:

$$V_h = \frac{E_h^2}{2} + \frac{\bar{F}_h\tilde{\varphi}_h^2}{2\eta_h} \tag{54}$$

where $\bar{F}_h \in \Re^+$ is a constant to be ingeniously chosen such that $\bar{F}_h \leq F_h(\bar{\upsilon}_h)$when $\tilde{\varphi}_h\dot{\tilde{\varphi}}_h \geq 0$ or else $\bar{F}_h > F_h(\bar{\upsilon}_h)$. Then, we get $\bar{F}_h\tilde{\varphi}_h\dot{\tilde{\varphi}}_h/\eta_h \leq F_h(\bar{\upsilon}_h)\tilde{\varphi}_h\dot{\tilde{\varphi}}_h/\eta_h$.

Taking time derivative along (54), $\dot{V}_h$ is given by

$$\dot{V}_h = E_h\dot{E}_h + \frac{\bar{F}_h}{\eta_h}\tilde{\varphi}_h\dot{\tilde{\varphi}}_h \leq E_h\dot{E}_h + \frac{F_h(\bar{\upsilon}_h)}{\eta_h}\tilde{\varphi}_h\dot{\tilde{\varphi}}_h \tag{55}$$

By utilizing (49) and (53), $\dot{V}_h$ further becomes

$$\begin{aligned}\dot{V}_h &\leq -K_h E_h^2 + F_h(\bar{\upsilon}_h)E_h\boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \\ &\quad +F_h(\bar{\upsilon}_h)E_h\varepsilon_h - \frac{1}{2}F_h(\bar{\upsilon}_h)E_h^2\hat{\varphi}_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \\ &\quad -F_h(\bar{\upsilon}_h)\frac{E_h^2}{2\rho_h^2} + \frac{1}{2}F_h(\bar{\upsilon}_h)E_h^2\tilde{\varphi}_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \\ &\quad -\frac{2F_h(\bar{\upsilon}_h)\tilde{\varphi}_h\hat{\varphi}_h}{\eta_h} \\ &= -K_h E_h^2 + F_h(\bar{\upsilon}_h)E_h\boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + F_h(\bar{\upsilon}_h)E_h\varepsilon_h \\ &\quad -\frac{1}{2}F_h(\bar{\upsilon}_h)E_h^2\varphi_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \\ &\quad -F_h(\bar{\upsilon}_h)\frac{E_h^2}{2\rho_h^2} - \frac{2F_h(\bar{\upsilon}_h)\tilde{\varphi}_h\hat{\varphi}_h}{\eta_h}\end{aligned} \tag{56}$$

Because $E_h\boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \leq \frac{E_h^2}{2}\left\|\boldsymbol{W}_h^{*\mathrm{T}}\boldsymbol{\psi}_h(\boldsymbol{\ell}_h)\right\|^2 + \frac{1}{2} = \frac{E_h^2}{2}\left\|\boldsymbol{W}_h^*\right\|^2\left\|\boldsymbol{\psi}_h(\boldsymbol{\ell}_h)\right\|^2 + \frac{1}{2} = \frac{E_h^2}{2}\varphi_h\boldsymbol{\psi}_h^T(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + \frac{1}{2}$, $2\tilde{\varphi}_h\hat{\varphi}_h \geq \tilde{\varphi}_h^2 - \varphi_h^2$ and $E_h\varepsilon_h \leq |E_h|\,\varepsilon_{hM} = 2\left|\frac{E_h}{2}\right|\varepsilon_{hM} \leq \frac{E_h^2}{4} + \varepsilon_{hM}^2$, Eq. (56) is further calculated as

$$\begin{aligned}\dot{V}_h &\leq -K_h E_h^2 + F_h(\bar{\upsilon}_h) \\ &\quad \times\left[\frac{E_h^2}{2}\varphi_h\boldsymbol{\psi}_h^T(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) + \frac{1}{2}\right] \\ &\quad +F_h(\bar{\upsilon}_h)\left(\frac{E_h^2}{4} + \varepsilon_{hM}^2\right) \\ &\quad -\frac{1}{2}F_h(\bar{\upsilon}_h)E_h^2\varphi_h\boldsymbol{\psi}_h^{\mathrm{T}}(\boldsymbol{\ell}_h)\boldsymbol{\psi}_h(\boldsymbol{\ell}_h) \\ &\quad -F_h(\bar{\upsilon}_h)\frac{E_h^2}{2\rho_h^2} - \frac{F_h(\bar{\upsilon}_h)}{\eta_h}\left(\tilde{\varphi}_h^2 - \varphi_h^2\right) \\ &= -K_h E_h^2 - \frac{F_h(\bar{\upsilon}_h)}{\eta_h}\tilde{\varphi}_h^2 + \frac{F_h(\bar{\upsilon}_h)E_h^2}{4} \\ &\quad -F_h(\bar{\upsilon}_h)\frac{E_h^2}{2\rho_h^2} + F_h(\bar{\upsilon}_h)\varepsilon_{hM}^2 \\ &\quad +\frac{F_h(\bar{\upsilon}_h)}{\eta_h}\varphi_h^2 + \frac{F_h(\bar{\upsilon}_h)}{2}\end{aligned} \tag{57}$$

Let $\rho_h = \sqrt{2}$ and then we conclude from (57) that

$$\begin{aligned}\dot{V}_h &\leq -K_h E_h^2 - \frac{F_h(\bar{\upsilon}_h)}{\eta_h}\tilde{\varphi}_h^2 + F_h(\bar{\upsilon}_h)\varepsilon_{hM}^2 \\ &\quad +\frac{F_h(\bar{\upsilon}_h)}{\eta_h}\varphi_h^2 + \frac{F_h(\bar{\upsilon}_h)}{2}\end{aligned} \tag{58}$$

Define the following compact sets:

$$\begin{aligned}\Omega_{E_h} &= \left\{E_h\,\middle|\,|E_h| \leq \sqrt{\left(F_h(\bar{\upsilon}_h)\varepsilon_{hM}^2 + \frac{F_h(\bar{\upsilon}_h)}{\eta_h}\varphi_h^2 + \frac{F_h(\bar{\upsilon}_h)}{2}\right)/K_h}\right\} \\ \Omega_{\tilde{\varphi}_h} &= \left\{\tilde{\varphi}_h\,\middle|\,|\tilde{\varphi}_h| \leq \sqrt{\left(\varepsilon_{hM}^2 + \frac{\varphi_h^2}{\eta_h} + \frac{1}{2}\right)/\left(\frac{1}{\eta_h}\right)}\right\}\end{aligned} \tag{59}$$

If $E_h \notin \Omega_{E_h}$ or $\tilde{\varphi}_h \notin \Omega_{\tilde{\varphi}_h}$, then $\dot{V}_h < 0$. Hence, E_h and $\tilde{\varphi}_h$ are semi-globally uniformly ultimately bounded. Furthermore, when we design infinitely large K_h, the velocity tracking error can converge to an arbitrarily small value. This is the end of the proof. □

Remark 2 For each subsystem, only one FWNN is applied to approximate the unknown term. Hence, disturbance rejection performance is guaranteed for the addressed controller and meanwhile the computational costs are low.

Remark 3 The existing studies [10,20–23] require that all the control design parameters K_{V1} and K_h must be greater than unknown positive constants. In this paper, novel roust terms υ_{V4} and υ_{h4} are developed such that $K_{V1} \in \Re^+$ and $K_h \in \Re^+$ can ensure the stability of closed-loop control system, which infers that larger stable regions than the previous methodologies [10,20–23] are achieved in this study.

4 Simulation results

This section presents the simulation test of the proposed hybrid controller (HB) in comparison with a traditional back-stepping control (TBC) approach [26] to show its superiority in velocity and altitude tracking performance. The input vectors of FWNNs are $\boldsymbol{\ell}_V = V$ and $\boldsymbol{\ell}_h = \boldsymbol{x} = [\gamma, \theta, Q]^{\mathrm{T}}$ with $V \in [7700\ \text{ft/s}, 8700\ \text{ft/s}]$, $\gamma \in [-5°, 5°]$, $\theta \in [-10°, 10°]$, $Q \in [-10°/\text{s}, 10°/\text{s}]$. Other design parameters are chosen as: $K_{V1} = 0.5$, $K_{V2} = 0.8$, $\eta_V = 0.1$, $\kappa_h = 12$, $\mu_h = 7$, $K_h = 50$, $\eta_h = 25$, N=10. To test the robustness performance, we suppose that all the aerodynamic coefficients (i.e., C_{T}^0, $C_{\mathrm{T}}^{\alpha^i}$, $C_{\mathrm{D}}^{\delta_{\mathrm{e}}^i}$, $C_{\mathrm{D}}^{\alpha^i}$, C_{D}^0, $C_{\mathrm{L}}^{\delta_{\mathrm{e}}}$, C_{L}^{α}, C_{L}^0, $C_{M,\alpha}^0$, $C_{M,\alpha}^{\alpha^i}$, c_{e}, N_1^0, $N_1^{\alpha^i}$, $N_2^{\alpha^i}$, N_2^0, $N_2^{\delta_{\mathrm{e}}}$ and $\beta_j\ (h, \bar{q})$, where i=1,2, j=1,2, …, 8.) are uncertain. A maximum uniform variation within 40% of the nominal value is considered by defining

$$C = \begin{cases} C_0, & 0\ \text{s} \le t \le 50\ \text{s} \\ C_0\left[1 + 0.4\sin(0.1\pi t)\right], & \text{else} \end{cases} \tag{60}$$

where C represents the value of uncertain coefficient mentioned above and C_0 means the nominal value of C.

The obtained simulation results, presented in Figs. 3, 4, 5, 6, 7 8, show that the proposed control strategy can provide better tracking of reference commends in contrast to TBC in the presence of seriously parametric uncertainties. Figures 3 and 4 reveal that velocity tracking error and altitude tracking error obtained by HB are smaller than the ones provided by TBC, and this superiority is more obvious when parameters are uncertain (50 s $< t \le$ 80 s). Hence, the exploit controller exhibits better robustness performance than TBC. For both controllers, Figs. 5 and 6 show that the responses of Φ and δ_{e}, γ, θ and Q are smooth and there is no high frequency chattering. It is noticed from Fig. 7 that the flexible states of applying HC are smoother than utilizing TBC. The estimations of $||\boldsymbol{W}_V^*||$ and $||\boldsymbol{W}_h^*||$, presented in Fig. 8, indicate that $||\hat{\boldsymbol{W}}_V||$ and $||\hat{\boldsymbol{W}}_h||$ can turn themselves along with the variations of uncertain parameters, which means that desired FWNN approximations are achieved and robustness performance can be guaranteed for the studied controllers.

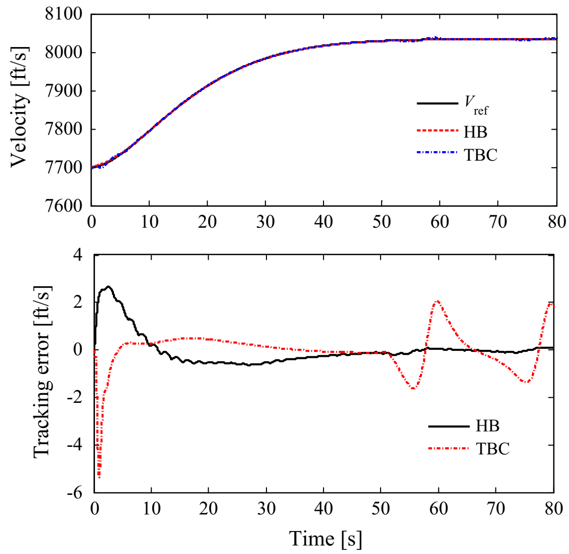

Fig. 3 Velocity tracking performance

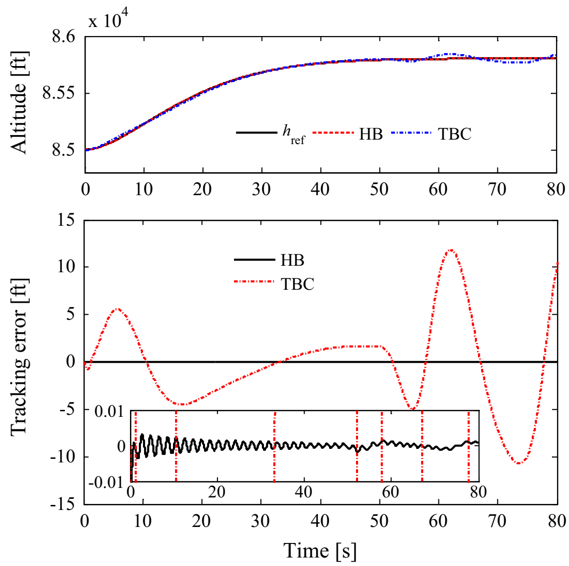

Fig. 4 Altitude tracking performance

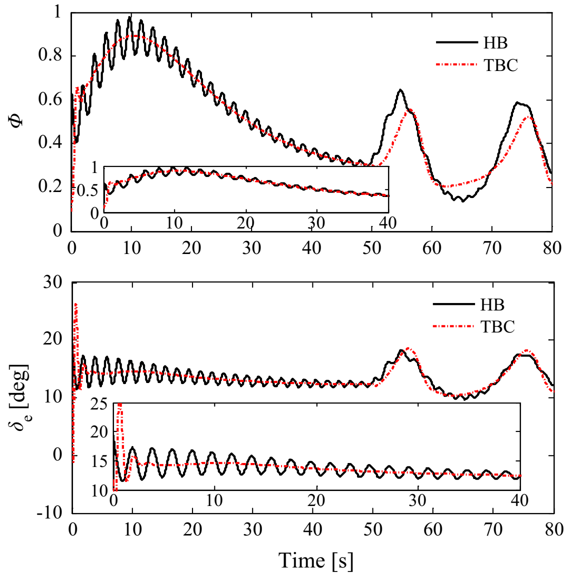

Fig. 5 Control inputs

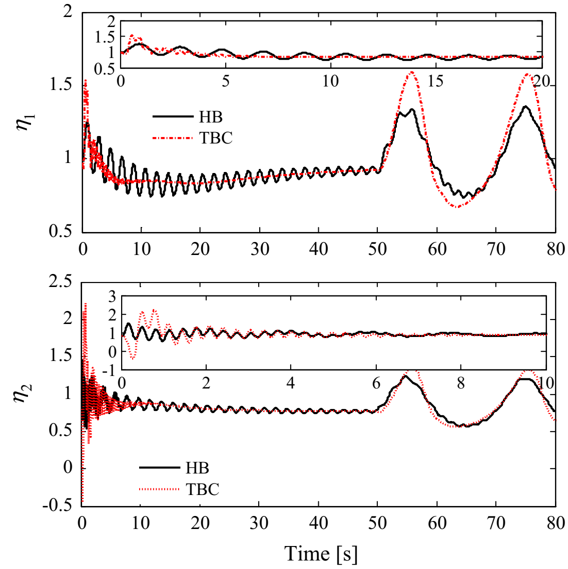

Fig. 7 The flexible states

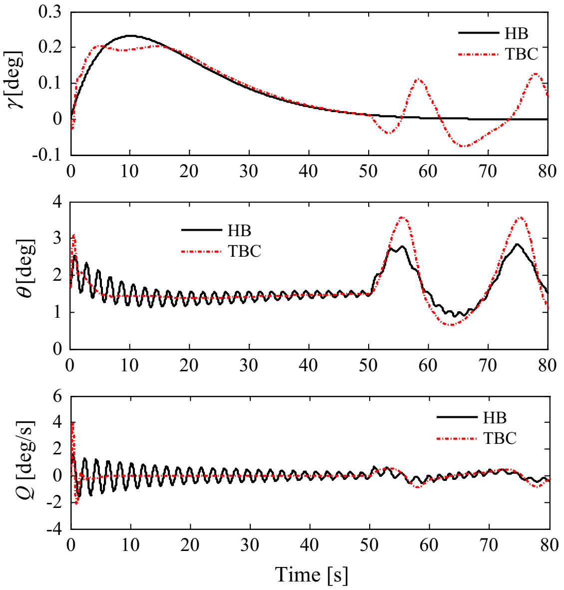

Fig. 6 Attitude angles

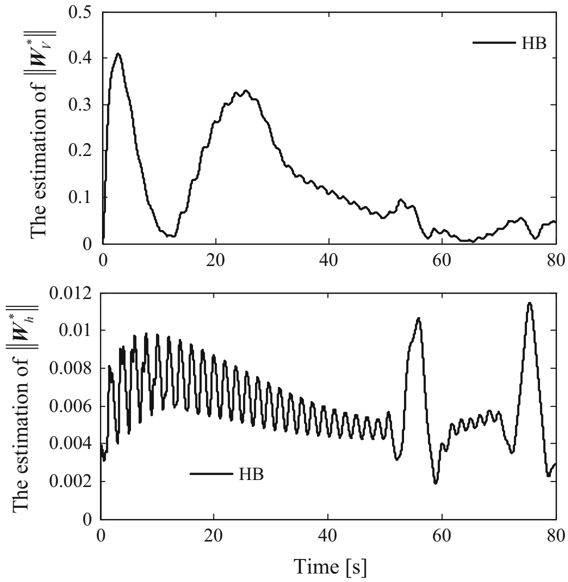

Fig. 8 Estimations of $||\boldsymbol{W}_V^*||$ and $||\boldsymbol{W}_h^*||$

5 Conclusions

This study investigates a direct nonaffine hybrid controller for HFVs. For velocity subsystem and altitude subsystem, direct nonaffine controllers are addressed without model simplifications, extending the previous indirect nonaffine control approaches. By a fusion of FWNN approximation and pseudocontrol strategy, both the nonaffine dynamics and parametric uncertainties are well handled. Further, advanced regulation laws are exploited for online learning parameters to reduce computational load and guarantee real-time performance. In addition, the stable regions of closed-loop control system are broadened via exploiting robust terms. Finally, numerical simulation results are presented to validate the effectiveness and superiority of the developed control scheme.

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Compliance with ethical standards

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References

1. Bertin, J., Cummings, R.: Fifty years of hypersonics: where we've been, where we're going. Progr. Aerosp. Sci. **39**, 511–536 (2003)
2. Bu, X., He, G., Wang, K.: Tracking control of air-breathing hypersonic vehicles with non-affine dynamics via improved neural back-stepping design. ISA Trans. **75**, 88–100 (2018)
3. Bolender, M.A., Doman, D.B.: Nonlinear longitudinal dynamical model of an air-breathing hypersonic vehicle. J. Spacecr. Rockets **44**(2), 374–387 (2007)
4. Morelli, E.: Flight-test experiment design for characterizing stability and control of hypersonic vehicles. J. Guidance Control Dyn. **32**(3), 949–959 (2009)
5. Bu, X., Wu, X., Huang, J., Wei, D.: A guaranteed transient performance-based adaptive neural control scheme with low-complexity computation for flexible air-breathing hypersonic vehicles. Nonlinear Dyn. **84**, 2175–2194 (2016)
6. Fiorentini, L., Serrani, A., Bolender, M., Doman, D.: Nonlinear robust adaptive control of flexible air-breathing hypersonic vehicles. J. Guidance Control Dyn. **32**(2), 401–416 (2009)
7. Parker, J., Serrani, A., Yurkovich, S., Bolender, M., Doman, D.: Control-oriented modeling of an air-breathing hypersonic vehicle. J. Guidance Control Dyn. **30**(3), 856–869 (2007)
8. Mu, C., Ni, Z., Sun, C., He, H.: Air-breathing hypersonic vehicle tracking control based on adaptive dynamic programming. IEEE Trans. Neural Netw. Learn. Syst. (2016). https://doi.org/10.1109/TNNLS.2016.2516948
9. Bu, X., Wu, X., Wei, D., Huang, J.: Neural-approximation-based robust adaptive control of flexible air-breathing hypersonic vehicles with parametric uncertainties and control input constraints. Inf. Sci. **346–347**, 29–43 (2016)
10. Xu, B.: Robust adaptive neural control of flexible hypersonic flight vehicle with dead-zone input nonlinearity. Nonlinear Dyn. **80**, 1509–1520 (2015)
11. Wu, H., Liu, Z., Guo, L.: Robust L_∞-gain fuzzy disturbance observer-based control design with adaptive bounding for a hypersonic vehicle. IEEE Trans. Fuzzy Syst. **22**(6), 1401–1412 (2014)
12. Shen, Q., Jiang, B., Cocquempot, V.: Fuzzy logic system-based adaptive fault-tolerant control for near-space vehicle attitude dynamics with actuator faults. IEEE Trans. Fuzzy Syst. **21**(2), 289–300 (2013)
13. Wu, H., Feng, S., Liu, Z., Guo, L.: Disturbance observer based robust mixed H_2/H_∞ fuzzy tracking control for hypersonic vehicles. Fuzzy Sets Syst. **306**, 118–136 (2017)
14. Zhang, Y., Li, R., Xue, T., Lei, Z.: Exponential sliding mode tracking control via back-stepping approach for a hypersonic vehicle with mismatched uncertainty. J. Frankl. Inst. **353**, 2319–2343 (2016)
15. Wang, J., Wu, Y., Dong, X.: Recursive terminal sliding mode control for hypersonic flight vehicle with sliding mode disturbance observer. Nonlinear Dyn. **81**, 1489–1510 (2015)
16. An, H., Liu, J., Wang, C., Wu, L.: Approximate back-stepping fault-tolerant control of the flexible air-breathing hypersonic vehicle. IEEE/ASME Trans. Mechatron. **21**(3), 1680–1691 (2016)
17. Wu, G., Meng, X.: Nonlinear disturbance observer based robust backstepping control for a flexible air-breathing hypersonic vehicle. Aerosp. Sci. Technol. **54**, 174–182 (2016)
18. Bu, X., Wu, X., Huang, J., Ma, Z., Zhang, R.: Minimal-learning-parameter based simplified adaptive neural back-stepping control of flexible air-breathing hypersonic vehicles without virtual controllers. Neurocomputing **175**, 816–825 (2016)
19. An, H., Liu, J., Wang, C., Wu, L.: Disturbance observer-based antiwindup control for air-breathing hypersonic vehicles. IEEE Trans. Ind. Electron. **63**(5), 3038–3049 (2016)
20. Bu, X., Xiao, Y., Wang, K.: A prescribed performance control approach guaranteeing small overshoot for air-breathing hypersonic vehicles via neural approximation. Aerosp. Sci. Technol. **71**, 485–498 (2017)
21. Bu, X., Wang, Q., Zhao, Y., He, G.: Concise neural non-affine control of air-breathing hypersonic vehicles subject to parametric uncertainties. Int. J. Aerosp. Eng. (2017). https://doi.org/10.1155/2017/1374932
22. Bu, X., Wei, D., Wu, X., Huang, J.: Guaranteeing preselected tracking quality for air-breathing hypersonic non-affine models with an unknown control direction via concise neural control. J. Frankl. Inst. **353**, 3207–3232 (2016)
23. Bu, X.: Guaranteeing prescribed output tracking performance for air-breathing hypersonic vehicles via non-affine back-stepping control design. Nonlinear Dyn. **91**, 525–538 (2018)
24. Bu, X., Xiao, Y.: Prescribed performance-based low-computational fuzzy control of a hypersonic vehicle using non-affine models. Adv. Mech. Eng. **10**, 1–12 (2018)
25. Davanipour, M., Tehrani, R., Shabani-nia F.: Self-turning PID control of liquid level system based on fuzzy wavelet neural network model. In: 24th Iranian Conference on Electrical Engineering (ICEE), 10-12 May 2016. https://doi.org/10.1109/IranianCEE.2016.7585575
26. Bu, X., Wu, X., Zhang, R.: Tracking differentiator design for the robust backstepping control of a flexible air-breathing hypersonic vehicle. J. Frankl. Inst. **352**(4), 1739–1765 (2015)