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Global robust tracking control for a class of cascaded nonlinear systems using a reduced-order extended state observer

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Abstract This article considers the global robust tracking control problem via output feedback for a class of nonlinear systems subjected to dynamic uncertainties and nonvanishing disturbances. A reduced-order extended state observer is firstly designed to estimate the unmeasured states and to compensate the external disturbances. Then, we propose a deadzone-based tracking control scheme, which could make the system output track any desired reference signal with small tracking error arbitrarily, and keep all signals in the closed-loop system bounded. It is shown that the parameter drift instability may be avoided using the proposed method through a numerical example. Finally, a fan speed system is used to demonstrate the effectiveness of the control strategy.

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1 Introduction

The global output feedback tracking control for nonlinear systems is an important and actively studied problem. The asymptotic output tracking aims to design a feedback law, such that the tracking error between the plant output and a prescribed smooth reference signal converges to zero as time approaches infinity; see, for instance, [1–4]. In many practical applications, because of the severe uncertainties or the less information on the reference signal, the asymptotic tracking is hardly realized. In such a case, the practical tracking could be an alternative with tracking error asymptotic to a ball of arbitrary prescribed radius $\lambda > 0$. The constant λ is called the prescribed accuracy, and hence, practical tracking is also known as the λ-tracking. Mainly because of weaker conditions and less information on reference signals, the practical tracking has received a lot of attention during the recent years, such as [5–13], and the references therein.

It is known that the disturbances would deteriorate the control performance and even destabilize the whole system. As noted in [14], the nonvanishing disturbances may result in the parameter drift instability. As a result, the disturbance rejection is a fundamental issue in control theory [15–28]. The active disturbance rejection control (ADRC) is a new control strategy proposed recently by Han [29] in dealing with the systems with large uncertainty. Extended state observer (ESO) is the most important part of ADRC and plays an important role in estimating the unmeasured states as well as the external disturbances. Through some kind of ESO, the "total disturbance" is considered as an extended state and then the estimation of the "total disturbance" is canceled in feedback. This estimation/cancelation feature makes this technique capable of eliminating the severe uncertainties and recently becomes an effective tool in engineering applications [30].

In this article, we use the idea of ESO to investigate the global robust practical tracking control problem for a class of nonlinear systems with nonvanishing disturbances. Technically, the external disturbances are firstly considered as an extended state, and then a reduced-order extended state observer (RESO) is constructed for the augmented system. In the procedure of control design, a deadzone together with some kind of pseudosign function in [9] is inserted into the update law of the dynamic gain to avoid its infinite increasing. Finally, we use two examples to illustrate our control scheme. It is noted that previous work on this practical system concentrates on the set-point tracking control, that is the tracking of *constant* reference signals like [28,31–33]. The results are improved in [34], where the *sine*-type time-varying references are allowed in the context of output regulation. Here, in lieu of the assumption that the reference signals belong to a family of trajectories like the *constant* [28,31–33] or *sine* type in [34], we propose a novel speed controller realizing the speed tracking control for any bounded continuously differentiable reference signals.

Our main work consists of the following aspects.

(i) By designing a RESO, the practical tracking control problem is solved for a class of nonlinear cascaded system (1) in the presence of nonlinear dynamic uncertainties and less restrictive nonvanishing disturbances.
(ii) A numerical example is provided here to demonstrate that the parameter drift instability may happen due to the additive external nonvanishing disturbances. This phenomenon can be avoided using the control scheme developed in this paper.
(iii) As an application in practical systems, it is shown that the proposed control scheme could realize the practical tracking for the fan speed system in the presence of unknown load/drag torque with unmeasured armature current. This further improves the existing results [28,31–34].

2 Problem statement and assumptions

In this article, we study the following class of cascade nonlinear systems

$$\begin{aligned} \dot{z} &= \eta(z, y, t), \\ \dot{x}_i &= x_{i+1} + \Delta_i(z, y, u),\ i = 1, \ldots, n-1, \\ \dot{x}_n &= u + d(t) + \Delta_n(z, y, u), \\ y &= x_1, \end{aligned} \tag{1}$$

where $z \in R^r$ represents the dynamic uncertainty, $x = (x_1, \ldots, x_n) \in R^n$ are the system states, u is the input, y is the output. The time-varying and continuous function $d(t)$ represents unknown parameters or disturbances. For existence and uniqueness of solutions, the uncertain functions $\eta(\cdot)$ and $\Delta_i(\cdot)(1 \le i \le n)$ are locally Lipschitz in (z, y). The states $(x_2, \ldots, x_n)$ as well as z of the z-subsystem are not assumed to be measurable. For the controlled system (1), our control task is to solve the global practical tracking problem.

Throughout the paper, we make the following assumptions.

Assumption 1 There exists a continuously differentiable, positive definite proper function $V_0(z)$, and positive constants $c_i (i = 1, \ldots, 4)$, such that

$$c_1 \|z\|^2 \le V_0(z) \le c_2 \|z\|^2, \tag{2}$$

$$\frac{\partial V_0}{\partial z} \eta(z, y, t) \le -c_3 \|z\|^2 + c_4 \left(1 + |y|^{p_0}\right), \tag{3}$$

where p_0 is any known integer in N^*(the set of natural numbers).

Assumption 2 For $i = 1, \ldots, n$, there exist unknown positive constants δ_{i1} and δ_{i2}, such that

$$|\Delta_i(z, y, u)| \le \delta_{i1} \|z\|^k + \delta_{i2} \left(1 + |y|^{p_i}\right), \tag{4}$$

where k and p_i are any known integers in N^*.

Assumption 3 The unknown external disturbance $d(t)$ together with its first time derivative $\dot{d}(t)$ satisfies the following properties:

$$|d(t)| \le \bar{d}, \quad \left|\dot{d}(t)\right| \le \bar{d}, \tag{5}$$

where $\bar{d} > 0$ is a unknown constant.

Assumption 4 The reference signal $y_d(t)$ is continuously differentiable and bounded. Specifically, there exists an unknown constant $\varpi > 0$ satisfying

$$|y_d(t)| \leq \varpi, \quad |\dot{y}_d(t)| \leq \varpi, \ \forall\, t \geq 0. \tag{6}$$

Remark 1 Assumption 1 is an ISpS-like condition, and the similar assumption can be found in [35,36]. The structural information of the z subsystem is unknown, and only the constant p_0 is known apriori. In [14], the global regulation problem is studied under the assumption of $\Delta_i(\cdot)$ only depending on (z, y) and vanishing at the origin. Most recently, in [37], by skillfully inserting some kind of deadzone function into the control design, we realize the global practical tracking control under the nonvanishing nonlinearities. However, it does not involve the input disturbances in [37]. Here, we further consider the disturbance rejection problem in the presence of input additive disturbances with the help of the extended state observer technique (ESO). It will be shown in Example 4.1 that this is an interesting control problem.

Remark 2 Assumption 3 shows that the external disturbance $d(t)$ and its time derivative $\dot{d}(t)$ are bounded by some unknown constants. This assumption is much weaker than the existing closely related results, such as [15,19,20,27,28]. For example, $d(t)$ is not required to be L_2 as in [19]. This relaxation allows the constant disturbances such as [27]. The disturbances $d(t)$ and $\dot{d}(t)$ are not assumed to have any vanishing properties like in [20]. In addition, we also remove the restriction of $\dot{d}(t)\in L_2$ in [28]. Here, it does only require that the unknown terms of $d(t)$ and $\dot{d}(t)$ are bounded. This is a more relaxed version of noise signals [30].

Denote $p = \max\{(k+1)p_0, p_1, \ldots, p_n\}$ and the tracking error $\xi_1 = y - y_d$, then we have the following lemma, whose proof is provided in Appendix A.

Lemma 1 *For $i = 1, \ldots, n$, there exist positive constants δ_i^* such that*

$$|\Delta_i(z, y, u)| \leq \delta_i^* \|z\|^{k+1} + \delta_i^* \left(1 + |\xi_1|^p\right). \tag{7}$$

The following kind of deadzone function is used in our paper, which is helpful to depress the parameter drift instability [14]. For arbitrary prescribed $\lambda > 0$, define the deadzone function $d_{\frac{\lambda}{2}}(\cdot) : R \to [0, \infty)$ parameterized by λ as follows

$$d_{\frac{\lambda}{2}}(s) = \begin{cases} |s| - \frac{\lambda}{2}, & \text{if } |s| > \frac{\lambda}{2}, \\ 0, & \text{if } |s| \leq \frac{\lambda}{2}. \end{cases} \tag{8}$$

It is known that $d_{\frac{\lambda}{2}}(s) \geq 0$ is continuous but not differentiable at $\pm\frac{\lambda}{2}$. Nonetheless, with $i \geq 2$ an integer, $d^i_{\frac{\lambda}{2}}(s)$ is continuously differentiable on R.

3 Robust tracking control design and main result

In this section, we develop a systematic design procedure using the backstepping method.

3.1 Robust tracking control design

We construct the following RESO.

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} + L_{i+1} y - L_i(\hat{x}_2 + L_2 y), \\ &\quad i = 2, \ldots, n-1 \\ \dot{\hat{x}}_n &= u + \hat{x}_{n+1} + L_{n+1} y - L_n(\hat{x}_2 + L_2 y), \\ \dot{\hat{x}}_{n+1} &= -L_{n+1}(\hat{x}_2 + L_2 y), \end{aligned} \tag{9}$$

where $L_i (i = 2, \ldots, n)$ are design parameters. Defining the error variables

$$\tilde{x}_i = x_i - \hat{x}_i - L_i y, \ i = 2, \ldots, n+1, \tag{10}$$

together with (1) and (9), one has

$$\begin{aligned} \dot{\tilde{x}}_i &= \tilde{x}_{i+1} - L_i \tilde{x}_2 + \Delta_i(z, y, u) \\ &\quad - L_i \Delta_1(z, y, u), \ i = 2, \ldots, n, \\ \dot{\tilde{x}}_{n+1} &= -L_{n+1} \tilde{x}_2 - L_{n+1} \Delta_1(z, y, u) + \dot{d}(t), \end{aligned} \tag{11}$$

which can be further written into the compact form

$$\dot{\tilde{x}} = A \tilde{x} + \Delta(z, y, u) + b\, \dot{d}(t), \tag{12}$$

with

$$\tilde{x} = \begin{bmatrix} \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \\ \tilde{x}_{n+1} \end{bmatrix}, A = \begin{bmatrix} -L_2 & \\ \vdots & I_{n-1} \\ -L_{n+1} & 0 \ \cdots \ 0 \end{bmatrix},$$

$$\Delta(\cdot) = \begin{bmatrix} \Delta_2(\cdot) - L_2 \Delta_1(\cdot) \\ \vdots \\ \Delta_n(\cdot) - L_n \Delta_1(\cdot) \\ -L_{n+1}\, \Delta_1(\cdot) \end{bmatrix}, b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Choose parameters $L_i (i = 2, \ldots, n+1)$ such that A is asymptotically stable, and then there exists a positive definite matrix Q satisfying

$$A^{\mathrm{T}} Q + Q A = -I. \tag{13}$$

For the error system (12), we have the following result. The proof is provided in Appendix B.

Lemma 2 *Choose the positive definite function $V_{\tilde{x}} = \tilde{x}^{\mathrm{T}} Q \tilde{x}$, then its time derivative along (12) satisfies*

$$\dot{V}_{\tilde{x}} \leq -\frac{1}{2}\tilde{x}^{\mathrm{T}}\tilde{x} + \Theta_{\tilde{x},z}\|z\|^{2(k+1)} + \Theta_{\tilde{x},\xi_1}\left(1 + d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right), \quad (14)$$

where $\Theta_{\tilde{x},z}$ and $\Theta_{\tilde{x},\xi_1}$ are two positive constants.

In what follows, the controller design is provided in a step-wise strategy.

Step 1 Consider the function V_1 defined by

$$V_1 = \frac{1}{p+1} d_{\frac{\lambda}{2}}^{p+1}(\xi_1). \quad (15)$$

It can be verified that the time derivation of V_1 satisfies

$$\dot{V}_1 = d_{\frac{\lambda}{2}}^{p}(\xi_1)\mathrm{sign}(\xi_1) \times \left(\tilde{x}_2 + \hat{x}_2 + L_2 y + \Delta_1(z, y, u) - \dot{y}_d\right). \quad (16)$$

In view of $d_{\frac{\lambda}{2}}^{p}(\xi_1) \geq 0$, from Lemma 1 and Assumption 4, we have

$$d_{\frac{\lambda}{2}}^{p}(\xi_1)\mathrm{sign}(\xi_1)\left(\tilde{x}_2 + \Delta_1(z, y, u) - \dot{y}_d\right) \leq d_{\frac{\lambda}{2}}^{p}(\xi_1)\Big(\|\tilde{x}\| + \delta_1^*\|z\|^{k+1} + \delta_1^*(1 + |\xi_1|^p) + \varpi\Big). \quad (17)$$

Applying the similar method in proving Lemma 1, one can find a positive constant ρ_1 such that

$$\delta_1^*(1 + |\xi_1|^p) + \varpi \leq \rho_1(1 + \xi_1^{2p}), \quad (18)$$

and furthermore

$$d_{\frac{\lambda}{2}}^{p}(\xi_1)\left(\delta_1^*(1 + |\xi_1|^p) + \varpi\right) \leq \rho_1 d_{\frac{\lambda}{2}}^{p}(\xi_1)(1 + \xi_1^{2p}). \quad (19)$$

As a result, (16) becomes

$$\dot{V}_1 \leq d_{\frac{\lambda}{2}}^{p}(\xi_1)\mathrm{sign}(\xi_1)(\hat{x}_2 + L_2 y) + \rho_1 d_{\frac{\lambda}{2}}^{p}(\xi_1)(1 + \xi_1^{2p}) + d_{\frac{\lambda}{2}}^{p}(\xi_1)\left(\|\tilde{x}\| + \delta_1^*\|z\|^{k+1}\right). \quad (20)$$

We take $\hat{x}_2 + L_2 y$ as the control input, and ϑ_1 is the virtual control law with the error variable $\xi_2 = \hat{x}_2 + L_2 y - \vartheta_1$. Choose the first virtual control law and the updating law of the form

$$\vartheta_1 = -\chi\left(1 + \xi_1^{2p}\right)\mathrm{sig}_{\frac{\lambda}{2},n}(\xi_1), \quad (21)$$

$$\dot{\chi} = \Gamma d_{\frac{\lambda}{2}}^{p}(\xi_1)\left(1 + \xi_1^{2p}\right), \quad (22)$$

where $\Gamma > 0$ is a design constant, and $\mathrm{sig}_{\frac{\lambda}{2},n}(\cdot)$ is a pseudosign function as defined in [9]. It can be directly verified that

$$d_{\frac{\lambda}{2}}^{p}(\xi_1)\mathrm{sig}_{\frac{\lambda}{2},n}(\xi_1)\mathrm{sign}(\xi_1) = d_{\frac{\lambda}{2}}^{p}(\xi_1). \quad (23)$$

Considering

$$d_{\frac{\lambda}{2}}^{p}(\xi_1)\mathrm{sign}(\xi_1)\xi_2 \leq d_{\frac{\lambda}{2}}^{p}(\xi_1)|\xi_2|, \quad (24)$$

then with (21)–(24), we obtain

$$\dot{V}_1 \leq -\frac{1}{\Gamma}(\chi - \rho_1)\dot{\chi} + d_{\frac{\lambda}{2}}^{p}(\xi_1)\left(|\xi_2| + \|\tilde{x}\| + \delta_1^*\|z\|^{k+1}\right). \quad (25)$$

Step 2 Let $V_2 = \frac{1}{2}\xi_2^2$. Taking the time derivative of V_2 yields

$$\dot{V}_2 = \xi_2\left(\hat{x}_3 + L_3 y + L_2(\tilde{x}_2 + \Delta_1(z, y, u)) - \frac{\partial\vartheta_1}{\partial\chi}\dot{\chi}\right) - \xi_2\frac{\partial\vartheta_1}{\partial\xi_1}\left(\tilde{x}_2 + \hat{x}_2 + L_2 y + \Delta_1(z, y, u) - \dot{y}_d\right). \quad (26)$$

From Lemmas 1–2, by completing the squares, the uncertain terms in (26) can be handled as follows

$$\xi_2 L_2\tilde{x}_2 \leq \frac{1}{4}\|\tilde{x}\|^2 + \xi_2^2 L_2^2, \quad (27)$$

$$-\xi_2\frac{\partial\vartheta_1}{\partial\xi_1}\tilde{x}_2 \leq \frac{1}{4}\|\tilde{x}\|^2 + \xi_2^2\left(\frac{\partial\vartheta_1}{\partial\xi_1}\right)^2, \quad (28)$$

$$\xi_2 L_2\Delta_1(z, y, u) \leq \frac{1}{4}\delta_1^{*2}\|z\|^{2(k+1)} + \xi_2^2 L_2^2 + \frac{1}{4}\delta_1^{*2} + 2\xi_2^2 L_2^2(1 + \xi_1^{2p}), \quad (29)$$

$$-\xi_2\frac{\partial\vartheta_1}{\partial\xi_1}\left(\Delta_1(z, y, u) - \dot{y}_d\right) \leq \frac{1}{4}\delta_1^{*2}\|z\|^{2(k+1)} + \frac{1}{4}\delta_1^{*2} + \frac{1}{4}\varpi^2 + 2\xi_2^2\left(\frac{\partial\vartheta_1}{\partial\xi_1}\right)^2 + 2\xi_2^2\left(\frac{\partial\vartheta_1}{\partial\xi_1}\right)^2(1 + \xi_1^{2p}). \quad (30)$$

Considering $d_{\frac{\lambda}{2}}^{p}(\xi_1)$ may be only a continuous function, inspired by [6], we choose a known smooth function $\varphi(\xi_1)$ such that

$$\varphi(\xi_1) \geq \Gamma d_{\frac{\lambda}{2}}^{p}(\xi_1)(1 + \xi_1^{2p}), \quad (31)$$

which results in

$$-\xi_2\frac{\partial\vartheta_1}{\partial\chi}\dot{\chi} \le |\xi_2||\frac{\partial\vartheta_1}{\partial\chi}|\varphi(\xi_1) \le \frac{1}{4}+\xi_2^2\left(\frac{\partial\vartheta_1}{\partial\chi}\right)^2\varphi^2(\xi_1). \tag{32}$$

Define $\psi_2(\chi,\xi_1,\hat{x}_2)=\xi_2\Big(2L_2^2\xi_2(2+\xi_1^{2p})+(\frac{\partial\vartheta_1}{\partial\xi_1})^2(3+2(1+\xi_1^{2p}))+(\frac{\partial\vartheta_1}{\partial\chi})^2\varphi^2(\xi_1)\Big)$. Consequently, we get

$$\begin{aligned}&\xi_2L_2(\tilde{x}_2+\Delta_1(z,y,u))-\xi_2\frac{\partial\vartheta_1}{\partial\chi}\dot{\chi}\\&\quad-\xi_2\frac{\partial\vartheta_1}{\partial\xi_1}(\tilde{x}_2+\Delta_1(z,y,u)-\dot{y}_d)\\&\quad\le\frac{1}{4}+\frac{2}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2+\frac{2}{4}\delta_1^{*2}\|z\|^{2(k+1)}\\&\quad\quad+\frac{2}{4}\|\tilde{x}\|^2+\xi_2\psi_2(\chi,\xi_1,\hat{x}_2).\end{aligned} \tag{33}$$

Therefore, in view of (33), (26) becomes

$$\begin{aligned}\dot{V}_2\le{}&\xi_2\Big(\hat{x}_3+L_3y-\frac{\partial\vartheta_1}{\partial\xi_1}(\hat{x}_2+L_2y)\\&+\psi_2(\chi,\xi_1,\hat{x}_2)\Big)+\frac{1}{4}+\frac{2}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2\\&+\frac{2}{4}\delta_1^{*2}\|z\|^{2(k+1)}+\frac{2}{4}\|\tilde{x}\|^2.\end{aligned} \tag{34}$$

Choose the virtual control law ϑ_2 of the form

$$\begin{aligned}\vartheta_2={}&-\mu_2\xi_2-L_3y+\frac{\partial\vartheta_1}{\partial\xi_1}(\hat{x}_2+L_2y)\\&-\psi_2(\chi,\xi_1,\hat{x}_2).\end{aligned} \tag{35}$$

Let $\xi_3=\hat{x}_3-\vartheta_2$, and a direct substitution yields to

$$\begin{aligned}\dot{V}_2\le{}&-\mu_2\xi_2^2+\xi_2\xi_3+\frac{1}{4}+\frac{2}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2\\&+\frac{2}{4}\|\tilde{x}\|^2+\frac{1}{4}\delta_1^{*2}\|z\|^{2(k+1)}.\end{aligned} \tag{36}$$

Define $\Omega_{c,2}=\frac{1}{4}+\frac{2}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2$, $\Omega_{\tilde{x},2}=\frac{2}{4}$, $\Omega_{z,2}=\frac{1}{4}\delta_1^{*2}$, and we have

$$\begin{aligned}\dot{V}_2\le{}&-\mu_2\xi_2^2+\Omega_{c,2}+\Omega_{\tilde{x},2}\|\tilde{x}\|^2\\&+\Omega_{z,2}\|z\|^{2(k+1)}+\xi_2\xi_3.\end{aligned} \tag{37}$$

Step i ($3\le i\le n$): By an induction argument, assuming that the virtual control laws $\vartheta_j(1\le j\le i-1)$ have been designed, and with $\xi_{j+1}=\hat{x}_{j+1}-\vartheta_j(1\le j\le i-1)$, the time derivative of the following function

$$V_{i-1}=\sum_{j=2}^{i-1}\frac{1}{2}\xi_j^2 \tag{38}$$

satisfies

$$\begin{aligned}\dot{V}_{i-1}\le{}&-\sum_{j=2}^{i-1}\mu_j\xi_j^2+\xi_{i-1}\xi_i+\Omega_{c,i-1}\\&+\Omega_{\tilde{x},i-1}\|\tilde{x}\|^2+\Omega_{z,i-1}\|z\|^{2k},\end{aligned} \tag{39}$$

with design parameters $\mu_j>0(2\le j\le i-1)$.

In the sequel, one shows that the property (39) also holds in *Step i*. Let $\xi_{i+1}=\hat{x}_{i+1}-\vartheta_i$, and we consider the function

$$V_i=V_{i-1}+\frac{1}{2}\xi_i^2. \tag{40}$$

In view of $\dot{\xi}_i$, as in *Step 2*, by completing the squares, the following holds

$$-\xi_i\frac{\partial\vartheta_{i-1}}{\partial\chi}\dot{\chi}\le\frac{1}{4}+\xi_i^2\left(\frac{\partial\vartheta_{i-1}}{\partial\chi}\right)^2\varphi^2(\xi_1), \tag{41}$$

$$-\xi_i\frac{\partial\vartheta_{i-1}}{\partial\xi_1}\tilde{x}_2\le\frac{1}{4}\|\tilde{x}\|^2+\xi_i^2\left(\frac{\partial\vartheta_{i-1}}{\partial\xi_1}\right)^2, \tag{42}$$

and

$$\begin{aligned}&-\xi_i\frac{\partial\vartheta_{i-1}}{\partial\xi_1}(\Delta_1(z,y,u)-\dot{y}_d)\\&\quad\le\frac{1}{4}\delta_1^{*2}\|z\|^{2(k+1)}+\frac{1}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2\\&\quad\quad+2\xi_i^2\left(\frac{\partial\vartheta_{i-1}}{\partial\xi_1}\right)^2+2\xi_i^2\left(\frac{\partial\vartheta_{i-1}}{\partial\xi_1}\right)^2(1+\xi_1^{2p}).\end{aligned} \tag{43}$$

Define $\psi_i(\chi,\xi_1,\hat{x}_2,\ldots,\hat{x}_i)=\xi_i\Big((\frac{\partial\vartheta_{i-1}}{\partial\xi_1})^2(3+2(1+\xi_1^{2p}))+(\frac{\partial\vartheta_{i-1}}{\partial\chi})^2\varphi^2(\xi_1)\Big)$, and one has

$$\begin{aligned}&-\xi_i\frac{\partial\vartheta_{i-1}}{\partial\chi}\dot{\chi}-\xi_i\frac{\partial\vartheta_{i-1}}{\partial\xi_1}(\tilde{x}_2+\Delta_1(z,y,u)-\dot{y}_d)\\&\quad\le\frac{1}{4}+\frac{1}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2+\frac{1}{4}\|\tilde{x}\|^2\\&\quad\quad+\frac{1}{4}\delta_1^{*2}\|z\|^{2(k+1)}+\xi_i\psi_i(\chi,\xi_1,\hat{x}_2,\ldots,\hat{x}_i).\end{aligned} \tag{44}$$

Accordingly, the function V_i satisfies

$$\begin{aligned}\dot{V}_i\le{}&-\sum_{j=2}^{i-1}\mu_j\xi_j^2+\xi_i\Big(\vartheta_i+\xi_{i-1}-L_i(\hat{x}_2+L_2y)\\&+L_{i+1}y-\frac{\partial\vartheta_{i-1}}{\partial\xi_1}(\hat{x}_2+L_2y)\\&-\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j+\psi_i(\chi,\xi_1,\hat{x}_2,\ldots,\hat{x}_i)\Big)\\&+\Omega_{z,i-1}\|z\|^{2(k+1)}+\frac{1}{4}\delta_1^{*2}\|z\|^{2(k+1)}\end{aligned}$$

$$+\Omega_{\tilde{x},i-1}\|\tilde{x}\|^2+\frac{1}{4}\|\tilde{x}\|^2+\Omega_{c,i-1}+\frac{1}{4}+\frac{1}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2+\xi_i\xi_{i+1}. \quad (45)$$

Take the virtual control ϑ_i as

$$\vartheta_i=-\xi_{i-1}-\mu_i\xi_i-L_{i+1}y+L_i(\hat{x}_2+L_2y)+\frac{\partial\vartheta_{i-1}}{\partial\xi_1}(\hat{x}_2+L_2y)+\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j-\psi_i(\chi,\xi_1,\hat{x}_2,\ldots,\hat{x}_i), \quad (46)$$

with $\mu_i>0$, then we get

$$\dot{V}_i\le-\sum_{j=2}^{i}\mu_j\xi_j^2+\Omega_{c,i}+\Omega_{\tilde{x},i}\|\tilde{x}\|^2+\Omega_{z,i}\|z\|^{2(k+1)}+\xi_i\xi_{i+1}, \quad (47)$$

with $\Omega_{c,i}=\Omega_{c,i-1}+\frac{1}{4}+\frac{1}{4}\delta_1^{*2}+\frac{1}{4}\varpi^2$, $\Omega_{\tilde{x},i}=\Omega_{\tilde{x},i-1}+\frac{1}{4}$, $\Omega_{z,i}=\Omega_{z,i-1}+\frac{1}{4}\delta_1^{*2}$.

In particular, when $i=n$, the real control input u appears. Similar to (46) (using $i=n$, and $u+\hat{x}_{n+1}+L_{n+1}y=\vartheta_n$), we obtain the actual control law of the form

$$u=-\xi_{n-1}-\mu_n\xi_n-\hat{x}_{n+1}-L_{n+1}y+L_n(\hat{x}_2+L_2y)+\sum_{j=1}^{n-1}\frac{\partial\vartheta_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j+\frac{\partial\vartheta_{n-1}}{\partial\xi_1}(\hat{x}_2+L_2y)-\psi_n(\chi,\xi_1,\hat{x}_2,\ldots,\hat{x}_n), \quad (48)$$

such that the time derivative of the function

$$V_n=\sum_{j=2}^{n}\frac{1}{2}\xi_j^2 \quad (49)$$

satisfies

$$\dot{V}_n\le-\sum_{j=2}^{n}\mu_j\xi_j^2+\Omega_{c,n}+\Omega_{\tilde{x},n}\|\tilde{x}\|^2+\Omega_{z,n}\|z\|^{2(k+1)}. \quad (50)$$

This completes the controller design procedure. In the next subsection, it will be shown that the designed control law could achieve the control task.

3.2 Main results

Before the main result is presented, we first state the following facts. The proofs can be found in Appendices C and D.

Lemma 3 *Choose the function $U_z(z)=(V_0(z))^{k+1}$, then its time derivative along the z subsystem satisfies*

$$\dot{U}_z(z)\le-\bar{c}_3\|z\|^{2(k+1)}+c_5\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right), \quad (51)$$

where $\bar{c}_3$ and c_5 are two positive constants.

Lemma 4 *There exists a positive constant ρ_2 such that for any $\lambda>0$, the following holds*

$$d_{\frac{\lambda}{2}}^{p}(\xi_1)\left(1+d_{\frac{\lambda}{2}}^{p}(\xi_1)\right)\le\rho_2 d_{\frac{\lambda}{2}}^{p}(\xi_1)(1+\xi_1^{2p}). \quad (52)$$

Now we give the main result in this paper.

Theorem 1 *Suppose that the investigated system (1) and the reference signal $y_d(t)$ satisfy Assumptions 1–4. When the λ-tracker designed in (48) is applied to (1), for every initial conditions $z(0)\in R^r$, $x(0)\in R^n$, all the closed-loop signals are well defined and bounded on $[0,\infty)$, and moreover, the system output can realize the global robust λ-tracking control of any desired reference signal $y_d(t)$ with prescribed accuracy λ, i.e., for any given $\lambda>0$, there exists a finite time $T_\lambda>0$, such that*

$$|y(t)-y_d(t)|<\lambda,\ \forall\, t\ge T_\lambda. \quad (53)$$

Proof The proof can be carried out from the following two aspects. We first demonstrate the boundedness of all signals in closed loop on $[0,\infty)$, and then prove the λ-tracking property (53).

To this end, we consider the following Lyapunov function

$$V=V_n+l_1V_{\tilde{x}}+l_2U_z, \quad (54)$$

with some constants $l_1>0, l_2>0$. In view of (50) and Lemmas 2–3, one can get

$$\dot{V}\le-\sum_{j=2}^{n}\mu_j\xi_j^2-\left(\frac{1}{2}l_1-\Omega_{\tilde{x},n}\right)\|\tilde{x}\|^2-\left(\bar{c}_3l_2-\Omega_{z,n}-l_1\Theta_{\tilde{x},z}\right)\|z\|^{2(k+1)}+\Omega_{c,n}+(\Theta_{\tilde{x},\xi_1}l_1+c_5l_2)\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right). \quad (55)$$

In view of $\bar{c}_3>0$, the constants l_1, l_2 and l_3 can be chosen such that

$$\frac{1}{2}l_1-\Omega_{\tilde{x},n}\ge1,\ \bar{c}_3l_2-\Omega_{z,n}-l_1\Theta_{\tilde{x},z}\ge1, \quad (56)$$

$$\Omega_{c,n}+(\Theta_{\tilde{x},\xi_1}l_1+c_5l_2)\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right)$$

$$\le l_3\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right). \tag{57}$$

Let $c = \min\{2\mu_i(i = 2,\ldots,n), \frac{1}{l_1\lambda_{\max}(Q)}, \frac{1}{l_2c_2^k}\}$, where $\lambda_{\max}(Q)$ denotes the maximum eigenvalue of the matrix Q, according to (55), (56) and (57), we get

$$\begin{aligned}\dot V &\le -\sum_{j=2}^{n}\mu_j\xi_j^2 - \|z\|^{2(k+1)} - \|\tilde x\|^2 \\ &\quad + l_3\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right) \\ &\le -cV + l_3\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right).\end{aligned} \tag{58}$$

By means of Lemma 2 in [6], one can find a constant $\bar c > 0$, such that

$$\begin{aligned}&\int_0^t V^{\frac12}(\tau)d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))d\tau \\ &\le \bar c\int_0^t d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))\left(1+d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))\right)d\tau.\end{aligned} \tag{59}$$

Furthermore, using Lemma 4, we have

$$\begin{aligned}&\int_0^t V^{\frac12}(\tau)d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))d\tau \\ &\le \bar c\rho_2\int_0^t d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))\left(1+\xi_1^{2p}(\tau)\right)d\tau \\ &= \frac{\bar c\rho_2}{\Gamma}\int_0^t \dot\chi(\tau)d\tau.\end{aligned} \tag{60}$$

In view of (54), there exists a constant $l_4 > 0$ satisfying

$$|\xi_2| + \|\tilde x\| + \delta_1^*\|z\|^{k+1} \le l_4V^{\frac12}. \tag{61}$$

As a result of $d_{\frac{\lambda}{2}}^{p}(\xi_1) \ge 0$, one have

$$d_{\frac{\lambda}{2}}^{p}(\xi_1)\left(|\xi_2| + \|\tilde x\| + \delta_1^*\|z\|^{k+1}\right) \le l_4V^{\frac12}d_{\frac{\lambda}{2}}^{p}(\xi_1). \tag{62}$$

Considering (25), we derive that

$$\dot V_1 \le -\frac{1}{\Gamma}(\chi-\rho_1)\dot\chi + l_4V^{\frac12}d_{\frac{\lambda}{2}}^{p}(\xi_1). \tag{63}$$

Integrating both sides of (63), and from (60) we get

$$\begin{aligned}V_1(\xi_1(t)) &\le V_1(\xi_1(0)) - \frac{1}{\Gamma}\int_0^t(\chi-\rho_1)\dot\chi(\tau)d\tau \\ &\quad + \int_0^t l_4V^{\frac12}(\tau)d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))d\tau \\ &\le V_1(\xi_1(0)) - \frac{1}{\Gamma}\int_0^t(\chi-\rho_1)\dot\chi(\tau)d\tau \\ &\quad + \frac{l_4\bar c\rho_2}{\Gamma}\int_0^t\dot\chi(\tau)d\tau.\end{aligned} \tag{64}$$

After some simple calculations, there holds

$$0 \le V_1(\xi_1(t)) \le -\frac{1}{2\Gamma}\chi^2(t) + \frac{\rho_1+\bar c\rho_2l_4}{\Gamma}\chi(t) + C, \tag{65}$$

with some constant $C = V_1(\xi_1(0)) + \frac{1}{2\Gamma}\chi^2(0) - \frac{\rho_1+\bar c\rho_2l_4}{\Gamma}\chi(0)$.

Suppose that $[0, t_f)$ is the maximal interval of existence of the solutions. We will show that the variable $\chi(t)$ is bounded on $[0, t_f)$. Otherwise, if $\chi(t)$ is unbounded, and then, in terms of $\dot\chi(t) \ge 0$, it concludes that $\chi(t)$ tends to ∞ as $t\to t_f$. As a result, there exists $t^*\in[0, t_f)$ such that $\chi(t^*) \ge 1$. Dividing by $\chi(t)$ on time interval $[t^*, t_f)$ in (65) results in

$$0 \le -\frac{1}{2\Gamma}\chi(t) + \frac{\rho_1+\bar c\rho_2l_4}{\Gamma} + \frac{C}{\chi(t)},\ \ \forall\, t\in[t^*, t_f). \tag{66}$$

Since $\chi(t)$ is unbounded, this will lead to a contradiction $0 < -\infty$. Therefore, $\chi(t)$ is bounded. Consequently, $V_1(\xi_1)$ and $\xi_1(t)$ are bounded. From Assumption 4, we can conclude that y or x_1 is bounded. From (58), since $\xi_1(t)$ is bounded, it concludes that V is bounded and then, $z(t), \xi_i(t)(i = 2,\ldots,n)$ are bounded, which further guarantees that $\tilde x(t)$ is bounded. Considering the boundedness of $\xi_1(t)$ and $\chi(t)$, we know ϑ_1 is bounded. From $\xi_2 = \hat x_2+L_2y-\vartheta_1$, it is clear that $\hat x_2$ is bounded. Using a recursive method, it can be concluded that $\hat x_i(i = 3,\ldots,n+1)$ are bounded. From $\tilde x_i = x_i-(\hat x_i+L_iy)(i = 2,\ldots,n+1)$, we know x_i is bounded. In view of (48), the control law u is also bounded on $[0, t_f)$. Therefore, we have established the boundedness of all the signals in closed loop, and hence $t_f = \infty$.

Since $\chi(t)$ is bounded and monotonely nondecreasing on $[0,\infty)$, $\lim_{t\to\infty}\chi(t)$ exists and is finite, which implies

$$\begin{aligned}\int_0^\infty\dot\chi(\tau)d\tau &= \int_0^\infty\Gamma d_{\frac{\lambda}{2}}^{p}(\xi_1(\tau))\left(1+\xi_1^{2p}(\tau)\right)d\tau \\ &= \lim_{t\to\infty}\chi(t)-\chi(0) < \infty.\end{aligned} \tag{67}$$

It is not difficult to prove the uniformly continuous property of $\dot\chi(t)$ according to (22). Furthermore, it follows by Barbalat's lemma in [38] that

$$\lim_{t\to\infty}\dot{\chi}(t)=\lim_{t\to\infty}\Gamma d^{p}_{\frac{\lambda}{2}}(\xi_1(t))\left(1+\xi_1^{2p}(t)\right)=0. \tag{68}$$

In view of $1+\xi_1^{2p}(t)\geq 1$, one can obtain

$$\lim_{t\to\infty} d^{p}_{\frac{\lambda}{2}}(\xi_1(t))=0. \tag{69}$$

Considering $d_{\frac{\lambda}{2}}(\xi_1(t))=\max\left\{|\xi_1(t)|-\frac{\lambda}{2},0\right\}$, for any $\lambda>0$, there exists a finite time $T_\lambda>0$, such that when $t>T_\lambda$, the following holds

$$|\xi_1(t)|=|y(t)-y_d(t)|<\lambda,\ \forall\, t>T_\lambda. \tag{70}$$

This completes the proof of Theorem 1. □

4 Examples and discussion

In this section, we use two examples to illustrate our robust tracking control strategy. It is shown that using the proposed control approach, the parameter drift phenomenon can be avoided in Example 4.1, and the λ-tracking can be realized for the fan control system in Example 4.2.

Example 4.1 We consider the following simple system

$$\dot{x}=u+y+d(t),$$
$$y=x, \tag{71}$$

where $d(t)$ is the time-varying disturbance. In the case of $d(t)=0$, the system (71) degenerates to the considered system (1) in [14], with $r=1$, $z=0$, $\Delta_1(z,y,u)=y$. Using the control methodology in [14], the controller can be designed as follows

$$u=-\chi y,\ \dot{\chi}=\Gamma y^2. \tag{72}$$

Consider the Lyapunov function $V=\frac{1}{2}y^2$ whose time derivative along solutions of (71), (72) is

$$\dot{V}=-\chi y^2+y^2. \tag{73}$$

Using a contradiction argument, it can be proved that y and χ are bounded. Moreover, according to Barbalat's lemma, it further follows that $y(t)$ converges to the origin as t tends to infinity.

In the case of $d(t)\neq 0$, then the system (71) falls into our investigated system (1) with nonvanishing disturbance $\Delta_1(z,y,u)=y+d(t)$. The controller (72) still could guarantee the convergence of $y(t)$. In fact, we choose the following Lyapunov function candidate

$$V=\frac{1}{2}y^2+\frac{1}{2\Gamma}(\chi-2)^2, \tag{74}$$

then, in view of (71) and (72), we get

$$\dot{V}=-y^2+yd(t)\leq -\frac{1}{2}y^2+\frac{1}{2}d^2(t). \tag{75}$$

This shows that the system (71) is input-to-state stable (ISS) with state y and input $d(t)$ [14]. As a result, if $d(t)\to 0$ as $t\to\infty$, then $y(t)\to 0$ as $t\to\infty$. However, in such case, the other variable χ may drift to infinity as t goes to infinity. To illustrate this point, as suggested in [39], we choose $d(t)$ in the form of

$$d(t)=(1+t)^{-\frac{1}{8}}\left(1-(1+t)^{-\frac{1}{4}}-\frac{3}{8}(1+t)^{-\frac{5}{4}}\right) \tag{76}$$

and $\Gamma=\frac{1}{4}$, $\chi(0)=1$, $y(0)=1$, then

$$y(t)=(1+t)^{-\frac{3}{8}}\to 0,\quad t\to\infty, \tag{77}$$

but

$$\chi(t)=(1+t)^{\frac{1}{4}}\to\infty,\quad t\to\infty, \tag{78}$$

which destabilizes the closed-loop system.

However, considering the system (71) in the form of system (1) with $n=1$ and $p=1$, using the control scheme developed in Sect. 3, we design the following controller

$$u=-\chi(1+y^2)\mathrm{sig}_{\frac{\lambda}{2},1}(y),\ \dot{\chi}=\Gamma d_{\frac{\lambda}{2}}(y)(1+y^2), \tag{79}$$

which could guarantee the signals in closed-loop system bounded on $[0,\infty)$. The stability analysis can be done in the same way as in the proof of Theorem 1.

Example 4.2 In this example, we apply our output feedback tracking control strategy into the fan speed tracking control. The dynamics of a fan driven by a DC motor is described by [31,32]

$$J\dot{\upsilon}=k_1 I-\tau_L-\tau_D(\upsilon),$$
$$L\dot{I}=u_o-k_2\upsilon-RI+d(t),$$
$$y=\upsilon, \tag{80}$$

where υ is the fan speed viewed as the output, I is the unmeasured armature current, τ_{L} is an uncertain constant load torque, $\tau_{\mathrm{D}}(\upsilon)$ is an uncertain drag torque, u_o is the armature voltage which is considered as the input, and J, L, k_1, k_2, R are known positive constants. The function $d(t)$ represents some external disturbances. Like in [31], we assume here that $\tau_D(\upsilon)=\kappa\upsilon$ with $\kappa>0$ a possibly unknown constant.

Remark 3 In [28,31–33], the proposed control schemes could realize the set-point tracking control of *constant* reference signals for this system (80). In [34], we consider the asymptotic tracking control for some smooth time-varying signals generated by an autonomous exosystem using the internal model principle. However, this framework severely limits the class of exogenous signals to be some kind of sinusoid references. Here, we remove the assumption that the reference signals that are *constants* in [31–33] or *sinusoids* in [34], and allows it to be any bounded time-varying trajectory. Additionally, although the current I is not measured, a preliminary feedback is needed in [31–34]. As a result, the actual control voltage u_o could not be worked out using these control schemes. In our recent work [28], assuming the parameters to be known a priori, the voltage u_o could be derived by designing a current observer. However, the work [28] still focuses on the *constant* reference signals. It is shown that our current work removes the above drawbacks: (1) the speed tracking control can be achieved for any time-varying references satisfying Assumption 4; (2) the actual control voltage u_o could be worked out by skillful coordinates changes (81) without the assistance of a current observer in [28].

In what follows, we give the control design procedure for (80). First, we define the following new state variables:

$$x_1 = \upsilon,\ x_2 = \frac{R}{L}x_1 + \frac{k_1}{J}I,\ u = \frac{k_1}{JL}u_o. \tag{81}$$

In view of (80) and (81), by some direct calculations, we have

$$\begin{aligned} \dot{x}_1 &= x_2 - \frac{R}{L}x_1 - \frac{1}{J}(\tau_L + \tau_D(y)) \\ \dot{x}_2 &= u - \frac{k_1k_2}{JL}x_1 - \frac{R}{JL}(\tau_L + \tau_D(y)) + \frac{k_1}{JL}d(t) \\ y &= x_1, \end{aligned} \tag{82}$$

which falls into the investigated system (1) with $z = 0$, $\Delta_1(y) = -\frac{R}{L}x_1 - \frac{1}{J}(\tau_{\rm L} + \tau_{\rm D}(y))$, $\Delta_2(y) = -\frac{k_1k_2}{JL}x_1 - \frac{R}{JL}(\tau_L + \tau_D(y))$, and the external disturbance $\frac{k_1}{JL}d(t)$.

Using the proposed control scheme in Sect. 3, we design the following λ-tracking controller for (82)

$$\begin{aligned} u = &-\hat{x}_3 - \mu_2\xi_2 - L_3y + L_2(\hat{x}_2 + L_2y) \\ &+ \frac{\partial\vartheta_1}{\partial\xi_1}\hat{x}_2 - \psi_2(\chi, \xi_1, \hat{x}_2) \end{aligned} \tag{83}$$

with ϑ_1, χ defined as in (21), (22) and $\psi_2(\chi, \xi_1, \hat{x}_2) = \xi_2\left(\frac{1}{2}(\frac{k_1}{J})^2(\frac{\partial\vartheta_1}{\partial\xi_1})^2 + (\frac{\partial\vartheta_1}{\partial\xi_1})^2 2(1+\xi_1^2)\right) + (\frac{\partial\vartheta_1}{\partial\xi_1})^2 + (\frac{\partial\vartheta_1}{\partial\xi_1})^2\varphi^2(\xi_1)$. According to (81), we work out the actual control voltage for the fan speed system (80)

$$u_o = \frac{JL}{k_1}u. \tag{84}$$

For simulation purposes, the tracked reference signal $y_d(t)$ is chosen as

$$y_d(t) = \begin{cases} 0, & 0 \le t \le 10 \\ 0.5 + 0.5\sin\left(\frac{\pi}{10}(t-15)\right), & 10 \le t \le 20 \\ 1, & 20 \le t \le 40 \\ 1.5 + 0.5\sin\left(\frac{\pi}{20}(t-50)\right), & 40 \le t \le 60 \\ 2, & t \ge 60. \end{cases} \tag{85}$$

The external disturbance is $d(t) = 0.5\sin(t)$. The parameter values in (80) are set to $J = 1, L = 1, k_1 = 1, k_2 = 1, R = 10, \kappa = 1$. The design parameters and initial condition are chosen as $\mu_2 = 5, \lambda = 0.1, \Gamma = 1.5, \varpi = 2, L_2 = 3, L_3 = 2$, and $\tilde{x}_2(0) = 1, x_1(0) = 0.5, \hat{x}_2(0) = 1, \chi(0) = 1.5$, and the functions are $\varphi(\xi_1) = (1+\xi_1^2)^2$ and

$$\mathrm{sig}_{\frac{\lambda}{2},2}(\xi_1) = \begin{cases} \mathrm{sign}(\xi_1), & |\xi_1| \ge \frac{\lambda}{2}, \\ \frac{15}{8}\frac{2}{\lambda}\xi_1 - \frac{15}{4}\left(\frac{2}{\lambda}\xi_1\right)^3 + \frac{3}{8}\left(\frac{2}{\lambda}\xi_1\right)^5, & |\xi_1| < \frac{\lambda}{2}. \end{cases}$$

The simulation results are shown in Fig. 1, from which one can see the fan speed υ can realize the λ-tracking control for the reference signal y_d with the tracking error asymptotic to an interval $[-0.1, 0.1]$. Figure 1 also shows the current I and its estimate $\hat{I}$ as well as the control voltage u_o are bounded, which demonstrate the efficacy of the presented control scheme.

5 Conclusions

In this paper, by designing a RESO, the global robust practical tracking control problem is investigated for a class of nonlinear systems with the additive nonvanishing disturbances and dynamic uncertainties. With less information on the reference signal, the designed robust tracking controller could guarantee that the tracking error asymptotic to the interval $[-\lambda, \lambda]$ with arbitrary prescribed λ after a finite time. The computer simulation demonstrates its effectiveness by its application into the fan speed control system. However, a restrictive condition is that the uncertain nonlinearities need to satisfy the polynomial growth of the output. How to

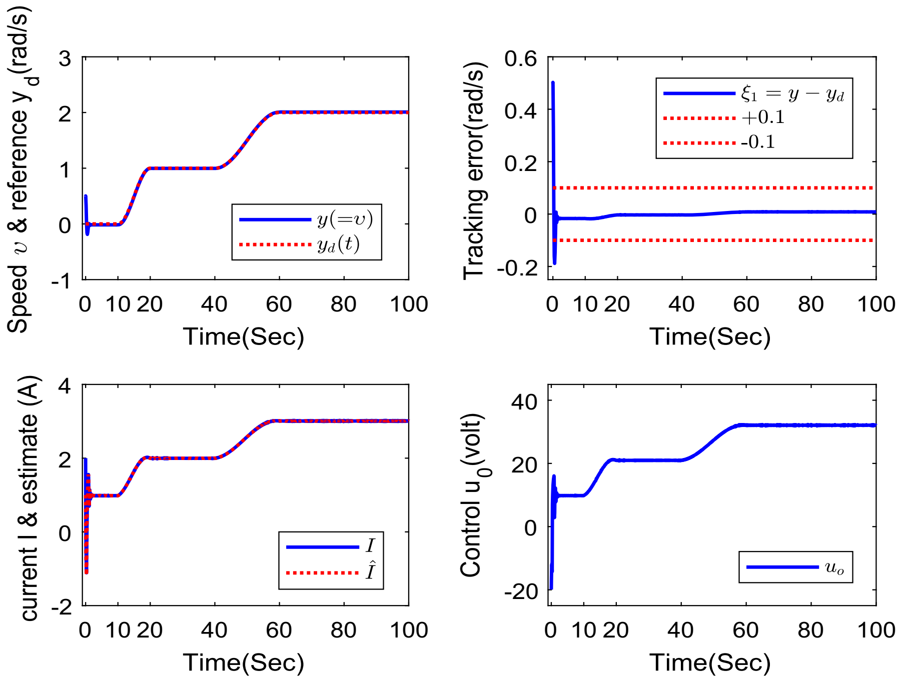

Fig. 1 The responses of closed-loop system (81)–(85)

remove this assumption is an interesting topic in the future research.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix A: Proof of Lemma 1

In accordance with Assumptions 2–4, we have

$$\left|\Delta_i(z, y, u)\right| \le \delta_{i1}\|z\|^k + \delta_{i2} + \delta_{i2}\cdot 2^{p_i-1}\left(|\xi_1|^{p_i} + \varpi^{p_i}\right). \quad (86)$$

Using the Young's inequality in [9], the following holds

$$\delta_{i1}\|z\|^k \le \delta_{i1}\left(\frac{1}{k+1} + \frac{k}{k+1}\|z\|^{k+1}\right) \le \delta_{i1}\|z\|^{k+1} + \delta_{i1}, \quad (87)$$

and

$$|\xi_1|^{p_i} + \varpi^{p_i} \le (\varpi^{p_i} + 1)\left(1 + |\xi_1|^p\right). \quad (88)$$

As a consequence,

$$|\Delta_i(z, y, u)| \le \delta_{i1}\|z\|^{k+1} + \delta_{i1} + \delta_{i2} + \delta_{i2}\cdot 2^{p_i-1}(\varpi^{p_i} + 1)\left(1 + |\xi_1|^p\right). \quad (89)$$

Taking a positive constant $\delta_i^* = \delta_{i1} + \delta_{i2} + \delta_{i2}\cdot 2^{p_i-1}(\varpi^{p_i} + 1)$, then the lemma is proved.

Appendix B: Proof of Lemma 2

The time derivative of $V_{\tilde{x}}$ along the solutions of (12) satisfies

$$\dot{V}_{\tilde{x}} = \dot{\tilde{x}}^{\mathrm{T}} Q\tilde{x} + \tilde{x}^{\mathrm{T}} Q\dot{\tilde{x}} = -\|\tilde{x}\|^2 + 2\tilde{x}^{\mathrm{T}} Q\Delta(z, y, u) + 2\tilde{x}^{\mathrm{T}} Q\, b\, \dot{d}(t). \quad (90)$$

By completing the squares, one have the following calculations:

$$2\tilde{x}^{\mathrm{T}} Q\Delta(z, y, u) \le \frac{1}{4}\|\tilde{x}\|^2 + 4\|Q\|^2\|\Delta(z, y, u)\|^2 \le \frac{1}{4}\|\tilde{x}\|^2 + l\sum_{i=1}^{n}\Delta_i^2(z, y, u) \quad (91)$$

with $l = \max\left\{8\|Q\|^2, 4\|Q\|^2\left(2\sum_{i=2}^{n} L_i^2 + L_{n+1}^2\right)\right\}$. According to Lemma 1, we have

$$\Delta_i^2(z, y, u) \le 2\delta_i^{*2}(\|z\|^{2(k+1)} + 2\left(1 + |\xi_1|^{2p}\right), \quad i = 1, \ldots, n. \quad (92)$$

Then, we further get

$$2\tilde{x}^{\mathrm{T}} Q\Delta(z, y, u) \le \frac{1}{4}\|\tilde{x}\|^2 + 2l\sum_{i=2}^{n}\delta_i^{*2}\|z\|^{2(k+1)} + 2nl\left(1 + |\xi_1|^{2p}\right). \quad (93)$$

Additionally, using the completion of squares again, the following holds

$$2\,\tilde{x}^{\mathrm{T}} Q\, b\, \dot{d}(t) \le \frac{1}{4}\|\tilde{x}\|^2 + 4\|Q\|^2 \bar{d}^{\,2}. \tag{94}$$

Combining the above analysis, we have

$$\dot{V}_{\tilde{x}} \le -\frac{1}{2}\|\tilde{x}\|^2 + 2l\sum_{i=2}^{n}(\delta_i^{*2}) \cdot \|z\|^{2(k+1)} + 2nl\left(1+|\xi_1|^{2p}\right) + 4\|Q\|^2\bar{d}^{\,2}. \tag{95}$$

Next, we will prove that there exists a positive constant σ such that

$$(1+\xi_1^{2p}) \le \sigma\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right),\ \forall\,\lambda > 0. \tag{96}$$

If $|\xi_1| \le \frac{\lambda}{2}$, from the definition of $d_{\frac{\lambda}{2}}(\cdot)$, we know $d_{\frac{\lambda}{2}}^{2p}(\xi_1) = 0$, and the left-hand side satisfies

$$1+\xi_1^{2p} \le 1+\left(\frac{\lambda}{2}\right)^{2p}. \tag{97}$$

If $|\xi_1| > \frac{\lambda}{2}$, using the completion of squares again, the following holds

$$1+\xi_1^{2p} = 1+\left(|\xi_1| - \frac{\lambda}{2} + \frac{\lambda}{2}\right)^{2p} \le \left(1+2^{2p-1}\left(\frac{\lambda}{2}\right)^{2p} + 2^{2p-1}\right)\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right). \tag{98}$$

Therefore, (96) holds with $\sigma = \max\left\{1+(\frac{\lambda}{2})^{2p}, 1+2^{2p-1}(\frac{\lambda}{2})^{2p}+2^{2p-1}\right\} = 1+2^{2p-1}(\frac{\lambda}{2})^{2p}+2^{2p-1}$. Let $\Theta_{\tilde{x},z} = 2l\sum_{i=2}^{n}\delta_i^{*2}$, $\Theta_{\tilde{x},\xi_1} = \max\{2nl\sigma, 4\|Q\|^2\bar{d}^{\,2}\}$, then the lemma is proved.

Appendix C: Proof of Lemma 3

In view of Assumption 1 and $U_z(z) = (V_0(z))^{k+1}$, it can be verified that

$$\dot{U}_z(z) \le -(k+1)c_1^k c_3\|z\|^{2(k+1)} + (k+1)c_2^k c_4\|z\|^{2k}\left(1+|y|^{p_0}\right). \tag{99}$$

For any $\varepsilon > 0$, using the Young's inequality, we obtain

$$\|z\|^{2k}(1+|y|^{p_0}) \le \varepsilon\frac{2k}{2k+2}\|z\|^{2(k+1)} + \varepsilon^{-k}\frac{2}{2k+2}(1+|y|^{p_0})^{k+1}. \tag{100}$$

In view of the definition of p, it can be shown that

$$(1+|y|^{p_0})^{k+1} \le \frac{5}{4}2^{k+1}(1+2^{p-1}+2^{p-1}M^p)(1+|\xi_1|^{2p}). \tag{101}$$

According to (96), we further have

$$(1+|y|^{p_0})^{k+1} \le \varrho\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right) \tag{102}$$

with $\varrho = \frac{5}{4}2^{k+1}(1+2^{p-1}+2^{p-1}M^p)\sigma$.

As a result,

$$(k+1)c_2^k c_4\|z\|^{2k}\left(1+|y|^{p_0}\right) \le \varepsilon^{-k}c_2^k\bar{c}_4\varrho\left(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)\right) + \varepsilon k c_2^k c_4\|z\|^{2(k+1)}. \tag{103}$$

Let $\overline{c}_3 = (k+1)c_1^k c_3 - \varepsilon k c_2^k c_4$, $c_5 = \varepsilon^{-k}c_2^k c_4\varrho$. Choose $\varepsilon > 0$ appropriately satisfying $\overline{c}_3 > 0$. This fact together with (99)–(101) implies that

$$\dot{U}_z \le -\overline{c}_3\|z\|^{2k} + c_5(1+d_{\frac{\lambda}{2}}^{2p}(\xi_1)). \tag{104}$$

Appendix D: Proof of Lemma 4

The proof can be established from the following two cases.

Case one: $|\xi_1| \le \frac{\lambda}{2}$. It is clear that the lemma holds for any $\rho_2 > 0$, because of $d_{\frac{\lambda}{2}}^p(\xi_1) = 0$.

Case two: $|\xi_1| > \frac{\lambda}{2}$. In view of $d_{\frac{\lambda}{2}}^p(\xi_1) = \left(|\xi_1| - \frac{\lambda}{2}\right)^p > 0$, it suffices to prove there exists $\rho_2 > 0$ such that

$$1+d_{\frac{\lambda}{2}}^p(\xi_1) \le \rho_2(1+\xi_1^{2p}). \tag{105}$$

In fact, as the left-hand side of (105), using the Young's inequality, we have

$$1+d_{\frac{\lambda}{2}}^p(\xi_1) \le \frac{3}{2}\left(1+2^{p-1}+2^{p-1}(\frac{\lambda}{2})^p\right)(1+\xi_1^{2p}). \tag{106}$$

Take $\rho_2 = \frac{3}{2}\left(1+2^{p-1}+2^{p-1}(\frac{\lambda}{2})^p\right)$, and one has (105). Consequently, the proof is completed.

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