

Effect of Lévy noise on the networks of Izhikevich neurons

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Abstract Lévy noise plays a significant role in jump-noise processes found in neurons. We perform numerical simulations of synaptically coupled Izhikevich networks under the effect of general non-Gaussian Lévy noise. Firing dynamics of an all-to-all coupled Izhikevich network and two excitatory coupled Izhikevich networks with differing adaptation properties are studied in response to applied Lévy noise. Whole parameter space of Lévy noise is investigated by changing its characteristic exponent, skewness, scale and mean parameters. The simulation results show that the noise generated from α -stable Lévy distribution causes significant dynamical changes in the firing pattern of neuronal network. Different firing patterns exhibited by Izhikevich neurons such as tonic spiking, chattering, intrinsic bursting, low threshold spiking and fast spiking are studied in the network level with and without Lévy noise. In all cases, neurons are found to be irregularly spiking when Lévy noise with characteristic exponent α equal to 0.5 is applied. Lévy noise with this particular value of α as 0.5 causes the Izhikevich network to have a dynamical behaviour independent of topology and heterogeneity of the network. At this value of characteristic exponent, density of the stable Lévy distribution in standard parameterisation is maximum. Other parameters of Lévy noise do not influence the way in which a particular neuron is firing. The effects of Lévy

noise on the correlation between individual neurons and on the networks are investigated using statistical measures.

Keywords Izhikevich model · Lévy noise · Neuron networks · Nonlinear dynamics · Synaptic coupling

1 Introduction

In recent times, the spiking neuron models have been subjected to intense research in the field of neuronal science owing to the fact that neurons carry information among each other using their firing rates [1]. Examples of commonly investigated spiking neuron models are Hodgkin–Huxley-type model, FitzHugh–Nagumo model and Hindmarsh–Rose model [2]. Also, several hybrid neuron models have been introduced which combine continuous spike-generation mechanisms and discontinuous resetting process after spiking. An example of nonlinear hybrid neuron model is the Izhikevich neuron model which can generate different types of bifurcation and is capable of reproducing most of the spiking activities observed in the actual neural systems by tuning a few parameters. This two-dimensional model has been shown to capture a wide range of cellular firing characteristics in various parts of mammalian brain. It is a biologically plausible neuron model and can reproduce spiking and bursting behaviour of known types of neocortical and thalamic neurons [3,4].

It is well known that the brain can produce fluctuations with complex scaling properties. Using the sta-

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tistical properties of these fluctuations, evolution of the brain dynamics can be studied [5]. The fluctuations during a brain activity can be considered as a random motion. In the continuous limit, these fluctuations can also be approximated as the motion of a diffusing macroscopic particle [6,7]. This framework can be well modelled by a generalised Langevin differential equation consisting of Lévy noise with deterministic and stochastic components [8,9].

Though the stochastic effect is often regarded as unwanted as it damages the extraction of useful information, in certain nonlinear systems it is realised that the presence of internal or external noise can enhance the response of the dynamics of the signal [10]. The noise in dynamical systems has wide applications in various fields of physics, biology, geology and neurology [11–16]. Usually, the random excitations of noise are assumed to be of Gaussian white noise kind which can only describe random small fluctuations without big jumps. But it has been investigated that small disturbances are present in the biological and neurological activities and those disturbances are always combined with discontinuous unpredictable jumps of random nature. Also, the statistical characteristics of the random noise significantly deviate from the Gaussian distribution [17–19]. Therefore, it is necessary to consider the non-Gaussian disturbance in the studies of biological systems. Lévy noise can model such non-Gaussian noise having both small and big random fluctuations [20,21]. Since the paths of Lévy processes may contain random jump discontinuities of arbitrary size occurring at arbitrary random times, Lévy processes form a rich class of stochastic processes having applications in many different areas of science [22].

Another advantage of Lévy noise model is that it includes not only pure-diffusion and pure-jump models but also jump-diffusion models [23,24], and it is much better than a pure-diffusion neuron model with additive Gaussian noise in describing the evolution of membrane potential of a neuron. Pure-diffusion neuron models require large number of synapses with small membrane effects due to the small coupling coefficients or the synaptic weights [25,26]. On the contrary in real scenarios, often fewer synaptic inputs are produced near the postsynaptic neuron's trigger zone, and these inputs produce impulses in noise amplitudes [27,28]. Thus, standard Gaussian noise does not faithfully represent the dynamics of realistic neuron models but Lévy noise does. The general Lévy model is favourable in

engineering applications as well [29–31]. Lévy noise can also effectively represent the random noises due to external radiation-induced magnetic flux of electrical fields when applied on neuron networks [32–34].

Lévy noise can greatly describe the complex biological and neuronal environment [35,36]. One of the simplest creatures *Escherichia coli* has been studied based on biological fluctuation framework using the Lévy walks model [37]. Dubkov and Kharcheva [38] have examined the steady-state time characteristics of anomalous diffusion in the form of Lévy flights in a symmetric bistable quadratic potential. Xu et al. have investigated stochastic resonance phenomenon in a FitzHugh–Nagumo system induced by an additive Lévy noise [39]. Wu et al. [40] have investigated the electrical activities in an improved Hindmarsh–Rose model excited by the external electromagnetic radiation of Lévy noise. They have observed that decreasing of the characteristic exponent of Lévy noise improves the firing of Hindmarsh–Rose neuron. They have chosen arbitrary values for characteristic exponent and skewness parameter of the noise. In this paper, complete parameter space of Lévy noise is investigated, and the Lévy noise with particular characteristic exponent is found to increase the firing activity of neurons. Cai et al. [41] have considered the dynamics of escape in the stochastic FitzHugh–Nagumo (FHN) neuronal model driven by symmetric α -stable Lévy noise. Zhang et al. [42] have done a study on low-dimensional reduction for a slow–fast data assimilation system with non-Gaussian α -stable Lévy noise via stochastic averaging. Zhan and Liu [43] have studied dynamical response of Morris–Lecar system with electromagnetic induction (EMI) and Gaussian white noise. Jin et al. [44] have explored how noise can affect the synchronisation and stability of a neuronal network. Transition of electrical activities in neurons induced by electromagnetic radiation has been also explored by researchers [45,46]. But the influence of Lévy noise on the networks of Izhikevich neurons models has not been investigated. Therefore, the study of effect of non-Gaussian Lévy noise on the neurons of such model is relevant.

2 Neuron model and method

2.1 Izhikevich neuron model

The Izhikevich model consists of two parts: a two-dimensional system of ordinary differential equations

and a reset mechanism. The differential equations model the subthreshold behaviour of the system and generate the upstroke of the action potential, while the reset mechanism is responsible for producing the downstroke of the action potential. V and U are the model variables. V is the membrane potential. U is the current which represents the effect of the slow currents on spike generation and allows for spike- frequency adaptation [3,4]. The model is represented as

While $V < V_{\text{peak}}$:

$$C\dot{V} = k(V - V_R)(V - V_T) - U + I_{\text{ext}} \tag{2.1}$$

$$\dot{U} = a[b(V - V_R) - U] \tag{2.2}$$

When V crosses a certain value called V_{peak} from below, V and U are updated as

$$V \leftarrow c, \quad U \leftarrow U + d \tag{2.3}$$

C is the membrane capacitance, V_R is the resting membrane potential, I_{ext} is the applied current (the effect of externally injected and synaptic currents), V_T is the voltage threshold when $b = 0$ and $I_{\text{ext}} = 0$, V_{peak} is the spike cut-off value, k is a scaling factor that affects the value of dV/dt and the spike width, a is the time constant of the adaptation current U , b describes the sensitivity of the adaptation current to subthreshold fluctuations of the membrane potential V , c is the voltage reset value, d describes the difference between outward and inward currents activated during the spike which affects the after-spike behaviour.

When neuron models of Izhikevich kind are coupled together in large networks, they show bifurcations that isolated neuron model cannot generate [47]. A feasible method of analysing large networks is through population density equations. In population density methods, a time-varying population density function is defined initially whose evolution is determined by a conservation law. Population density equation is simplified using moment closure methods and a sequence of approximations as described in the literature [48,49]. Finally, a system of ODEs is achieved which can be analysed numerically. Equations (2.1)–(2.3) are rewritten as [48]:

$$C\dot{V}_i = k(V_i - V_T)(V_i - V_R) - U_i + I_{\text{ext}} \tag{2.4}$$

$$\dot{U}_i = \frac{\eta(V_i - V_R) - U_i}{\tau_U} \tag{2.5}$$

$$\dot{s} = -\frac{s}{\tau_{\text{syn}}} + s_{\text{jump}}j(t) \tag{2.6}$$

$$V_i(t_{\text{spike}}^-) = V_{\text{peak}} \Rightarrow V_i(t_{\text{spike}}^+) = V_{\text{reset}}$$

$$\text{and } U_i(t_{\text{spike}}^+) = U_i(t_{\text{spike}}^-) + U_{\text{jump}} \tag{2.7}$$

for $i = 1, 2, \dots N$. The parameter values are chosen as follows. $C = 250$ pF, $k = 2.5$ nS/mV, $V_R = -65$ mV, $V_T = V_R + 40 - \frac{\eta}{k} = 41.7$ mV, $V_{\text{peak}} = 30$ mV, $V_{\text{reset}} = -55$ mV, $U_{\text{jump}} = 200$ pA, $\tau_U = 100$ ms, $\eta = -1$ nS, $s_{\text{jump}} = 0.8$, $\tau_{\text{syn}} = 2$ ms.

2.2 The Lévy noise

Let ξ denotes the time-dependent Lévy noise and obeys the probability density function $L_{\alpha,\beta}(\xi; \sigma, \mu)$, whose characteristic function is [50]:

$$\Phi(k) = \int_{-\infty}^{+\infty} d\xi e^{ik\xi} L_{\alpha,\beta}(\xi; \sigma, \mu) \tag{2.8}$$

Therefore, for $\alpha \in (0, 1) \cup (1, 2]$,

$$\Phi(k) = \exp \left[i\mu k - \sigma^\alpha |k|^\alpha \left(1 - i\beta sgn(k) \tan \frac{\pi\alpha}{2} \right) \right] \tag{2.9}$$

and for $\alpha = 1$,

$$\Phi(k) = \exp \left[i\mu k - \sigma |k| \left(1 + i\beta sgn(k) \frac{\pi}{2} \ln |k| \right) \right] \tag{2.10}$$

here $\alpha \in (0, 2]$ is known as the characteristic exponent, and it denotes the stability index that describes an asymptotic power law of the Lévy distribution. The constant β ($\beta \in [-1, 1]$) is the asymmetry or skewness parameter. σ ($\sigma \in (0, \infty)$) is the scale parameter, and μ ($\mu \in R$) is the mean parameter. The noise intensity is denoted as $D = \sigma^\alpha$, and hence, the Lévy process can also be represented as $L_{\alpha,\beta}(\xi; D, \mu)$. When $\alpha = 0.5$, the stable distribution is called α -stable Lévy distribution.

We analyse the effect of Lévy noise on Izhikevich neurons using the approach mentioned in Sect. 2.1. The paper is organised as follows: we consider two neural networks that are deterministic and homogeneous. The first example, studied in Sect. 2.3, is a homogeneous all-to-all coupled Izhikevich network. Section 2.4 considers two Izhikevich networks that are all-to-all coupled both internally and externally and each network is split into two populations, one strongly adapting and

the other weakly adapting. The parameters are chosen so as to match the neuronal experimental results from research papers [47, 48].

2.3 The single all-to-all coupled Izhikevich network with Lévy noise

We consider a network of N identical neurons of the Izhikevich model [Eqs. (2.4)–(2.6)], with all-to-all coupling. The coupling is modelled as synaptic [51]. The postsynaptic neuron generates a postsynaptic potential by integrating the inputs from the neighbouring presynaptic neuron. The synaptic current in the j th neuron is modelled by

$$I_{\text{syn},j} = g_{\text{syn}} (V_j - E_{\text{rnr}}) \sum_{i=1, i \neq j}^N s_{j,i}(t, V_i) \quad (2.11)$$

where g_{syn} is the maximal synaptic conductance and E_r is the synaptic reversal potential. $s_{j,i}$ is the gating variable of the synapse from the i th neuron to the j th, and it is not allowed to exceed 1.

When V_i crosses 0 from below:

$$s_{j,i} \rightarrow s_{j,i} + S_{\text{max}} \quad (2.12)$$

where S_{max} is a synaptic update parameter and is chosen to be 0.8.

Otherwise:

$$\tau_{\text{syn}} \frac{ds_{j,i}}{dt} = -s_{j,i} \quad (2.13)$$

Also, the total effect of current in the neuronal system is taken as

$$I_{\text{ext}} = I_{\text{app}} - I_{\text{syn},j} \quad (2.14)$$

Therefore, dimensionless form of the single all-to-all coupled Izhikevich network with synaptic coupling and Lévy noise is given by

$$\dot{v}_i = v_i(v_i - \alpha) - u_i + I + g(e_r - v_i)s + \xi \quad (2.15)$$

$$\dot{u}_i = a(bv_i - u_i) \quad (2.16)$$

$$\dot{s} = -\frac{s}{\tau_s} + s_{\text{jump}}j(t) \quad (2.17)$$

$$v_i(t_{\text{spike}}^-) = v_{\text{peak}} \Rightarrow v_i(t_{\text{spike}}^+) = v_{\text{reset}} \\ \text{and } u_i(t_{\text{spike}}^+) = u_i(t_{\text{spike}}^-) + u_{\text{jump}} \quad (2.18)$$

for $i = 1, \dots, N$, where $v_i = 1 + \frac{V_i}{|V_R|}$, $u_i = \frac{U_i}{k|V_R|^2}$. The dimensionless parameters are given as follows:

$$\alpha = 1 + \frac{V_T}{|V_R|} = 0.62, \quad b = \frac{\eta}{k|V_R|} = 0.006, \\ I = \frac{I_{\text{app}}}{k|V_R|^2} = 0.14, \quad g = \frac{g_{\text{syn}}}{k|V_R|} = 1.23, \quad a = \left(\frac{C}{\tau_U k|V_R|} \right) = 0.015, \quad \tau_s = \frac{\tau_{\text{sync}} k|V_R|}{C} = 1.3, \quad u_{\text{jump}} = \frac{U_{\text{jump}}}{k|V_R|^2} = 0.0189, \quad I_{\text{app}} = 1500 \text{ pA}, \quad g_{\text{syn}} = 300 \text{ nS}, \\ N = 1000.$$

2.4 Excitatory coupled pair of Izhikevich networks with different adaptation properties with Lévy noise

Neuronal adaptation refers to the response profile when a stimulus is applied to a neuron. It indicates the ability of a neuron to adapt to a particular situation by changing its threshold. Strongly adapting neurons quickly modify their firing rate in response to a stimulus. The adaptation levels of the networks are determined and controlled by $\tau_{U,SA}$, $\tau_{U,WA}$, $I_{\text{app},SA}$, $I_{\text{app},WA}$ and synaptic gating variables s_j . We consider a pair of networks which are excitatory in nature, and they are all-to-all coupled. Each network consists of two distinct homogeneous populations: one is strongly adapting and the other is weakly adapting. Biological networks present samples for such cases. For example, pyramidal neurons in hippocampal area CA3 can be grouped into strongly adapting, weakly adapting and intrinsically bursting populations based on the amount of spike-frequency adaptation they exhibited [53]. Since there are two populations, four maximal synaptic conductances g_{mj} and two synaptic gating variables s_j are needed. Here j denotes the presynaptic network and m denotes the postsynaptic network, with $j = SA$ indicating the subnetwork with strong adaptation and $j = WA$ is the subnetwork with weak adaptation, and similarly for m .

The equations for the network are [48],

$$C \dot{V}_{i,m} = k (V_{i,m} - V_T) (V_{i,m} - V_R) - U_{i,m} + I_{\text{app},m} \\ + (1 - p) g_{\text{syn},mSA} (E_r - V_{i,m}) s_{SA} \\ + p g_{\text{syn},mWA} (E_r - V_{i,m}) s_{WA} \quad (2.19)$$

$$\dot{U}_{i,m} = \frac{\eta (V_{i,m} - V_R) - U_{i,m}}{\tau_{U,m}} \quad (2.20)$$

$$\dot{s}_{SA} = -\frac{s_{SA}}{\tau_{\text{syn}}} + s_{\text{jump}} j_{SA}(t) \quad (2.21)$$

$$\dot{s}_{WA} = -\frac{s_{WA}}{\tau_{\text{syn}}} + s_{\text{jump}} j_{WA}(t) \quad (2.22)$$

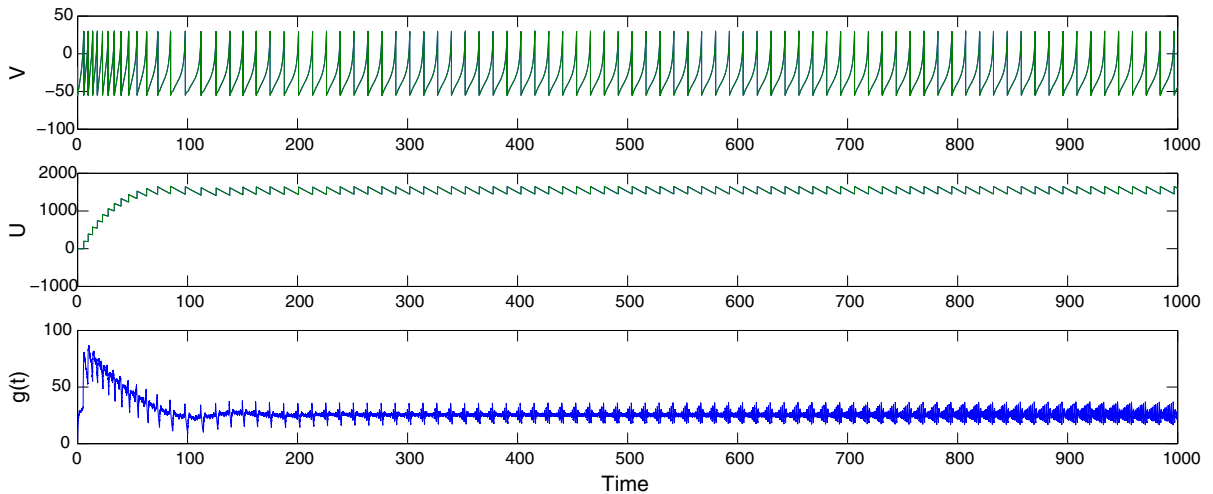


Fig. 1 Simulation of a neuron in the homogenous network of 1000 tonic spiking Izhikevich neurons without Lévy noise

$$j_{SA}(t) = \frac{1}{N_{SA}} \sum_{j=1}^{N_{SA}} \sum_{t_{j,k,SA} < t} \delta(t - t_{j,k,SA})$$

$$j_{WA}(t) = \frac{1}{N_{WA}} \sum_{j=1}^{N_{WA}} \sum_{t_{j,k,WA} < t} \delta(t - t_{j,k,WA})$$

for $i = 1, \dots, N_m$. Note that $p = N_{SA}/(N_{SA} + N_{WA})$ is the proportion of strongly adapting neurons in the network.

Equations (2.19)–(2.22) are converted into dimensionless form. There are separate population density equations for strongly and weakly adapting populations. Then moment closure reduced population density equations are coupled together by the two coupling variables and two adaptation variables [48]. Lévy noise is applied to the voltage variable of each neuron of Izhikevich population. The other parameter values are $\tau_{U,SA} = 100$ ms, $\tau_{U,WA} = 10$ ms, $I_{app,SA} = 1000\text{--}2000$ pA, $I_{app,WA} = 1200$ pA, $g_{syn,mj} = 200$ nS, $N_{SA} = 800$ and $N_{WA} = 200$.

3 Results

3.1 Dynamics of single all-to-all network with Lévy noise

For analysing the time series of the Izhikevich model network, Euler method is used. The whole parameter space of Lévy noise is investigated. It is found that when characteristic exponent α becomes 0.5, Lévy

noise causes the neuronal firing to be irregular in different types of firing activities shown by Izhikevich network such as tonic spiking, intrinsic bursting, chattering, low threshold spiking and fast spiking (refer Figs. 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14). Other parameter values of Lévy noise are varied in their respective ranges, but a significant change in neuronal firing is not observed. According to Nolan, density takes maximum value when α becomes 0.5 in the standard parameterisation of the stable Lévy distribution [52]. This property may be considered as the reason why Lévy noise with α equal to 0.5 triggers irregular firing in the neuronal network. Figure 15 shows the probability density function $f(x)$ of stable Lévy distribution for different values of α . Other parameters of Lévy distribution are chosen to be $\beta = 0, \sigma = 1, \mu = 0$. This figure is reproduced using the program stable.exe with permission from the website <http://fs2.american.edu/jpnolan/www/stable/>. When $\alpha = 0.5$, the stable density is found to be maximum. Therefore, maximum influence due to the Lévy noise on the neurons occurs when Lévy noise has the characteristic exponent $\alpha = 0.5$. Noise with other values of α has minimal effect on the neuronal firings.

In Figs. 1 and 2, membrane potential, adaptation current and synaptic conductance of a randomly selected tonic spiking neuron of all-to-all Izhikevich network are plotted without and with Lévy noise, respectively. The parameters of Lévy noise are chosen as $\alpha = 0.5, \beta = 1, \sigma = 1, \mu = 0$. The spiking activities are found to be different with and without Lévy noise. It

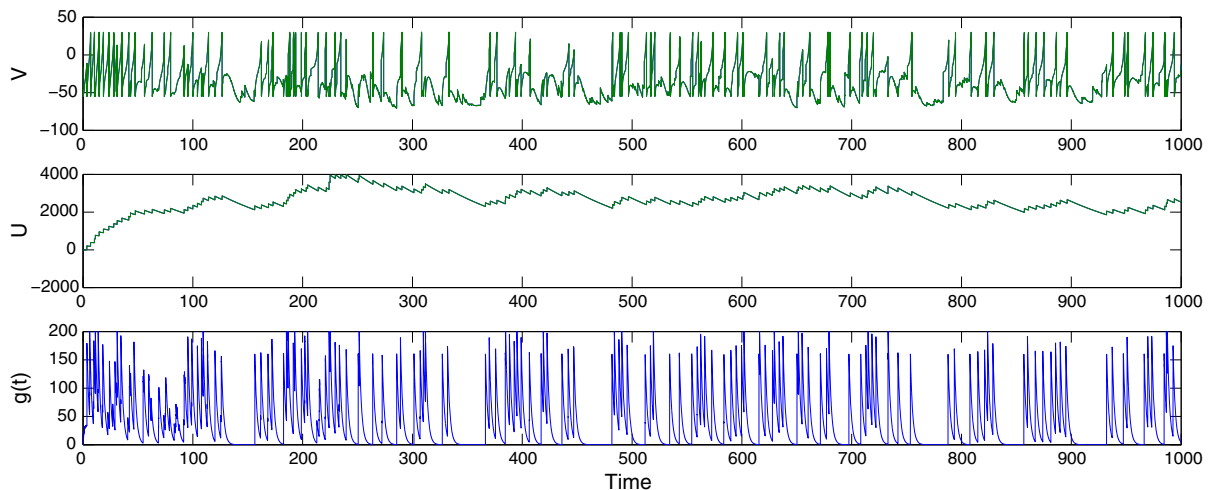


Fig. 2 Simulation of a neuron in the network of 1000 tonic spiking Izhikevich neurons with Lévy noise ($\alpha = 0.5$, $\beta = 1$, $\sigma = 1$, $\mu = 0$). Membrane potential V and synaptic conductance $g(t)$ exhibit irregular and random spiking behaviour. As time evolves,

the adaptation current U also behaves randomly. Lévy noise with $\alpha = 0.5$ causes additional spike resettings in the neurons of the network

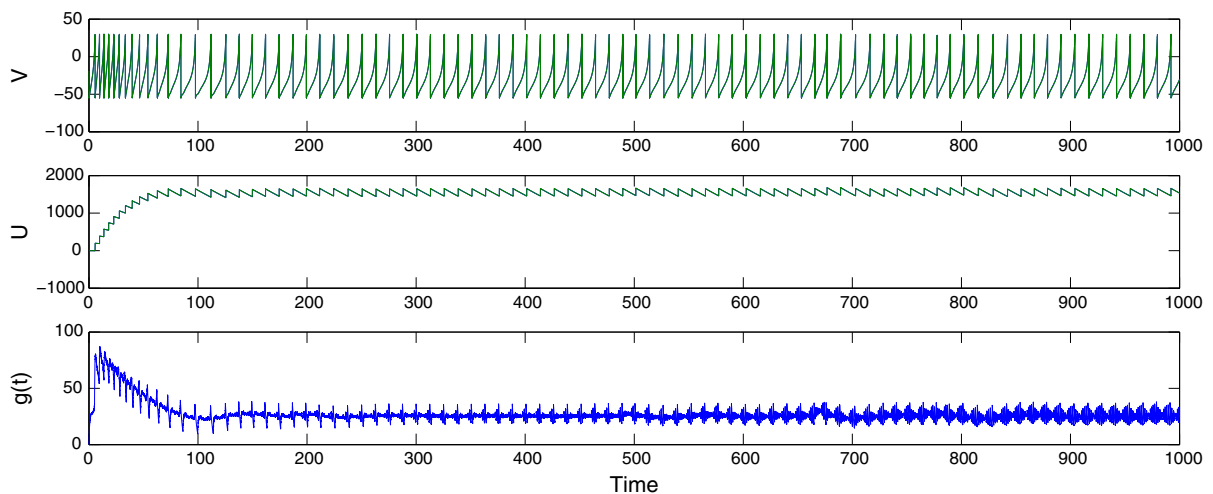


Fig. 3 Simulation of a neuron in the network of 1000 tonic spiking Izhikevich neurons with Lévy noise ($\alpha = 1$, $\beta = 1$, $\sigma = 1$, $\mu = 0$). This noise does not force the network to alter its dynamics

has been observed that only for the Lévy noise characterised by the α -stable distribution $L_{\alpha,\beta}(\xi; \sigma, \mu) = L_{0.5,\beta}(\xi; 1, 0)$, the activity modes show deviation from the noise-free case. The tonic spiking mode of the membrane potential has become aperiodic and irregular. It can be seen that the neurons which produced 2 spikes in a particular time interval have been excited to yield 3, 4 or more spikes in the same interval under the influence of Lévy noise with $\alpha = 0.5$. The spiking pattern

of membrane voltage shows transition into an irregular state. The adaptation current also exhibits a deviation from the fluctuation pattern shown when noise is absent. It can also be seen that synaptic conductance goes into a completely random and irregular pattern. For all other cases of parameter values of Lévy noise, even if they are varied within their respective limits, no significant difference in spiking activities is noticed. Figures 3 and 4 represent the cases where noise does not

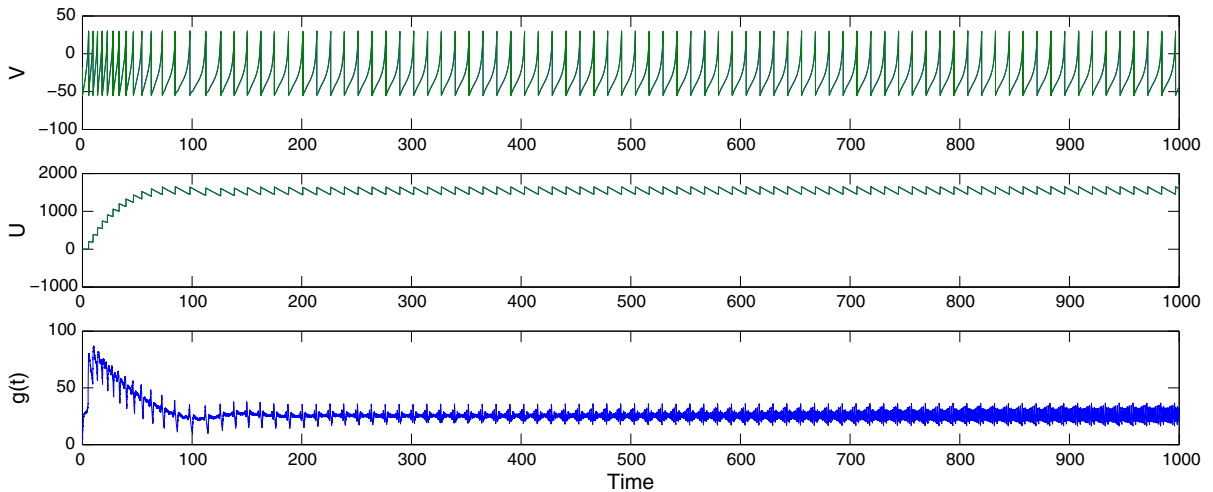


Fig. 4 Simulation of a neuron in the network of 1000 tonic spiking Izhikevich neurons with Lévy noise ($\alpha = 1.7, \beta = 0, \sigma = 1, \mu = 0$). The spiking dynamics of the network is similar to the case when noise is not applied

alter the spiking behaviour of the neurons and retains the dynamics of noise-free network. The phase space plots of the network without and with Lévy noise are represented in Figs. 5 and 6.

Also, the effect of Lévy noise is studied for other firing types shown by Izhikevich neuronal network, namely intrinsic bursting, chattering, fast spiking and low threshold spiking (Figs. 7, 8, 9, 10, 11, 12, 13, 14). In all cases, it is found that when α parameter of Lévy noise is 0.5, firings become random and irregular.

3.2 Dynamics of excitatory coupled pair of networks with Lévy noise

To study the response of excitatory coupled pair of Izhikevich networks towards the applied noise, neurons are randomly chosen from strongly and weakly adapting populations. Figure 16 represents the neuronal fluctuations without noise. Neuronal fluctuations for strongly adapting neuron are marked in red colour, while those for weakly adapting neuron are in green colour. The strongly adapting neuron population is bursting, while the weakly adapting population has an oscillatory non-bursting firing pattern. Figure 17 denotes the dynamics of the networks with Lévy noise having parameters $\alpha = 0.5, \beta = 1, \sigma = 1, \mu = 0$. Spikes have become irregular in nature. Completely random fluctuations appear in the membrane potential

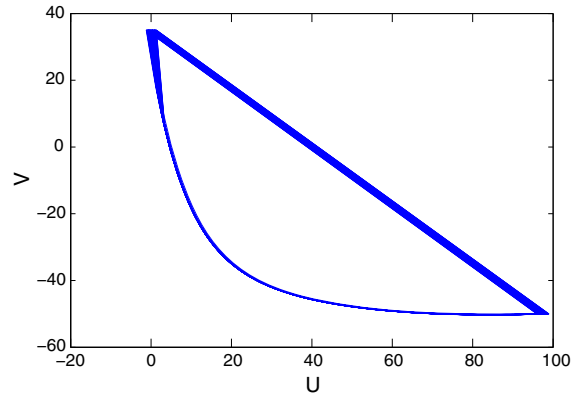


Fig. 5 Phase space plot of Izhikevich neuron

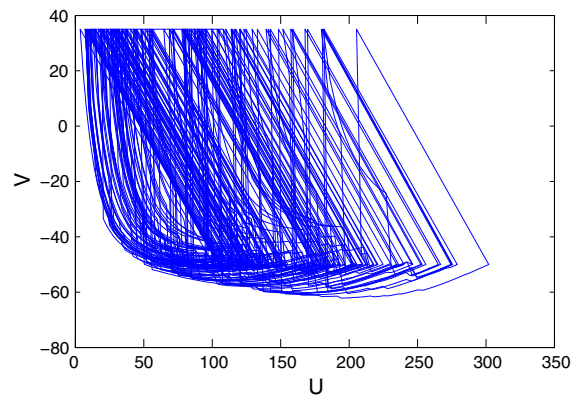


Fig. 6 Phase space plot of Izhikevich neuron with Lévy noise ($\alpha = 0.5$). The limit cycle behaviour of the neuron has been changed

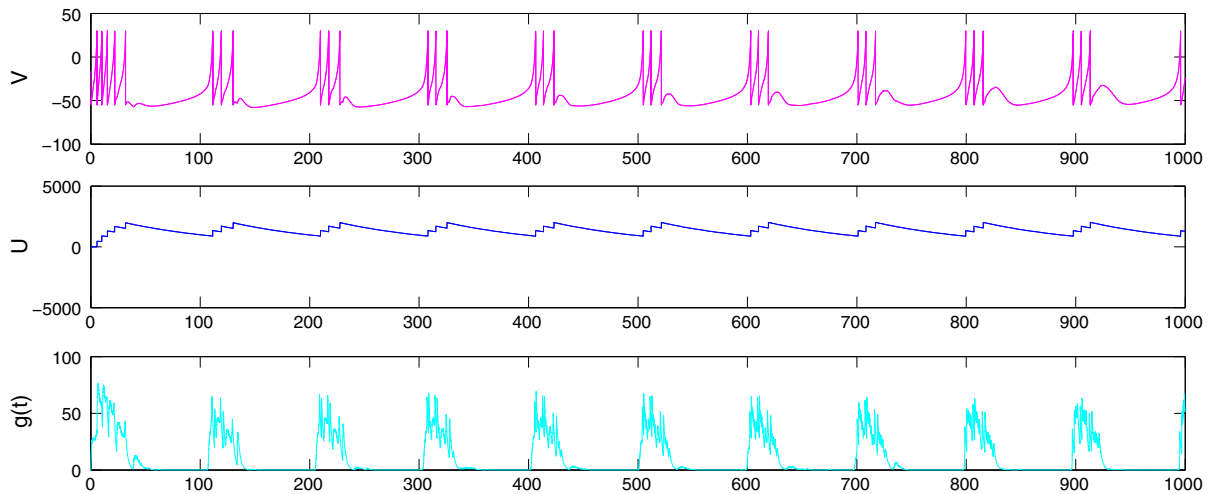


Fig. 7 Simulation of a neuron in the network of 1000 chattering Izhikevich neurons without Lévy noise. Similar behaviour is seen when a Lévy noise is applied with α other than 0.5

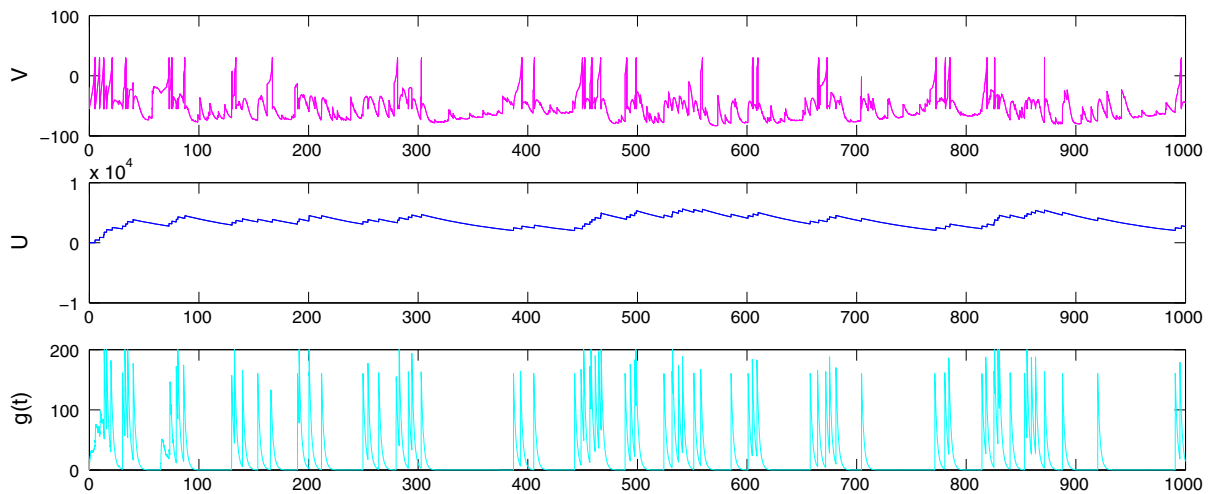


Fig. 8 Simulation of a neuron in the network of 1000 chattering Izhikevich neurons with Lévy noise ($\alpha = 0.5$, $\beta = 0$, $\sigma = 1$, $\mu = 0$). The spiking dynamics of the network have become irregular

and in the synaptic conductance variables of strongly adapting population. The spiking patterns of membrane potential of weakly adapting population have also undergone a change with a loss in periodicity. But the current of weakly adapting population is not showing a change in the firing pattern. The noise is found to have no effect on the overall behaviour of the networks for any other α parameter value of the Lévy noise. It is interesting to note that the strongly adapting neuronal population in coupled pair of heterogeneous networks under Lévy noise with $\alpha = 0.5$ (Fig. 17) shows spik-

ing patterns similar to those exhibited by the single homogenous network under the Lévy noise with same α (Fig. 2). It shows that Lévy noise with particular value of α equal to 0.5 causes the Izhikevich network to have a dynamical behaviour independent of topology and heterogeneity of the network. In cases where the networks are either noise-free or has Lévy noise with $\alpha \neq 0.5$ (Figs. 18, 19), spiking dynamics depend on the topology of the networks.

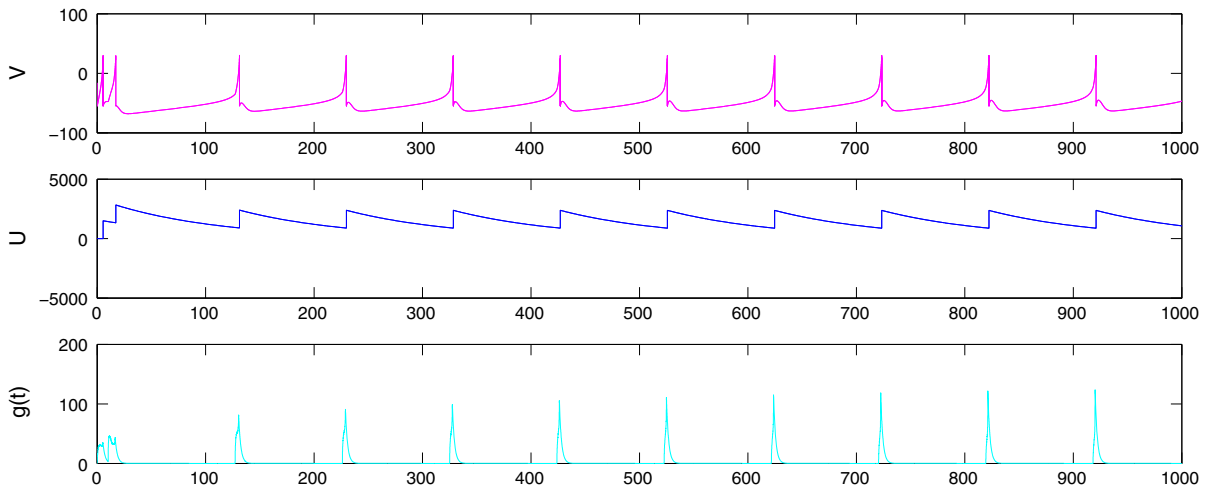


Fig. 9 Simulation of a neuron in the network of 1000 intrinsic bursting Izhikevich neurons without Lévy noise. When a Lévy noise is applied with α other than 0.5, same kind of firing is observed

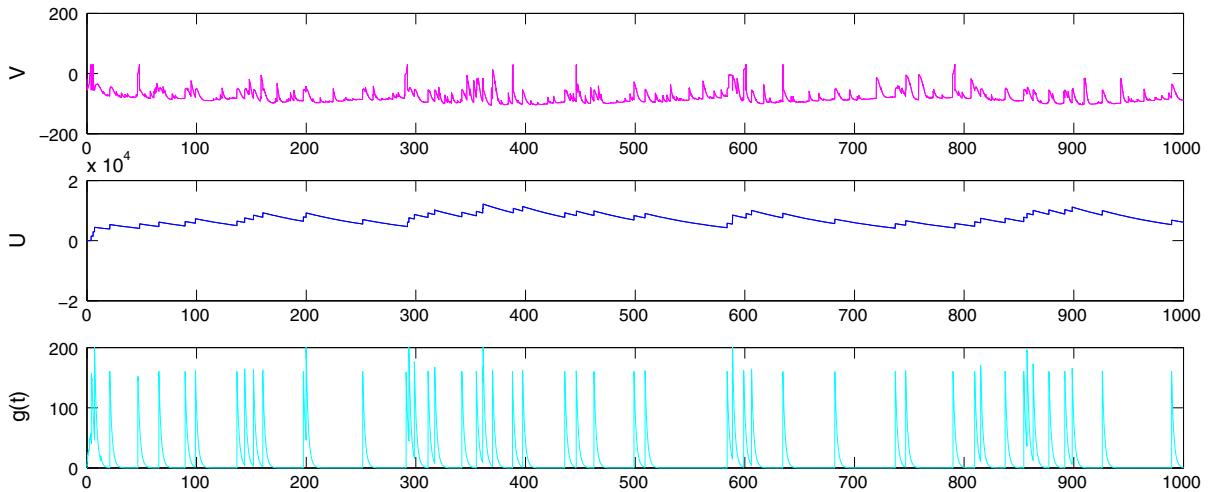


Fig. 10 Simulation of a neuron in the network of 1000 intrinsic bursting Izhikevich neurons with Lévy noise ($\alpha = 0.5, \beta = 0, \sigma = 1, \mu = 0$). Randomness is brought about by the Lévy noise with $\alpha = 0.5$

3.3 Statistical analysis

The irregularity and spiking variability of the neuronal fluctuations of the networks with Lévy noise ($\alpha = 0.5$) are further analysed using statistical methods. Analyses show that clustered neuronal networks behave in a random manner. Single all-to-all coupled Izhikevich network and the excitatory coupled pair of Izhikevich networks produce similar graphs for spike rate and variation measures. The analyses are carried out for tonic spiking neuron network for a time duration of 1000 ms.

3.3.1 Spiking rate

Spike rate is a measure of neuronal activity over a duration of time measured in Hz. They are commonly used to quantify a neural response over a given duration of time T . When a neuron’s spiking activity is represented as a vector of spike times $t_1, t_2, t_3 \dots t_N$, the rate is defined by

$$R = \frac{N}{T}$$

where N is the total number of spikes over a given duration T . Spike rates can be computed as a function

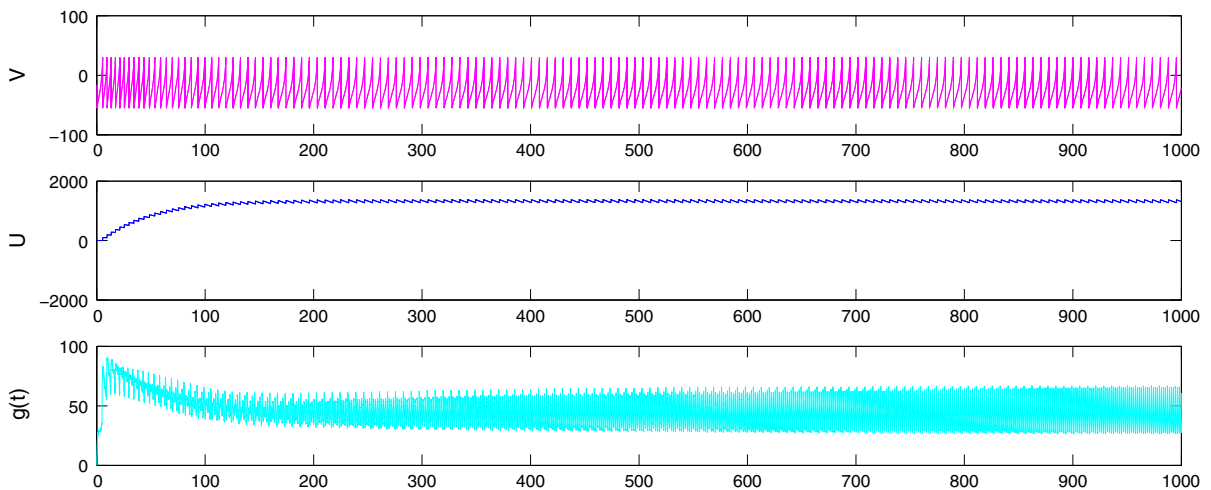


Fig. 11 Simulation of a neuron in the network of 1000 fast-spiking Izhikevich neurons without Lévy noise. Firing shows same behaviour when a Lévy noise is applied with α other than 0.5

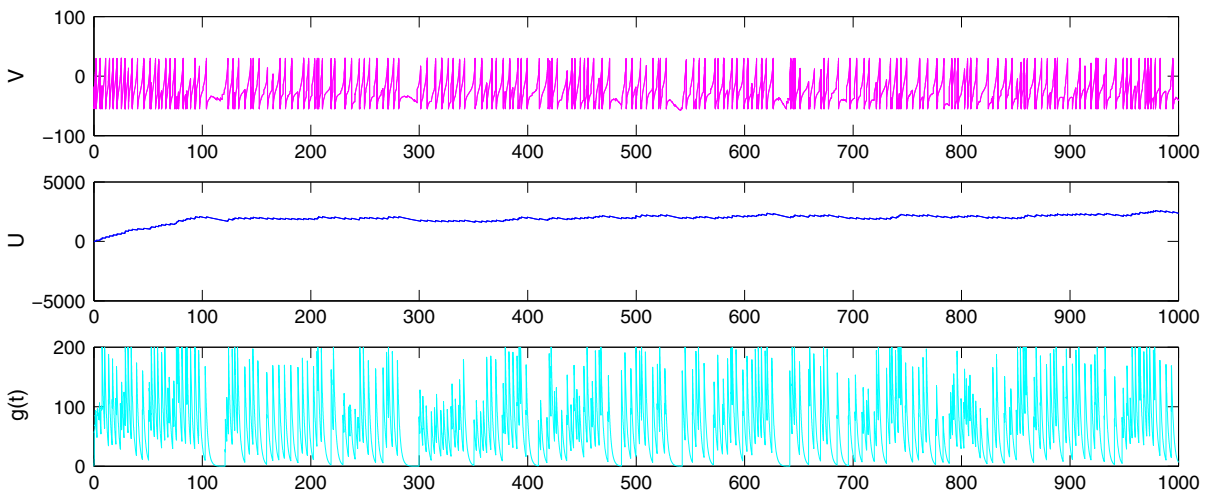


Fig. 12 Simulation of a neuron in the network of 1000 fast-spiking Izhikevich neurons with Lévy noise ($\alpha = 0.5$, $\beta = 0$, $\sigma = 1$, $\mu = 0$). Firing has gone irregular when Lévy noise with $\alpha = 0.5$ is applied

of single neuron across trials, or over a network, thus generating a distribution of rates with variability [54].

The activities of the 100 neurons randomly chosen from the Izhikevich population are plotted as a function of their firing rate. Without noise, neurons fire approximately at a constant rate of 0.162 (Fig. 20). It is observed that the population activity is significantly increased to 0.36–0.365 when the Lévy noise is applied (Fig. 21). Such increase is seen only when Lévy noise with the parameter $\alpha = 0.5$ is applied. This result is in agreement with the increased spiking behaviour of

both single homogeneous network (Fig. 2) and coupled pair of heterogeneous networks (Fig. 17) with Lévy noise having $\alpha = 0.5$. When the α value of Lévy noise is changed, spiking rate decreases to its value in the noise-free case.

3.3.2 Inter-spike intervals (ISIs)

An ISI measures the latency Δt between precise spike times. ISIs can be computed for every single spike (minus one) or a percentage of spikes. When a neuron's spiking activity is arranged in a vector of spike

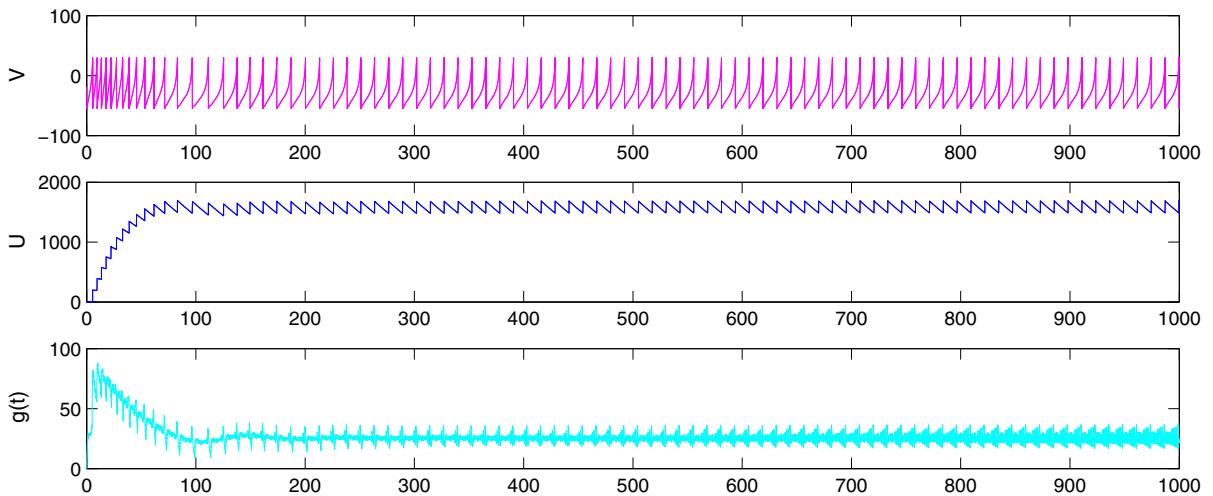


Fig. 13 Simulation of a neuron in the network of 1000 low- threshold-spiking Izhikevich neurons without Lévy noise. When a Lévy noise is applied with α other than 0.5, we get the same pattern

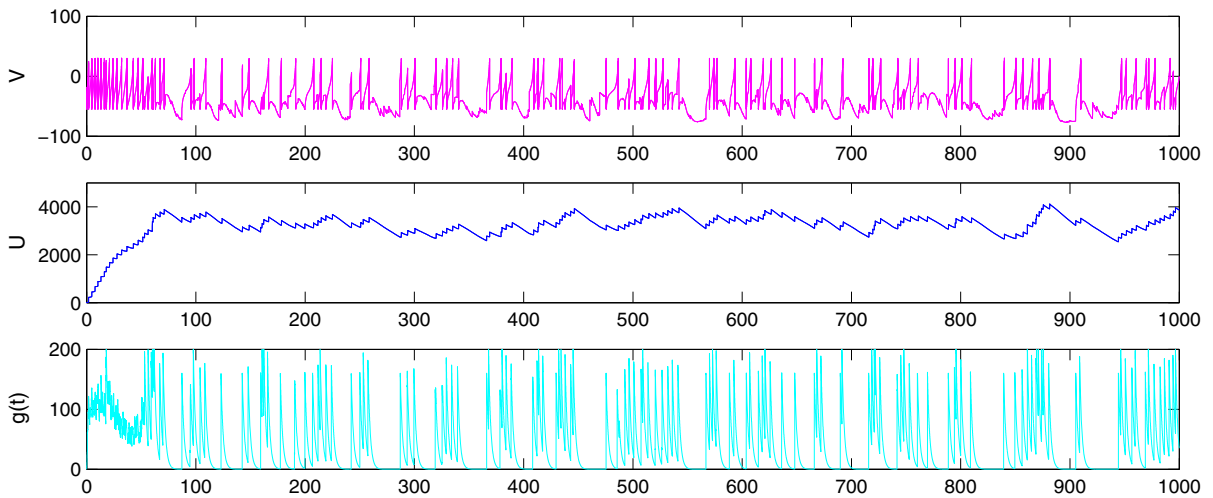


Fig. 14 Simulation of a neuron in the network of 1000 low- threshold-spiking Izhikevich neurons with Lévy noise ($\alpha = 0.5, \beta = 0, \sigma = 1, \mu = 0$)

times $t_1, t_2, t_3 \dots t_N$, the i th ISI is defined by

$$\Delta t_i = t_{i+1} - t_i$$

where t represents a given spike time. Intervals are then counted, generating a distribution of ISIs [54].

A bifurcation plot of ISIs is drawn against varying neuronal currents of the network under the Lévy noise with $\alpha = 0.5$ (Fig. 22). Noise is imposed on the neuronal network from time $t=0$ onwards. It is observed that chaotic nature is introduced from the beginning of time when the applied Lévy noise is having characteristic index α as 0.5.

3.3.3 Coefficient of variation (Cv)

The Cv is a measure of the dispersion in the ISI distribution. Using the vector of spike times $t_1, t_2, t_3 \dots t_N$, then Cv is defined by

$$Cv = \sqrt{\frac{1}{N-2} \sum_{i=1}^{N-1} (\Delta t_i - \bar{\Delta t})^2} / \bar{\Delta t}$$

where N is the total number of spikes, Δt is a given ISI, and $\bar{\Delta t}$ is defined by

$$\bar{\Delta t} = \frac{1}{N-1} \sum_{i=1}^{N-1} (\Delta t)$$

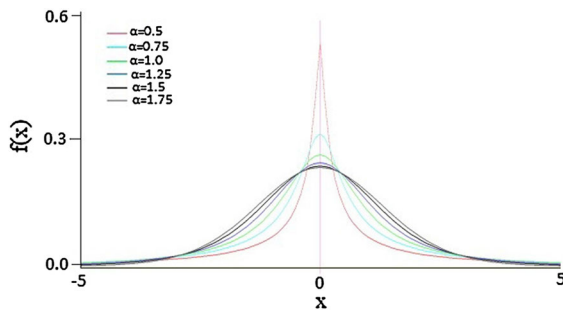


Fig. 15 Stable densities in the standard parameterisation of Lévy distribution ($\beta = 0, \sigma = 1, \mu = 0$). Graphs are drawn for $\alpha = 0.5, 0.75, 1.0, 1.25, 1.5, 1.75$

A Poisson process produces distributions with $Cv = 1$. When $Cv > 1$, it implies that a given spike train is less regular than a Poisson process with the same firing rate. Cv detects a global variability of a whole ISI sequence and is sensitive to the firing rate fluctuation which may occur in natural conditions. Cases where either noise is absent or Lévy noise ($\alpha \neq 0.5$) is applied, neurons in the network show a Cv value much less than 1 (Fig. 23). For Izhikevich networks having Lévy noise ($\alpha = 0.5$), the coefficient of vari-

ation of each neuron has a value greater than 1 which indicates the irregularity in the spiking nature of the neurons introduced by the particular α - stable Lévy noise (Fig. 24).

3.3.4 Autocorrelation

Autocorrelation gives a measure of how well a signal is correlated with itself. Using an autocorrelation, we quantify the self-similarity of a neuron signal as a function of the time lag τ [55]. Autocorrelation plot helps to check randomness in a data set. This randomness is ascertained by computing autocorrelations for data values at varying time lags. If random, such autocorrelations should be near zero for any and all time-lag separations. If non-random, then one or more of the autocorrelations will be significantly nonzero.

The neuronal fluctuations of Izhikevich networks with Lévy noise ($\alpha = 0.5$) show the autocorrelation diagram as in Fig. 26. They have self-similarity only when $\tau = 0$. It occurs when the Lévy noise with $\alpha = 0.5$ is added to the action potentials of the neurons in the network. When noise distribution takes other parameter values, such irregularity is lost (Fig. 25).

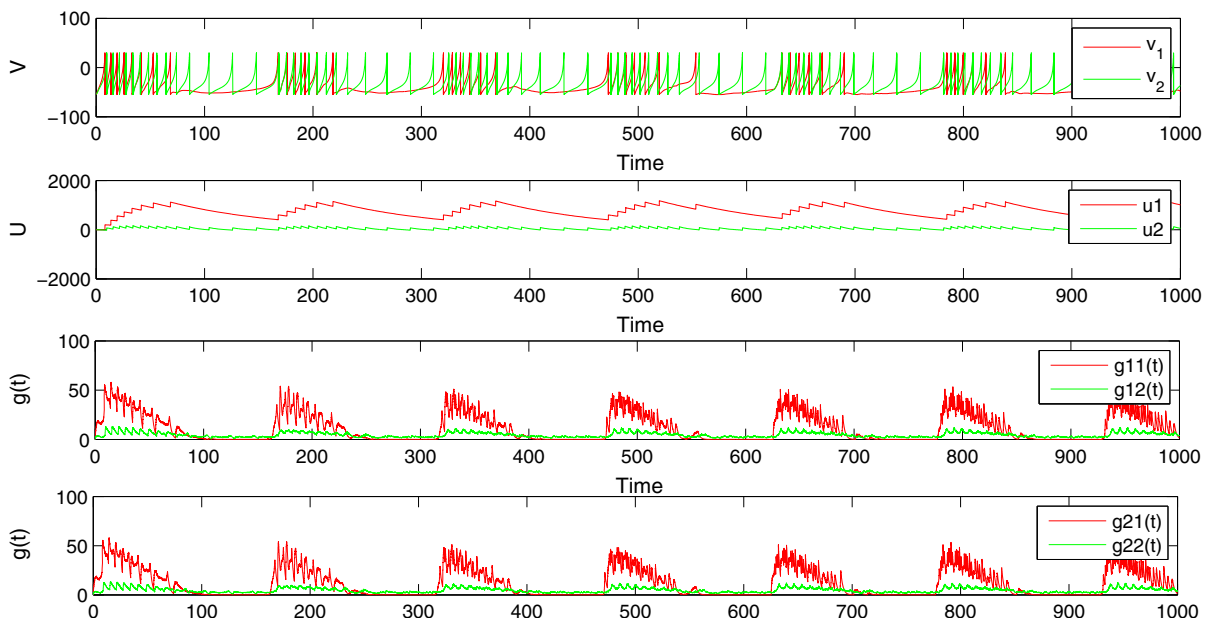


Fig. 16 Simulation of a network of Izhikevich neurons without Lévy noise. The network consists of two distinct populations: 800 strongly adapting neurons and 200 weakly adapting neurons. Dynamics of strongly adapting neuron are shown in red,

while that of weakly adapting neuron is in green. Approximately regular bursting dynamics can be seen for both kinds of neurons

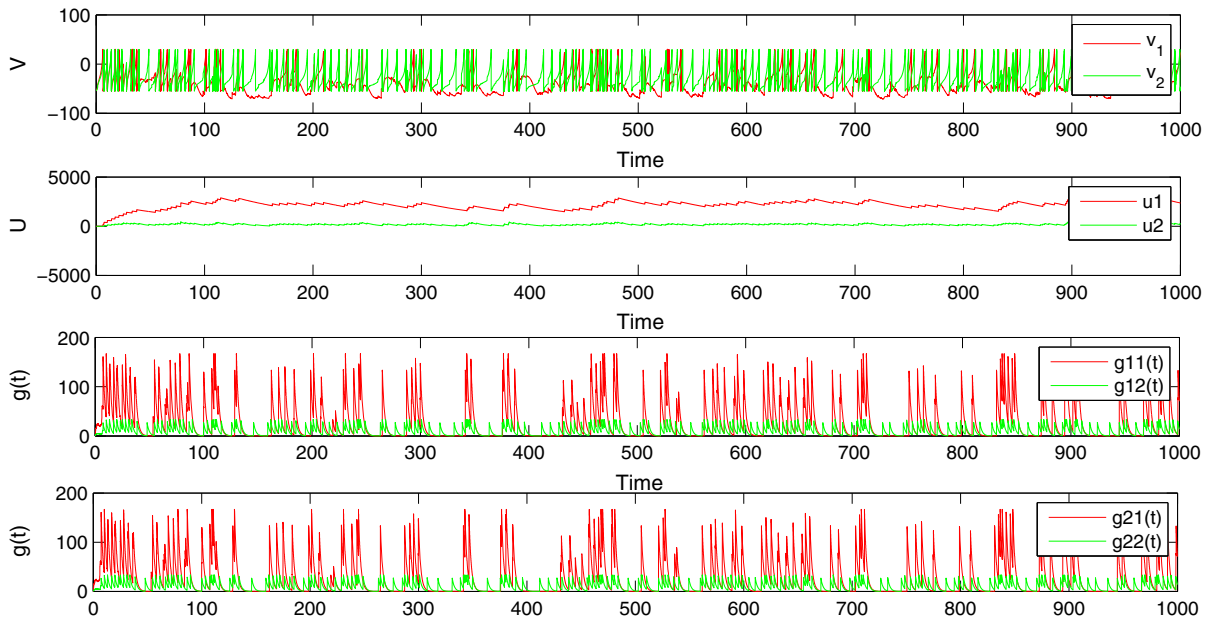


Fig. 17 Simulation of Izhikevich network with 800 strongly adapting neurons and 200 weakly adapting neurons. Lévy noise ($\alpha = 0.5, \beta = 1, \sigma = 1, \mu = 0$) is applied. The regularity of the dynamics has been lost. In the noise-free case, strongly and weakly adapting neurons had undergone bursting simultaneously. But when Lévy noise ($\alpha = 0.5, \beta = 1, \sigma = 1, \mu = 0$),

no such synchronisation is seen between neurons with different adaptations. Quiescent states in the firing patterns have undergone a resetting into spiking states. The whole dynamical behaviour of coupled pair of these heterogeneous networks under Lévy noise ($\alpha = 0.5$) matches with that of a single homogenous network under the same Lévy noise (refer Fig. 2)

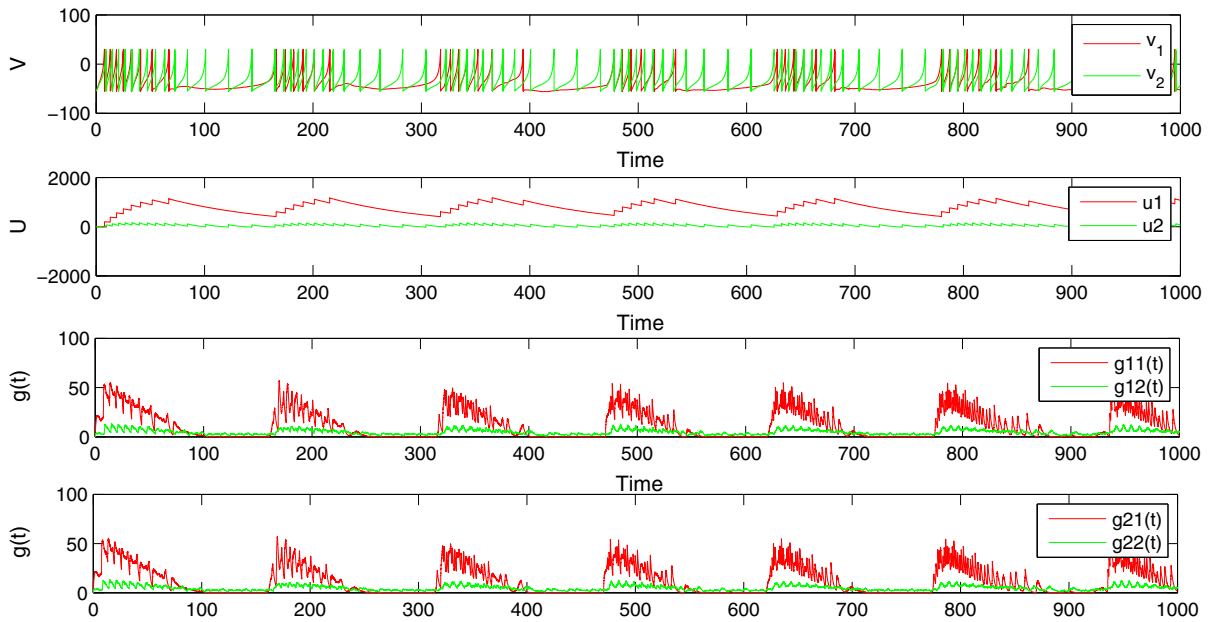


Fig. 18 Simulation of Izhikevich network of 800 strongly adapting neurons and 200 weakly adapting neurons. Lévy noise ($\alpha = 1, \beta = 1, \sigma = 1, \mu = 0$) is applied. This noise retains the behaviour of the noise-free neuronal networks

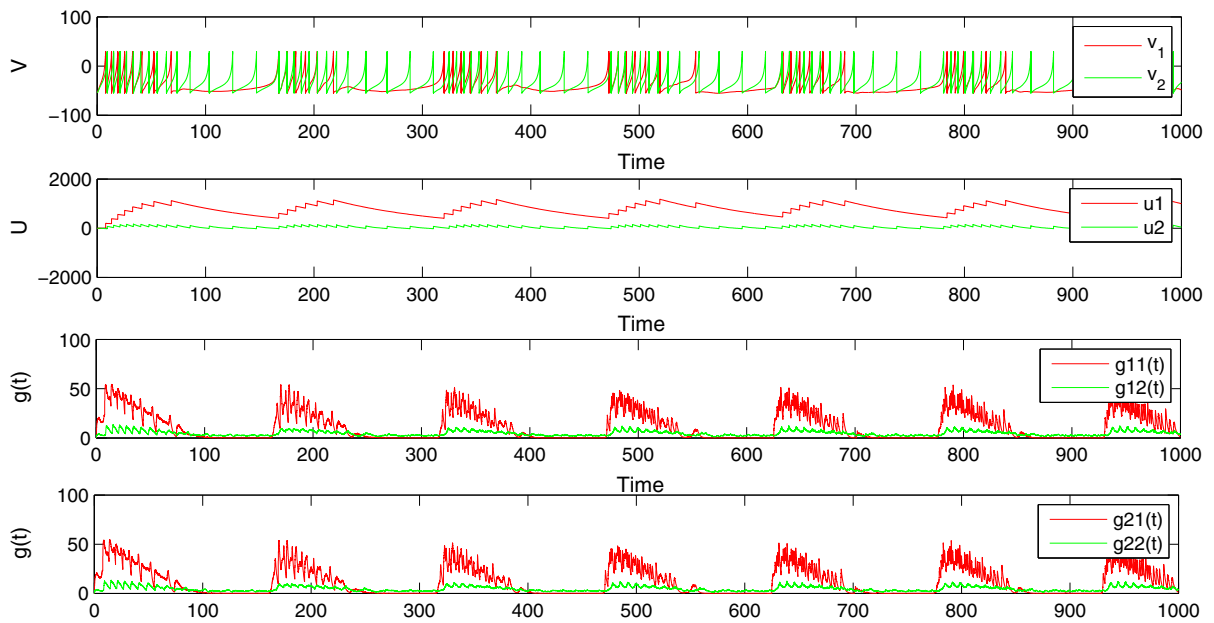


Fig. 19 Simulation of a network of Izhikevich neurons. The network consists of two distinct populations: 800 strongly adapting neurons and 200 weakly adapting neurons. Lévy noise ($\alpha =$

$1.7, \beta = 0, \sigma = 1, \mu = 0$) is applied. No change in the dynamical behaviour is brought about by the introduction of this noise

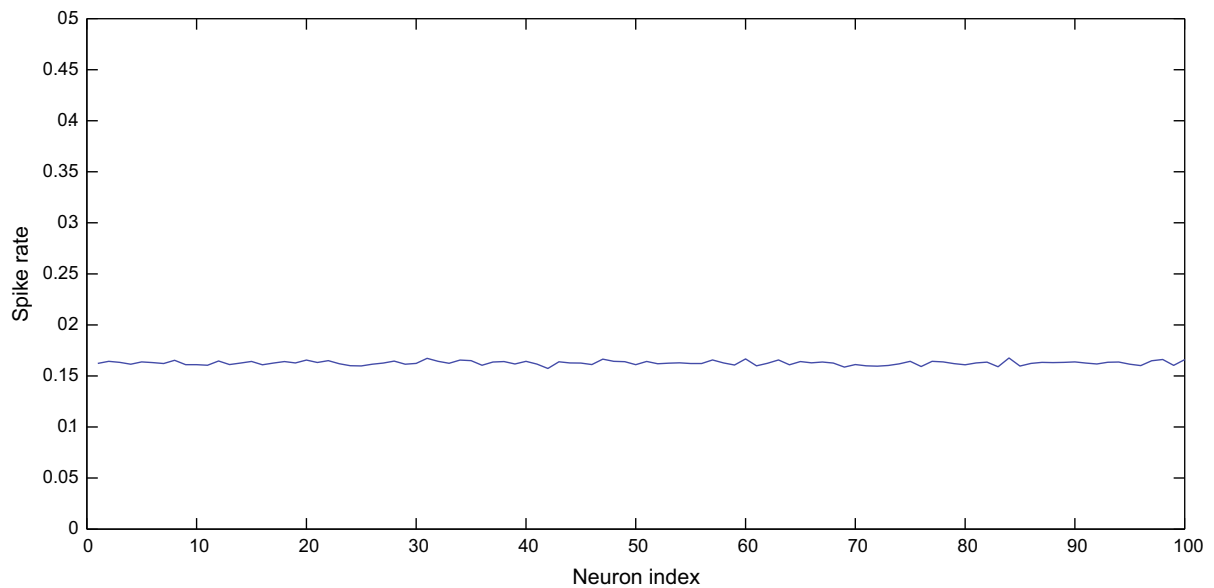


Fig. 20 Spiking rate graph of Izhikevich networks when Lévy noise is absent. The neurons fire at an almost constant rate of 0.162. Similar graph is obtained when a Lévy noise with $\alpha \neq 0.5$ is applied to the networks

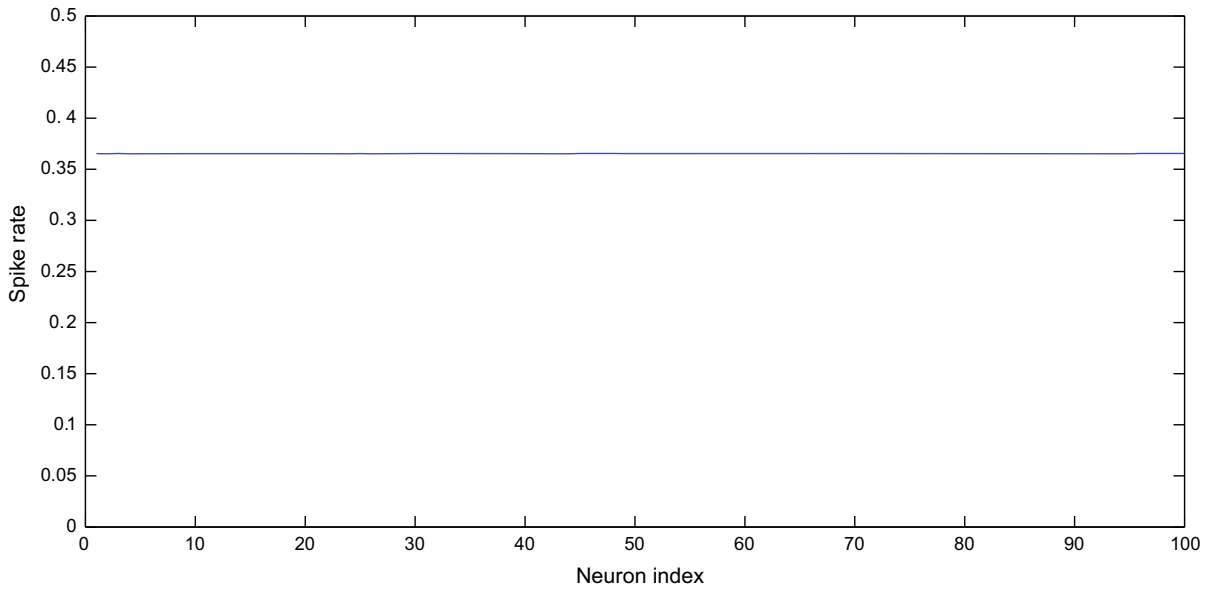


Fig. 21 Spiking rate graph when Lévy noise ($\alpha = 0.5, \beta = 1, \sigma = 1, \mu = 0$) is applied. The applied noise forces all the neurons to increase their firing so that their spiking rate is more than double in comparison with the noise-free neurons

Fig. 22 Bifurcation graph of ISI versus the current when Lévy noise ($\alpha = 0.5$) is applied

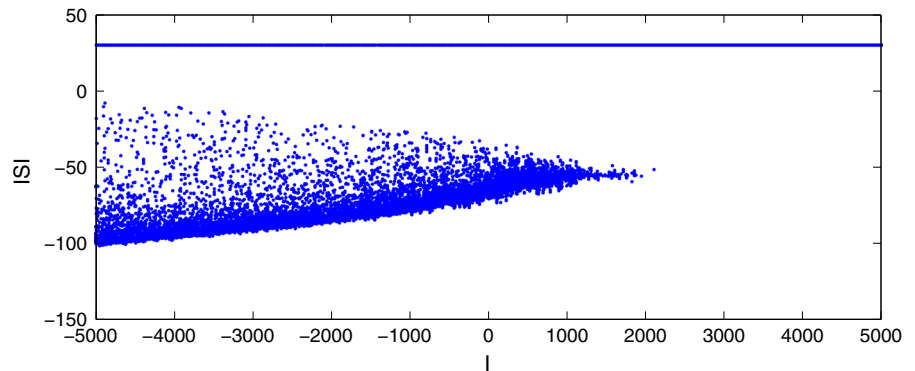


Fig. 23 Coefficient of variation graph of neurons when Lévy noise is absent. Cv values of the neurons lie in the range of 0.01–0.016. It is much less than 1. The Izhikevich neurons behave in a more or less periodic spiking manner in the absence of Lévy noise. Neurons produce similar graph with low values of Cv when Lévy noise ($\alpha \neq 0.5$) is applied

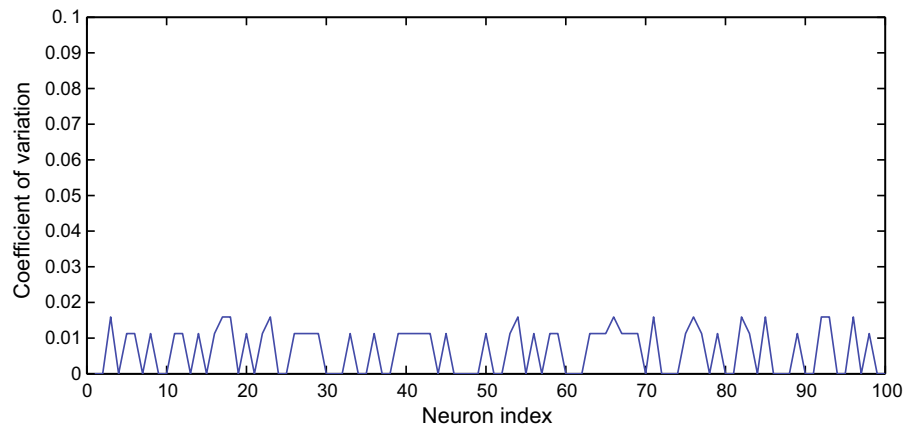


Fig. 24 Coefficient of variation graph of neurons with Lévy noise ($\alpha = 0.5$). Cv values of the neurons are increased to more than 1. It indicates that the spike trains produced by these neurons are random and irregular

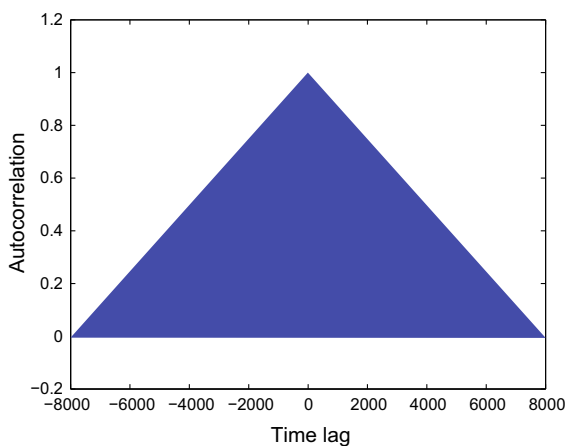
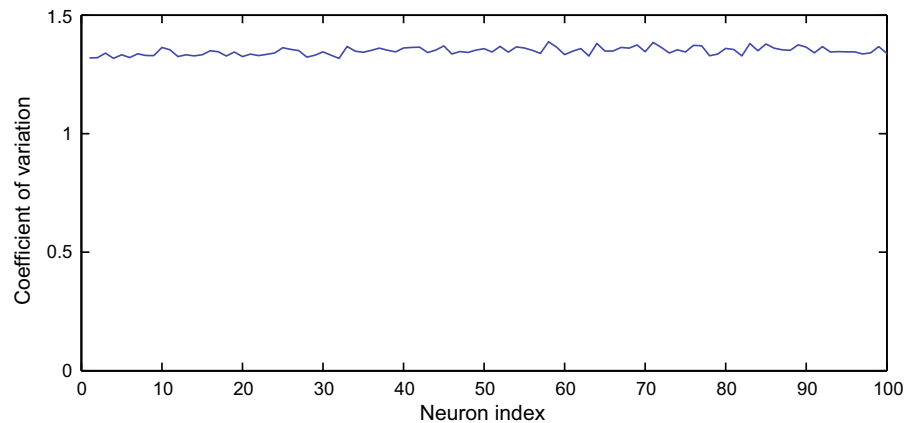


Fig. 25 Autocorrelation graph of the networks with no Lévy noise. Similar firing patterns exist in such networks. This kind of periodic nature exists also when Lévy noise ($\alpha \neq 0.5$) is present

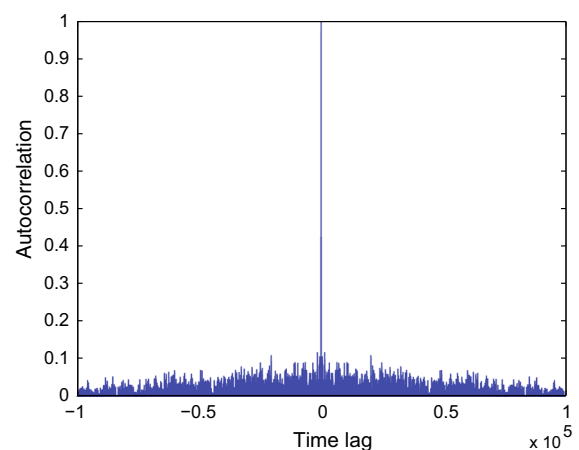


Fig. 26 Autocorrelation graph with Lévy noise ($\alpha = 0.5$). It shows the less periodical pattern and high variability of neuronal firings introduced by this α -stable noise

4 Discussion

The present work investigates the dimensionless stochastic Izhikevich model under the effect of non-Gaussian Lévy noise and examines the electrical activities of neurons. It is observed that the noise generated from Lévy noise distribution can cause the mode transition of electrical activities both in single Izhikevich network and in excitatory coupled pair of Izhikevich networks. When the noise produced from the Lévy distribution with parameter $\alpha = 0.5$ is applied to neuronal networks, the rest states of the electrical activities of neurons are excited to be firing states. The spiking states in a periodic electrical activity can be induced to be spiking states with high randomness. Also, the spiking states can be changed into a sort of bursting state. Irregularity

brought about by the Lévy noise depends only on the value taken by the characteristic exponent α . Even a slight deviation in the α parameter value of Lévy noise causes the neurons to revert to their more or less periodically spiking state. It is also noted that the strongly adapting neuronal population in coupled pair of heterogeneous networks under Lévy noise with $\alpha = 0.5$ and the single homogenous network under the Lévy noise with same α show similar spiking patterns. It shows that Lévy noise with particular value of α as 0.5 causes the Izhikevich network to have a dynamical behaviour independent of topology and heterogeneity of the network. When the networks are either noise-free or have Lévy noise with $\alpha \neq 0.5$, spiking dynamics depend on the topology of the networks. The statistical measure

also shows the increased firings of neurons under the Lévy noise with $\alpha = 0.5$.

Wu et al. [40] have studied the influence of Lévy noise on Hindmarsh–Rose (HR) model under electromagnetic induction. They have investigated that decrease in characteristic exponent from its upper boundary value 2 and increase in intensity of Lévy noise can heighten neuronal firings in HR neuron. Also the upward or downward skewing of the jumps of Lévy noise can alter HR neuron's firing dynamics. In this paper, we have explored that in the case of Izhikevich network an increase in neuronal firings is observed when characteristic exponent is set at $\alpha = 0.5$. But intensity, skewness and mean parameters of Lévy noise do not influence the dynamics of Izhikevich network.

In general, our studies establish that α -stable Lévy noise could improve electrical activity in a neuron individually and on the network level. This study can be extended to investigate the mean field approximation of the present network and to study the cumulative effect of electromagnetic radiation and Lévy noise on the network.

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