CrossMark

ORIGINAL PAPER

Adaptive practical stabilization of a class of uncertain nonlinear systems via sampled-data control

Jun Mao · **Zhengrong Xiang** ·
Guisheng Zhai · **Jian Guo**

Abstract In this paper, an adaptive practical stabilization problem is investigated for a class of nonlinear systems via sampled-data control. The systems under study possess uncertain dynamics and unknown gain functions. During sampled-data controller design procedure, a dynamic signal is introduced to dominate the unmeasured states existed in the external disturbances, and neural networks are adopted to approximate the unknown nonlinear functions. By choosing appropriate sampling period, the designed sampled-data controller can render all states of the resulting closed-loop system to be semi-globally uniformly ultimately bounded. Two examples are given to demonstrate feasibility and efficacy of the proposed methods.

Keywords Nonlinear systems · Adaptive practical stabilization · Sampled-data control · Dynamic signal · Neural networks

1 Introduction

It is well known that real-world systems are inevitable to contain uncertainties. Driven by practical require-

J. Mao · Z. Xiang (✉) · J. Guo
School of Automation, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China
e-mail: xiangzr@mail.njust.edu.cn

G. Zhai
Department of Mathematical Sciences, Shibaura Institute of Technology, Saitama 337-8570, Japan

ments and theoretical challenges, adaptive controller design of nonlinear systems has become an important research domain recently. An adaptive fault-tolerant tracking control problem of a class of nonlinear systems with multiple delayed state perturbations was investigated in [1]. Adaptive tracking and stabilization problems of the nonlinear large-scale systems with dynamic uncertainties was discussed in [2–5], where the inverse dynamics of the subsystems considered in [4,5] were required to be stochastic input-to-state stable. Finite-time stabilization of a class of switched stochastic nonlinear systems under arbitrary switchings was investigated in [6] by adopting the adding a power integrator technique, subsequently, a tracking control method was proposed in [7] for a class of switched stochastic nonlinear systems with unknown dead-zone input by using the common Lyapunov function method. Two additional factors of adaptive control in nonlinear systems were considered in [8,9], where time delay was considered in [8], and unmeasured dynamic uncertainties was considered in [9]. Adaptive fuzzy output-constrained tracking fault-tolerant control was focused in [10], where the barrier Lyapunov function was used to guarantee that all the signals in the closed-loop system were bounded in probability and the system outputs was constrained in a given compact set. Two novel adaptive finite-time control schemes were developed in [11,12] for high-order nonlinear systems, where the result presented in [11] was the first time to solve the finite-time control problem of the nonlinear systems

with uncertain time-varying control coefficients and unknown nonlinear perturbations. Under some appropriate assumptions, adaptive practical finite-time stabilization for a class of switched nonlinear systems in pure-feedback form was investigated in [13].

However, it should be emphasized that all the aforementioned results were based on continuous-time control. In the past decades, due to the remarkable development in digital technology, it becomes a commonly known standard that sampled-data controllers with analog-to-digital and digital-to-analog devices for interfacing are being digitally implemented into practical systems, such as aircraft systems [14] and multi-robot systems [15]. The three main approaches for the design of sampled-data controllers are (e.g., see [16–18]):

1. Design based on a continuous-time plant model plus a controller discretization (the CTD method);
2. Design based on the exact or approximate discrete-time plant model ignoring inter-sample behavior (respectively, the DTD method and the approximate DTD method);
3. Design based on the sampled-data model of the plant which takes into account the inter-sample behavior in the design procedure (the SDD method).

The majority of work for the sampled-data control of nonlinear systems uses the CTD method, and various sampled-data control algorithms for nonlinear systems depicted in continuous dynamics were presented in many existing literature, for example, two sampled-data control schemes were developed in [16,19] to realize global practical tracking of nonlinear systems, a systematic design scheme was developed in [20] to construct a linear sampled-data output feedback controller that semi-globally asymptotically stabilizes a class of uncertain systems with both higher-order and linear growth nonlinearities. By designing observers, sampled-data control methods were proposed for the nonlinear systems with improved maximum allowable transmission delay [21] and input delay [22]. An observer, which was featured with a special feedforward propagation structure, was constructed in [23] to estimate the unmeasured states of a class of uncertain nonlinear systems with uncontrollable and unobservable linearizations, and global sampled-data stabilization of the nonminimum-phase nonlinear systems were addressed in [24]. Using the small gain approach, a result concerning the sampled-data observer design for

a wide classes of nonlinear systems with delayed measurements was considered in [25]. Stabilization for the sampled-data systems under noisy sampling interval was investigated in [26]. A memory sampled-data control scheme which involved a constant signal transmission delay was proposed in [27] for the chaotic systems. Sampled-data stabilization and exponentially synchronization were, respectively, investigated in [28,29] for the T–S fuzzy systems and the Markovian neural networks. Reliable dissipative control and non-fragile H_∞ control for the Markov jump systems were focused in [30,31].

Although sampled-data control has received much attention, there are few works on sampled-data adaptive practical stabilization for uncertain nonlinear systems, which is mainly due to the complexity arising from the design of adaptive laws for sampled-data controller, that is, under sampled-data situation that system states can only be measured at the sampling points, adaptive laws cannot be constructed like continuous-time control by using system states which are measured everywhere. Thus, an issue naturally arises: under that situation, how to design an adaptive controller to practically stabilize such nonlinear systems by only using the sampled system states? In this paper, we will try to give an answer, and the main contributions can be summarized as follows:

(i) An adaptive practical stabilization problem is considered in this paper for a class of uncertain nonlinear systems via sampled-data control. Different from some existing nonlinear sampled-data control results, such as [16,19], our scheme can be obtained under a weaker assumption that the diffusion terms of the considered systems are not required to satisfy any linear growth and bounded conditions.

(ii) Unlike existing adaptive continuous-time control schemes of nonlinear systems, such as [2–5], the adaptive laws in this paper are designed based on the sampled signals due to that the system states can only be measured at the sampling points, and then, we use the sampling values of the adaptive parameters to construct the sampled-data controller to practically stabilize the considered nonlinear systems.

(iii) The sampling period in this paper is non-periodic, which is different from those in [18–24].

The remainder of the paper is organized as follows: in Sect. 2, we present the problem statements

and some preliminaries. An adaptive sampled-data controller design are presented in Sect. 3. The main theorem is shown in Sect. 4, where the stabilization proof is also obtained using the well-known Gronwall–Bellman inequality. Numerical simulations and discussions are shown in Sect. 5 to demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in Sect. 6.

Notations R denotes the set of all real numbers; R^n indicates the real n-dimension-al space; $R^{m \times n}$ denotes the real $m \times n$ matrix space; $\|\cdot\|$ denotes the Euclidean norm; K denotes the set of all functions: $R^+ \rightarrow R^+$, which are continuous, strictly increasing and vanish at zero; K_∞ denotes the set of all functions which are class K and unbounded; C^i denotes a set of functions whose ith-order derivatives are continuous and differentiable; the argument of the functions will be omitted throughout the paper for simplicity whenever no confusion arises, such as functions V, x_1 and z_2 to be used hereafter.

2 Problem statement and preliminaries

Consider the following uncertain nonlinear system:

$$\begin{cases} \dot{z}_0 = q(z_0, x_1), \\ \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) \\ \qquad + \Delta_i(\bar{x}_i, z_0), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n = g_n(\bar{x}_n)u + f_n(\bar{x}_n) + \Delta_n(\bar{x}_n, z_0), \\ y = x_1, \end{cases} \quad (1)$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ are the states which can only be measured at the sampling points $t_k, k = 1, 2, \ldots, +\infty$, and $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i, i = 1, 2, \ldots, n$; $z_0 \in R^{n_0}$ is an unmeasured state; $u \in R$ and $y \in R$ are the control input and output, respectively; $f_i(\cdot), g_i(\cdot), i = 1, 2, \ldots, n$, are unknown nonlinear smooth functions, and $f_i(0, \ldots, 0) = 0$; $\Delta_i(\cdot)$, $i = 1, 2, \ldots, n$, are external disturbances, and $\Delta_i(0, \ldots, 0) = 0$; $q(\cdot)$ is an unknown Lipschitz function.

Remark 1 In practical engineering, some nonlinear systems, such as thruster assisted position mooring systems [32], levitated ball systems [33], flexible crane systems [34] and single-link manipulator systems [35], can be described as or converted into system (1). Therefore, it is of importance to investigate system (1).

Remark 2 Note that, for strict-feedback nonlinear systems, the adaptive sampled-data stabilization problem has been addressed in [18] by designing the controller for the systems' Euler approximate discrete-time model; however, the diffusion terms of the systems considered in [18] are $\phi_i^T(\bar{x}_i)\theta, i = 1, 2, \ldots, n$, where θ is an unknown constant and $\phi_i(\cdot)$ is a known nonlinear function; furthermore, the control coefficients of the former $(i - 1)$ subsystems' equations in [18] are 1; thus, system (1) is more general than the systems investigated in [18], and the method proposed in [18] maybe invalid for system (1).

We make the following assumptions.

Assumption 1 [36] The external disturbances Δ_i $(\bar{x}_i, z_0), i = 1, 2, \ldots, n$, satisfy

$$|\Delta_i(\bar{x}_i, z_0)| \leqslant \phi_{i1}(\|z_0\|) + \phi_{i2}(\|\bar{x}_i\|), \quad (2)$$

where $\phi_{i1}(\cdot) \geqslant 0$ and $\phi_{i2}(\cdot) \geqslant 0$ are unknown continuous functions.

Assumption 2 There exist two unknown constants $g_i^{\min}$ and $g_i^{\max}$ such that $0 < g_i^{\min} \leqslant |g_i(\cdot)| \leqslant g_i^{\max}$. Without loss of generality, we assume $0 < g_i^{\min} \leqslant g_i(\cdot) \leqslant g_i^{\max}$.

Remark 3 It should be pointed out that continuous-time control of the strict-feedback nonlinear systems has been investigated in [4,7,12]; however, in those literature, the upper and lower bounds of the time-varying control coefficients $g_i(\bar{x}_i), i = 1, 2, \ldots, n$, are required to be known.

Assumption 3 [36] The dynamic $\dot{z}_0 = q(z_0, x_1)$ is exp-ISPS (exponential-input-state-practically stable), that is, there exists a C^1 function $V_0(z_0)$ such that

$$\bar{\alpha}_1(\|z_0\|) \leqslant V_0(z_0) \leqslant \bar{\alpha}_2(\|z_0\|), \quad (3a)$$

$$\frac{\partial V_0(z_0)}{\partial z_0} q(z_0, x_1) \leqslant -cV_0(z_0) + \Gamma(|x_1|) + d, \quad (3b)$$

where $\bar{\alpha}_1(\cdot), \bar{\alpha}_2(\cdot)$ are class K_∞ functions, $\Gamma(\cdot)$ is a smooth function with $\Gamma(0) = 0$, and $c > 0, d \geqslant 0$ are constants.

Remark 4 In the existing results on sampled-data control [19–24], the diffusion terms $f_i(\bar{x}_i), i = 1, 2, \ldots, n$, of the considered nonlinear systems are required to satisfy the linear growth condition; however, in system (1), such restriction for the diffusion terms has been relaxed.

Lemma 1 ([37], Gronwall–Bellman inequality) *Let $D := [a, b] \subset R$ be a real region. Suppose $\mu(t), \rho(t)$ and $\omega(t) \geqslant 0$ defined in D are real continuous functions. If $\mu(t)$ satisfies the following inequality*

$$\mu(t) \leqslant \rho(t) + \int_a^t \omega(s)\mu(s)\mathrm{d}s, \tag{4a}$$

the following holds for $t \in D$:

$$\mu(t) \leqslant \rho(t) + \int_a^t \rho(s)\omega(s) \exp\left(\int_s^t \omega(r)\mathrm{d}r\right)\mathrm{d}s. \tag{4b}$$

Lemma 2 [36] *If $V_0(z_0)$ is a C^1 function for $\dot{z}_0 = q(z_0, x_1)$ such that (4a) and (4b) hold, then, for any constant $\bar{c} \in (0, c)$, any initial instant $t_0 > 0$, any initial condition $z_0(t_0), \gamma_0 > 0$, for any continuous function $\bar{\gamma}(|x_1|)$ such that $\bar{\gamma}(|x_1|) \geqslant \Gamma(|x_1|)$, there exist a finite $T_0 = \max\{0, \ln(V_0(z_0)/\gamma_0)/(c - \bar{c})\} \geqslant 0$, a function $D(t, t_0) \geqslant 0$ defined for all $t \geqslant t_0$, and a signal described by*

$$\dot{v}(t) = -\bar{c}v(t) + \bar{\gamma}(|x_1(t)|) + d, v(t_0) = v_0, \quad \forall t \geqslant t_0, \tag{5}$$

such that $D(t, t_0) = 0$ for $t \geqslant t_0 + T_0$, and $D(t, t_0) = \max\{0, e^{-c(t-t_0)}V_0(z_0) - e^{-\bar{c}(t-t_0)}\gamma_0\}$ such that $V_0(z_0) \leqslant v(t) + D(t, t_0)$ for $\forall t \geqslant t_0$. Without loss of generality, we assume $\bar{\gamma}(|x_1|) = \Gamma(|x_1|)$.

Lemma 3 [38] *For any real-valued continuous function $f(x, y)$, where $x \in R^m, y \in R^n$, there exist smooth scalar functions $\varphi_1(x) \geqslant 0$ and $\varphi_2(y) \geqslant 0$, such that*

$$|f(x, y)| \leqslant \varphi_1(x) + \varphi_2(y). \tag{6}$$

Lemma 4 ([39], Young's inequality) *Let $x \in R^n, y \in R^n$, and $p > 1, q > 1$ be two constants, $(p - 1)(q - 1) = 1$, given any real number $\varepsilon > 1$, the following inequality holds:*

$$x^{\mathrm{T}}y \leqslant \frac{1}{p\varepsilon^p}\|x\|^p + \frac{\varepsilon^q}{q}\|y\|^q. \tag{7a}$$

Specially, when $p = q = 2$, we have

$$x^{\mathrm{T}}y \leqslant \frac{1}{2\varepsilon^2}\|x\|^2 + \frac{\varepsilon^2}{2}\|y\|^2. \tag{7b}$$

Lemma 5 [40] *Let $H(z)$ be a continuous function defined on a compact set Ω_z, then a neural network can be constructed as the following form to approximate it:*

$$H(z) = W^{*T}T(z) + D(z), \quad \forall z \in \Omega_z, \tag{8}$$

where $T(z) = [T_1(z), \ldots, T_l(z)]^{\mathrm{T}} \in R^l$ is the basic function vector with the node number $l \geqslant 1, W^* = [W_1^*, W_2^*, \ldots, W_l^*]^{\mathrm{T}} \in R^l$ is the ideal weight vector, $D(z)$ is the approximate error satisfying $|D(z)| \leqslant D_1^*$ with bound $D_1^* > 0$.

The basic function $T_i(z)$ is taken as the Gaussian function, which has the following form:

$$T_i(z) = \exp\left(-\frac{(z - c_i)^{\mathrm{T}}(z - c_i)}{\mu_i^{*2}}\right), \quad i = 1, 2, \ldots, l,$$

where c_i is the center of the radial basic function vector, and $\mu_i > 0$ is the width of the Gaussian function.

The value of the ideal weight vector W^* is determined by W that minimizes the approximate error $D(z)$ for all $z \in \Omega_z$:

$$W^* = \arg\min_{W \in R^l}\left\{\sup_{z \in \Omega_z}\left|H(z) - W^{\mathrm{T}}(z)T(z)\right|\right\}.$$

The objective of the paper is to design the following adaptive sampled-data controller

$$u(t) = u(t_k) = K(x_1(t_k), x_2(t_k), \ldots, x_n(t_k)),$$
$$\forall t \in [t_k, t_{k+1}) \tag{9}$$

for nonlinear system (1) such that all states of the resulting closed-loop system to be semi-globally uniformly ultimately bounded.

3 Adaptive sampled-data controller design

In this section, an adaptive sampled-data controller will be constructed by adopting the backstepping technique. The whole design procedure needs n steps, which depends on the following changes of coordinates: $z_i = x_i - \alpha_{i-1}, i = 2, 3, \ldots, n$, where α_{i-1} is a virtual control law.

Step 1 In the first step, we choose the following Lyapunov function candidate:

$$V_1 = \frac{x_1^2}{2g_1^{\min}} + \frac{1}{2\eta_1}\tilde{\theta}_1^2 + \frac{v^2}{2\bar{\lambda}_1}, \tag{10}$$

where $\eta_1 > 0, \bar{\lambda}_1 > 0$ are design parameters, $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1, \hat{\theta}_1$ is the estimate of θ_1, θ_1 will be defined later. From Assumption 1, the time derivative of V_1 satisfies

$$\dot{V}_1 \leqslant \frac{g_1^{\max}}{g_1^{\min}} |x_1| |x_2 - \alpha_1| + \frac{g_1(x_1)}{g_1^{\min}} x_1 \alpha_1 + \frac{x_1 f_1(x_1)}{g_1^{\min}}$$
$$+ \frac{|x_1|}{g_1^{\min}} \phi_{11}(\|z_0\|) + \frac{|x_1|}{g_1^{\min}} \phi_{12}(|x_1|) - \frac{1}{\eta_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1$$
$$- \frac{\bar{c}}{\bar{\lambda}_1} v^2 + \frac{|x_1|}{\bar{\lambda}_1} \Phi(|x_1|) v + g_{11}^* \sqrt{V_1}, \qquad (11)$$

where $g_{11}^* = d \sqrt{\frac{2}{\bar{\lambda}_1}}$, and $\Phi(\cdot) \geqslant 0$ is an unknown smooth function.

According to Assumption 3, Lemmas 2 and 3, we obtain, for $\forall t \geqslant t_0$,

$$|x_1| \phi_{11}(\|z_0\|) \leqslant |x_1| \phi_{11} \circ \bar{\alpha}_1^{-1}(v(t) + D(t, t_0))$$
$$\leqslant |x_1| (\varphi_{11}(v(t)) + \varphi_{12}(D(t, t_0))), \qquad (12)$$

where $\varphi_{11}(\cdot) \geqslant 0$ and $\varphi_{12}(\cdot) \geqslant 0$ are smooth functions. Because $D(t, t_0)$ is a bounded function for $\forall t \geqslant t_0$, there exists a constant $\theta_{12}^* > 0$ such that $\varphi_{12}(D(t, t_0)) \leqslant \theta_{12}^*$ holds, substituting (12) into (11) results in

$$\dot{V}_1 \leqslant \frac{g_1^{\max}}{g_1^{\min}} |x_1 z_2| + \frac{g_1(x_1)}{g_1^{\min}} x_1 \alpha_1 + |x_1| \bar{\phi}_1(x_1)$$
$$- \frac{1}{\eta_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 - \frac{\bar{c}}{\bar{\lambda}_1} v^2 + g_{11}^* \sqrt{V_1}, \qquad (13)$$

where $\bar{\phi}_1(X_1) = \frac{|f_1(x_1)|}{g_1^{\min}} + \frac{1}{g_1^{\min}}(\varphi_{11}(v) + \theta_{12}^* + \phi_{12}(|x_1|)) + \frac{v \Phi(|x_1|)}{\bar{\lambda}_1}$ with $X_1 = [x_1, v]^{\mathrm{T}}$.

As mentioned in Lemma 5, neural network can approximate any continuous function to any desired accuracy; thus, a neural network is constructed as the following form to approximate $\bar{\phi}_1(X_1)$:

$$\bar{\phi}_1(X_1) = H_1^{*\mathrm{T}} S_1(X_1) + \varepsilon_1(X_1), \qquad (14)$$

where H_1^* is an unknown weight vector, $S_1(X_1) = [S_{11}(X_1), S_{12}(X_1), \ldots, S_{1L_1}(X_1)]^{\mathrm{T}} \in R^{L_1}$ is a basic function vector and $S_{1i}(\cdot), i = 1, 2, \ldots, L_1$, are chosen as Gaussian functions, $\varepsilon_1(X_1)$ is the approximation error satisfying $|\varepsilon_1(X_1)| \leqslant \varepsilon_1^*$ with $\varepsilon_1^* > 0$.

Substituting (14) into (13), and using Lemma 4, we arrive at

$$\dot{V}_1 \leqslant \frac{g_1^{\max}}{g_1^{\min}} |x_1 z_2| + \frac{g_1(x_1)}{g_1^{\min}} x_1 \alpha_1 + x_1^2 \theta_1 - \frac{1}{\eta_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1$$
$$- \frac{\bar{c}}{\bar{\lambda}_1} v^2 + \sum_{i=1}^2 g_{1i}^* \sqrt{V_1}, \qquad (15)$$

where $\theta_1 = \|H_1^*\|^2 L$ and $g_{12}^* = \varepsilon_1^* \sqrt{2 g_1^{\min}} + \sqrt{2 g_1^{\min} \theta_1}$.

Choose a virtual control law and a "virtual adaptive law" for Step 1 as follows:

$$\alpha_1 = -x_1 \sqrt{1 + \hat{\theta}_1^2} - c_1 x_1, \qquad (16a)$$
$$\dot{\hat{\theta}}_1 = \eta_1 x_1^2 - \eta_1 \hat{\theta}_1, \qquad (16b)$$

where $c_1 > 0$ is a design parameter. Furthermore, we have the following relation from (10) and (16):

$$\frac{1}{\eta_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 \leqslant x_1^2 \tilde{\theta}_1 - \tilde{\theta}_1 \hat{\theta}_1 \leqslant x_1^2 \tilde{\theta}_1 - \theta_1 \sqrt{2 \eta_1 V_1} + \tilde{\theta}_1^2. \qquad (17)$$

Substituting (16) and (17) into (15) yields

$$\dot{V}_1 \leqslant \frac{g_1^{\max}}{g_1^{\min}} |x_1 z_2| - \tilde{\theta}_1^2 - \frac{\bar{c}}{\bar{\lambda}_1} v^2$$
$$+ (g_{11}^* + g_{12}^* + g_{13}^*) \sqrt{V_1} - c_1 x_1^2, \qquad (18)$$

where $g_{13}^* = \theta_1 \sqrt{2 \eta_1}$.

Step 2 Choose a Lyapunov function candidate as the following form:

$$V_2 = V_1 + \frac{z_2^2}{2 g_2^{\min}} + \frac{1}{2 \eta_2} \tilde{\theta}_2^2, \qquad (19)$$

where $\eta_2 > 0$ is a design parameter, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$, $\hat{\theta}_2$ is the estimate of θ_2, θ_2 will be defined later. The time derivative of V_2 satisfies

$$\dot{V}_2 \leqslant \dot{V}_1 + \frac{g_2^{\max}}{g_2^{\min}} |z_2| |x_3 - \alpha_2| + \frac{g_2(\bar{x}_2)}{g_2^{\min}} z_2 \alpha_2$$
$$+ \frac{g_2(\bar{x}_2)}{g_2^{\min}} |z_2| (\phi_{21}(\|z_0\|) + \phi_{22}(\|\bar{x}_2\|))$$
$$+ |z_2| \frac{g_1^{\max}}{g_1^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| |x_2| + \frac{|z_2|}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| |f_1(x_1)|$$
$$+ \frac{\eta_1 |z_2|}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \right| \left| \dot{\hat{\theta}}_1 \right| - \frac{1}{\eta_2} \tilde{\theta}_2 \dot{\hat{\theta}}_2$$
$$+ \frac{|z_2|}{g_2^{\min}} |f_2(\bar{x}_2)| + \frac{\eta_1 x_1^2 |z_2|}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \right|$$
$$+ \frac{|z_2|}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| (\phi_{11}(\|z_0\|) + \phi_{12}(|x_1|)). \qquad (20)$$

According to Assumption 3, Lemmas 2 and 3, we have, for $\forall t \geqslant t_0$,

$$
\begin{aligned}
|z_2| \phi_{21}(\|z_0\|) &\leqslant |z_2| \phi_{21} \circ \bar{\alpha}_1^{-1}(v(t) + D(t, t_0)) \\
&\leqslant |z_2| \varphi_{21}(v(t)) + |z_2| \varphi_{22}(D(t, t_0)),
\end{aligned}
\tag{21}
$$

where $\varphi_{21}(\cdot) \geqslant 0$ and $\varphi_{22}(\cdot) \geqslant 0$ are two smooth functions. Furthermore, there exists a constant $\theta_{22}^* > 0$ such that $\varphi_{22}(D(t, t_0)) \leqslant \theta_{22}^*$ holds for $\forall t \geqslant t_0$. Substituting (21) into (20) leads to

$$
\begin{aligned}
\dot{V}_2 \leqslant{}& \dot{V}_1 + \frac{g_2^{\max}}{g_2^{\min}} |z_2 z_3| + \frac{g_2(\bar{x}_2)}{g_2^{\min}} z_2 \alpha_2 + |z_2| \bar{\phi}_2(X_2) \\
&- \frac{1}{\eta_2} \tilde{\theta}_2 \dot{\hat{\theta}}_2 - \frac{g_1^{\max}}{g_1^{\min}} z_2^2 |x_1|,
\end{aligned}
\tag{22}
$$

where

$$
\begin{aligned}
\bar{\phi}_2(X_2) ={}& \frac{1}{g_2^{\min}}(\varphi_{21}(v) + \theta_{22}^* + \phi_{22}(\|\bar{x}_2\|)) \\
&+ \frac{g_1^{\max} |x_2|}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| + \frac{|f_1(x_1)|}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| \\
&+ \frac{g_1^{\max}}{g_2^{\min}} |z_2 x_1| \\
&+ \frac{1}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| (\varphi_{11}(v) + \theta_{12}^* + \phi_{12}(|x_1|)) \\
&+ \frac{\eta_1 x_1^2}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \right| + \frac{\eta_1}{g_2^{\min}} \left| \frac{\partial \alpha_1}{\partial x_1} \right| |\hat{\theta}_1| \\
&+ \frac{|f_2(\bar{x}_2)|}{g_2^{\min}}, \quad X_2 = [\bar{x}_2^{\mathrm{T}}, \hat{\theta}_1, v]^{\mathrm{T}}.
\end{aligned}
$$

From Lemma 5, we adopt the following neural network to approximate the unknown nonlinear continuous function $\bar{\phi}_2(X_2)$:

$$
\bar{\phi}_2(X_2) = H_2^{*T} S_2(X_2) + \varepsilon_2(X_2),
\tag{23}
$$

where H_2^* is an unknown weight vector, $S_2(X_2) = [S_{21}(X_2), S_{22}(X_2), \ldots, S_{2L_2}(X_2)]^{\mathrm{T}} \in R^{L_2}$ is a basic function vector, $\varepsilon_2(X_2)$ is the approximation error satisfying $|\varepsilon_2(X_2)| \leqslant \varepsilon_2^*$ with $\varepsilon_2^* > 0$.

Substituting (23) into (22), and using Lemma 4, we have

$$
\begin{aligned}
\dot{V}_2 \leqslant{}& -c_1 x_1^2 + \frac{g_2^{\max}}{g_2^{\min}} |z_2 z_3| + \frac{g_2(\bar{x}_2)}{g_2^{\min}} z_2 \alpha_2 + z_2^2 \theta_2 \\
&+ g_{21}^* \sqrt{V_2} - \frac{1}{\eta_2} \tilde{\theta}_2 \dot{\hat{\theta}}_2 \\
&+ \left(\frac{g_1^{\max}}{4} \sqrt{\frac{2}{g_1^{\min}}} + \sum_{i=1}^{3} g_{1i}^* \right) \sqrt{V_1} - \tilde{\theta}_1^2,
\end{aligned}
\tag{24}
$$

where $\theta_2 = \|H_2^*\|^2 L$ and $g_{21}^* = \varepsilon_2^* \sqrt{g_2^{\min}} + \sqrt{2 g_2^{\min} \theta_2}$.

Take a virtual control law and a "virtual adaptive law" for Step 2 as follows:

$$
\alpha_2 = -z_2 \sqrt{1 + \hat{\theta}_2^2} - c_2 z_2,
\tag{25a}
$$

$$
\dot{\hat{\theta}}_2 = \eta_2 z_2^2 - \eta_2 \hat{\theta}_2,
\tag{25b}
$$

where $c_2 > 0$ is a design parameter.

Substituting (25) into (24) results in

$$
\begin{aligned}
\dot{V}_2 \leqslant{}& -c_1 x_1^2 - c_2 z_2^2 + \frac{g_2^{\max}}{g_2^{\min}} |z_2 z_3| \\
&+ (g_{21}^* + g_{22}^*) \sqrt{V_2} + g_1^* \sqrt{V_1} - \tilde{\theta}_1^2 - \tilde{\theta}_2^2,
\end{aligned}
\tag{26}
$$

where $g_1^* = \sum_{i=1}^{3} g_{1i}^* + \frac{g_1^{\max}}{4} \sqrt{\frac{2}{g_1^{\min}}}$ and $g_{22}^* = \theta_2 \sqrt{2\eta_2}$.

Step $h (3 \leqslant h \leqslant n-1)$ Consider a Lyapunov function candidate as

$$
V_h = V_{h-1} + \frac{z_h^2}{2 g_h^{\min}} + \frac{1}{2\eta_h} \tilde{\theta}_h^2,
\tag{27}
$$

where $\eta_h > 0$ is a design parameter, $\tilde{\theta}_h = \theta_h - \hat{\theta}_h$, $\hat{\theta}_h$ is the estimate of θ_h, θ_h will be given later. From (18), (26) and Assumption 1, the time derivative of V_h satisfies

$$
\begin{aligned}
\dot{V}_h \leqslant{}& \dot{V}_{h-1} + \frac{g_h^{\max}}{g_h^{\min}} |z_h| |x_{h+1} - \alpha_h| + \frac{g_h(\bar{x}_h)}{g_h^{\min}} z_h \alpha_h \\
&+ \frac{|z_h|}{g_h^{\min}}(\phi_{h1}(\|z_0\|) + \phi_{h2}(\|\bar{x}_h\|)) \\
&+ \sum_{i=1}^{h-1} \frac{|f_i(\bar{x}_i)| |z_h|}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial x_i} \right| \\
&+ \sum_{i=1}^{h-1} \frac{g_h^{\max} |z_h|}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial x_i} \right| |x_{i+1}| + \frac{|z_h|}{g_h^{\min}} |f_h(\bar{x}_h)| \\
&+ \sum_{i=1}^{h-1} \frac{|z_h|}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial x_i} \right| (\phi_{i1}(\|z_0\|) + \phi_{i2}(\|\bar{x}_i\|)) \\
&+ \sum_{i=2}^{h-1} \frac{\eta_i z_i^2 |z_h|}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial \hat{\theta}_i} \right| - \frac{1}{\eta_h} \tilde{\theta}_h \dot{\hat{\theta}}_h \\
&+ \frac{\eta_1 x_1^2 |z_h|}{g_h^{\min}} \left| \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \right| + \sum_{i=1}^{h-1} \frac{\eta_i |z_h|}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial \hat{\theta}_i} \right| |\hat{\theta}_i|.
\end{aligned}
\tag{28}
$$

Using Assumption 3, Lemmas 1 and 2, we obtain, for $\forall t \geqslant t_0$,

$$
\begin{aligned}
|z_h| \phi_{h1}(\|z_0\|) &\leqslant |z_h| \phi_{h1} \circ \bar{\alpha}_1^{-1}(v(t) + D(t, t_0)) \\
&\leqslant |z_h| \varphi_{h1}(v(t)) + |z_h| \varphi_{h2}(D(t, t_0)),
\end{aligned}
\tag{29}
$$

where $\varphi_{h1}(\cdot) \geqslant 0$ and $\varphi_{h2}(\cdot) \geqslant 0$ are two smooth functions. Similar to Steps 1 and 2, there exists a constant $\theta_{h2}^* > 0$ such that $\varphi_{h2}(D(t, t_0)) \leqslant \theta_{h2}^*$ holds. Substituting (29) into (28) leads to

$$\dot{V}_h \leqslant \dot{V}_{h-1} + \frac{g_h^{\max}}{g_h^{\min}} |z_h z_{h+1}| + \frac{g_h(\bar{x}_h)}{g_h^{\min}} z_h \alpha_h$$
$$+ |z_h| \bar{\phi}_h(X_h) - \frac{g_{h-1}^{\max}}{g_{h-1}^{\min}} z_h^2 |z_{h-1}| - \frac{1}{\eta_h} \tilde{\theta}_h \dot{\hat{\theta}}_h, \tag{30}$$

where

$$\bar{\phi}_h(X_h) = \frac{1}{g_h^{\min}} \left(\varphi_{h1}(v) + \theta_{h2}^* + \phi_{h2}(\|\bar{x}_h\|) \right)$$
$$+ \sum_{i=1}^{h-1} \frac{g_i^{\max}}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial x_i} \right| |x_{i+1}| + \frac{\eta_1 x_1^2}{g_h^{\min}} \left| \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \right|$$
$$+ \frac{|f_h(\bar{x}_h)|}{g_h^{\min}} + \sum_{i=1}^{h-1} \frac{|f_i(\bar{x}_i)|}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial x_i} \right|$$
$$+ \sum_{i=1}^{h-1} \frac{1}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial x_i} \right| (\varphi_{i1}(v)$$
$$+ \theta_{i2}^* + \phi_{i2}(\|\bar{x}_i\|))$$
$$+ \frac{g_{h-1}^{\max}}{g_{h-1}^{\min}} |z_h| |z_{h+1}| + \sum_{i=2}^{h-1} \frac{\eta_i z_i^2}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial \hat{\theta}_i} \right|$$
$$+ \sum_{i=1}^{h-1} \frac{\eta_i}{g_h^{\min}} \left| \frac{\partial \alpha_{h-1}}{\partial \hat{\theta}_i} \right| \left| \dot{\hat{\theta}}_i \right|, X_h$$
$$= [\bar{x}_h^T, \hat{\theta}_1, \ldots, \hat{\theta}_{h-1}, v]^T.$$

To approximate the unknown nonlinear continuous function $\bar{\phi}_h(X_h)$, we adopt the following neural network:

$$\bar{\phi}_h(X_h) = H_h^{*T} S_h(X_h) + \varepsilon_h(X_h), \tag{31}$$

where H_h^* is an unknown ideal weight vector, $S_h(X_h) = [S_{h1}(X_h), S_{h2}(X_h), \ldots, S_{hL_h}(X_h)] \in R^{L_h}$ is a basic function vector, $\varepsilon_h(X_h)$ is the approximation error satisfying $|\varepsilon_h(X_h)| \leqslant \varepsilon_h^*$ with $\varepsilon_h^* > 0$.

Substituting (31) into (30), and using Lemma 4, we obtain

$$\dot{V}_h \leqslant -c_1 x_1^2 - \sum_{i=2}^{h-1} c_i z_i^2 + \frac{g_h^{\max}}{g_h^{\min}} |z_h z_{h+1}|$$
$$+ \frac{g_h(\bar{x}_h)}{g_h^{\min}} z_h \alpha_h + z_h^2 \theta_h$$
$$+ g_{h1}^* \sqrt{V_h} - \frac{1}{\eta_h} \tilde{\theta}_h \dot{\hat{\theta}}_h$$

$$+ \sum_{i=1}^{h-2} g_i^* \sqrt{V_i}$$
$$+ \left(\frac{g_{h-1}^{\max}}{4} \sqrt{\frac{2}{g_{h-1}^{\min}}} + \sum_{i=1}^{2} g_{h-1,i}^* \right) \sqrt{V_{h-1}}$$
$$- \sum_{i=1}^{h-1} \tilde{\theta}_i^2, \tag{32}$$

where $\theta_h = \|H_h^*\|^2 L$ and $g_{h1}^* = \varepsilon_h^* \sqrt{2g_h^{\min}} + \sqrt{2g_h^{\min}\theta_h}$.

Choose a virtual control law and a "virtual adaptive law" for Step h as follows:

$$\alpha_h = -z_h \sqrt{1 + \hat{\theta}_h^2} - c_h z_h, \tag{33a}$$
$$\dot{\hat{\theta}}_h = \eta_h z_h^2 - \eta_h \hat{\theta}_h, \tag{33b}$$

where $c_h > 0$ is a design parameter.

Substituting (33) into (32) yields

$$\dot{V}_h \leqslant -c_1 x_1^2 - \sum_{i=2}^{h} c_i z_i^2 + \frac{g_h^{\max}}{g_h^{\min}} |z_h z_{h+1}|$$
$$+ (g_{h1}^* + g_{h2}^*)\sqrt{V_h} + \sum_{i=1}^{h-1} g_i^* \sqrt{V_i} - \sum_{i=1}^{h} \tilde{\theta}_i^2, \tag{34}$$

where $g_{h-1}^* = \sum_{i=1}^{2} g_{h-1,i}^* + \frac{g_{h-1}^{\max}}{4} \sqrt{\frac{2}{g_{h-1}^{\min}}}$ and $g_{h2}^* = \theta_h \sqrt{2\eta_h}$.

Step n Consider the following Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2g_n^{\min}} z_n^2 + \frac{1}{2\eta_n} \tilde{\theta}_n^2, \tag{35}$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$, $\hat{\theta}_n$ is the estimate of θ_n, θ_n will be given later. From (18), (26) and (34), the time derivative of V_n satisfies

$$\dot{V}_n \leqslant \dot{V}_{n-1} + \frac{g_n^{\max}}{g_n^{\min}} |z_n| |u - \alpha_n| + \frac{g_n(\bar{x}_n)}{g_n^{\min}} z_n$$
$$+ \frac{|z_n|}{g_n^{\min}} (\phi_{n1}(\|z_0\|) + \phi_{n2}(\|\bar{x}_n\|))$$
$$+ \sum_{i=1}^{n-1} \frac{g_i^{\max} |z_n|}{g_n^{\max}} \left| \frac{\partial \alpha_{n-1}}{\partial x_i} \right| |x_{i+1}|$$
$$+ \sum_{i=1}^{n-1} \frac{|z_n|}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial x_i} \right| |f_i(\bar{x}_i)| - \frac{1}{\eta_n} \tilde{\theta}_n \dot{\hat{\theta}}_n$$

$$+ \sum_{i=1}^{n-1} \frac{|z_n|}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial x_i} \right| (\phi_{i1}(\|z_0\|) + \phi_{i2}(\|\bar{x}_i\|))$$

$$+ \sum_{i=1}^{n-1} \frac{\eta_i |z_n|}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i} \right| \left| \hat{\theta}_i \right|$$

$$+ \frac{\eta_1 x_1^2 |z_n|}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i} \right| + \sum_{i=2}^{n} \frac{\eta_i |z_n| z_i^2}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i} \right|$$

$$+ \frac{|z_n|}{g_n^{\min}} |f_n(\bar{x}_n)| . \tag{36}$$

Under Assumption 1, for $\forall t \geqslant t_0$, the following inequality holds:

$$\begin{aligned} |z_n| \phi_{n1}(\|z_0\|) &\leqslant |z_n| \phi_{n1} \circ \bar{\alpha}_1^{-1} (v(t) + D(t, t_0)) \\ &\leqslant |z_n| \varphi_{n1}(v(t)) + |z_n| \varphi_{n2}(D(t, t_0)), \end{aligned} \tag{37}$$

where $\varphi_{n1}(\cdot) \geqslant 0$ and $\varphi_{n2}(\cdot) \geqslant 0$ are smooth functions.

Note that $D(t, t_0)$ is a bounded function for $\forall t \geqslant t_0$, then we can obtain $\varphi_{n2}(D(t, t_0)) \leqslant \theta_{n2}^*$, where $\theta_{n2}^* > 0$ is a constant. Substituting (37) into (36) results in

$$\begin{aligned} \dot{V}_n &\leqslant \dot{V}_{n-1} + \frac{g_n^{\max}}{g_n^{\min}} |z_n| |u - \alpha_n| + \frac{g_n(\bar{x}_n)}{g_n^{\min}} z_n \alpha_n \\ &\quad + |z_n| \bar{\phi}_n(X_n) - \frac{1}{\eta_n} \tilde{\theta}_n \dot{\hat{\theta}}_n \\ &\quad - \frac{g_{n-1}^{\max}}{g_{n-1}^{\min}} z_n^2 |z_{n-1}| , \end{aligned} \tag{38}$$

where

$$\begin{aligned} \bar{\phi}_n(X_n) &= \sum_{i=1}^{n-1} \frac{1}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial x_i} \right| |f_i(\bar{x}_i)| \\ &\quad + \sum_{i=1}^{n-1} \frac{g_i^{\max}}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial x_i} \right| |x_{i+1}| + \frac{g_{n-1}^{\max}}{g_{n-1}^{\min}} |z_n z_{n-1}| \\ &\quad + \frac{|f_n(\bar{x}_n)|}{g_n^{\min}} + \frac{\eta_1 x_1^2}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i} \right| \\ &\quad + \sum_{i=2}^{n} \frac{\eta_i z_i^2}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i} \right| \\ &\quad + \sum_{i=1}^{n-1} \frac{1}{g_n^{\min}} \left| \frac{\partial \alpha_{n-1}}{\partial x_i} \right| (\varphi_{i1}(v) + \theta_{i2}^* \\ &\quad + \phi_{i2}(\|\bar{x}_i\|)) \\ &\quad + \frac{1}{g_n^{\min}} \left(\varphi_{n1}(v) + \theta_{n2}^* + \phi_{n2}(\|\bar{x}_n\|) \right) \end{aligned}$$

$$+ \frac{1}{g_n^{\min}} \sum_{i=1}^{n-1} \eta_i \left| \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i} \right| \left| \hat{\theta}_i \right|, X_n$$

$$= [\bar{x}_n^{\mathrm{T}}, \hat{\theta}_1, \ldots, \hat{\theta}_{n-1}, v]^{\mathrm{T}}.$$

As same as the previous steps, we adopt a neural network to approximate the unknown nonlinear continuous function $\bar{\phi}_n(X_n)$, that is,

$$\bar{\phi}_n(X_n) = H_n^{*T} S_n(X_n) + \varepsilon_n(X_n), \tag{39}$$

where H_n^* is an unknown ideal weight vector, $S_n(X_n) = [S_{n1}(X_n), S_{n2}(X_n), \ldots, S_{nL_n}(X_n)] \in R^{L_n}$ is a basic function vector, $\varepsilon_n(X_n)$ is the approximation error satisfying $|\varepsilon_n(X_n)| \leqslant \varepsilon_n^*$ with $\varepsilon_n^* > 0$.

Substituting (39) into (38), and using Lemma 4, we obtain

$$\begin{aligned} \dot{V}_n &\leqslant -c_1 x_1^2 - \sum_{i=2}^{n-1} c_i z_i^2 + \frac{g_n^{\max}}{g_n^{\min}} |z_n| |u - \alpha_n| \\ &\quad + \frac{g_n(\bar{x}_n)}{g_n^{\min}} z_n \alpha_n + z_n^2 \theta_n - \frac{1}{\eta_n} \tilde{\theta}_n \dot{\hat{\theta}}_n \\ &\quad + g_{n1}^* \sqrt{V_n} \\ &\quad + \left(\frac{g_{n-1}^{\max}}{4} \sqrt{\frac{2}{g_{n-1}^{\min}}} + \sum_{h=1}^{2} g_{n-1,h}^* \right) \\ &\quad \times \sqrt{V_{n-1}} - \sum_{i=1}^{n-1} \tilde{\theta}_i^2 , \end{aligned} \tag{40}$$

where $\theta_n = \|H_n^*\|^2 L$ and $g_{n1}^* = \varepsilon_n^* \sqrt{2 g_n^{\min}} + \sqrt{2 g_n^{\min} \theta_n}$.

A virtual control law and a "virtual adaptive law" for the last step are chosen as

$$\alpha_n = -z_n \sqrt{1 + \hat{\theta}_n^2} - c_n z_n, \tag{41a}$$

$$\dot{\hat{\theta}}_n = \eta_n z_n^2 - \eta_n \hat{\theta}_n, \tag{41b}$$

where $c_n > 0$ is a design parameter.

Substituting (41) into (40) results in

$$\begin{aligned} \dot{V}_n &\leqslant -c_1 x_1^2 - \sum_{i=2}^{n} c_i z_i^2 + \frac{g_n^{\max}}{g_n^{\min}} |z_n| |u - \alpha_n| \\ &\quad - \sum_{i=1}^{n} \tilde{\theta}_i^2 + \sum_{i=1}^{n} g_i^* \sqrt{V_i} , \end{aligned} \tag{42}$$

where $g_{n-1}^* = \sum_{i=1}^{2} g_{n-1,i}^* + \frac{g_{n-1}^{\max}}{4} \sqrt{\frac{2}{g_{n-1}^{\min}}}$ and $g_n^* = g_{n1}^* + \theta_n \sqrt{2 \eta_n}$.

Note that the states of system (1) are measurable at the sampling points, hence, from (16), (25), (33) and (41), for $\forall t \in [t_k, t_{k+1})$, $k = 0, 1, \ldots, +\infty$, we design the adaptive sampled-data controller as

$$
\begin{cases}
u(t) = u(t_k) \\
= -\sum_{i=1}^{n} \left[\prod_{h=i}^{n} \left(\sqrt{1 + \mu_h^2(t_k)} + c_h \right) \right] x_i(t_k), \\
\dot{\mu}_1(t) = \eta_1 x_1^2(t_k) - \eta_1 \mu_1(t), \\
\dot{\mu}_l(t) = \eta_l \varrho_l^2(t_k) - \eta_l \mu_l(t), \quad l = 2, 3, \ldots, n,
\end{cases}
\tag{43}
$$

where $\varrho_l(t_k) = x_l(t_k) + \varrho_{l-1}(t_k)\left(\sqrt{1 + \mu_{l-1}^2(t_k)} + c_{l-1}\right)$, $\varrho_1(t_k) = x_1(t_k)$.

For each sampling interval $[t_k, t_{k+1})$, relation (42) can be further represented as

$$
\dot{V}_n(t) \leqslant -\tilde{c} V_n(t) + \frac{g_n^{\max}}{g_n^{\min}} |z_n(t)| |u(t_k) - \alpha_n(t)| + \bar{g} \sqrt{V_n(t)}, \quad \forall t \in [t_k, t_{k+1}),
\tag{44}
$$

where

$$
\tilde{c} = \min \left\{ 2g_1^{\min} c_1, 2g_2^{\min} c_2, \ldots, 2g_n^{\min} c_n, 2\eta_1, 2\eta_2, \ldots, 2\eta_n, 2\bar{c} \right\} \text{ and } \bar{g} = \sum_{i=1}^{n} g_i^*.
$$

Remark 5 It should be mentioned that the term $x_1^2 \theta$ in (15) was necessary for the adaptive sampled-data controller design since such term can deal with the uncertainties existed in the first step with virtual controller (16) [see (14)–(18)], and such way to deal with the uncertainties was motivated by the existing literature on the adaptive continuous-time control, such as [2–4].

Remark 6 In comparison with the design of adaptive laws in continuous-time control, it can be seen from (43) that the adaptive laws were constructed just based on the sampled values of the system states, which means that the proposed adaptive design can better save the recourses and helps to reduce the computation burden of the controller.

4 Stability analysis

In what follows, the stability of the closed-loop system under sampled-data controller (43) will be demonstrated by using the Gronwall–Bellman inequality.

Theorem 1 *Suppose system (1) satisfies Assumptions 1–3, for the given allowable sampling period and* *any bounded initial condition, sampled-data controller (43) will render all states of the resulting closed-loop system to be semi-globally uniformly ultimately bounded.*

Proof Choose the following Lyapunov function candidate

$$
V = V_n + \sum_{i=1}^{n} \frac{\tilde{\mu}_i^2}{2\beta_i} = \frac{x_1^2}{2g_1^{\min}} + \sum_{i=2}^{n} \frac{z_i^2}{2g_i^{\min}} + \sum_{i=1}^{n} \frac{\tilde{\theta}_i^2}{2\eta_i} + \sum_{i=1}^{n} \frac{\tilde{\mu}_i^2}{2\beta_i} + \frac{v^2}{2\bar{\lambda}_1},
\tag{45}
$$

where $\tilde{\mu}_i = \mu_i - \theta_i$, $\beta_i > 0$ is a design parameter. $\square$

Let $p > 0$ be a constant, where $p \geqslant V(t_0)$; and define a compact set as

$$
\Omega \equiv \left\{ \frac{x_1^2}{2g_1^{\min}} + \sum_{i=2}^{n} \frac{z_i^2}{2g_i^{\min}} + \sum_{i=1}^{n} \frac{\tilde{\theta}_i^2}{2\eta_i} + \sum_{i=1}^{n} \frac{\tilde{\mu}_i^2}{2\beta_i} + \frac{v^2}{2\bar{\lambda}_1} \leqslant p \right\},
$$

next, we will demonstrate that $V(t) \leqslant p$ is an invariant set for $\forall t \geqslant t_0$ and $V(t_0) \leqslant p$.

Under the situation that $V(t) \leqslant p$, we know that the following derivations hold: according to (45), we have that $x_1(t), z_2(t), \ldots, z_n(t), \hat{\theta}_1(t), \hat{\theta}_2(t), \ldots, \hat{\theta}_n(t)$ and $v(t)$ are bounded, which implies that there exist constants $\Xi_1, \Xi_2, \ldots, \Xi_n, \bar{\Delta}_1, \bar{\Delta}_2, \ldots, \bar{\Delta}_n$ and $\bar{H}$ such that $|x_1(t)| \leqslant \Xi_1, |z_2(t)| \leqslant \Xi_2, \ldots, |z_n(t)| \leqslant \Xi_n, \left|\hat{\theta}_1(t)\right| \leqslant \bar{\Delta}_1, \left|\hat{\theta}_2(t)\right| \leqslant \bar{\Delta}_2, \ldots, \left|\hat{\theta}_n(t)\right| \leqslant \bar{\Delta}_n$ and $|v(t)| \leqslant \bar{H}$ hold; furthermore, according to (16), (25), (33) and (41), we know that the virtual control laws are all bounded, noting that $z_i = x_i - \alpha_{i-1}, i = 2, 3, \ldots, n$, we can obtain the boundedness of $x_1(t), x_2(t), \ldots, x_n(t)$, i.e., there exist constants $\sigma_2^*, \sigma_3^*, \ldots, \sigma_n^*$ such that $|x_2(t)| \leqslant \sigma_2^*, |x_3(t)| \leqslant \sigma_3^*, \ldots, |x_n(t)| \leqslant \sigma_n^*$ hold; then, for the sampling points, we also have $|x_1(t_k)| \leqslant \Xi_1, |x_2(t_k)| \leqslant \sigma_2^*, \ldots, |x_n(t_k)| \leqslant \sigma_n^*$.

On the other hand, under the situation that $V(t) \leqslant p$, the following derivations also hold: from (43), we know that $\mu_1(t)$ is bounded by the boundedness of $x_1(t_k)$, then the boundedness of $\mu_1(t_k)$ can also be guaranteed, which means that $|\mu_1(t)| \leqslant \bar{\mu}_1$ and $|\mu_1(t_k)| \leqslant \bar{\mu}_1$ hold, where $\bar{\mu}_1 > 0$ is a constant; since

$$
\varrho_2(t_k) = x_2(t_k) + x_1(t_k)\left(\sqrt{1 + \mu_1^2(t_k)} + c_1\right),
$$

we can obtain the boundedness of $\varrho_2(t_k)$, that is, $|\varrho_2(t_k)| \leqslant \lambda_2$ holds, where $\lambda_2 > 0$ is a constant; according to (43) again, we have the boundedness of $\mu_2(t)$, equally, $\mu_2(t_k)$ is bounded, which means that $|\mu_2(t)| \leqslant \bar{\mu}_2$ and $|\mu_2(t_k)| \leqslant \bar{\mu}_2$ hold, where $\bar{\mu}_2 > 0$ is a constant; furthermore, we know that $\varrho_3(t_k)$ is bounded due to the boundedness of $x_3(t_k)$, $\mu_3(t_k)$ and $\varrho_2(t_k)$, i.e., $|\rho_3(t_k)| \leqslant \lambda_3$ holds for a constant $\lambda_3 > 0$; using (43) repeatedly, we can obtain the boundedness of $\varrho_3(t_k), \varrho_4(t_k), \ldots, \varrho_n(t_k), \mu_3(t_k), \ldots, \mu_n(t_k)$, that is, there exist constants $\lambda_3, \lambda_4, \ldots, \lambda_n, \bar{\mu}_3, \bar{\mu}_4, \ldots, \bar{\mu}_n$ such that $|\varrho_3(t_k)| \leqslant \lambda_3, \ldots, |\varrho_n(t_k)| \leqslant \lambda_n, |\mu_3(t)| \leqslant \bar{\mu}_3, \ldots, |\mu_n(t)| \leqslant \bar{\mu}_n, |\mu_3(t_k)| \leqslant \bar{\mu}_3, \ldots, |\mu_n(t_k)| \leqslant \bar{\mu}_n$ hold; furthermore, since $x(t)$ and $v(t)$ are bounded, we know that there exists a constant $H^* > 0$ such that

$$\sqrt{3} \sum_{i=1}^{n} |f_i(\bar{x}_i(t))|$$
$$+ \sqrt{3} \sum_{i=1}^{n} \left(\varphi_{i1}(v(t)) + \theta_{i2}^* + \phi_{i2}(\|\bar{x}_i(t)\|) \right) \leqslant H^*.$$

holds; then according to (44) and (45), the time derivative of $V(t)$ satisfies the following relation:

$$\dot{V}(t) \leqslant -\tilde{c}V(t) + \frac{g_n^{\max}}{g_n^{\min}} |z_n(t)| |u(t_k) - \alpha_n(t)|$$
$$+ \bar{g}\sqrt{V(t)} + \frac{\eta_1 x_1^2(t_k)}{\beta_1} |\tilde{\mu}_1(t)|$$
$$+ \sum_{i=2}^{n} \frac{\eta_i \rho_i^2(t_k)}{\beta_i} |\tilde{\mu}_i(t)| - \sum_{i=1}^{n} \frac{\eta_i}{\beta_i} \tilde{\mu}_i^2(t)$$
$$+ \sum_{i=1}^{n} \frac{\eta_i \theta_i}{\beta_i} |\tilde{\mu}_i(t)|$$
$$\leqslant -\tilde{c}V(t) + \frac{g_n^{\max}}{g_n^{\min}} |z_n(t)| |u(t_k) - \alpha_n(t)|$$
$$+ d^*\sqrt{V(t)}, \quad \forall t \in [t_k, t_{k+1}), \tag{46}$$

where $d^* = \bar{g} + \Xi_1^2 \eta_1 \sqrt{\frac{2}{\beta_1}} + \sum_{i=2}^{n} \lambda_i^2 \eta_i \sqrt{\frac{2}{\beta_i}} + \sum_{i=1}^{n} \eta_i \theta_i \sqrt{\frac{2}{\beta_i}}$.

Denote $\xi(t) = [\bar{x}_n^{\mathrm{T}}(t), \hat{\theta}_1(t), \hat{\theta}_2(t), \ldots, \hat{\theta}_n(t), \mu_1(t), \ldots, \mu_n(t)]^{\mathrm{T}}$, then we conclude from (16), (25), (33), (41) and (43) that $\dot{\xi}(t) = \Psi(\xi(t), \xi(t_k)), \forall t \in [t_k, t_{k+1})$. further, from Assumptions 1 and 2, for $\forall t \in [t_k, t_{k+1})$, we obtain

$$\|\xi(t) - \xi(t_k)\| \leqslant \sqrt{n} \int_{t_k}^{t} \|\Psi(\xi(s), \xi(t_k))\| \mathrm{d}s$$
$$\leqslant \int_{t_k}^{t} (g^*\sqrt{3n(n-2)} + \alpha) \|\xi(s) - \xi(t_k)\| \mathrm{d}s$$
$$+ (t - t_k)\beta^*(\|\xi(t_k)\| + 1), \tag{47}$$

where $\alpha = 2n \max \left\{ \sqrt{\eta_1}, \sqrt{\eta_2}, \ldots, \sqrt{\eta_n} \right\}, g^* = \max \left\{ g_1^{\max}, \ldots, g_n^{\max} \right\}, \beta^* = \max \left\{ \Theta_1, \Theta_2 \right\}$ with

$$\begin{cases} \Theta_1 = g^*\sqrt{3n(n-2)} \\ \quad + g^*\sqrt{3n} \left(\sum_{i=1}^{n} \prod_{h=i}^{n} \sqrt{1 + \bar{\Delta}_h^2} + c_h \right), \\ \Theta_2 = \sqrt{n} \left(2\sqrt{2}\Xi_1^2 + \sum_{i=2}^{n} \sqrt{2}\Xi_i^2 + \sum_{i=2}^{n} \sqrt{2}\lambda_i^2 \right) \\ \quad + H^*\sqrt{n}. \end{cases}$$

Furthermore, according to Lemma 4, we have the following relation:

$$\int_{t_k}^{t} \left(g^*\sqrt{3n(n-2)} \right) \|\xi(s) - \xi(t_k)\| \mathrm{d}s$$
$$\leqslant \Theta_3(t - t_k) (\|\xi(t_k)\| + 1), \quad \forall t \in [t_k, t_{k+1}), \tag{48}$$

where $\Theta_3 = \max \left\{ g^*\sqrt{6n(n-2)}, g^*\sqrt{6n(n-2)} \left(\Xi_1 + \sum_{i=2}^{n} \sigma_i^* + \sum_{i=1}^{n} \bar{\mu}_i \right) \right\}$.

Substituting (48) into (47), we obtain, for $\forall t \in [t_k, t_{k+1})$,

$$\|\xi(t) - \xi(t_k)\| \leqslant \Theta^*(t - t_k)(\|\xi(t_k)\| + 1)$$
$$+ \alpha \int_{t_k}^{t} \|\xi(s) - \xi(t_k)\| \mathrm{d}s, \tag{49}$$

where $\Theta^* = \Theta_3 + \beta^*$.

Using Lemma 1, relation (49) can be further rewritten as

$$\|\xi(t) - \xi(t_k)\|$$
$$\leqslant (t - t_k)\Theta^*(\|\xi(t_k)\| + 1)$$
$$+ \int_{t_k}^{t} \alpha^2(s - t_k)\Theta^* (\|\xi(t_k)\| + 1)e^{\alpha(t-s)}\mathrm{d}s$$
$$\leqslant \bar{\alpha} \left(\sqrt{V(t_k)} + 1 \right) \left(e^{\alpha(t-t_k)} - 1 \right), \forall t \in [t_k, t_{k+1}), \tag{50}$$

where $\bar{\alpha} = \max \left\{ \sqrt{2g_1^{\min}}, \sum_{i=2}^{n} \sigma_i^* + \sum_{i=1}^{n} \bar{\mu}_i + \Theta^* \right\}$.

Noting that $|x_i(t) - x_i(t_k)| \leqslant \|\xi(t) - \xi(t_k)\|, i = 1, 2, \ldots, n$, hold for each sampling period, then for

$\forall t \in [t_k, t_{k+1})$, we can deduce from (43) that

$$|u(t_k) - \alpha_n(t)| \leqslant \frac{\tilde{c}}{2g_n^{\max} \bar{\alpha}} \sqrt{\frac{g_n^{\min}}{2}} \|\xi(t) - \xi(t_k)\|$$
$$+ 2b^* \left(\Xi_1 + \sum_{i=2}^n \sigma_i^* + \sum_{i=1}^n \bar{\Delta}_i \right.$$
$$\left. + \sum_{i=1}^n \bar{\mu}_i \right) + \Theta_4 \left(\Xi_1 + \sum_{i=2}^n \sigma_i^* \right), \tag{51}$$

where $b^* = \sqrt{n} \max\{b_1^*, \ldots, b_n^*\}, b_i^* = \prod_{h=i}^n$ $\left(\sqrt{1 + \bar{\Delta}_h^2} + c_i \right), i = 1, 2, \ldots, n;$ and $\Theta_4 > 0$ is a constant.

From (45), (50) and (51), for $\forall t \in [t_k, t_{k+1})$, we have
$$\frac{g_n^{\max}}{g_n^{\min}} |z_n(t)| |u(t_k) - \alpha_n(t)|$$
$$\leqslant \frac{\tilde{c}}{2} (e^{\alpha(t-t_k)} - 1)(\sqrt{V(t_k)} + 1)\sqrt{V(t)} + \bar{d}\sqrt{V(t)}, \tag{52}$$

where $\bar{d} = g_n^{\max} \left(2b^* \left(\Xi_1 + \sum_{i=2}^n \sigma_i^* + \sum_{i=1}^n \bar{\Delta}_i + \sum_{i=1}^n \bar{\mu}_i \right) + \Theta_4 \left(\Xi_1 + \sum_{i=2}^n \sigma_i^* \right) \right) \sqrt{\frac{2}{g_n^{\min}}}$.

Substituting (52) into (46) results in

$$\dot{V}(t) \leqslant -\tilde{c}V(t) + \frac{\tilde{c}}{2}\sqrt{V(t)}\sqrt{V(t_k)}(e^{\alpha(t-t_k)} - 1)$$
$$+ \bar{d}\sqrt{V(t)} + d^*\sqrt{V(t)}$$
$$+ \frac{\tilde{c}}{2}\sqrt{V(t)}(e^{\alpha(t-t_k)} - 1), \quad \forall t \in [t_k, t_{k+1}). \tag{53}$$

Let $W(t) = \sqrt{V(t)}$, from (53), for $\forall t \in [t_k, t_{k+1})$, a straightforward calculation leads to

$$\dot{W}(t) \leqslant -\frac{1}{2}\tilde{c}W(t) + \frac{\tilde{c}}{4}(e^{\alpha T_k} - 1)W(t_k)$$
$$+ \frac{\tilde{c}}{4}(e^{\alpha T_k} - 1) + \frac{1}{2}(d^* + \bar{d}), \tag{54}$$

where $T_k = t_{k+1} - t_k$ is the sampling period. Choose a constant λ_0 satisfying

$$\lambda_0 \geqslant \max\{\beta_0, 1\}, \tag{55}$$

where $\beta_0 = \frac{\tilde{c}}{\tilde{c}\sqrt{p} - 2(d^* + \bar{d})} > 0;$ and the allowable sampling period T_k is given as

$$0 < T_k \leqslant \frac{1}{\alpha}\ln\left(1 + \frac{1}{\lambda_0}\right), \quad k = 0, 1, \ldots, +\infty. \tag{56}$$

Define
$$\begin{cases} \sigma_1 = \frac{1}{2}\tilde{c}, \\ \sigma_{2k} = \frac{\tilde{c}}{4}(e^{\alpha T_k} - 1), \\ \sigma_{3k} = \frac{\tilde{c}}{4}(e^{\alpha T_k} - 1) + \frac{1}{2}(d^* + \bar{d}), \\ k = 0, 1, \ldots, +\infty, \end{cases} \tag{57}$$

then we can obtain the following relations by substituting (55) and (56) into (57):

$$\sigma_{2k} \leqslant \frac{\tilde{c}}{4}, \quad \sigma_{3k}$$
$$\leqslant \frac{\tilde{c}}{4\lambda_0} + \frac{1}{2}(d^* + \bar{d}), \quad k = 0, 1, \ldots, +\infty, \tag{58}$$

under which, for $\forall t \in [t_0, t_1)$ and $W(t_0) \leqslant \sqrt{p}$, it can be concluded that

$$\dot{W}(t) \leqslant -\sigma_1 W(t) + \sigma_{20} W(t_0) + \sigma_{30}$$
$$\leqslant -\frac{1}{2}\tilde{c}W(t) + \frac{\tilde{c}}{4}\sqrt{p} + \frac{\tilde{c}}{4\lambda_0} + \frac{1}{2}(d^* + \bar{d}). \tag{59}$$

When $V(t) = p, \forall t \in [t_0, t_1)$, then from (59), we have $\dot{W}(t) \leqslant 0$, which means that $\{W(t) \leqslant \sqrt{p}\}$ is a invariant set for the first sampling period, that is, $V(t) \leqslant p$ holds for the interval $[t_0, t_1)$. Due to that $W(t)$ is a continuous function for $\forall t \geqslant t_0$, we can obtain $W(t_1^-) = W(t_1) \leqslant \sqrt{p}$; hence, for $\forall t \in [t_1, t_2)$, applying the same analysis approach which is used for the first interval $[t_0, t_1)$, we can know that $V(t) \leqslant p$ also holds, and we equally have $W(t_2^-) = W(t_2) \leqslant \sqrt{p}$. Following this line of argument, we finally know that $W(t) \leqslant \sqrt{p}$ holds for every sampling period, that is, $V(t) \leqslant p$ holds for $\forall t \geqslant t_0$, which has proved that $V(t) \leqslant p$ is an invariant set for $\forall t \geqslant t_0$ and $V(t_0) \leqslant p$. Substituting (56) into (54), and solving the inequality, we obtain, for $\forall t \in [t_k, t_{k+1})$,

$$W(t) \leqslant W(t_k) \left(e^{-\sigma_1(t-t_k)} + \frac{\sigma_{2k}}{\sigma_1}\left(1 - e^{-\sigma_1(t-t_k)}\right) \right)$$
$$+ \frac{1 - e^{-\sigma_1(t-t_k)}}{\sigma_1}\sigma_{3k}. \tag{60}$$

From (57) and (58), (60) can be further rewritten as

$$W(t) \leqslant Q^*(t - t_k)W(t_k) + \frac{\sigma_{3k}}{\sigma_1}, \quad \forall t \in [t_k, t_{k+1}), \tag{61}$$

where $Q^*(t - t_k) = \frac{1}{2} + \frac{1}{2}e^{-\sigma_1(t-t_k)}$. Substituting $t = t_{k+1}$ into (61) results in

$$W(t_{k+1}) \leqslant Q^*(T_k)W(t_k) + \frac{\sigma_{3k}}{\sigma_1}, \quad k = 0, 1, \ldots, +\infty. \tag{62}$$

Obviously, $Q^*(T_k) \in (0, 1)$, and we can deduce from (62) that

$$
\begin{aligned}
W(t_{k+1}) &\leqslant \prod_{i=0}^{k} Q^*(T_i) W(t_0) \\
&\quad + \left(\frac{1}{2\lambda_0} + \frac{d^* + \bar{d}}{\tilde{c}} \right) \left(1 + \sum_{i=1}^{k-1} \prod_{h=i}^{k} Q^*(T_h) \right) \\
&\leqslant Q^{*k+1}(T^*) W(t_0) \\
&\quad + \left(\frac{1}{2\lambda_0} + \frac{d^* + \bar{d}}{\tilde{c}} \right) \left(1 + \sum_{i=1}^{k-1} \prod_{h=i}^{k} Q^*(T^*) \right) \\
&\leqslant Q^{*k+1}(T^*) W(t_0) \\
&\quad + \left(\frac{1}{2\lambda_0} + \frac{d^* + \bar{d}}{\tilde{c}} \right) \left(\frac{1 - Q^{*k+1}(T^*)}{1 - Q^*(T^*)} \right),
\end{aligned}
\tag{63}
$$

where $T^* > 0$ is a constant.

From (63), it is easy to obtain that

$$
\lim_{k \to \infty} W(t_{k+1}) \leqslant \left(\frac{1}{2\lambda_0} + \frac{d^* + \bar{d}}{\tilde{c}} \right) \left(\frac{1}{1 - Q^*(T^*)} \right),
\tag{64}
$$

i.e.,

$$
\lim_{t \to \infty} W(t) \leqslant W^*,
\tag{65}
$$

where $W^* = \left(\frac{1}{2\lambda_0} + \frac{d^* + \bar{d}}{\tilde{c}} \right) \left(\frac{1}{1 - Q^*(T^*)} \right)$. Using (45) and (65), we get

$$
\lim_{t \to \infty} |x_1(t)| \leqslant \sqrt{2 g_1^{\min} W^*}, \quad \lim_{t \to \infty} |\tilde{u}_i(t)|
$$
$$
\leqslant \sqrt{2\beta_i} W^*, \quad i = 1, 2, \ldots, n, \tag{66a}
$$
$$
\lim_{t \to \infty} |z_i(t)| \leqslant \sqrt{2 g_i^{\min} W^*}, \quad i = 2, 3, \ldots, n, \tag{66b}
$$
$$
\lim_{t \to \infty} \left| \hat{\theta}_i(t) \right| \leqslant |\theta_i| + \sqrt{2\eta_i} W^*, \quad i = 1, 2, \ldots, n. \tag{66c}
$$

Further, for $i = 2, 3, \ldots, n$, it can be deduced from (43) and (66) that

$$
\begin{aligned}
\lim_{t \to \infty} |x_i(t)| &\leqslant \sqrt{2 g_i^{\min}} W^* \\
&\quad + \sqrt{2 g_{i-1}^{\min}} W^* \left(\sqrt{1 + \left(|\theta_{i-1}| + \sqrt{2\eta_{i-1}} W^* \right)^2} \right. \\
&\quad \left. + c_{i-1} \right),
\end{aligned}
\tag{67}
$$

Finally, from Assumption 3 and (66), we have the boundedness of $z_0(t)$ as $t \to +\infty$. The proof of Theorem 1 is completed.

Remark 7 Compared with the periodic sampling considered in the existing literature [18–24], the sampling period in this paper can be chosen freely within an allowable constant [see (56)], which means that the designer can increase the sampling frequency in the initial running stage to make the system to be stable as soon as possible, and decrease the sampling frequency to reduce the computation burden when the system tends to be stable; thus, for the real systems, non-periodic sampling has more advantages than the periodic sampling.

Remark 8 For nonlinear system (1), the adaptive sampled-data controller can be constructed as follows:
Step 1: Determine the constant α from the parameters $\eta_i > 0, i = 1, 2, \ldots, n$, and choose λ_0 satisfies (55), then work out the allowable sampling period $T_k, k = 0, 1, \ldots, +\infty$.
Step 2: The adaptive sampled-data controller (43) can be constructed with the design parameters $c_i > 0, i = 1, 2, \ldots, n$, and the sampled values of the system states.

Remark 9 Motivated by the existing literature on sampled-data control, such as [41–47], the proposed sampled-data control method will be extended to solve the stabilization problem of the Markovian jump systems in the form of (1); furthermore, by choosing appropriate Lyapunov–Krasovskii functional, the proposed sampled-data control method will also be extended to solve the stabilization problem of the systems in the form of (1) with time-varying delays.
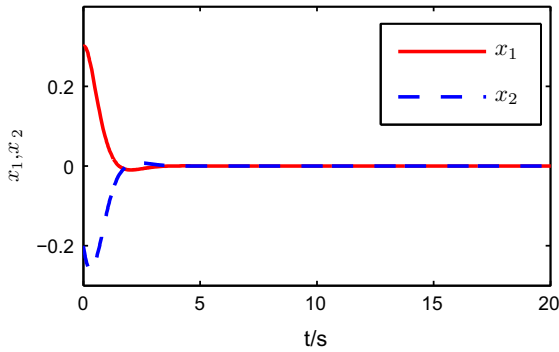
5 Simulation examples

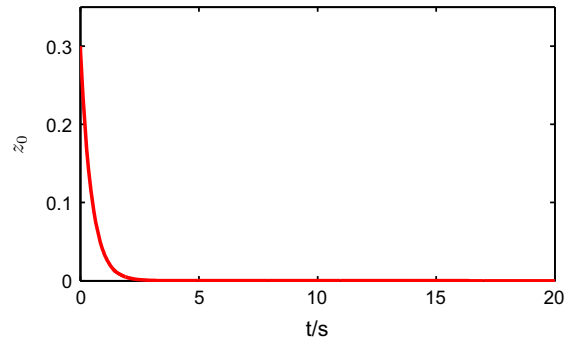In this section, two examples will be provided to show effectiveness of the proposed results.

Example 1 Consider the following second-order nonlinear system

$$
\begin{cases}
\dot{z}_0 = -2z_0 + x_1^2 \sin(x_1), \\
\dot{x}_1 = (\sin(x_1) + 2)x_2 \\
\quad + \sin(x_1^2)\cos(x_1) + \sin(x_1) + \Delta_1(x_1, z_0), \\
\dot{x}_2 = (\sin(x_1 x_2) + 3)u \\
\quad + \sin(x_1 x_2) + \sin(x_1)\cos(x_2) + \Delta_2(\bar{x}_2, z_0), \\
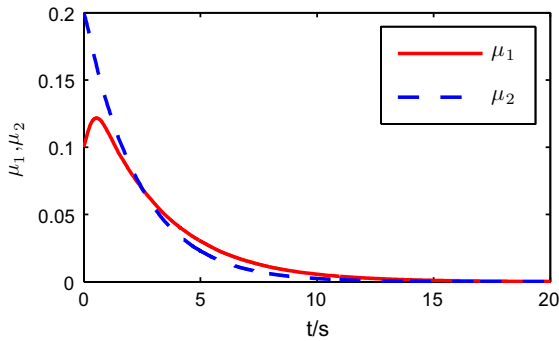y = x_1,
\end{cases}
\tag{68}
$$

where $\Delta_1(x_1, z_0) = z_0^2 \sin(z_0) + \cos^2(x_1^2)\sin^2(z_0)$ and $\Delta_2(\bar{x}_2, z_0) = z_0^2 \cos(z_0) + \sin(x_1^2 x_2^2)\cos(x_2)$.

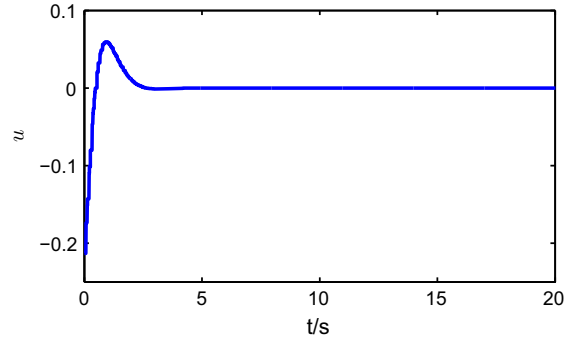**Fig. 1** Trajectories of x_1 (solid line) and x_2 (dashed line)

Fig. 2 Trajectories of μ_1 (solid line) and μ_2 (dashed line)

Fig. 3 Trajectory of z_0

Fig. 4 Control input u

From (43), for $\forall t \in [t_k, t_{k+1})$, we construct the sampled-data controller as

$$
\begin{aligned}
u(t) = & -x_1(t_k)\left(\sqrt{1 + \mu_2^2(t_k)} + c_2\right) \\
& \times \left(\sqrt{1 + \mu_1^2(t_k)} + c_1\right) \\
& - x_2(t_k)\left(\sqrt{1 + \mu_2^2(t_k)} + c_2\right),
\end{aligned} \tag{69}
$$

where $c_1 = 0.2$, $c_2 = 0.3$, and $\mu_1(t_k)$, $\mu_2(t_k)$ can be obtained from the following equations:

$$
\begin{cases}
\dot{\mu}_1(t) = \eta_1 x_1^2(t_k) - \eta_1 \mu_1(t), \\
\dot{\mu}_2(t) = \eta_2 \varrho_2^2(t_k) - \eta_2 \mu_2(t), & \forall t \in [t_k, t_{k+1}),
\end{cases} \tag{70}
$$

where $\eta_1 = 0.3$, $\eta_2 = 0.4$ and $\varrho_2(t_k) = x_1(t_k) + x_1(t_k)\sqrt{1 + \mu_2^2(t_k)} + c_1 x_1(t_k)$.

Simulation results are shown in Figs. 1, 2, 3 and 4, where $x(0) = [0.3, -0.2]^{\mathrm{T}}$, $\mu_1(0) = 0.1$, $\mu_2(0) = 0.2$ and $z_0(0) = 0.3$, the sampling period is chosen randomly in the interval (0, 0.08 s). It can be seen from Fig. 1 that all states of the resulting closed-loop system converge to a neighborhood of the origin, which

demonstrates effectiveness of the proposed sampled-data control method.

Example 2 In the example, a popular benchmark of application example will be provided, that is, the stabilization of an one-link manipulator actuated by a brush dc (BDC) motor, the dynamics of a one-link manipulator actuated by a BDC motor can be expressed as follows [48]:

$$
\begin{cases}
\dot{z}_0(t) = -z_0(t) + 2q^2(t)\cos(q(t)), \\
D\ddot{q}(t) + B\dot{q}(t) + N\sin(q(t)) \\
\quad = I(t) + \Delta_{I1}(q(t), \dot{q}(t), z_0(t)), \\
M\dot{I}(t) = -QI(t) - K_m\dot{q}(t) + V_E(t) \\
\quad\quad + \Delta_{I2}(q(t), \dot{q}(t), I(t), z_0(t)),
\end{cases} \tag{71}
$$

where $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$ are the link angular position, velocity, and acceleration, respectively; $I(t)$ is the motor current; $\Delta_{I1}(\cdot)$ is the additive bounded disturbance; $\Delta_{I2}(\cdot)$ is the additive bounded voltage disturbance; V_E is the input control voltage; M is the armature inductance; Q is the armature resistance; K_m is the back-emf coefficient; z_0 is an unmeasured states;

$$
\begin{cases}
D = \dfrac{J}{K_\tau} + \dfrac{mL_0^2}{3K_\tau} + \dfrac{2M_0 R_0^2}{5K_\tau}, \\[2mm]
N = \dfrac{mL_0 G}{2K_\tau} + \dfrac{M_0 L_0 G}{K_\tau}, \; B = \dfrac{B_0}{K_\tau},
\end{cases}
$$

J is rotor inertia; R_0 is the radius of the load; B_0 is the coefficient of viscous friction at the joint; m is the link mass; L_0 is the link length; G is the gravity coefficient, and K_τ is the coefficient which characterizes the electromechanical conversion of armature current to torque; $q(t)$, $\dot{q}(t)$ and $I(t)$ can only be measured at the sampling points.

In the simulation, the design parameters are selected as $J = 1.252 \times 10^{-3} \text{ kg m}^2$, $M = 15.0 \times 10^{-3} H$, $L_0 = 0.205 \text{ m}$, $Q = 3 \; \Omega$, $m = 0.506 \text{ kg}$, $M_0 = 0.423 \text{ kg}$, $B_0 = 15.25 \times 10^{-3} \text{ N m s/rad}$, $K_\tau = 1.5 \times 10^{-3} \text{ N m/A}$, and $K_m = 0.7 \text{ N m/A}$.

Now, setting $x_1(t) = q(t)$, $x_2(t) = \dot{q}(t)$ and $x_3(t) = I(t)$, then (71) can be expressed as

$$
\begin{cases}
\dot{z}_0(t) = -z_0(t) + 2x_1^2(t)\cos(x_1(t)), \\
\dot{x}_1(t) = x_2(t), \\
\dot{x}_2(t) = \dfrac{1}{D}x_3(t) + \dfrac{B}{D}x_2(t) \\
\qquad\quad - \dfrac{N\sin(x_1(t))}{D} + \dfrac{\Delta_{I1}(\bar{x}_2(t), z_0(t))}{D}, \\
\dot{x}_3(t) = \dfrac{1}{M}V_E(t) - \dfrac{Q}{M}x_3(t) \\
\qquad\quad - \dfrac{K_m}{M}x_2(t) + \dfrac{\Delta_{I2}(\bar{x}_3(t), z_0(t))}{M},
\end{cases} \tag{72}
$$

where $\Delta_{I1}(\bar{x}_2, z_0) = z_0^2\sin(x_2 z_0) + \sin(x_1^2 x_2^2)$, $\Delta_{I2}(\bar{x}_3, z_0) = z_0^2\cos(x_1 x_2)\sin^2(x_3) + \sin(x_1 x_2)$.

To solve the stabilization problem of system (72), we use the following sampled-data controller:

$$
\begin{cases}
\bar{\alpha}_1(t_k) = -x_1(t_k)\sqrt{1 + \mu_1^2(t_k)} - c_1 x_1(t_k), \\
\bar{\alpha}_2(t_k) = -\bar{z}_2(t_k)\sqrt{1 + \mu_2^2(t_k)} - c_2 \bar{z}_2(t_k), \\
V_E(t) = -\bar{z}_3(t_k)\sqrt{1 + \mu_3^2(t_k)} - c_3 \bar{z}_3(t_k), \\
\quad \forall t \in [t_k, t_{k+1}),
\end{cases} \tag{73}
$$

where $\bar{z}_i = x_i - \bar{\alpha}_{i-1}$, $i = 2, 3$, and $c_1 = 0.2$, $c_2 = 0.3$, $c_3 = 0.1$.

According to (43), the adaptive laws are taken as

$$
\begin{cases}
\dot{\mu}_1(t) = \eta_1 x_1^2(t_k) - \eta_1 \mu_1(t), \\
\dot{\mu}_2(t) = \eta_2 \varrho_2^2(t_k) - \eta_2 \mu_2(t), \\
\dot{\mu}_3(t) = \eta_3 \varrho_3^2(t_k) - \eta_3 \mu_3(t), \quad \forall t \in [t_k, t_{k+1}),
\end{cases} \tag{74}
$$

where $\eta_1 = 0.3$, $\eta_2 = 0.4$, $\eta_3 = 0.2$ and

$$
\begin{cases}
\varrho_1(t_k) = x_1(t_k), \\
\varrho_2(t_k) = x_2(t_k) + \varrho_1(t_k)\sqrt{1 + \mu_1^2(t_k)} + \varrho_1(t_k)c_1, \\
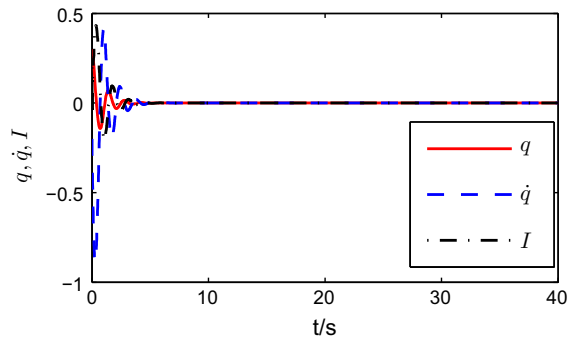\varrho_3(t_k) = x_3(t_k) + \varrho_2(t_k)\sqrt{1 + \mu_2^2(t_k)} + \varrho_2(t_k)c_2.
\end{cases}
$$

Fig. 5 Trajectories of q (solid line), $\dot{q}$ (dashed line), and I (dotted line)
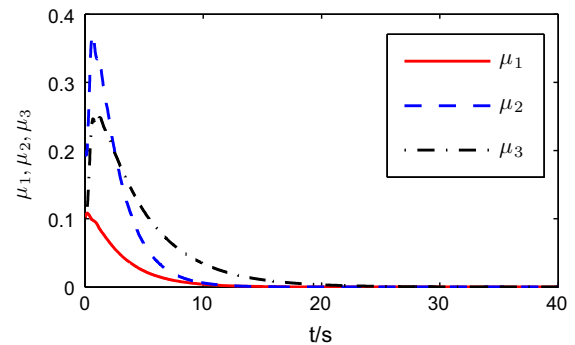

**Fig. 6** Trajectories of μ_1 (solid line), μ_2 (dashed line), and μ_3 (dotted line)

Simulation results are shown in Figs. 5, 6, 7 and 8 with initial conditions $x(0) = [0.3, -0.2, 0.2]^\text{T}$, $\mu_1(0) = 0.1$, $\mu_2(0) = 0.2$, $\mu_3(0) = 0.1$ and $z_0(0) = 0.3$, the sampling period can be chosen randomly in the interval $(0, 0.06 \text{ s})$. It can be seen from Fig. 5 that the link angular position $q(t)$, velocity $\dot{q}(t)$ and motor current $I(t)$ of system (71) converge to a neighborhood of the origin, which shows effectiveness of the proposed methods.

6 Conclusions

In this paper, an adaptive sampled-data control technique was developed to practically stabilize a class of nonlinear systems in strict-feedback structure with uncertain functions which were not required to satisfy any linear growth condition. During adaptive sampled-data controller design, neural networks were employed to approximate the unknown nonlinear functions, then an adaptive sampled-data controller was constructed by
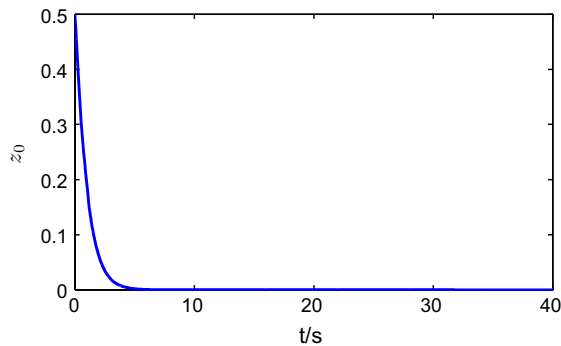
Fig. 7 Trajectory of z_0
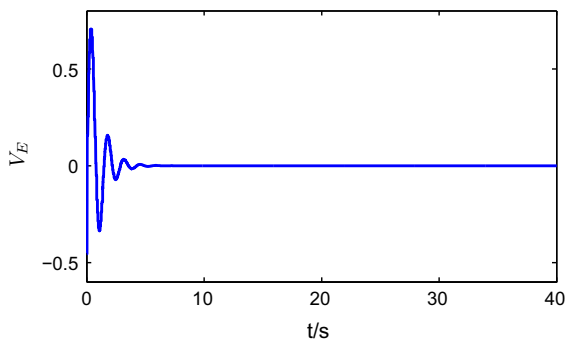
Fig. 8 Control input V_E

utilizing the virtual control laws and "virtual adaptive laws" which were obtained via the backstepping technique. With the help of Gronwall–Bellman inequality, it was demonstrated that the proposed sampled-data controller can make all states of the resulting closed-loop system to be semi-globally uniformly ultimately bounded with appropriate choice of the sampling period. Finally, two examples were provided to show validness of the obtained results. In addition, the proposed results can be extended to the switched nonlinear systems and multi-agent systems, which is our future work.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Wu, L.B., Yang, G.H.: Robust adaptive fault-tolerant tracking control of multiple time-delays systems with mismatched parameter uncertainties and actuator failures. Int. J. Robust Nonlinear Control **25**(16), 2922–2938 (2015)
2. Wang, H., Liu, X., Chen, B., Zhou, Q.: Adaptive fuzzy decentralized control for a class of pure-feedback large-scale nonlinear systems. Nonlinear Dyn. **75**(3), 449–460 (2014)
3. Wang, C., Wen, C., Guo, L.: Decentralized output-feedback adaptive control for a class of interconnected nonlinear systems with unknown actuator failures. Automatica **71**, 187–196 (2016)
4. Wang, Q., Wei, C.: Decentralized robust adaptive output feedback control of stochastic nonlinear interconnected systems with dynamic interactions. Automatica **54**, 124–134 (2015)
5. Zhang, T., Xia, X.: Decentralized adaptive fuzzy output feedback control of stochastic nonlinear large-scale systems with dynamic uncertainties. Inf. Sci. **315**, 17–38 (2015)
6. Huang, S., Xiang, Z.: Finite-time stabilization of a class of switched stochastic nonlinear systems under arbitrary switching. Int. J. Robust Nonlinear Control **26**(10), 2136–2152 (2016)
7. Wang, F., Chen, B., Zhang, Z., Lin, C.: Adaptive tracking control of uncertain switched stochastic nonlinear systems. Nonlinear Dyn. **84**(4), 2099–2109 (2016)
8. Miao, X., Li, L.: Adaptive observer-based control for uncertain nonlinear stochastic systems with time-delay. J. Franklin Inst. **353**(14), 3595–3609 (2016)
9. Zhang, T., Xia, X.: Adaptive output feedback tracking control of stochastic nonlinear systems with dynamic uncertainties. Int. J. Robust Nonlinear Control **25**(9), 1282–1300 (2015)
10. Li, Y., Mao, Z., Tong, S.: Adaptive fuzzy output-constrained fault-tolerant control of nonlinear stochastic large-scale systems with actuator faults. IEEE Trans. Cybern. **47**(9), 2362–2376 (2017)
11. Chen, Y., Du, H., He, Y., Jia, R.: Finite-time tracking control for a class of high-order nonlinear systems and its applications. Nonlinear Dyn. **76**(2), 1133–1140 (2014)
12. Li, D., Cao, J.: Global finite-time output feedback synchronization for a class of high-order nonlinear systems. Nonlinear Dyn. **82**(2), 1027–1037 (2015)
13. Mao, J., Huang, S., Xiang, Z.: Adaptive practical finite-time stabilization for switched nonlinear systems in pure-feedback form. J. Franklin Inst. **354**(10), 3971–3994 (2017)
14. Ma, J., Ni, S., Jie, W., Dong, W.: Deterministic sampling strong tracking filtering algorithms: fast detection and isolation for aircraft actuator fault. Control Theory Appl. **32**(6), 734–743 (2015)
15. Liu, Z., Wang, L., Wang, J., Dong, D., Hu, X.: Distributed sampled-data control of nonholonomic multi-robot systems with proximity networks. Automatica **77**, 170–179 (2017)
16. Wang, Z., Zhai, J., Ai, W., Fei, S.: Global practical tracking for a class of uncertain nonlinear systems via sampled-data control. Appl. Math. Comput. **260**, 257–268 (2015)
17. Nešić, D., Teel, A.R., Kokotović, P.V.: Sufficient conditions for stabilization of sampled-data nonlinear systems

via discrete-time approximations. Syst. Control Lett. **38**(4), 259–270 (1999)

18. Wu, B., Ding, Z.: Sampled-data adaptive control of a class of nonlinear systems. Int. J. Adapt. Control Signal Process. **25**, 1050–1060 (2011)

19. Chu, H., Qian, C., Liu, R., Li, S.: Global practical tracking for a class of nonlinear systems via linear sampled-data control. Int. J. Control **88**(9), 2667–2672 (2015)

20. Zhang, C., Jia, R., Qian, C., Li, S.: Semi-global stabilization via linear sampled-data output feedback for a class of uncertain nonlinear systems. Int. J. Robust Nonlinear Control **25**(13), 2041–2061 (2015)

21. Zhang, D., Shen, Y.: Global output feedback sampled-data stabilization for upper-triangular nonlinear systems with improved maximum allowable transmission delay. Int. J. Robust Nonlinear Control **27**(2), 212–235 (2017)

22. Du, H., Qian, C., He, Y., Cheng, Y.: Global sampled-data output feedback stabilisation of a class of upper-triangular systems with input delay. IET Control Theory Appl. **7**(10), 1437–1446 (2013)

23. Qian, C., Du, H., Li, S.: Global stabilization via sampled-data output feedback for a class of linearly uncontrollable and unobservable systems. IEEE Trans. Autom. Control **61**(12), 4088–4093 (2016)

24. Lin, W., Wei, W., Ye, G.: Global stabilization of a class of nonminimum-phase nonlinear systems by sampled-data output feedback. IEEE Trans. Autom. Control **61**(10), 3076–3082 (2016)

25. Tarek, A.A., Iasson, K., Francoise, L.L.: Global exponential sampled-data observers for nonlinear systems with delayed measurements. Syst. Control Lett. **62**(7), 539–549 (2013)

26. Shen, B., Wang, Z., Huang, T.: Stabilization for sampled-data systems under noisy sampling interval. Automatica **63**, 162–166 (2016)

27. Liu, Y., Park, J.H., Guo, B.Z., Sun, Y.: Further results on stabilization of chaotic systems based on fuzzy memory sampled-data control. IEEE Trans. Fuzzy Syst. (2017). https://doi.org/10.1109/TFUZZ.2017.2686364

28. Wang, B., Cheng, J., Ai-Barakati, A., Fardoun, H.M.: A mismatched membership function approach to sampled-data stabilization for T–S fuzzy systems with time-varying delayed signals. Sig. Process. **140**, 161–170 (2017)

29. Cheng, J., Park, J.H., Karimi, H.R., Shen, H.: A flexible terminal approach to sampled-data exponentially synchronization of Markovian neural networks with time-varying delayed signals. IEEE Trans. Cybern. (2017). https://doi.org/10.1109/TCYB.2017.2729581

30. Shen, H., Su, L., Wu, Z.G., Park, J.H.: Reliable dissipative control for Markov jump systems using an event-triggered sampling information scheme. Nonlinear Anal. Hybird Syst. **25**, 41–59 (2017)

31. Shen, H., Men, Y., Wu, Z.G., Park, J.H.: Non-fragile H_∞ control or fuzzy Markovian jump systems under fast sampling singular perturbation. IEEE Trans. Syst. Man Cybern. Syst. (2017). https://doi.org/10.1009/TSMC.2017.2758381

32. He, W., Zhang, S., Ge, S.S.: Robust adaptive control of a thruster assisted position mooring system. Automatica **50**(7), 1843–1851 (2014)

33. Krstic, M., Kanellakopoulos, L., Kokotovic, P.V.: Nonlinear and Adaptive Control Design. Wiley, New York, NY (1995)

34. He, W., Zhang, S., Ge, S.S.: Adaptive control of a flexible crane system with the boundary output constraint. IEEE Trans. Ind. Electron. **61**(8), 4126–4133 (2014)

35. Li, T.S., Wang, D., Feng, G., Tong, S.C.: A DSC approach to robust a NN tracking control for strict-feedback nonlinear systems. IEEE Trans. Syst. Man Cybern. B Cybern. **40**(3), 915–927 (2010)

36. Jiang, Z.P., Praly, L.: Design of robust adaptive controllers for nonlinear systems with dynamic uncertainties. Automatica **34**(7), 825–840 (1998)

37. Apostol, T.: Mathematical Analysis. Addison-Wesley, New Jersey (1974)

38. Lin, W., Qian, C.: Adaptive control of nonlinearly parameterized systems: the smooth feedback case. IEEE Trans. Autom. Control **47**(8), 1249–1266 (2002)

39. Hardy, G.H., Littlewood, J.E., Pslya, G.: Inequalities. Cambridge University Press, Cambridge (1952)

40. Sanner, R.M., Slotine, J.J.E.: Gaussian networks for direct adaptive control. IEEE Trans. Neural Netw. **3**(6), 837 (1991)

41. Cheng, J., Park, J.H., Zhang, L., Zhu, Y.: An asynchronous operation approach to event-triggered control for fuzzy Markovian jump systems with general switching policies. IEEE Trans. Fuzzy Syst. (2016). https://doi.org/10.1109/TFUZZ.2016.2633325

42. Cheng, J., Park, J.H., Karimi, H.R., Zhao, X.: Static output feedback control for nonhomogeneous Markovian jump systems with asychronous time delays. Inf. Sci. **399**, 219–238 (2017)

43. Liang, K., Dai, M., Shen, H., Wang, J., Wang, Z., Chen, B.: $L_2 - L_\infty$ synchronization for singularly perturbed complex networks with semi-Markov jump topology. Appl. Math. Comput. **321**, 450–462 (2018)

44. Lee, T.H., Wu, Z.G., Park, J.H.: Synchronization of a complex dynamical network with coupling time-varying delays via sampled-data control. Appl. Math. Comput. **219**(3), 1354–1366 (2012)

45. Lee, T.H., Park, J.H.: Improved criteria for sampled-data synchronization of chaotic Lur'e systems using two new approaches. Nonlinear Anal. Hybrid Syst. **24**, 132–145 (2017)

46. Lee, T.H., Park, J.H.: Stability analysis of sampled-data systems via free-matrix-based time-dependent discontinuous Lyapunov approach. IEEE Trans. Autom. Control **62**(7), 3653–3657 (2017)

47. Liu, Y., Guo, B.Z., Park, J.H., Lee, S.M.: Event-based reliable dissipative filtering for T–S fuzzy systems with asynchronous constraints. IEEE Trans. Fuzzy Syst. (2017). https://doi.org/10.1109/TFUZZ.2017.2762633

48. Carroll, J.J., Schneider, M., Dawson, D.M.: Integrator backstepping techniques for the tracking control of permanent magnet brush DC motors. IEEE Trans. Ind. Appl. **31**(2), 248–255 (1995)