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ORIGINAL PAPER

A time-specified nonsingular terminal sliding mode control approach for trajectory tracking of robotic airships

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Abstract The robotic airships provide potential aerial platforms for various applications and require robust trajectory tracking to support these tasks. A time-specified nonsingular terminal sliding mode control (TS-NTSMC) scheme is proposed to address the problem of trajectory tracking for robotic airships, which can avoid the singularity problem and specify the convergence time of terminal sliding mode control. First, the problem of trajectory tracking of robotic airships is formulated. Second, a nonsingular terminal sliding manifold consisting of pre-specified nonlinear functions is proposed, and the TS-NTSMC law is designed for trajectory tracking. Time-specified convergence and stability of the closed-loop system can be guaranteed by Lyapunov theory. Finally, compared experimental simulations are given to illustrate the advantages of TS-NTSMC against NTSMC.

Keywords Trajectory control · Terminal sliding mode control · Time-specified convergence · Robotic airship

1 Introduction

The robotic airship, a typical aerostatic aircraft, performs as a potential platform for various applications such as surveillance, earth observation, environment monitoring, disaster guard [1–5]. To achieve these tasks, effective and robust trajectory tracking is quite necessary. However, dynamics nonlinearity and parameter variation bring on major difficult of the control design.

There are many control schemes and approaches proposed for reference, such as dynamic inversion approach [6], back-stepping technique [7–11], feedback linearization approach [12], artificial neural network [13], sliding mode control (SMC) [14–16]. Among these control approaches, SMC provides a promising tool of control design for nonlinear systems, due to its intrinsic property of insensitivity to parameter variations and external disturbances [17–20]. However, the conventional SMC suffers an obvious drawback, i.e., asymptotic error convergence of the closed-loop system, due to the use of linear sliding surfaces [21,22]. Fortunately, a new type of SMC called TSMC was developed to deal with the problem of infinite time convergence. The TSMC employs nonlinear sliding surfaces instead of linear sliding surfaces, which guarantees the finite time error convergence [23–27]. However, TSMC faces a major problem called "singularity problem," which has been overcome by nonsingular terminal sliding mode control (NTSMC) [28,29]. However, NTSMC needs further research to solve the following problem: the time taken to reach the terminal sliding manifold strongly depended on the error dynamics of the systems.

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In this paper, a TS-NTSMC is proposed for trajectory tracking of robotic airships. A new nonsingular terminal sliding manifold using pre-specified nonlinear functions is proposed to overcome the singularity problem and time-depended problem of TSMC. The time taken to reach the manifold from any initial state and the time taken to reach the equilibrium point in the sliding mode can be guaranteed to be a specified time. The proposed time-specified nonsingular terminal sliding manifold is then applied to design the trajectory control law. Experimental simulations are presented to validate the proposed control approach. In practice, parameter variations and external disturbance are the most important factors that affect the control performance and stability of the closed-loop system. The proposed TS-NTSMC can address these problems effectively, due to its intrinsic property of insensitivity to parameter variations and external disturbances.

The main contributions of this paper are listed as follows.

(1) A TS-NTSMC approach with nonsingular sliding manifolds consisting of pre-specified nonlinear functions is proposed for trajectory tracking of robotic airships, which address the infinite time convergence problem of SMC and the singularity problem of TSMC.
(2) In comparison with NTSMC, the proposed TS-NTSMC can specify the finite time to obtain faster convergence.
(3) The specified finite time convergence and stability of the closed-loop system can be guaranteed by Lyapunov theory, and the validity of the proposed approach is conformed via experimental simulations.

The rest of this paper is organized as follows. Section 2 formulates the problem of trajectory tracking. Section 3 demonstrates the design of NTSMC and TS-NTSMC. In Sect. 4, experimental simulations illustrate the performance of the designed control law. Finally, Sect. 5 gives the conclusions.

2 Modeling and formulation

A low-altitude airship equipped with full-actuated actuators is investigated in the current paper. The motion equations of the airship are expressed as follows [30–33]

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{V} \tag{1}$$

$$\boldsymbol{M}_\eta \ddot{\boldsymbol{\eta}} + \boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} + \boldsymbol{G}_\eta = \boldsymbol{u} \tag{2}$$

where $\boldsymbol{V} = [u, v, w, p, q, r]^T \in \mathbb{R}^6$ are the translational velocities and angular velocities, $\boldsymbol{\eta} = [x, y, z, \theta, \psi, \phi]^T \in \mathbb{R}^6$ are the position and Euler angles, as depicted in Fig. 1, $\boldsymbol{u} = \left[F_u, F_v, F_w, \tau_p, \tau_q, \tau_r\right]^T \in \mathbb{R}^6$ denotes the control inputs, $\boldsymbol{M}_\eta$, $\boldsymbol{N}_\eta$ and $\boldsymbol{G}_\eta$ are the inertial matrix, the terms of nonlinear dynamics and the terms of buoyancy and gravitational forces and moments [34,35], respectively.

The problem of trajectory tracking can be stated as [35,36]: given any initial states $\boldsymbol{\eta}_0$ and $\boldsymbol{V}_0$, consider the kinematics and dynamics equations given by (1) and (2), respectively, design an appropriate control law to drive the robotic airship approach to and track the desired trajectory in finite time, i.e., $\lim_{t\to t_f} \left\|\boldsymbol{\eta} - \boldsymbol{\eta}_d\right\| = 0$, where $\boldsymbol{\eta}$ denotes the generalized trajectory, $\boldsymbol{\eta}_d$ denotes the desired generalized trajectory, and t_f denotes the time taken to approach the desired trajectory from the initial state.

3 Control design for trajectory tracking

3.1 Nonsingular terminal sliding mode control

The control design of NTSMC consists of the following steps: first, define the tracking errors; second, define the nonsingular terminal sliding manifold; third, design the control input based on the defined sliding manifold; and finally, prove the finite time convergence and stability of the closed-loop system. The block diagram of NTSMC is depicted in Fig. 2.

The tracking error between the actual trajectory and the desired trajectory is defined as

$$\boldsymbol{e} = \boldsymbol{\eta} - \boldsymbol{\eta}_d \tag{3}$$

where $\boldsymbol{\eta}_d = [x_d, y_d, z_d, \theta_d, \psi_d, \varphi_d]^T$.

The nonsingular terminal sliding manifold based on the nonlinear combination of tracking errors and its derivative is defined as [26]

$$\boldsymbol{s} = \boldsymbol{e} + \boldsymbol{c}\dot{\boldsymbol{e}}^{a/b} \tag{4}$$

where $\boldsymbol{c} = \mathrm{diag}(c_1, c_2, c_3, c_4, c_5, c_6)$ is a designed matrix, and all the diagonal elements of $\boldsymbol{c}$ are positive real numbers, $\dot{\boldsymbol{e}}^{a/b} = \left[\dot{e}_1^{a/b}, \dot{e}_2^{a/b}, \dot{e}_3^{a/b}, \dot{e}_4^{a/b}, \dot{e}_5^{a/b}, \dot{e}_6^{a/b}\right]^T$, a and b are both positive odd integers satisfying $1 < a/b < 2$.

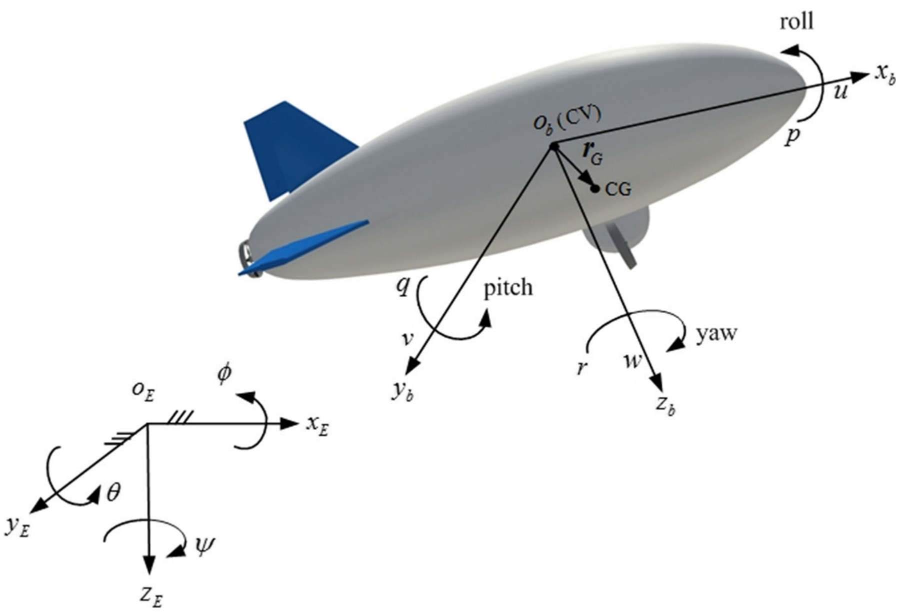

Fig. 1 Sketch of reference frames and motion variables of the airship

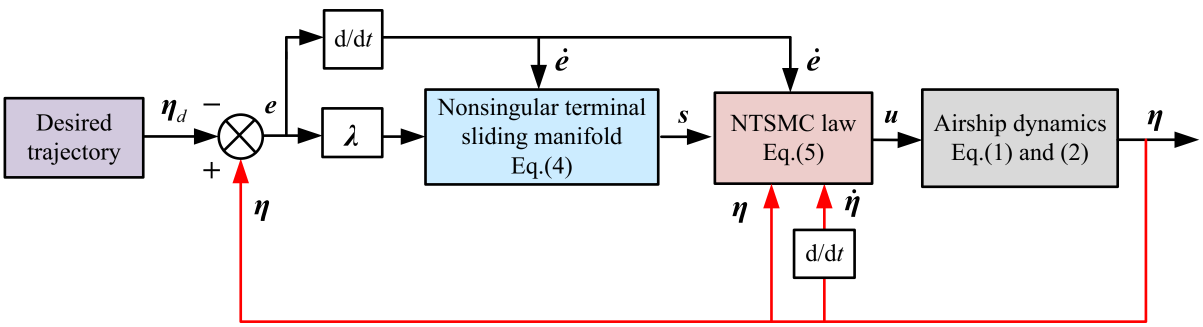

Fig. 2 Block diagram of NTSMC

The NTSMC law is designed as follows [35]

$$\begin{aligned}\boldsymbol{u} &= \boldsymbol{M}_\eta \ddot{\boldsymbol{\eta}}_d + \boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} + \boldsymbol{G}_\eta \\ &\quad - \frac{b}{a}\boldsymbol{M}_\eta \boldsymbol{c}^{-1}\mathrm{diag}\left(\dot{\boldsymbol{e}}^{2-a/b}\right) \\ &\quad - \frac{\left[\boldsymbol{s}^T \boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\boldsymbol{M}_\eta^{-1}\right]^T}{\left\|\boldsymbol{s}^T \boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\boldsymbol{M}_\eta^{-1}\right\|^2} \\ &\quad \times \kappa\,\|\mathbf{s}\|\left\|\mathbf{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\boldsymbol{M}_\eta^{-1}\right\| \end{aligned} \tag{5}$$

where κ is a designed parameter and $\kappa > 0$.

Theorem 1 *For the airship kinematics and dynamics given by (1) and (2), respectively, if the nonsingular terminal sliding manifold is given by (4) and the NTSMC law is designed as (5), then the nonsingular terminal sliding manifold given by (4) will be reached in finite time and the tracking errors will converge to zero in finite time.*

Proof Select the following Lyapunov function candidate

$$V = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{s} \tag{6}$$

Differentiating (6) and using (4) yield

$$\dot{V} = \boldsymbol{s}^T\dot{\boldsymbol{s}} = \boldsymbol{s}^T\left[\dot{\boldsymbol{e}} + \frac{a}{b}\boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\ddot{\boldsymbol{e}}\right] \tag{7}$$

Differentiating (3) with respect to time twice and using (1) and (2), it is obtained that

$$\begin{aligned}\ddot{\boldsymbol{e}} &= \ddot{\boldsymbol{\eta}} - \ddot{\boldsymbol{\eta}}_d = \boldsymbol{M}_\eta^{-1}\left(\boldsymbol{u} - \boldsymbol{N}_\eta\dot{\boldsymbol{\eta}} - \boldsymbol{G}_\eta\right) \\ &= \boldsymbol{M}_\eta^{-1}\left[-\frac{a}{b}\boldsymbol{M}_\eta\boldsymbol{c}^{-1}\mathrm{diag}\left(\dot{\boldsymbol{e}}^{2-a/b}\right)\right]\end{aligned}$$

$$+\boldsymbol{M}_\eta^{-1}\left[-\frac{\left[\boldsymbol{s}^T\boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\boldsymbol{M}_\eta^{-1}\right]^T}{\left\|\boldsymbol{s}^T\boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\boldsymbol{M}_\eta^{-1}\right\|^2}\cdot\right.$$
$$\left.\kappa\left\|\boldsymbol{s}\right\|\left\|\boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\boldsymbol{M}_\eta^{-1}\right\|\right] \tag{8}$$

Substituting (8) into (7) yields

$$\dot{V}=-\frac{a}{b}\kappa\left\|\boldsymbol{s}\right\|\left\|\boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{p/q-1}\right)\right.$$
$$\left.\boldsymbol{M}_\eta^{-1}\right\|\leq-\left|\frac{a}{b}\right|\left|\kappa\right|\left\|\boldsymbol{s}\right\|\left\|\boldsymbol{c}\ \mathrm{diag}\left(\dot{\boldsymbol{e}}^{a/b-1}\right)\right.$$
$$\left.\boldsymbol{M}_\eta^{-1}\right\|<0,\quad (\boldsymbol{s}(t)\neq 0) \tag{9}$$

According to (9), it is known that if $\boldsymbol{s}=0$ then $\dot{V}=0$.

The following equation is derived from (4):

$$\lim_{t\to t_f}\boldsymbol{s}=\lim_{t\to t_f}\left[\boldsymbol{e}+\boldsymbol{c}\dot{\boldsymbol{e}}^{a/b}\right]=\lim_{t\to t_f}\left[(\boldsymbol{\eta}-\boldsymbol{\eta}_d)\right.$$
$$\left.+\boldsymbol{c}\left(\dot{\boldsymbol{\eta}}-\dot{\boldsymbol{\eta}}_d\right)^{a/b}\right]=0 \tag{10}$$

where $t_f=t_r+t_s$, t_r is the time when s reaches zero, and t_s is the finite time which is expressed as

$$t_s=-c_i^{a/b}\int_{e_i(t_r)}^{0}\frac{de}{e_i^{-b/a}(t)}=\frac{ac_i^{b/a}}{a-b}\left[e_i\left(t_r\right)\right]^{1-b/a} \tag{11}$$

Since all the diagonal elements of $\boldsymbol{c}$ are positive real numbers, then we obtained

$$\lim_{t\to t_f}\boldsymbol{\eta}=\boldsymbol{\eta}_d,\quad \lim_{t\to t_f}\dot{\boldsymbol{\eta}}=\dot{\boldsymbol{\eta}}_d \tag{12}$$

Remark 1 The NTSMC law designed as (5) guarantees Lyapunov stability of the closed-loop system. If $\boldsymbol{s}(t)=\boldsymbol{e}+\boldsymbol{c}\dot{\boldsymbol{e}}(t)^{a/b}=0$ are satisfied, then the tracking errors given by (3) will converge to zero in finite time.

3.2 Time-specified nonsingular terminal sliding mode control

A TS-NTSMC method with specified finite time was proposed in the current subsection. The detailed design of TS-NTSMC is given as follows [37–39].

Step 1: Define $\boldsymbol{x}_1=\boldsymbol{\eta}$ and $\boldsymbol{x}_2=\dot{\boldsymbol{\eta}}$, and (2) can be expressed as

$$\begin{cases}\boldsymbol{x}_1=\boldsymbol{\eta}\\ \boldsymbol{x}_2=\boldsymbol{M}_\eta^{-1}\boldsymbol{u}-\boldsymbol{M}_\eta^{-1}N_\eta\boldsymbol{\eta}-\boldsymbol{M}_\eta^{-1}\boldsymbol{G}_\eta\end{cases} \tag{13}$$

Algorithm: TS-NTSMC

Input:
1) desired trajectory $\boldsymbol{\eta}_d$
2) actual velocities $\boldsymbol{V}$ and actual trajectory $\eta$of the airship
3) model parameters of the airship

Output: The control law for trajectory tracking

Step 1: Definition of tracking error
a) Compute the tracking error $\boldsymbol{e}=\boldsymbol{\eta}-\boldsymbol{\eta}_d$;
b) Define the desired state $\boldsymbol{X}_d$ and system state $\boldsymbol{X}$;
c) Define the following error $\boldsymbol{E}(t)=\boldsymbol{X}-\boldsymbol{X}_d=\left[\boldsymbol{e}^T,\ \dot{\boldsymbol{e}}^T\right]^T$;

Step 2: Design of the nonsingular terminal sliding manifold
a) Select the nonlinear function $\boldsymbol{\zeta}_i(t)$;
b) Select the designed parameter $\boldsymbol{\Pi}$;
c) Design the nonsingular terminal sliding manifold $\boldsymbol{s}=\boldsymbol{\Pi}\boldsymbol{E}(t)-\boldsymbol{\Pi}\boldsymbol{\xi}(t)$;

Step 3: Design of the control inputs
a) Select the designed parameter γ;
b) Design the control input$\boldsymbol{u}$;
c) Select a Lyapunov function candidate V;
d) Prove finite time convergence and stability of the closed-loop system;

Step 4: Termination
If the tolerance of control error is satisfied, terminate the algorithm and output$\boldsymbol{u}$. Otherwise, go to **step 2**.

The error between the desired state$\boldsymbol{X}_d=\left[\boldsymbol{x}_{1d}^T,\ \boldsymbol{x}_{2d}^T\right]^T$ and the system state $\boldsymbol{X}=\left[\boldsymbol{x}_1^T,\ \boldsymbol{x}_2^T\right]^T$ is defined as follows

$$\boldsymbol{E}(t)=\boldsymbol{X}-\boldsymbol{X}_d=\left[\boldsymbol{e}^T,\ \dot{\boldsymbol{e}}^T\right]^T \tag{14}$$

where $\boldsymbol{e}=\boldsymbol{x}_1-\boldsymbol{x}_{1d}=\boldsymbol{\eta}-\boldsymbol{\eta}_d$ is the tracking error between the desired trajectory and the actual trajectory.

Step 2: Define the following time-specified nonsingular terminal sliding manifold

$$\boldsymbol{s}=\boldsymbol{\Pi}\boldsymbol{E}(t)-\boldsymbol{\Pi}\boldsymbol{\xi}(t) \tag{15}$$

where $\boldsymbol{\Pi}=[\boldsymbol{\lambda}_1,\ \boldsymbol{\lambda}_2]$, $\boldsymbol{\lambda}_i=\mathrm{diag}(\lambda_{i1},\ \lambda_{i2},\ \lambda_{i3},\lambda_{i4},\ \lambda_{i5},\ \lambda_{i6})$, $\lambda_{ij}>0$ $(i=1,2;\ j=1,2,3,4,5,6)$ are designed parameters; $\boldsymbol{\xi}(t)=\left[\boldsymbol{\zeta}(t)^T,\ \dot{\boldsymbol{\zeta}}(t)^T\right]^T$ in which $\boldsymbol{\zeta}(t)=[\zeta_1(t),\ \zeta_2(t),\ \zeta_3(t),\ \zeta_4(t),\ \zeta_5(t),\ \zeta_6(t)]^T$and $\zeta_i(t)\ (i=1,2,3,4,5,6)$ satisfy the following assumption.

Assumption 1 [41]: $\zeta_i(t):\mathbb{R}_+\to\mathbb{R}$, $\zeta_i(t)\in\mathbb{C}^2[0,\infty)$, $\dot{\zeta}_i(t)\in L^\infty$ and $\ddot{\zeta}_i(t)\in L^\infty$ are bounded within $[0,\ t_s]$ for a constant t_s $(t_s>0)$, $\zeta_i(0)=e_i(0)$, $\dot{\zeta}_i(0)=\dot{e}_i(0)$ and $\ddot{\zeta}_i(0)=\ddot{e}_i(0)$, where e_i $(i=1,2,3,4,5,6)$ are the elements of the tracking error.

Based on **Assumption 1**, $\zeta_i(t)$ is selected as follows [40]

$$\zeta_i(t) = \begin{cases} \sum_{n=0}^{2} \frac{1}{n!} e_j^{(n)}(0) t^n + \sum_{n=0}^{2} \left(\sum_{n=0}^{2} \frac{k_{jl}}{T^{k-l+3}} ej^{(l)}(0) \right) t^{n+3}, & 0 \le t \le t_s; \\ 0, & t > t_s. \end{cases} \tag{16}$$

where $n = 0,\ 1,\ 2, k_{jl}(j, l = 0,\ 1,\ 2)$ is a real number.

The parameters k_{jl} can be obtained based on the **Assumption 1**. Since $\zeta_i(t_s) = e_i(t_s)$, $\dot{\zeta}_i(t_s) = \dot{e}_i(t_s)$ and $\ddot{\zeta}_i(t_s) = \ddot{e}_i(t_s)$, then we obtained

$$\begin{cases} 1 + k_{00} + k_{10} + k_{20} = 0 \\ 1 + k_{01} + k_{11} + k_{21} = 0 \\ 0.5 + k_{02} + k_{12} + k_{22} = 0 \end{cases}, \quad \begin{cases} 3k_{00} + 4k_{10} + 5k_{20} = 0 \\ 1 + 3k_{01} + 4v_{11} + 5v_{21} = 0 \\ 1 + 3k_{02} + 4k_{12} + 5k_{22} = 0 \end{cases}, \quad \begin{cases} 6k_{00} + 12k_{10} + 20k_{20} = 0 \\ 6k_{01} + 12k_{11} + 20k_{21} = 0 \\ 1 + 6k_{02} + 12k_{12} + 20k_{22} = 0 \end{cases} \tag{17}$$

Solving (17) yields

$$\begin{cases} k_{00} = -10 \\ k_{10} = 15 \\ k_{20} = -6 \end{cases}, \quad \begin{cases} k_{01} = -6 \\ k_{11} = 8 \\ k_{21} = -3 \end{cases}, \quad \begin{cases} k_{02} = -1.5 \\ k_{12} = 1.5 \\ k_{22} = -0.5 \end{cases} \tag{18}$$

Substituting (18) into (16) yields

$$p_i(t) = \begin{cases} e_i + \dot{e}_i t + \frac{1}{2}\ddot{e}_i t^2 - \left(\frac{10}{t_s^3} e_i + \frac{6}{t_s^2}\dot{e}_i + \frac{3}{2t_s}\ddot{e}_i\right) t^3 + \left(\frac{15}{t_s^4} e_i + \frac{8}{t_s^3}\dot{e}_i + \frac{3}{2t_s^2}\ddot{e}_i\right) t^4 \\ - \left(\frac{6}{t_s^5} e_i + \frac{3}{t_s^4}\dot{e}_i + \frac{1}{2t_s^3}\ddot{e}_i\right) t^5 & , (0 \le t \le t_s) \\ 0 & , (t > t_s) \end{cases} \tag{19}$$

Step 3: Design the TS-NTSMC law as follows

$$\boldsymbol{u} = \boldsymbol{M}_\eta \ddot{\boldsymbol{\eta}}_d + \boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} + \boldsymbol{G}_\eta + \boldsymbol{M}_\eta \ddot{\boldsymbol{\xi}}(t) - \boldsymbol{M}_\eta \boldsymbol{\lambda}_2^{-1} \boldsymbol{\lambda}_1 \left(\dot{\boldsymbol{e}} - \dot{\boldsymbol{\xi}}(t)\right) - \gamma \frac{\boldsymbol{\lambda}_2^T \boldsymbol{s}}{\left\|\boldsymbol{\lambda}_2^T \boldsymbol{s}\right\|} \tag{20}$$

where $\gamma > 0$ is a designed parameter.

Theorem 2 *For the nonlinear systems described by (13), if the terminal sliding manifold is given by (15) and the control law is designed as (20), then the nonsingular terminal sliding manifold given by (15) will be reached in finite time and the tracking errors will converge to zero in finite time.*

Proof A candidate Lyapunov function is chosen as follows

$$V = \frac{1}{2}\boldsymbol{s}^T \boldsymbol{s} \tag{21}$$

Differentiating (21) and using (13), (15) and (20) yields

$$\begin{aligned} \dot{V} &= \boldsymbol{s}^T \dot{\boldsymbol{s}} \\ &= \boldsymbol{s}^T \left(\boldsymbol{\Pi}\dot{\boldsymbol{E}}(t) - \boldsymbol{\Pi}\dot{\boldsymbol{\xi}}(t)\right) = \boldsymbol{s}^T \left[\boldsymbol{\lambda}_2 \left(\ddot{\boldsymbol{e}} - \ddot{\boldsymbol{\xi}}(t)\right) + \boldsymbol{\lambda}_1 \left(\dot{\boldsymbol{e}} - \dot{\boldsymbol{\xi}}(t)\right)\right] \\ &= \boldsymbol{s}^T \left[\boldsymbol{\lambda}_2 \left(\boldsymbol{M}_\eta^{-1}\boldsymbol{u} - \boldsymbol{M}_\eta^{-1}\boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} - \boldsymbol{M}_\eta^{-1}\boldsymbol{G}_\eta - \ddot{\boldsymbol{\eta}}_d - \ddot{\boldsymbol{\xi}}(t)\right) + \boldsymbol{\lambda}_1 \left(\dot{\boldsymbol{e}} - \dot{\boldsymbol{\xi}}(t)\right)\right] \end{aligned} \tag{22}$$

Substituting (20) into (22) yields

$$\begin{aligned} \dot{V} &= \boldsymbol{s}^T \left[\boldsymbol{\lambda}_2 \left(\boldsymbol{M}_\eta^{-1}\boldsymbol{u} - \boldsymbol{M}_\eta^{-1}\boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} - \boldsymbol{M}_\eta^{-1}\boldsymbol{G}_\eta - \ddot{\boldsymbol{\eta}}_d - \ddot{\boldsymbol{\xi}}(t)\, s\right) + \boldsymbol{\lambda}_1 \left(\dot{\boldsymbol{e}} - \dot{\boldsymbol{\xi}}(t)\right)\right]\phi \\ &= \boldsymbol{s}^T \left\{\boldsymbol{\lambda}_2 \left[\boldsymbol{M}_\eta^{-1} \left(\boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} + \boldsymbol{G}_\eta + \boldsymbol{M}_\eta \ddot{\boldsymbol{\eta}}_d + \boldsymbol{M}_\eta \ddot{\boldsymbol{\xi}}(t) - \boldsymbol{M}_\eta \boldsymbol{\lambda}_2^{-1}\boldsymbol{\lambda}_1 \left(\dot{\boldsymbol{e}} - \dot{\boldsymbol{\xi}}(t)\right) - \gamma \frac{\boldsymbol{\lambda}_2^T \boldsymbol{s}}{\left\|\boldsymbol{\lambda}_2^T \boldsymbol{s}\right\|}\right) - \boldsymbol{M}_\eta^{-1}\boldsymbol{N}_\eta \dot{\boldsymbol{\eta}} - \boldsymbol{M}_\eta^{-1}\boldsymbol{G}_\eta - \ddot{\boldsymbol{\eta}}_d - \ddot{\boldsymbol{\xi}}(t)\right] + \boldsymbol{\lambda}_1 \left(\dot{\boldsymbol{e}} - \dot{\boldsymbol{\xi}}(t)\right)\right\} \\ &= -\gamma \frac{\boldsymbol{s}^T \boldsymbol{\lambda}_2 \boldsymbol{\lambda}_2^T \boldsymbol{s}}{\left\|\boldsymbol{\lambda}_2^T \boldsymbol{s}\right\|} \\ &= -\gamma \left\|\boldsymbol{\lambda}_2^T \boldsymbol{s}\right\| < 0, \ (\|\boldsymbol{s}\| \ne 0) \end{aligned} \tag{23}$$

Remark 2 The Lyapunov stability of the closed-loop system has been proved via (21)–(23). The nonlinear function $\zeta_i(t)$ given by (16) is employed to construct the nonsingular terminal sliding manifold, and the specified finite time convergence of error dynamics can be guaranteed.

Table 1 Model parameters of the robotic airship

Model parameters	Value
m	9.5 kg
m_u	1.2 kg
m_v	7.5 kg
m_w	7.5 kg
I_x	2.2 kgm^2
I_y	19 kgm^2
I_z	19.2 kgm^2
I_p	0 kgm^2
I_q	9.1 kgm^2
I_r	9.1 kgm^2
x_c	0 m
y_c	0 m
z_c	-0.05 m

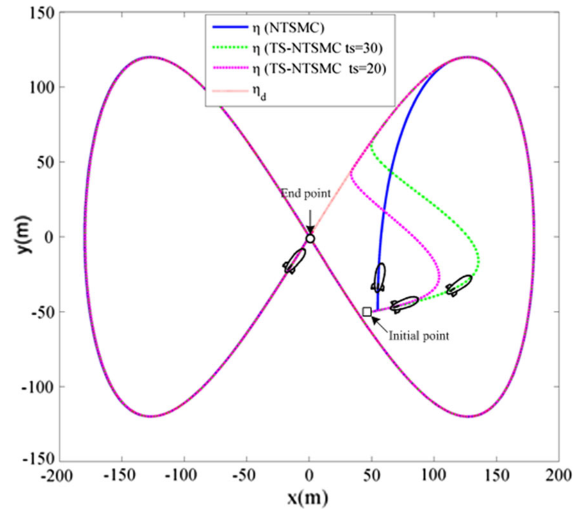

Fig. 3 Desired trajectory and actual trajectory

4 Experimental simulations

In the current section, experimental simulations have been conducted to verify the performance of the designed control law. The main parameters of the airship are listed in Table 1, and it is assumed that all the parameters are uncertain with a random error of the order of 15% and the disturbances in lateral direction are $\tau_d = 0.2\cos(\pi/100t)\,N$.

Simulations were performed by using NTSMC and TS-NTSMC (with different specified time, i.e., t_s=20 s and t_s=30 s), respectively. The initial position and velocity of the robotic airship are given by

$$\boldsymbol{\eta}_0 = [50\,\text{m}, -50\,\text{m}, 10\,\text{m}, 0.01\,\text{rad}, 0.01\,\text{rad}, 0.01\,\text{rad}]^T,$$
$$\boldsymbol{V}_0 = [15\,\text{m}/s, 2.5\,\text{m/s}, 0\text{m/s}, 0.001\,\text{rad/s}, 0.001\,\text{rad/s}, 0\,\text{rad/s}]^T,$$

and the robotic airship is required to track the following desired trajectory

$$\boldsymbol{\eta}_d = [180\sin(0.01t)\ \text{m},\ 120\sin(0.02t)\ \text{m}, 10\,\text{m}, 0\,\text{rad}, 0.02\,\text{rad}, 0\,\text{rad}]^T.$$

The designed parameters of NTSMC are listed as follows:

$$\boldsymbol{c} = \text{diag}(2, 2, 2, 2, 2, 2), \quad a = 5, \quad b = 3.$$

The designed parameters of TS-NTSMC are listed as follows:

$$\gamma = 10, \quad \boldsymbol{\lambda}_1 = diag(2, 2, 2, 5, 5, 5), \quad \boldsymbol{\lambda}_2 = diag(0.001, 0.002, 0.002, 0.01, 0.01, 0.01).$$

Simulation results of planar tracking are shown in Figs. 3, 4, and 5. The desired trajectory is plotted in red dotted line, the actual trajectory under NTSMC is plotted in blue solid line, and the actual trajectory under TS-NTSMC with specified time t_s=20 s and t_s=30 s is plotted in green dot dash line and magenta double scribing line, respectively, as shown in Fig. 3. The airship from an initial position (50m, $-$50m, 10 m) approaches to and tracks the desired trajectory, which illustrates the effectiveness of the designed control laws. The tracking errors are shown in Figs. 4 and 5. As depicted in Fig. 4, the tracking errors are all decreased to 0m in finite time. Figure 5 illustrates the detailed curve of tracking errors. The tracking errors under NTSMC converge to 0m with finite time t_s = 338.12 s, and tracking errors under TS-NTSMC are decreased to 0m with specified time t_s =20s and t_s =30 s, respectively. Figure 5 demonstrates that the convergence time of tracking errors under TS-NTSMC can be specified by choosing appropriate parameters of the function given by (25). The transition curves of nonsingular terminal sliding manifolds are given in Figs. 6 and 7, and all the manifolds approach to zero in finite time. The nonsingular terminal sliding manifolds of TS-NTSMC have faster convergence and better dynamic performance. The simulation results demonstrate the effectiveness of the designed NTSMC law and TS-NTSMC law for trajectory tracking of robotic airships and verify that

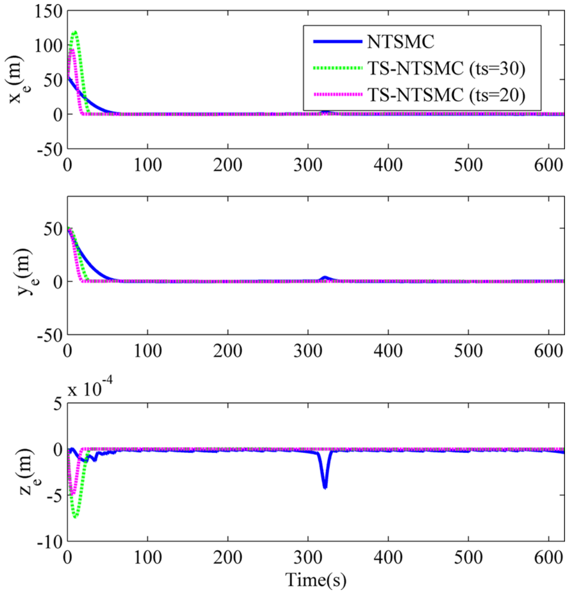

Fig. 4 Tracking errors

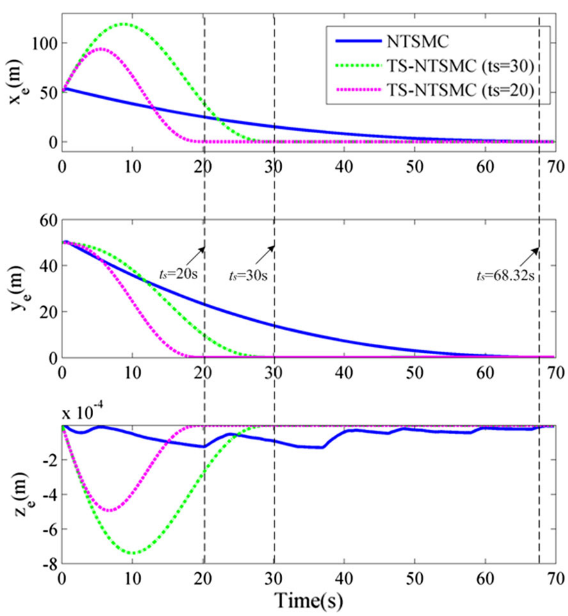

Fig. 5 Tracking errors within 70 s

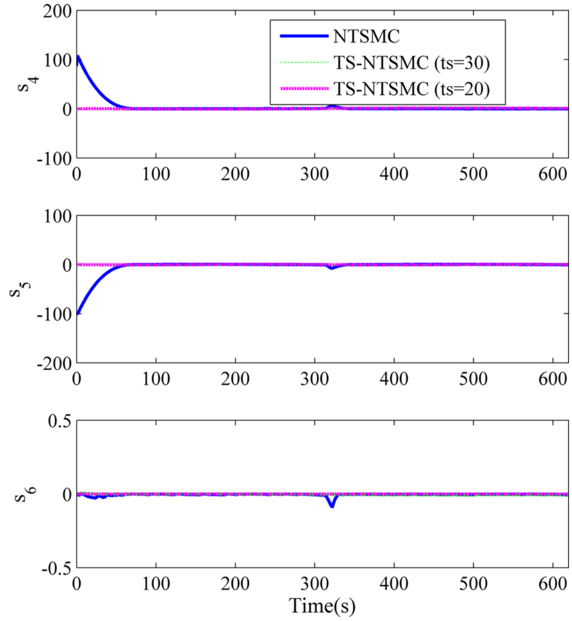

Fig. 6 Sliding manifold ($s_1 - s_3$)

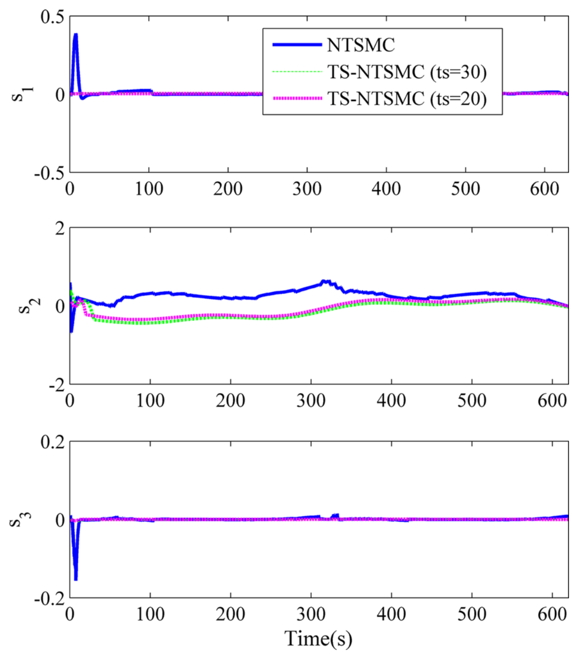

Fig. 7 Sliding manifold ($s_4 - s_6$)

the TS-NTSMC has the superiority of specifying the convergence time to obtain faster convergence.

Remark 3 The proposed TS-NTSMC has the following advantages compared to the NTSMC: (1) the TS-NTSMC addresses the infinite time convergence problem of SMC and the singularity problem of TSMC by using a nonsingular terminal sliding manifold consisting of pre-specified nonlinear functions, (2) the TS-NTSMC can specify the convergence time to obtain faster convergence, and (3) the TS-NTSMC has better dynamic performance.

5 Conclusions

In this paper, a novel TS-NTSMC approach is proposed for trajectory tracking of robotic airships, in which a nonsingular terminal sliding manifold is employed to design the NTSMC law to guarantee specified finite time stability. The effectiveness of the proposed NTSMC and TS-NTSMC was demonstrated via simulation tests, and the superiority of TS-NTSMC was illustrated via the compared simulation results. Future work will involve hardware-in-the-loop simulations and experimental tests of the designed controller.

The proposed TS-NTSMC provides an effective and promising approach for control system design of robotic airships, which is the key important system to support the various applications, such as safety monitoring, reconnaissance, earth observation, broadcasting relays, telecommunication, scientific exploration.

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Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest.

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