

Abundant lump and lump–kink solutions for the new (3+1)-dimensional generalized Kadomtsev–Petviashvili equation

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Abstract By utilizing the Hirota’s bilinear form and symbolic computation, abundant lump solutions and lump–kink solutions of the new (3+1)-dimensional generalized Kadomtsev–Petviashvili equation are derived in this work. Meanwhile, the interaction between lump solutions and the exponential function is also investigated. The dynamic properties of these obtained lump and interaction solutions are analyzed and described in figures by selecting appropriate parameters.

Keywords Lump solutions · Hirota’s bilinear form · Lump–kink solutions · New generalized Kadomtsev–Petviashvili equation

1 Introduction

Nonlinear evolution equations (NLEEs) and their solutions play an important role in almost all branches of physics [1–4]. To further understand these nonlinear phenomena, a variety of powerful methods are proposed to look for exact solutions of NLEEs [5–12],

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such as Hirota direct method [13–17], *exp* function method [18], three-wave approach [19–21], etc.

Recently, the research about lump solutions has become the hot spot since lump solutions were firstly proposed [22,23], which appeared in a lot of interesting physically relevant situations [24–38]. In this paper, by utilizing the Hirota’s bilinear form, we will discuss the following new (3+1)-dimensional generalized Kadomtsev–Petviashvili (ngKP) equation [39,40]:

$$u_{ty} + u_{tx} + u_{tz} - u_{zz} + 3(u_x u_y)_x + u_{xxx} = 0. \quad (1)$$

Wazwaz [39] obtained the multiple soliton solutions of the ngKP equation. Liu [41] presented new exact periodic solitary-wave solutions for the ngKP equation. As far as we know, lump solutions of the ngKP equation have not been discussed.

The organization of this paper is as follows: Sect. 2 obtains the lump solutions of the ngKP equation based on the Hirota’s bilinear form of Eq. (1). Section 3 presents the lump–kink solutions of Eq. (1) and shows the dynamic properties of these obtained solutions described in figures by selecting appropriate parameters. Finally, the conclusions are given.

2 Lump solutions for the ngKP equation

Utilizing the variable substitution $u = 2(\ln \xi)_x$, we get the following Hirota’s bilinear form

$$(D_t D_x + D_t D_y + D_t D_z + D_x^3 D_y - D_z^2) \xi \cdot \xi = 0. \quad (2)$$

This is equivalent to

$$(\xi_{xxy} + \xi_{tx} + \xi_{ty} + \xi_{tz} - \xi_{zz}) \xi - 3\xi_{xxy}\xi_x + 3\xi_{xy}\xi_{xx} - \xi_y \xi_{xx} - \xi_t \xi_x - \xi_t \xi_y - \xi_t \xi_z + \xi_z^2 = 0. \tag{3}$$

To look for lump solutions of Eq. (3), we make the following hypothesis

$$\begin{aligned} g &= x\alpha_1 + y\alpha_2 + z\alpha_3 + t\alpha_4 + \alpha_5, \\ h &= x\alpha_6 + y\alpha_7 + z\alpha_8 + t\alpha_9 + \alpha_{10}, \\ \xi &= g^2 + h^2 + \alpha_{11}, \end{aligned} \tag{4}$$

where $\alpha_i (1 \leq i \leq 11)$ are real constants to be determined later. Substituting Eq. (4) into Eq. (3), we can derive the following form of solutions for the parameters $\alpha_i (1 \leq i \leq 11)$ by the Mathematical software

Case (1)

$$\begin{aligned} \alpha_4 &= \frac{\alpha_6(\alpha_6\alpha_3^2 - 2\alpha_1\alpha_8\alpha_3 - \alpha_6\alpha_8^2)}{(\alpha_1 + \alpha_2 + \alpha_3)(\alpha_1^2 + \alpha_6^2)}, \quad \alpha_9 = \frac{-\alpha_1\alpha_4}{\alpha_6}, \\ \alpha_7 &= [\alpha_6\alpha_8^3 + [\alpha_6^2 - \alpha_1(\alpha_1 + \alpha_2 - \alpha_3)]\alpha_8^2 + \alpha_3(4\alpha_1 + 2\alpha_2 + \alpha_3)\alpha_6\alpha_8 + \alpha_3^2[\alpha_1(\alpha_1 + \alpha_2 + \alpha_3) - \alpha_6^2]] / [\alpha_6\alpha_3^2 - 2\alpha_1\alpha_8\alpha_3 - \alpha_6\alpha_8^2], \\ \alpha_{11} &= [3(\alpha_1^2 + \alpha_6^2)^2[-\alpha_6^2\alpha_8^3 + \alpha_6(\alpha_1(\alpha_1 + 2\alpha_2 - \alpha_3) - \alpha_6^2)\alpha_8^2 - \alpha_3[(4\alpha_1 + 2\alpha_2 + \alpha_3)\alpha_8^2 - 2\alpha_1^2\alpha_2]\alpha_8 + \alpha_3^2\alpha_6[\alpha_6^2 - \alpha_1(\alpha_1 + 2\alpha_2 + \alpha_3)]]] / [(\alpha_1\alpha_3 + \alpha_6\alpha_8)^2(-\alpha_6\alpha_3^2 + 2\alpha_1\alpha_8\alpha_3 + \alpha_6\alpha_8^2)], \end{aligned} \tag{5}$$

where $(\alpha_1\alpha_3 + \alpha_6\alpha_8)^2(-\alpha_6\alpha_3^2 + 2\alpha_1\alpha_8\alpha_3 + \alpha_6\alpha_8^2) \neq 0, \alpha_6 \neq 0, (\alpha_1 + \alpha_2 + \alpha_3)(\alpha_1^2 + \alpha_6^2) \neq 0$.

Case (2)

$$\begin{aligned} \alpha_2 &= \alpha_1 \left(-\frac{\alpha_1\alpha_3^2}{\alpha_4\alpha_6^2} - 1 \right) - \alpha_3, \quad \alpha_8 = -\frac{\alpha_1\alpha_3}{\alpha_6}, \\ \alpha_7 &= \frac{\alpha_1\alpha_3(\alpha_4 - \alpha_3)}{\alpha_4\alpha_6} - \alpha_6, \\ \alpha_9 &= \frac{\alpha_4\alpha_6}{\alpha_1}, \quad \alpha_{11} = -\frac{3(\alpha_1^2 + \alpha_6^2)(\alpha_1\alpha_3^2 + \alpha_4\alpha_6^2)}{\alpha_3^2\alpha_4}, \end{aligned} \tag{6}$$

where $\alpha_1\alpha_6\alpha_3\alpha_4 \neq 0$.

Case (3)

$$\begin{aligned} \alpha_8 &= \frac{\alpha_1\alpha_3\alpha_6 + \epsilon_1\sqrt{-\alpha_1^2((\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 - \alpha_3^2)(\alpha_1^2 + \alpha_6^2)}}{\alpha_1^2}, \\ \alpha_9 &= \frac{\alpha_4\alpha_6}{\alpha_1}, \\ \alpha_7 &= \frac{\epsilon_1(-\alpha_4 + 2\alpha_3)\sqrt{-\alpha_1^2((\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 - \alpha_3^2)(\alpha_1^2 + \alpha_6^2)}}{\alpha_1^2\alpha_4} + \frac{-2\alpha_4\alpha_6\alpha_1^2 + (2\alpha_3^2 - (\alpha_2 + 2\alpha_3)\alpha_4)\alpha_6\alpha_1}{\alpha_1^2\alpha_4}, \\ \alpha_{11} &= -[3(\alpha_1^2 + \alpha_6^2)[\alpha_2\alpha_4\alpha_1^3 - 2\alpha_4\alpha_6^2\alpha_1^2 + (2\alpha_3^2 - (\alpha_2 + 2\alpha_3)\alpha_4)\alpha_6^2\alpha_1 + \epsilon_1(-\alpha_4 + 2\alpha_3)\alpha_6\sqrt{-\alpha_1^2((\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 - \alpha_3^2)(\alpha_1^2 + \alpha_6^2)}]] / [\alpha_1^2\alpha_4((\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 - \alpha_3^2)], \end{aligned} \tag{7}$$

where $\epsilon_1 = \pm 1, \alpha_1\alpha_4 \neq 0, (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 - \alpha_3^2 < 0$.

Case (4)

$$\begin{aligned} \alpha_9 &= \frac{\alpha_3\alpha_8 + \epsilon_2\sqrt{(\alpha_3^2 - (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)(\alpha_8^2 + (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)}}{\alpha_1 + \alpha_2 + \alpha_3}, \\ \alpha_7 &= \frac{\alpha_3\alpha_8 - \epsilon_2\sqrt{(\alpha_3^2 - (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)(\alpha_8^2 + (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)}}{\alpha_4} - (\alpha_6 + \alpha_8), \\ \alpha_{11} &= -[3(\alpha_1^2 + \alpha_6^2)[\alpha_1\alpha_2\alpha_4 + \alpha_6[\alpha_3\alpha_8 - \alpha_4(\alpha_6 + \alpha_8) - \epsilon_2\sqrt{(\alpha_3^2 - (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)(\alpha_8^2 + (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)}]] / [\alpha_4((\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 - \alpha_3^2)], \end{aligned} \tag{8}$$

where $(\alpha_3^2 - (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4)(\alpha_8^2 + (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4) > 0, \epsilon_2 = \pm 1, (\alpha_1 + \alpha_2 + \alpha_3)\alpha_4 \neq 0$.

Substituting Eqs. (5)–(8) into Eq. (4) and the transformation $u = 2(\ln \xi)_x$, we can obtain four classes lump solutions of the ngKP equation.

These obtained lump solutions given from Eqs. (5) to (8) have the equal form but differ from each other only by the coefficients. Therefore, the plots of these solutions also have properties in common with each other. As an example, we will take Case (1) to illustrate their dynamic properties through three-dimensional plots and contour plots (see Figs. 1, 2).

3 Lump–kink solutions for the ngKP equation

In this section, we will investigate the interaction solution between the lump solution and exponential function, called a lump–kink solution. Adding the exponential function to Eq. (4)

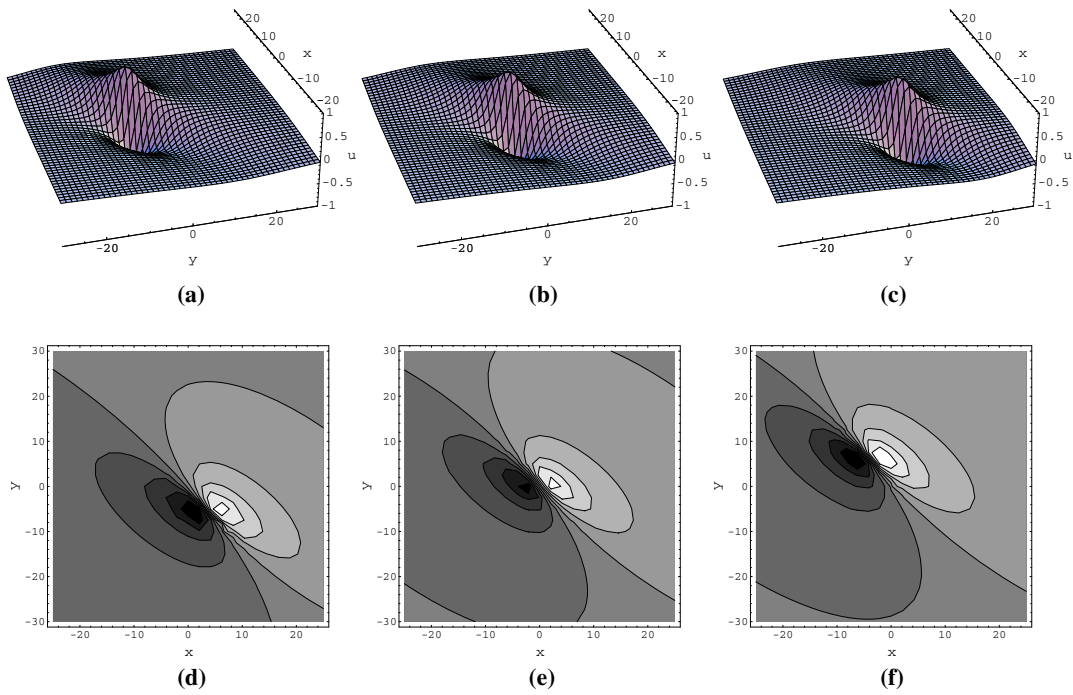


Fig. 1 Plots of the lump solution of Eq. (1) for $\alpha_1 = 3, \alpha_2 = \alpha_3 = 2, \alpha_5 = \alpha_6 = 0, \alpha_7 = 1, \alpha_8 = -2, \alpha_{10} = -1, z = 0$, when $t = -10$ in **a, d**, $t = 0$ in **b, e** and $t = 10$ in **c, f**

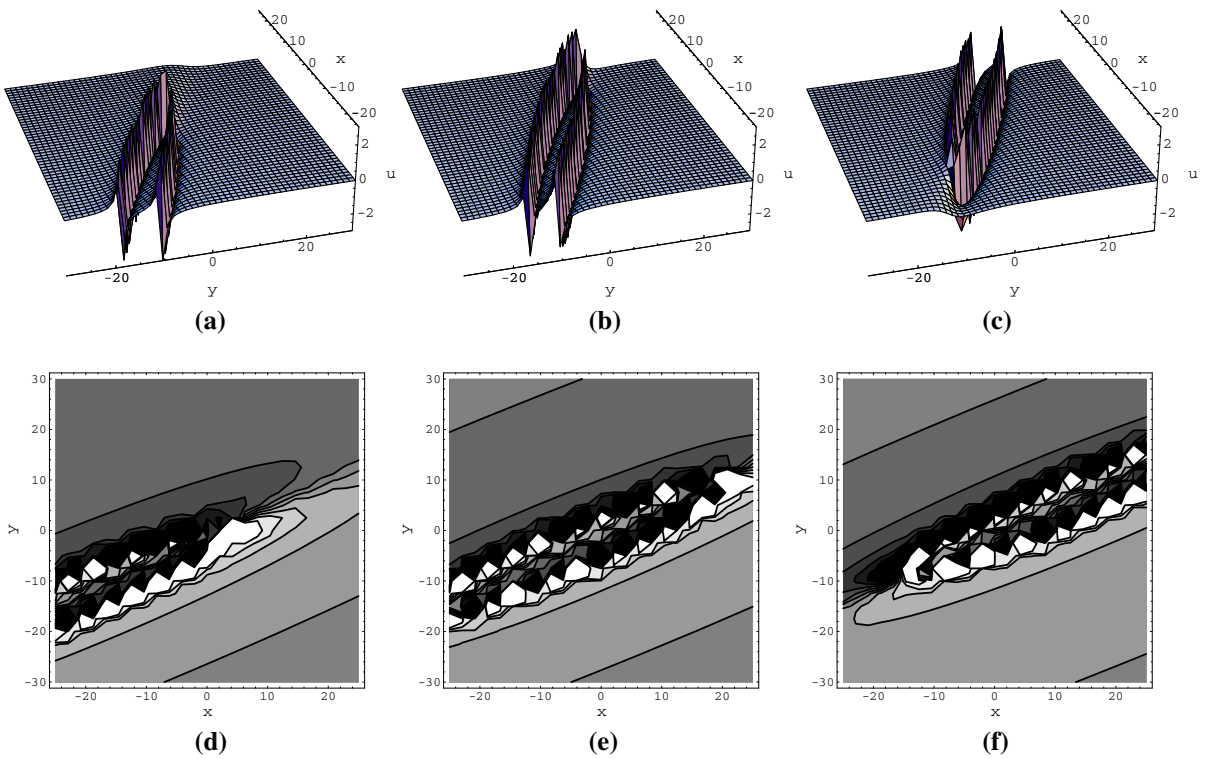


Fig. 2 Plots of the lump solution of Eq. (1) for $\alpha_2 = 2, \alpha_1 = \alpha_3 = 1, \alpha_5 = 14, \alpha_6 = -7, \alpha_7 = -1, \alpha_8 = -2, \alpha_{10} = 12, z = 0$, when $t = -40$ in **a, d**, $t = 0$ in **b, e** and $t = 40$ in **c, f**

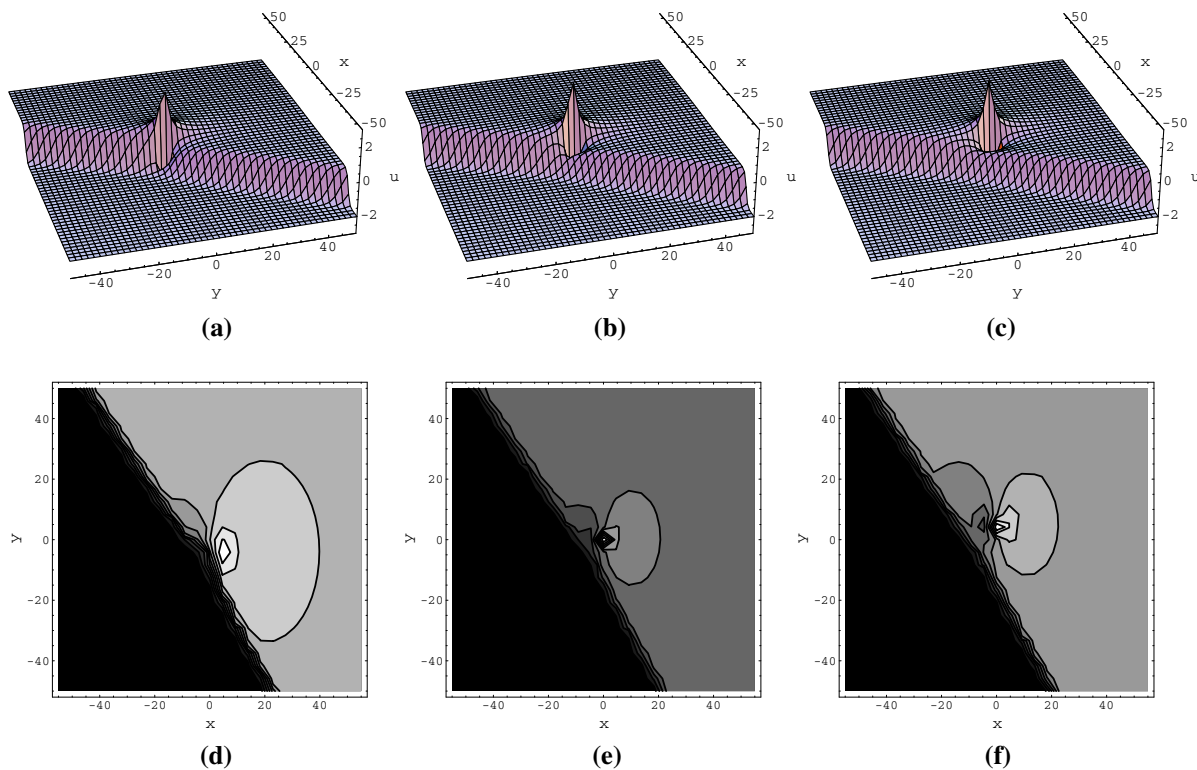


Fig. 3 Plots of the lump-link solution of Eq. (1) for $\alpha_2 = 2, \alpha_3 = 25, \alpha_5 = -5, k_1 = -1, k = 5, \epsilon_3 = 1, \epsilon_4 = -1, \alpha_{10} = 10, z = 0$, when $t = -2$ in **a, d**, $t = 0$ in **b, e** and $t = 2$ in **c, f**

$$\begin{aligned}
 g &= x\alpha_1 + y\alpha_2 + z\alpha_3 + t\alpha_4 + \alpha_5, \\
 h &= x\alpha_6 + y\alpha_7 + z\alpha_8 + t\alpha_9 + \alpha_{10}, \\
 l &= e^{xk_1 + yk_2 + zk_3 + tk_4}, \\
 \xi &= g^2 + h^2 + l + \alpha_{11},
 \end{aligned}
 \tag{9}$$

where $\alpha_i (1 \leq i \leq 11)$, k and $k_i (i = 1, 2, 3, 4)$ are real undetermined constant. Substituting Eq. (9) into Eq. (3), we have

$$\begin{aligned}
 \alpha_1 &= -\alpha_3, \alpha_4 = -\frac{\alpha_6^2}{\alpha_2}, \alpha_8 = -\alpha_6, \alpha_7 = \frac{\alpha_2\alpha_3}{\alpha_6}, \\
 \alpha_9 &= -\frac{\alpha_3\alpha_6}{\alpha_2}, \\
 k_2 &= \frac{-3\alpha_2^2k_1^5 + 2(3k_1^2 - 2)\alpha_6^2k_1 + \epsilon_3\sqrt{9(k_1^2 - 4)\alpha_2^4k_1^8 + 4(3k_1^2 + 4)\alpha_2^2\alpha_6^2k_1^4}}{(2 - 3k_1^2)^2\alpha_6^2 - 9k_1^4\alpha_2^2}, \\
 \alpha_6 &= \epsilon_4\frac{3}{2}k_1^2\alpha_2, k_4 = \frac{k_3^2 - k_1^3k_2}{k_1 + k_2 + k_3}, \alpha_{11} = 0, k_3 = -\frac{3}{2}k_1^2k_2,
 \end{aligned}
 \tag{10}$$

with the following restrictive conditions

- (1) : $\alpha_2 \neq 0, k_1 < 0, k_1^2 \neq \frac{4}{3}, k_1 + k_2 + k_3 \neq 0,$
 $\epsilon_3 = 1, \epsilon_4 = \pm 1,$
- (2) : $\alpha_2 \neq 0, k_1 > 0, k_1^2 \neq \frac{4}{3}, k_1 + k_2 + k_3 \neq 0,$
 $\epsilon_3 = -1, \epsilon_4 = \pm 1.$

Substituting Eqs. (9) and (10) into the transformation $u = 2(\ln \xi)_x$, the lump-kink solutions of Eq. (1) can be obtained as follows

$$u = \frac{2(2\alpha_1 g + 2\alpha_6 h + lk_1)}{g^2 + h^2 + l}, \quad (11)$$

where g, h, l satisfy Eq. (10) with some restrictive conditions.

Three-dimensional plots and corresponding contour plots of the lump–kink solutions (11) are shown in Fig. 3 when $t = -2, 0, 2$. Figure 3 describes the interaction between the obtained lump solutions and the exponential function.

4 Conclusion

In this work, by utilizing the Hirota's bilinear formulation, abundant lump solutions of the ngKP equation are given with some restrictive conditions. By adding an exponential function to Eq. (4), we obtain the lump–kink solutions of the ngKP equation. The dynamic properties of these obtained lump solutions are shown in Figs. 1 and 2 by selecting appropriate parameters. The interaction between the obtained lump solutions and the exponential function is described by some three-dimensional plots and corresponding contour plots in Fig. 3.

Compliance with ethical standards

Conflicts of interest The authors declare that there is no conflict of interests regarding the publication of this article.

Ethical standard The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

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