

The incentive effect of venture capital in bilateral partnership systems with the bias mono-stable Cobb–Douglas utility

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Abstract Partnerships, between multiple sides that share goals and strive for mutual benefit, are ubiquitous both between and within the enterprises, and competition and cooperation are the fundamental characteristics in partnership systems. As the inherent effect of capital-product switching applied together with stochastic fluctuations of internal and external environments, the effects compete and cooperate to make the system achieve global optimum in the statistical sense. Thus motivated, we establish an over-damped nonlinear Langevin equation to describe the dynamical behaviors subject to the bias mono-stable Cobb–Douglas utility under the wealth-constraint condition. Based on linear response theory, we derive the performance indexes, including output signal-to-noise ratio, stationary unit risk return, systematic risk and bilateral risk, and stochastic resonance (SR) and reverse SR phenomena are observed by the simulations. Finally, we

introduce one true example to explain the actual phenomenon observed from the practice. The purpose in this paper is to develop a quantitative method and associated prototype system beg the questions of how the external venture capital incents the partners especially associated with partnership success and what roles the internal and external risks play, respectively.

Keyword Stochastic resonance (SR) · Reverse stochastic resonance (RSR) · Bias mono-stable nonlinear system · Cobb–Douglas utility (CDU) potential · Linear response theory (LRT)

1 Introduction

Partnership system can be defined as the purposive strategic relationships between two or more sides that share compatible goals, strive for mutual benefit, and acknowledge a high level of mutual interdependence [1,2]. They join efforts, including their time, technologies, resources, etc., to achieve goals that each side, acting alone, could not attain easily [3,4]. From the micro-perspective of intra-enterprise, the system can be decomposed into two essential aspects, competition and cooperation, and it is randomly evolved within the utility equilibria of each side. Actually, the process contains many kinds of intrinsic periodic structures, which lead to the different research and development (R&D) and market transformation cycles. Simultaneously, motivated by external venture capital, the rhythm

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synchronism between the internal and external environments can affect or magnify the efficiency of capital-product switching.

Extensive academic literature shows that researchers are gradually becoming aware of this important managerial concern, nevertheless little guidance has emerged on how to better ensure partnership success quantitatively from stochastic dynamics, and how to measure the risk of partnership systems. Another neglected aspect is the role of venture capital in financing the partnership systems, especially for small and medium-sized enterprises (SMEs) seeking to grow rapidly [5]. Even in most cases, it guarantees the continuation to the next developing stage, and is sufficient to keep the entrepreneurs moral hazard problem under control [6,7]. Therefore, the presence of venture capital accompanied by the processes of capital-product switching, profitably leads to the enterprise randomly hopping between local optimums, and the system probably achieves the global optimum and shows the stochastic resonance (SR) behavior, strictly speaking, this holds true only in the statistical sense [8]. It is generally accepted that nonlinearity, periodic force and random fluctuation are the essential ingredients for the phenomenon, which was first studied in bistable system [9]. Then, it was also been observed when, at variance to the archetypal setting, either the noise fluctuation is multiplicative, or even both periodic force and random fluctuation are additive [10–12].

Based on the above description, we are inspired to establish a mono-stable nonlinear Langevin equation to describe the dynamical behaviors of partnership systems, which are subject to cooperative relationship and competitive condition [13,14]. In reality, a variety of complex behaviors have been explained by the equation and are of relevance in diverse actual areas, including physical [15–17], biological [18,19], electronic [20,21], and industrial systems [22,23]. Luo et al. [24] considered the SR phenomena in a polynomial asymmetric mono-stable system based on the adiabatic approximation theory. Agudov et al. [25] investigated the piecewise-type mono-stable system driven by additive mix of periodic force and white Gaussian noise and showed the similar behaviors as the non-monotonic dependence of signal-to-noise ratio (SNR) on input noise intensity in bistable systems. Serajian et al. [22,23] investigated the motion equations of railway vehicles in a tangent track, and analytically studied the bifurcation, nonlinear lateral stability and

hunting behaviors by bifurcation theory and Bogoliubov method. Yao et al. [26] proposed the bias mono-stable system driven by a periodic rectangular signal and uncorrelated noises by using SNR in the adiabatic limit. Khovanov et al. [19] described the dynamical mechanism of the excitable neuronal systems, and compared the responses of a mono-stable resonator and mono-stable integrator to different stochastic inputs. In recent years, Arathi et al. [27] considered an anharmonic mono-stable oscillator where period doubling and chaotic orbits coexist with a large amplitude periodic orbit for a wide external driving frequency. Leng et al. [17] analyzed the response of pulse series with different half-peak width in the mono-stable system. Duan et al. [28] presented the output characteristics of a linear duffing mono-stable system. Lin et al. developed the theoretical results of a linear frequency-modulated (LFM) mono-stable system driven by aperiodic force, and parameter-induced SR behaviors are observed from the view of SNR.

In this paper, we consider a new nonlinear dynamical model with mono-stable Cobb–Douglas utility (CDU) potential in bilateral partnership systems. The first goal of the present work is to demonstrate that such mono-stable CDU potential in over-damped nonlinear system can lead to the SR behaviors based on the definitions of output SNR, stationary unit risk return (URR) under the measures of two types of response risks, systematic risk and bilateral risk. As a next step, knowledge of mechanism of venture capital driving SR behaviors, bilateral competition and cooperation mechanism, associated with partnership success, could aid in the adjustment of partners, as well as in the on-going management of the external capital and risk controlling.

The remainder of this paper is structured as follows. In Sect. 2, we provide a brief overview of the mono-stable CDU potential. In Sect. 3, we establish a stochastic dynamical model for bilateral partnership systems. In Sect. 4, the performance measures are analyzed based on linear response theory (LRT). The computational experiments and their results are presented and discussed in Sect. 5. A brief conclusion follows in Sect. 6.

2 Bias mono-stable Cobb–Douglas utility

We start considering an enterprise, which is characterized by the bilateral partnership system, typically

including technology-side and marketing-side. It is supposed that the enterprise is established with registered capital C_R , and the real operating capital is from the internal paid-in capital C_P and external venture capital C_V , and enterprise scale m is made up of development elements (i.e., manpower, facilities, material resources, etc), and it is reflected in all the capital inputs C_S , which can be allocated and adjusted between both sides. It is noted that, in the following description, C_R is used to be the normalization factor, that is, $c_0 \triangleq \frac{C_S}{C_R}$ is considered as normalized capital-constraint relevant to the scale at the start-up stage, and the enterprise raises internal investment $A_0 \triangleq \frac{C_P}{C_R}$ and recruits external capital $A_1 \triangleq \frac{C_V}{C_R}$ to pursue the successful development of new product, which consists of a business opportunity during capital-to-product period, when technology-side plays the key role and does the most work of new product development. This profitability can be translated into the final return if the project is successfully developed during product-to-capital period, when marketing-side plays the leading role. That is, the achievement depends on the bilateral efforts from technology-side and marketing-side, which perform the cooperative relationship. Here the efforts, each side can achieve, constitute the core competence, technological ability and market expanding ability. Specifically, based on the respective efforts at the cost of bilateral capital inputs, the product reaches a cash-out stage at which it yields an expected return, which meets certain input-return relationship in the statistical sense [13]. That is, whether during capital-to-product or product-to-capital period, the achievement requires the effort a from technology-side (who has the original idea, patents or some exclusive knowledge of new product) and b from marketing-side (who has some previously accumulated business expertise, contacts and reputation that are relevant at this stage), and the final return R to input efforts a and b from both sides can be quantitatively described by the following Cobb–Douglas production function [8, 14]:

$$R(a, b) = K a^\alpha b^\beta, \tag{1}$$

where K is the total factor productivity. It is the portion of return not explained by the amount of inputs used in partnership systems, and its level is determined by how efficiently and intensely the inputs are utilized [29]. $\alpha, \beta \in (0, 1)$ are the return elasticities, which measure the cooperative return to the change of input efforts from both sides. Further, if $\alpha + \beta = 1$, R has

constant return to inputs, which refers to an enterprise property that examines change in return subsequent to proportional change in all inputs, e.g., an increase of 1% in both a and b causes R to increase by 1%. However, if $\alpha + \beta < 1$, return increases by less than that proportional change, that is, there is decreasing return to inputs, and if $\alpha + \beta > 1$, return to inputs is increasing. Actually, Eq. (1) is widely used to represent the relationship of an output to inputs, and here it captures the bilateral complementarity under perfect competition condition. In order to motivate and obtain the efforts a and b from both sides, essential development elements are involved to satisfy the requirements, including manpower, facilities, material resources, etc. Based on the analysis in literature [13], the costs are, respectively, given by

$$V_T(a) = \frac{1}{2u} a^2, \quad V_M(b) = \frac{1}{2v} b^2, \tag{2}$$

where $u, v > 0$ are the ability level factors, which perform inversely proportional with the costs. As to partnership system, in which (a, b) is a cooperative pair jointly leading to the success return R , the return shall be shared according to the bilateral ownership structure $(\theta, 1 - \theta)$, which is a result that follows from ex ante competition between both sides [30], and specifies the share θ of R is given to the technology-side, and the share $1 - \theta$ is given to the marketing-side. Each side cares about the maximization of expected profit net after deducting the cost of his effort, and thus the utility functions, technology-side's $U_T(a, b)$ and marketing-side's $U_M(a, b)$, can be, respectively, expressed as

$$\begin{cases} U_T(a, b) = \theta R(a, b) - V_T(a), \\ U_M(a, b) = (1 - \theta) R(a, b) - V_M(b), \end{cases} \tag{3}$$

where $\theta \in (0, 1)$ is the ownership ratio of technology-side. Subsequently, we involve the new variable $x \triangleq a - b$ to describe the bilateral cooperation state, and suppose the bilateral capital costs (2) satisfy the following boundary condition of competitive capital-constraint [31]:

$$V_T(a) + V_M(b) = c_0, \tag{4}$$

where c_0 is the normalized capital costs of development elements at the start-up stage. Based on the initial resource allocation between $V_T(a_0)$ and $V_M(b_0)$ under condition (4), it could stimulate both sides to be willing to make efforts (a_0, b_0) , which corresponds to $x_0 = a_0 - b_0$ as initial cooperative state. During capital-to-product period, including the start-up and

product improvement stages, in order to stimulate the technology-side to make more effort, the essential elements should be adjusted to firstly ensure the R&D requirements, which leads to increasing a and decreasing b , and accordingly the variable x increases until it resides into the maximum utility of technology-side $U_T(a, b)$. Although it is cooperatively decided by both sides, maybe it has deviated from the maximum utility of marketing-side $U_M(a, b)$, and thus there exists the high rupture risk of bilateral cooperation in partnership system. Conversely, during product-to-capital period, including the market expansion and cash-out stages, the enterprise has to cut down on technology investment to ensure marketing expenses, which causes x to decrease until it resides into the maximum utility of marketing-side. Obviously, the defined variable x reflects the bilateral cooperative level under competitive capital-constraint, and here it is employed to describe the enterprise state.

Furthermore, substituting $x = a - b$ into Eqs. (2) and (4), we obtain the function relations $a(x)$ and $b(x)$ dependent on the state variable x , that is,

$$\begin{cases} a(x) = \frac{\sqrt{2uv(u+v)c_0 - uvx^2} + ux}{u+v}, \\ b(x) = \frac{\sqrt{2uv(u+v)c_0 - uvx^2} - vx}{u+v}, \end{cases} \quad (5)$$

with $x \in \mathcal{C} = [-\sqrt{2vc_0}, \sqrt{2uc_0}]$. Further, substituting Eq. (5) into Eq. (3), we obtain the bilateral utilities, $U_T(x) = U_T(a(x), b(x))$ and $U_M(x) = U_M(a(x), b(x))$ with respect to x , and the curves are plotted in Fig. 1. It is shown that each side expects the enterprise can be stabilized at the state of maximum utility U_T^{opt} or U_M^{opt} (known as the local equilibria), respectively, at $x = x_T$ or x_M . Therefore, the dynamical evolution of x should be subject to the utility potential with the form $U_T(x)$ (blue solid line) or $U_M(x)$ (green solid line), which plays the role of inherent regulation on bilateral competition and cooperation relationship in partnership system. That is, any deviation from the maximum utility of one side could result in internal force from the side to drive the state back to its own local equilibrium. Besides, it is noted that, based on the assumption of inevitable informational asymmetry, it is unrealistic to consider the perfect cooperation with total utility $U_P(x) \triangleq U_T(x) + U_M(x)$ (red dashed line), unless the non-profit or public-private partnerships with more social responsibility are considered [32,33].

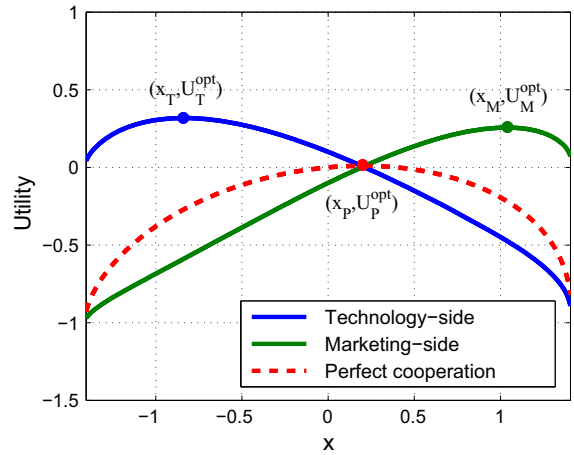


Fig. 1 The utility curves of each side in bilateral partnership system with $c_0 = 1.0$, $K = 1.0$, $\alpha = 0.6$, $u = 1.0$, $v = 1.0$ and $\theta = 0.6$

Without loss of generality, here we suppose the technical-side holds the ownership with $\theta > 0.5$. Under the conditions of constant returns $\alpha + \beta = 1$ and capital-constraint (4), the partnership system is subject to the following CDU potential:

$$U(x) = \begin{cases} \theta K a^\alpha(x) b^{1-\alpha}(x) - \frac{1}{2a} a^2(x), & x \in \mathcal{C}, \\ -\infty, & x \notin \mathcal{C}, \end{cases} \quad (6)$$

which has the asymmetric mono-stable structure with one bias well as shown in Fig. 2. In order to facilitate the description from Langevin dynamics of particle motion in mono-stable system, the utility curves are reversely plotted. As θ increases, in Fig. 2a, the well depth increases, and the position shifts to the right value of x . The bias decreases, and the impact of θ gradually comes to disappear as x tends to the boundary values. With the increase in α , in Fig. 2b, the depth has an opposite tendency as θ , and the position shows a little shift to the right. As x tends to the right boundary value, the utility increases with increasing α . Furthermore, in Fig. 2c, with increasing u , the depth and position have the same tendency as θ , while the utility is sensitive to u as x tends to the right boundary value. In Fig. 2d, as c_0 increases, the depth increases and the position shifts remarkably to the left, thus the bias increases. Besides, it is clearly observed from the subgraphs that the utility of marketing-side stay low, even seriously deviate from its own maximum value at the position of local optimum of the technology-side. For the enterprise trapping into the deeper well of mono-stable CDU potential, it becomes more difficult to escape from local optimum

of technology-side to the global optimum in the statistical sense, which makes the marketing-side unwilling to apply the effort he deserves, thus the rupture risk of bilateral cooperation increases if without appropriate driving forces or stochastic fluctuations.

3 System model

From the perspective of dynamics, we consider the partnership system as a Brownian particle and analyze fluctuation-dissipation behaviors to demonstrate four types of evolving effects: (i) macroscopic driving effect with the gradient form of mono-stable CDU utility potential field, which represents the inherent regulation and results in the driving force to equilibrium position, (ii) periodic driving force from internal investment and external capital and related with the inherent industry discipline of four-stage life cycle, i.e., start-up, product improvement, market expansion and cash-out return, (iii) stochastic fluctuations from the changes of internal and external environments, i.e., configuration instability of essential elements, uncertainty of venture capital, etc, and (iv) damping effect with the viscous force, which results from inter-enterprise contradictions, disorders, cooperations and competitions because of the change of state x , and can be supposed to be proportional to the change rate based on Stokes' law.

As a generic system model by putting all the evolving effects together, we can use the nonlinear stochastic differential equation to describe the dynamical behaviors. That is, the enterprise is regarded as a Brownian particle moving in potential $U(x)$, which is driven by periodic force $f(t)$, and perturbed by the internal and external additive noise fluctuations $\xi(t) + \varepsilon(t)$. Therefore, we start to describe the system model from the following over-damped Langevin equation:

$$\gamma \frac{d}{dt}x(t) = \frac{\partial}{\partial x}U(x) + f(t) + \xi(t) + \varepsilon(t), \tag{7}$$

where $x(t)$ is the state variable reflecting the cooperation and competition process at time t . $\gamma > 0$ is the damping constant of friction, and it can be renormalized and normally set to unity without loss of generality [11, 12]. Inherent driving effect $f(t)$ follows the process of capital-product switching, which can be generally divided into four stages: start-up, product improvement, market expansion and cash-out return, thus $f(t) = A \cos(\Omega t)$, with $A = A_0 + A_1$ and

$\Omega = 2\pi/T$, is employed to describe the periodic force, which involves the normalized internal investment $A_0 \triangleq \frac{C_p}{C_R}$, external capital $A_1 \triangleq \frac{C_V}{C_R}$, and switching cycle T . Here $U(x)$ is the bias mono-stable CDU potential described by Eq. (6), and the partial derivative $\frac{\partial}{\partial x}U(x)$ indicates the marginal cooperative productivity at which $U(x)$ changes with respect to state $x(t)$ during the capital-product process. We treat the additional noise $\xi(t)$ as the risk of internal environment, and it satisfies the fluctuation dissipation relationship [34]. The term $\varepsilon(t)$ is understood as external risk involved by venture capital environment no matter from venture capitalists or cash pooling, and it is modeled as the additional white Gaussian noise (WGN).

In this paper, we assume that the internal and external noise terms have different origins, thus $\xi(t)$ and $\varepsilon(t)$ are uncorrelated noises with zero mean, and they can be characterized by their variance:

$$\left\langle \begin{bmatrix} \xi(t) \\ \varepsilon(t) \end{bmatrix} \begin{bmatrix} \xi(s) & \varepsilon(s) \end{bmatrix} \right\rangle = \delta(t-s) \cdot \begin{bmatrix} 2\sigma_0 & 0 \\ 0 & 2\sigma_1 \end{bmatrix}, \tag{8}$$

where σ_0 and σ_1 are the risk levels of internal and external environments, respectively, and driving noise $\zeta(t) = \xi(t) + \varepsilon(t)$ has the statistical properties: $\langle \zeta(t) \rangle = 0$ and $\langle \zeta(t)\zeta(s) \rangle = 2\sigma\delta(t-s)$ with $\sigma = \sigma_0 + \sigma_1$. Further, if the introduced external capital C_V is from N independent sources, $C_j, j = 1, 2, \dots, N$, we have the portfolio: $A_1 = \sum_{j=1}^N c_j \triangleq \sum_{j=1}^N \frac{C_j}{C_R}$, and the risks of unit capital, normalized by C_R , correspond to WGNs $\varepsilon_{1j}(t)$, respectively, with the unit risk level $\sigma_{1j}, j = 1, 2, \dots, N$, thus we have $\sigma_1 = \sum_{j=1}^N c_j^2 \sigma_{1j}$. Obviously, for fixed A_1 , the external risk σ_1 is related to the number N of sources and their risk profiles.

In the presence of periodic capital-product effect, the system (7) leads to the following associated Fokker-Planck equation for a stationary Markov process [35]:

$$\frac{\partial}{\partial t}P(x, t) = -\frac{\partial}{\partial x} [U'_{\text{eff}}(x)P(x, t)] + \sigma \frac{\partial^2}{\partial x^2}P(x, t), \tag{9}$$

in which $P(x, t)$ is the probability distribution function (PDF) for the process $x(t)$, which only fluctuates around its local equilibrium of the bias mono-stable CDU potential, and it has the initial and boundary conditions $P(x, 0) = \delta(x)$ and $P(\pm \infty, t) = 0$. Due to the time-dependent effect $f(t)$, the system has the effective potential function as follows:

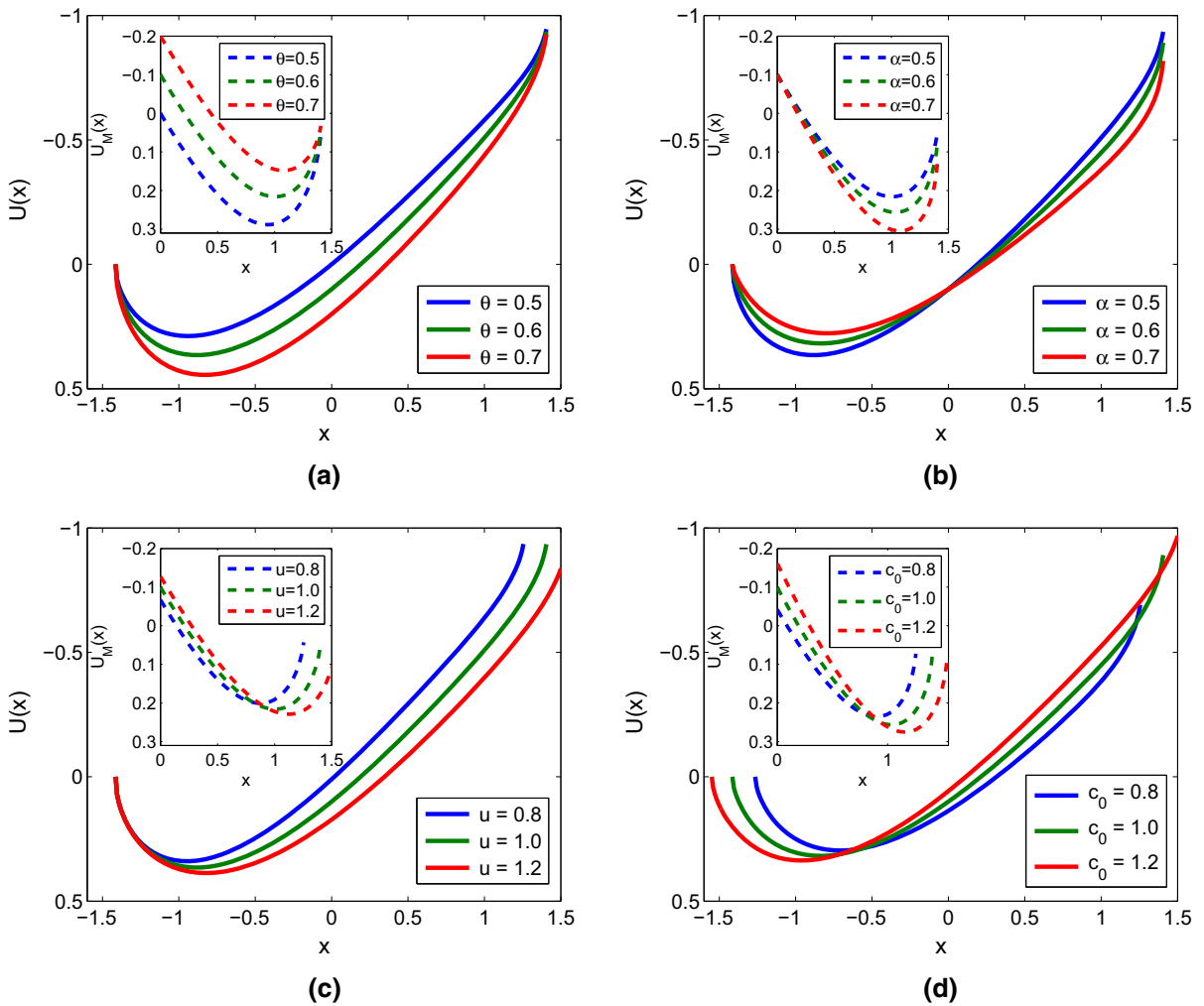


Fig. 2 The curves of mono-stable CDU potential, respectively, with different parameters θ , α , u and c_0 . **a** $c_0 = 1.0$, $K = 1.0$, $\alpha = 0.6$, $u = 1.0$, $v = 1.0$; **b** $c_0 = 1.0$, $K = 1.0$, $u = 1.0$,

$v = 1.0$, $\theta = 0.6$; **c** $c_0 = 1.0$, $K = 1.0$, $\alpha = 0.6$, $v = 1.0$, $\theta = 0.6$; **d** $K = 1.0$, $\alpha = 0.6$, $u = 1.0$, $v = 1.0$, $\theta = 0.6$

$$U_{\text{eff}}(x, t) = \theta K a^\alpha(x) b^{1-\alpha}(x) - \frac{1}{2u} a^2(x) + A \cos(\Omega t) \cdot x. \tag{10}$$

Furthermore, it is natural to assume that the driving frequency Ω is so small that there is enough time for the enterprise to reach the local equilibrium during one driving period, as shown in Fig. 3, thus we make the assumption that the system satisfies the adiabatic approximation condition [36,37], and the asymptotic long-time distribution function can be derived from Eq. (9). Figure 3 also clearly shows the driving effect of product-capital switching on bias mono-stable CDU potential, that is, at the stage with $f(t) = 0$, the

enterprise state easily traps into the maximum utility of $U_{\text{eff}}(x) = U(x)$, which is shown in Fig. 3a, and seriously deviates from the marketing-side utility. In the process of product-to-capital, the enterprise has to adjust and transfer more resources to inspire the marketing-side to further effort, which is achieved by the continuous input of operating capital from $f(t)$, and accordingly causes the equilibrium position of $U_{\text{eff}}(x)$ to shift to the right with time t , and approach that of $U_M(x)$ with the increase of $f(t)$, i.e., at $t \in (\frac{T}{2}, T]$, and it reaches the maximum effect at $t = T$ with $U_{\text{eff}}(x) = U(x) + Ax$, as shown in Fig. 3b. Conversely,

in the process of capital-to-product, i.e., at $t \in (0, \frac{T}{2}]$, technology-side takes more resources and make further effort at the cost of marketing-side utility. It is observed from Fig. 3c that the equilibrium position of $U_{\text{eff}}(x)$ shifts to the left and further deviates from that of $U_M(x)$ with the decrease of $f(t)$, and it reaches the maximum effect at $t = \frac{T}{2}$ with $U_{\text{eff}}(x) = U(x) - Ax$. Therefore, the driving effect makes $U_{\text{eff}}(x)$ show two time-dependent local equilibria, as shown in Fig. 3d.

In perspective of particle hopping, the system describes the process of over-damped particle oscillation mainly subject to the bias mono-stable CDU potential of the technology-side. In the absence of driving force, the PDF of the unperturbed process satisfies the following Fokker–Planck equation [35]:

$$\frac{\partial}{\partial t} P_0(x, t) = -\frac{\partial}{\partial x} [U'(x)P_0(x, t)] + \sigma \frac{\partial^2}{\partial x^2} P_0(x, t), \tag{11}$$

with boundary conditions $P_0(\pm\infty, t) = 0$. The stationary distribution will be established in the system with time:

$$P_0^{\text{st}}(x) = M \cdot e^{U(x)/\sigma}, \tag{12}$$

where M is the normalization constant. Let us consider the CDU potential (6), which has a deltoid initial probability distribution at $x = 0$, $P_0(x, 0) = \delta(x)$. With time, the particles diffuse, which leads to $P_0(x, t)$ becoming indistinct, and ultimately, the stationary bias unimodal distribution is established, as shown in Fig. 4.

When a stationary distribution is reached in the system, the power spectral density (PSD) of the output process $x(t)$ is equal to

$$S_0(\omega) = \frac{1}{\pi} \int_0^\infty R_0(\tau) \cos(\omega\tau) d\tau, \tag{13}$$

where $R_0(\tau) = \langle x(t)x(t + \tau) \rangle$ is the correlation function of the output stationary process of the unperturbed system. Using Fourier transform defined as $\tilde{R}_0(\omega) = \int_0^\infty R_0(\tau)e^{-j\omega\tau} d\tau$, one can rewrite the PSD as

$$S_0(\omega) = \frac{1}{\pi} \text{Re} \left\{ \tilde{R}_0(\omega) \right\}. \tag{14}$$

For determining the spectrum of the process at the output of perturbed system, we consider the case that the nonzero driving force is harmonic and small compared with the size of the system. Based on linear response theory (LRT) [38], when harmonic force $f(t)$ and additional noise $\zeta(t)$ with arbitrary intensity act on the system, the output is the sum of noise and periodic components, and PSD has the following form:

$$S(\omega) = S_0(\omega) + \frac{1}{4} A^2 |\chi(\omega)|^2 \cdot [\delta(\omega - \Omega) + \delta(\omega + \Omega)], \tag{15}$$

where $\chi(\omega)$ is the system susceptibility, which is the Fourier transform of linear response $h(\tau)$, that is,

$$\chi(\omega) = \int_{-\infty}^\infty h(\tau)e^{-j\omega\tau} d\tau, \tag{16}$$

respectively, with the modulus $|\chi(\omega)| = \sqrt{(\text{Re}\chi(\omega))^2 + (\text{Im}\chi(\omega))^2}$ and the associated phase $\phi_\chi = \arctan\{\text{Re}\chi(\omega)/\text{Im}\chi(\omega)\}$. According to the fluctuation-dissipative theorem [34], the nonlinear response function can be obtained via the parameters of unperturbed system, and it is given by

$$h(\tau) = -\frac{\rho(\tau)}{\sigma} \cdot \frac{d}{d\tau} R_0(\tau), \tag{17}$$

where $\rho(\tau)$ is a Heaviside function. Since the linear response function exists only for $\tau > 0$ in the actual systems, integration in Eq. (16) can be performed from 0 to $+\infty$, and we have

$$\begin{aligned} \chi(\omega) &= \int_0^\infty h(\tau)e^{-j\omega\tau} d\tau \\ &= \frac{1}{\sigma} \left(R_0(0) - j\omega \tilde{R}_0(\omega) \right), \end{aligned} \tag{18}$$

in which the correlation function for $\tau = 0$ in the unperturbed system can be expressed as

$$R_0(0) = \langle x \rangle^2 + \langle x^2 \rangle. \tag{19}$$

Here, $\langle x \rangle$ and $\langle x^2 \rangle$ are, respectively, the mean and variance values of the output process, which can be obtained from the known stationary distribution given by

$$\begin{cases} \langle x \rangle = \int_{-\infty}^\infty x P_0^{\text{st}}(x) dx, \\ \langle x^2 \rangle = \int_{-\infty}^\infty x^2 P_0^{\text{st}}(x) dx - \langle x \rangle^2, \end{cases} \tag{20}$$

respectively. Based on the method proposed in [25], the approximated estimates of the output correlation function and PSD of an unperturbed system, is given by

$$\begin{cases} \hat{R}_0(\tau) = \langle x \rangle^2 + \langle x^2 \rangle e^{-\tau/\tau_0}, \\ \hat{S}_0(\omega) = \frac{\tau_0}{\pi(1 + \omega^2\tau_0^2)} \cdot \langle x^2 \rangle + \delta(\omega) \cdot \langle x \rangle^2, \end{cases} \tag{21}$$

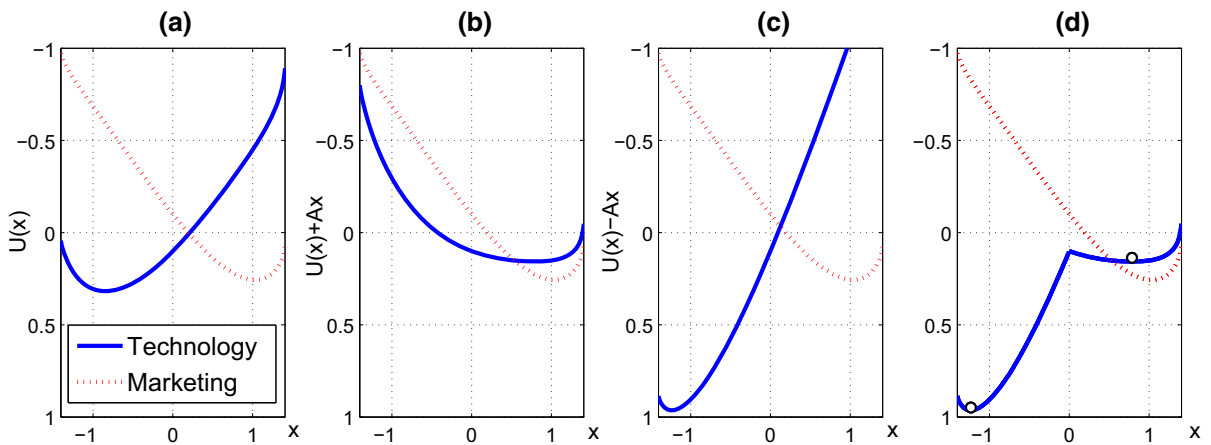
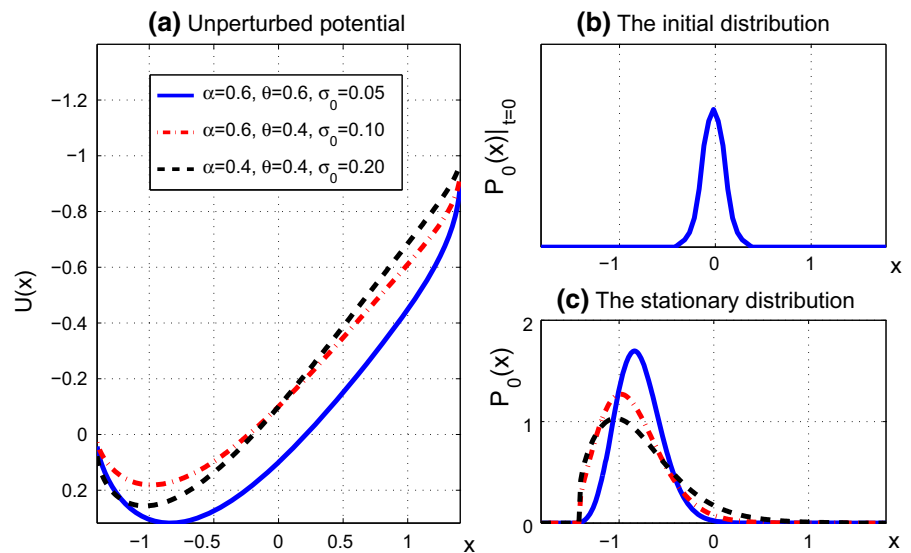


Fig. 3 The effective CDU potential in perturbed partnership system with $c_0 = 1.0$, $K = 1.0$, $\alpha = 0.6$, $u = 1.0$, $v = 1.0$, $\theta = 0.6$ and $A = 0.5$

Fig. 4 The stationary bias distribution in unperturbed partnership systems



where τ_0 is the exact correlation time [39], and can be calculated by

$$\tau_0 = \frac{\int_{-\infty}^{\infty} e^{-U(s)/\sigma} \left[\int_s^{\infty} x e^{U(x)/\sigma} dx \right]^2 ds}{\sigma \int_s^{\infty} x^2 e^{U(x)/\sigma} dx}, \tag{22}$$

for arbitrary potential $U(x)$. Furthermore, in the limit $\omega \rightarrow 0$, one can show, by means of standard arguments, that the susceptibility in Eq. (18) satisfies the following version of fluctuation dissipation theorem [11]:

$$|\hat{\chi}(\omega)|_{\omega \rightarrow 0} = \frac{\langle x^2 \rangle - \langle x \rangle^2}{\sigma}. \tag{23}$$

Actually, this approximate method in Eqs. (21) and (23) replaces initial nonlinear system (7) by an equivalent linearized one in the statistical sense [12].

4 Performance measures

In this section, we derive the performance measures of proposed model (7) to describe the statistic characteristics of competition and cooperation process in bilateral partnership systems. By submitting Eqs. (21)–(23) into Eq. (15), we have the following approximate form:

$$S(\omega) = \frac{\pi}{4} A_{\text{out}}^2 \cdot [\delta(\omega - \Omega) + \delta(\omega + \Omega)] + \sigma_B^2 \cdot \delta(\omega) + \sigma_S^2(\omega), \tag{24}$$

respectively, with

$$\begin{cases} A_{\text{out}} &= \frac{A}{\sqrt{\pi}\sigma} [\langle x^2 \rangle - \langle x \rangle^2], \\ \sigma_B^2 &= \langle x \rangle^2, \\ \sigma_S^2(\omega) &= \frac{\tau_0}{\pi(1+\omega^2\tau_0^2)} \langle x^2 \rangle. \end{cases} \quad (25)$$

Notice that the power spectrum can naturally divide into three parts: the periodic output, which is a delta function at the driving frequency Ω and $-\Omega$, and A_{out} is the amplitude of system stationary response; the zero-frequency output, which is produced by the asymmetry of bias mono-stable CDU potential, that is, σ_B is the measure index of asymmetric risk, including internal bilateral competition and cooperation, and it will disappear if the potential is symmetric with $\langle x \rangle = 0$; and the broadband noise output $\sigma_S(\omega = \Omega)$, which is the measure index of systematic risk, which is important for target enterprise to create strategies for introducing venture capital portfolios based on the profile. Here, two types of risks are also important for understanding valuation model, which describes the relationship between risk and expected return for risky investments.

In order to investigate the stationary dynamical behaviors in bilateral partnership systems, the output SNR is employed via the system susceptibility, and has the following approximated form for the case of sufficiently small but finite driving frequency Ω :

$$\lambda = \frac{A^2(1 + \Omega^2\tau_0^2)}{2\tau_0\sigma^2\langle x^2 \rangle} [\langle x^2 \rangle - \langle x \rangle^2]^2, \quad (26)$$

where $A = A_0 + A_1$ and $\sigma = \sigma_0 + \sigma_1$. Meanwhile, the stationary URR can be defined as

$$\begin{aligned} \eta &= \frac{A_{\text{out}}}{\sigma_B + \sigma_S(\omega = \Omega)} \\ &= \frac{A}{\sqrt{\pi}\sigma|\langle x \rangle| + \sigma\sqrt{\frac{\tau_0\langle x^2 \rangle}{(1+\Omega^2\tau_0^2)}}} [\langle x^2 \rangle - \langle x \rangle^2], \end{aligned} \quad (27)$$

which refines an investment return by measuring how much risk is involved in producing the return over a given period of time. Stationary URR is widely applied to individual securities, investment funds and portfolios [40, 41]. Furthermore, the system can be valued by the discounted analysis, which is a method of analyzing the partnership system using the concepts of the time value, and has been widely used in investment finance and corporate financial management [42]. Therefore, for the continuous cooperation and competition process $x(t)$ in bilateral partnership system, the discounted

process is defined as $x_D(t) = x(t)e^{-rt}$ with a known rate r . Then averaging $\langle x_D(t)x_D(t + \tau) \rangle$ with respect to t uniformly within one driving periodic $[0, T_\Omega]$, we obtain

$$\begin{aligned} R_x^D(\tau) &= \frac{1}{T_\Omega} \int_0^{T_\Omega} R_x(\tau)e^{-r(2t+\tau)} dt \\ &= \frac{1 - e^{-rT_\Omega}}{rT_\Omega} \cdot R_x(\tau)e^{-r\tau}, \end{aligned} \quad (28)$$

where $T_\Omega = 2\pi/\Omega$. Therefore, the PSD is given by

$$\begin{aligned} S_D(\omega) &= \int_0^\infty R_x^D(\tau)e^{-j\omega\tau} d\tau \\ &= \frac{1 - e^{-rT_\Omega}}{rT_\Omega} \int_0^\infty R_x(\tau)e^{-r\tau}e^{-j\omega\tau} d\tau. \end{aligned} \quad (29)$$

By using the convolution properties, we have

$$\begin{aligned} \frac{rT_\Omega}{1 - e^{-rT_\Omega}} S_D(\omega) &= \frac{\pi}{4} A_{\text{out}}^2 \\ &\left[\frac{2r}{r^2 + (\omega - \Omega)^2} + \frac{2r}{r^2 + (\omega + \Omega)^2} \right] \\ &+ \frac{2r\langle x \rangle^2}{r^2 + \omega^2} + \frac{\tau_0\langle x^2 \rangle}{\pi} \frac{r\tau_0 + 1}{(r\tau_0 + 1)^2 + \tau_0^2\omega^2}. \end{aligned} \quad (30)$$

Thus, the output SNR and stationary URR of discounted process $x_D(t)$ can be defined as

$$\begin{cases} \lambda_D = \frac{2A^2[(r\tau_0 + 1)^2 + \tau_0^2\Omega^2][\langle x^2 \rangle - \langle x \rangle^2]^2}{r\sigma^2\tau_0(r\tau_0 + 1)\langle x^2 \rangle}, \\ \eta_D = \frac{A}{\sqrt{\pi}\sigma|\langle x \rangle| + \sigma\sqrt{\frac{r\tau_0(r\tau_0 + 1)\langle x^2 \rangle}{2[(r\tau_0 + 1)^2 + \tau_0^2\Omega^2]}}} [\langle x^2 \rangle - \langle x \rangle^2], \end{cases} \quad (31)$$

respectively. Correspondingly, generalized SR can be also understood as the non-monotonic behaviors of λ_D or η_D . Moreover, when comparing two or more potential investments, investors should always compare the same risk measures to each different investment to get a relative performance perspective. If different investments have the same return or utility, the one that has the lowest risk will have the better risk return. However, considering that different risk measurements give investors very different analytical results, it is important to be clear on what type of risk return is being considered. Besides, target enterprise should also consider the scales of venture capital, and the acceptability of external risk under the bilateral structure of competition and cooperation. Sometimes, this funding can be the capital portfolio, which is provided by a large number of venture capitalists with different risk levels. Therefore, it is of great reference value to quantitatively create

strategies to control the portfolio risk for maximizing the return on risk-adjusted capital. It is noted that rapid economic changes are observed in special periods, thus sometime, $f(t)$ should be considered as the periodic force with relative larger Ω . Although, according to adiabatic approximation theory [43], SR is first studied and observed in small parameters, under the large parameter conditions, Leng et al. [44,45] proposed scale transformation methods to analyze the dynamical behaviors, and the high-frequency driving force can be conventional processed by the pre-proceeder, which maps the parameters into the small parameters with the scale transform factor. Therefore, only the cases of low-frequency driving force are discussed. Besides, some important events can lead to brusque variations in external capital environment, where external risk $\varepsilon(t)$ can be modeled as the Lévy noise, and Lévy noise driving SR systems can be analyzed by applying the methods proposed in Refs. [46,47].

Considering the bilateral partnership system in the regime of normal capital environment, the underlying mechanism of dynamical behaviors can be explored as follows: In the absence of periodic effect, the unperturbed system traps into the equilibrium of technology-side utility with very high probability as shown in Figs. 3a and 4 shows that it even seriously deviates from the equilibrium of marketing-side utility, which leads to the high-risk of bilateral cooperation. When motivated by the periodic effect from internal or external capital, the equilibrium position of perturbed system periodically moves with the change of driving cycle, which includes four stages as mentioned previously, and it is seen from Fig. 3c that the equilibrium has approached the utility optimum of marketing-side, which is needed fitly to play an important role at the cash-out stage. Obviously, the introduced venture capital changes the evolution state and leads to the decrease in unfair competition of insufficient resource, thus it makes the marketing-side willing to apply more effort, and the rupture risk of bilateral cooperation decreases.

Based on SR theory, as the inherent effect of capital-product switching applied together with stochastic fluctuation from internal or external environment, external venture capital might magnify the switching efficiency, and competes and cooperates to make the system achieve global optimum in the statistical sense. Because of the fluctuation independence hypothesis of different origins, the external risk plays an important role as internal risk on the effects, and thus controlling

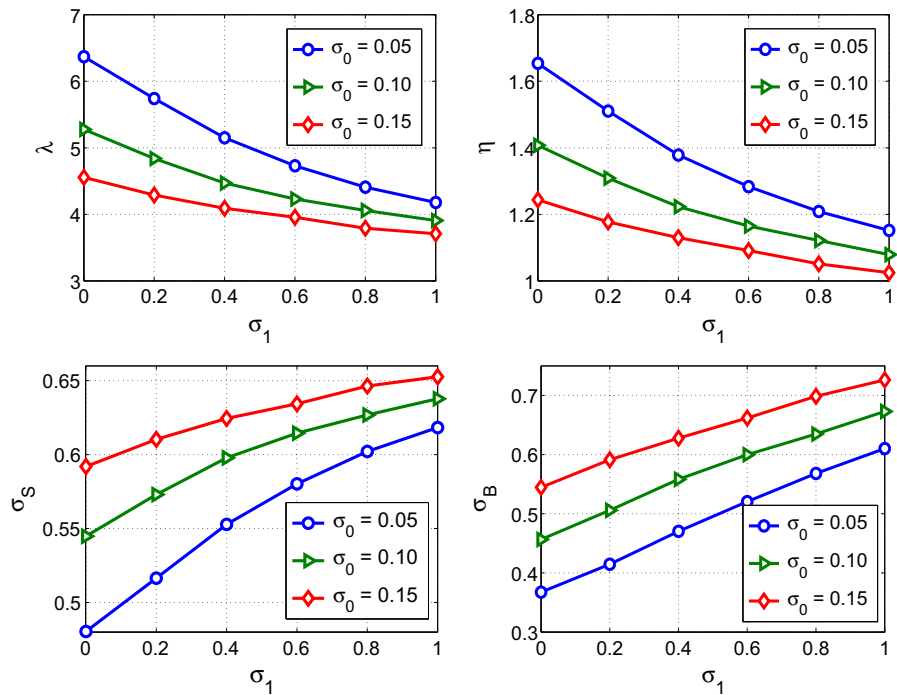
the internal risk and creating strategies to optimize the external risk have the equivalent effect on improving the performance of partnership systems. Furthermore, for surplus venture capital, the equilibrium gradually deviates from the optimum of marketing-side again. It raises the unfair competition of surplus resource, and the fluctuation only has the effect on disordering the normal periodic operation of capital-product switching, which leads to the increase in systematic risk.

5 Simulation results and discussion

In this section, we present the simulation results to investigate the stationary dynamical behaviors of bilateral partnership systems, and observe the incentive effects, including the output SNR λ , stationary URR η , systematic risk σ_S and bilateral risk σ_B , which are, respectively, plotted as a function of the system internal or external parameters. Here, we consider the most common annual periodic, thus only the cases of periodic force with low-frequency $\Omega \approx 0.0172$ are discussed. Moreover, the friction coefficient is set to unity by normalization, thus the parameters are comparably set to be the relative values, and the units are not considered. If not specified, the potential parameters are set to $c_0 = 1.0$, $K = 1.0$, $\alpha = 0.6$, $u = v = 1.0$ and $\theta = 0.6$, which show that technology-side holds the ownership and have $U(x) = -\infty$ for $|x| > \sqrt{2}$. Performance measures, described in Eqs. (25), (26) and (27), are obtained from the ensembles average over 1000 realizations of stochastic paths $x(t)$ by the classical second-order Runge–Kutta numerical method, and each one continues for 10 times driving periods.

In Fig. 5, we plot the curves of different performance measures of incentive effect as a function of the external risk level σ_1 in the bilateral partnership systems with sufficient internal capital investment $A_0 = 1.2$, which, respectively, involves internal risk level $\sigma_0 = 0.05$, 0.10 and 0.15. One can clearly conclude that the output SNR λ and stationary URR η follow the same trend, and monotonically decrease with increasing σ_1 for sufficient $A_0 = 1.2$. It is because of the fact that it is not necessary for the target enterprise to introduce venture capital, even $A_1 = 0.2$ is small, the increase in external risk σ_1 leads to the increase in systems disorder, that is, the external noise energy only has the effect on disordering the normal periodic operation of capital-product switching, thus, we observe the both increase of sys-

Fig. 5 The performance measures versus the external risk level in partnership systems with internal investment $A_0 = 1.2$ and risk level $\sigma_0 = 0.05, 0.10$ and 0.15



tematic risk σ_S and bilateral risk σ_B . Meanwhile, two types of risks increase as σ_0 increases, which further leads to the decrease of λ and η . Therefore, for enterprises with sufficient internal capital, the wise choice is not to recruit external capital, and this also accords with the intuitive understanding.

Naturally, we ignore the introduction of external capital $A_1 = 0$, to simply investigate the effect of internal capital A_0 and risk level σ_0 . Here we plot the curves of performance measures as a function of σ_0 in the system, respectively, with sufficient $A_0 = 1.0, 1.2$ and 1.4 . It is seen from Fig. 6 that λ monotonically decreases as σ_0 increases, and under the condition of sufficient internal capital, the smaller A_0 leads to the higher λ , but ideally, the system without internal risk $\sigma_0 = 0$ has the same output SNR. Meanwhile, it shows the same pattern that η monotonically decreases with increasing σ_0 . As to lower σ_0 , more sufficient internal capital, i.e., $A_0 = 1.4$, shows higher η . Conversely, as to higher internal risk, i.e., $\sigma_0 > 0.16$, less A_0 leads to the higher η , and the differences trend to disappear with increasing σ_0 . It is mainly because of the inverse relationship to σ_S and σ_B . Here we note that internal risk plays a crucial role on the stationary response under the condition of sufficient internal capital, and it is impor-

tant for enterprises to create strategies for investing the initial capital based on the internal risk profile.

On the other hand, we are also interested in the incentive effect of insufficient internal investment. Specially, what will happen if the enterprise fails or refuses to introduce external venture capital? In Fig. 7, we plot the curves of performance measures as a function of σ_0 in the system, respectively, with insufficient $A_0 = 0.4, 0.5$ and 0.6 . One can clearly conclude that the noticeable feature is the high and nearly constant σ_B , which is lower with the more investment. It also shows that the enterprise is always in the local optimum of the technology-side because of the insufficient capital, thus the marketing-side doesn't want to apply much effort, and the rupture risk of bilateral cooperation increases mainly due to unfair competition of insufficient resource. Therefore, the dominated higher σ_B also leads to lower and nearly constant η . As to σ_S , it monotonically increases with increasing σ_0 , and it is higher with the more internal capital A_0 . Naturally, it makes the output SNR have the opposite performance, that is, λ monotonically and obviously decreases with the increase of σ_0 , and it is lower with the more A_0 . Therefore, under the condition of insufficient internal investment and no external venture capital, controlling the internal risk only has the effect on decreasing sys-

Fig. 6 The performance measures versus the internal risk level in partnership systems, respectively, with internal investment $A_0 = 1.0, 1.2$ and 1.4 , and without introduced external capital $A_1 = 0, \sigma_1 = 0$

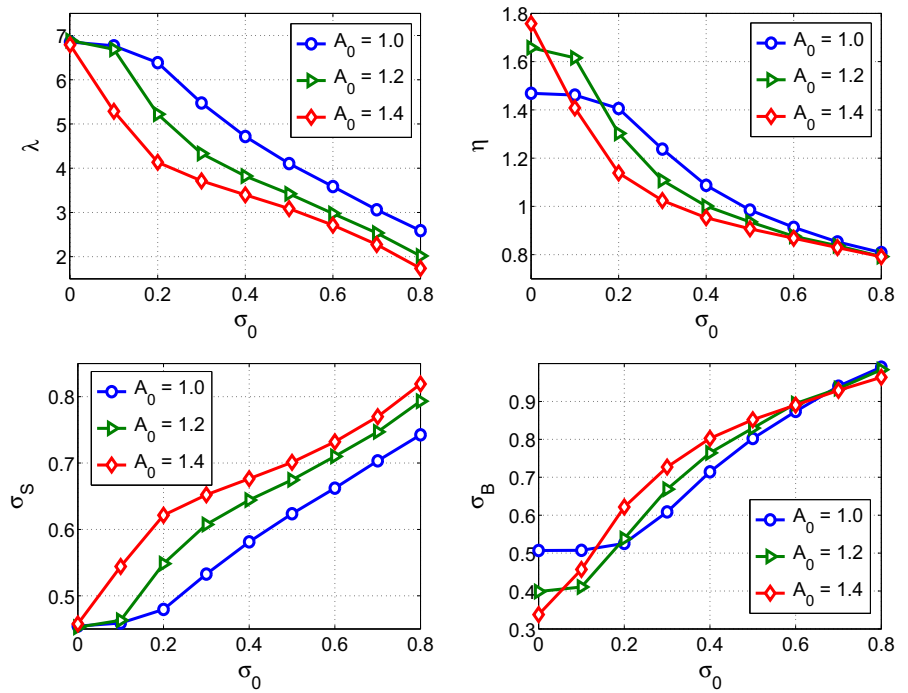
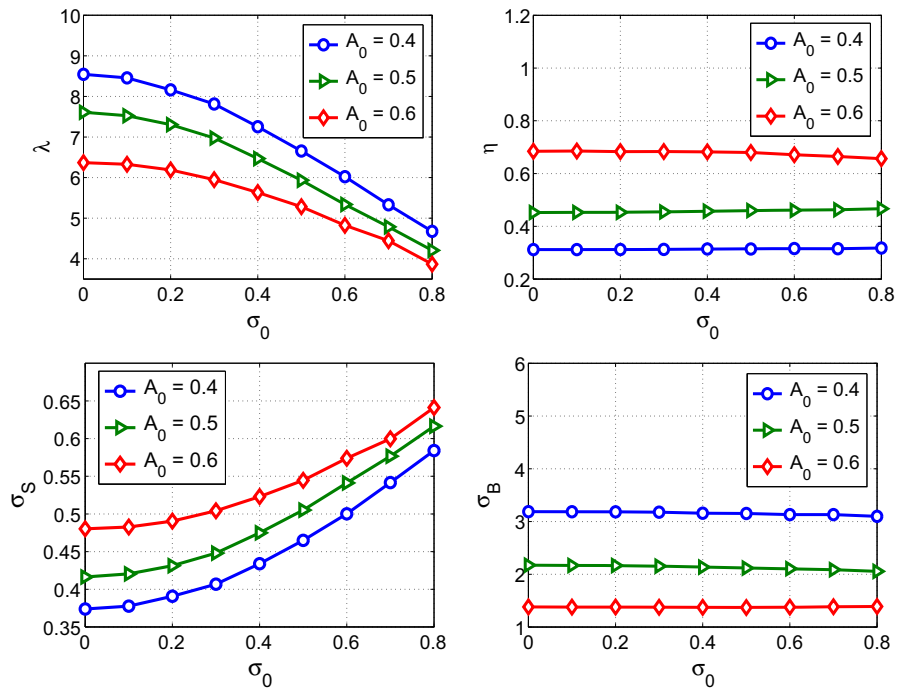


Fig. 7 The performance measures versus the internal risk level in partnership systems, respectively, with internal investment $A_0 = 0.4, 0.5$ and 0.6 , and without introduced external capital $A_1 = 0, \sigma_1 = 0$



tematic risk, while the decrease in dominated bilateral risk can only be accomplished by increasing the internal investment as much as possible, thus the stationary URR improves.

Moreover, considering the incentive effect of external risk level under the condition of insufficient internal investment $A_0 = 0.6$ with the internal risk level $\sigma_0 = 0.1$, in Fig. 8, we plot the curve of performance measures as a function of the external risk level σ_1 in the system, respectively, with different external venture capital $A_1 = 0.3, 0.4$ and 0.5 . It is seen that λ monotonically decreases as increasing σ_1 , but it is lower with the more A_1 when the external risk measure is greater than a certain level, i.e., $\sigma_1 > 0.2$ in this case. Because the two types of risks have the same tendency of monotonic increase, and it shows the same pattern as SNR that η decreases with increasing σ_1 . As to lower external risk, i.e., $\sigma_1 < 0.2$, more external capital, i.e., $A_1 = 0.5$, has higher η . Conversely, as to higher external risk, i.e., $\sigma_1 > 0.4$, lower internal investment leads to higher η . Here we note that external risk plays the same role as internal risk on different measures due to the risk independence of different origins, and thus controlling the internal risk and creating strategies to optimize the external risk have the equivalent effect of improving the performance of a partnership system.

Furthermore, considering the incentive effect of external capital with risk level $\sigma_1 = 0.1$ under the condition of insufficient internal investment $A_0 = 0.6$, respectively, with internal risk level $\sigma_0 = 0.1, 0.2$ and 0.3 , in Fig. 9, we plot the performance curves as a function of A_1 . One can clearly observe the non-monotonic behaviors of output SNR and stationary URR, and there exist a peak on each curve and we call it a resonance peak. Specially, λ firstly decreases, then increases and then decreases with the increase of A_1 . The value of the peak decreases with the increase of σ_0 , and the position shifts obviously from the small values of A_1 to the large values with increasing σ_0 . This phenomenon can be explained from the tendency of systematic risk, that is, it first increases, then decreases and then increases as A_1 increases, and naturally, it is higher with higher σ_0 . As to stationary URR, it is seen that η also shows the non-monotonic behavior, first increases and then decreases, with the increase of A_1 , and it is lower with higher σ_0 . Specially, the value of the peak decreases with the increase of σ_0 , and the position shifts obviously from the large values of A_1 to the small value. It is explained as follows, in partnership system with

$A_0 = 0.6$, increasing A_1 leads to the decrease in probability that the enterprise is in optimal state of the technology-side, in spite of the insufficient capital input $A_0 + A_1$. Here the introduction of small venture capital cannot fundamentally change the state, but decreases the unfair competition of insufficient resource, and this has made the marketing-side willing to apply more effort, thus the rupture risk measure σ_B of bilateral cooperation first decreases. While the input has reached a certain threshold, which is smaller with the higher σ_0 , the surplus venture capital instead raises the bilateral risk because of unfair competition of surplus resource, thus σ_B then increases. Here, the dominated σ_B at $A_1 < 0.2$ also makes η show monotonic increase at $A_1 < 0.2$.

In this case, we note that external venture capital A_1 plays a crucial role in order to achieve optimal λ or η , thus for any internal state $A_0 \in [0.2, 0.8]$ and $\sigma_0 \in [0.02, 0.2]$, we investigate, in Fig. 10, the optimal URR η_{opt} and related $(A_1)_{opt}$, which is accompanied by the unit risk level $\sigma_1 = 0.3$. As a whole, more venture capital should be introduced for less A_0 and higher σ_0 to pursuit the optimal URR. It is important for enterprises to create portfolio strategies of introducing venture capital, including determining the amount of capital and optimizing the portfolio risk, based on the internal investment and risk profile.

Based on the previous results and discussion, bilateral risk is a sensitive index to affect the stationary URR, especially in the system with insufficient $A_0 + A_1$, and it dominates the effect, compared with the systematic risk. Therefore, we are also interested in the effect of adjustable parameters. For technology-side share θ , we plot the performance curves in Fig. 11 as a function of θ in partnership system with internal investment $A_0 = 0.45$ and different external venture capital $A_1 = 0.40, 0.45$ and 0.50 , which correspond to the internal risk $\sigma_0 = 0.2$ and external risk $\sigma_1 = 0.1$, respectively. One can clearly observe the non-monotonic behaviors of λ with increasing θ , that is, there exist a valley on each curve and here we call it a reverse SR. Specially, the value of the resonance valley increases with the increase of A_1 , and the position shifts obviously from the small value of θ to the large value, but the inverse resonance behavior tends to fade and disappear with less A_1 . This phenomenon can be explained by the resonance behavior of σ_S . As to stationary URR, the SR and reverse SR phenomena coexist. Specially, the reverse SR behavior occurs at

Fig. 8 The performance measures versus the external risk level in partnership systems, respectively, with internal investment $A_0 = 0.6$ and risk level $\sigma_0 = 0.1$, and with external capital $A_1 = 0.3, 0.4$ and 0.5

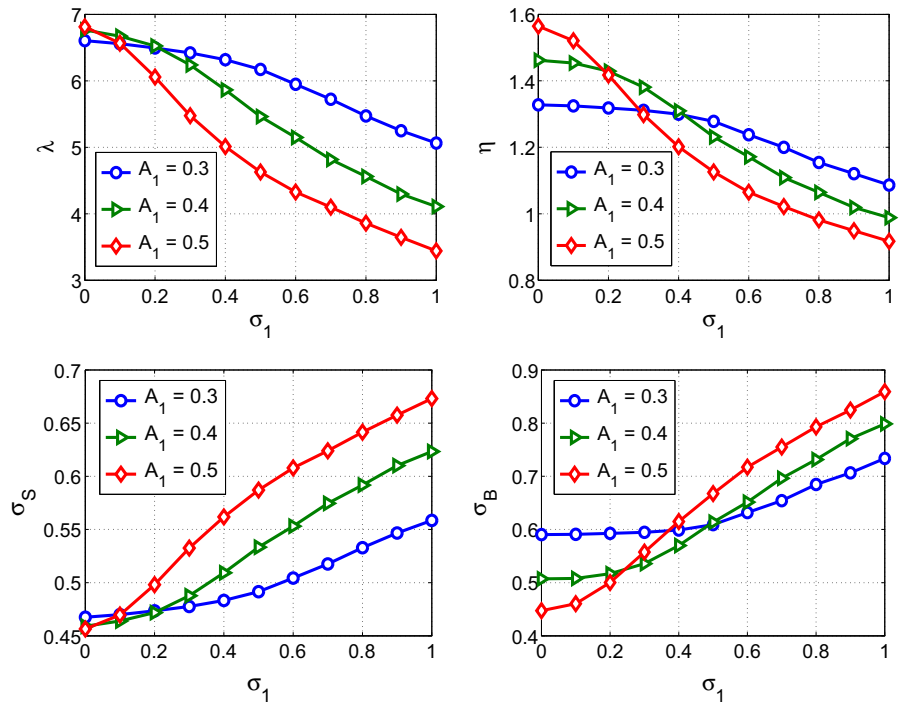
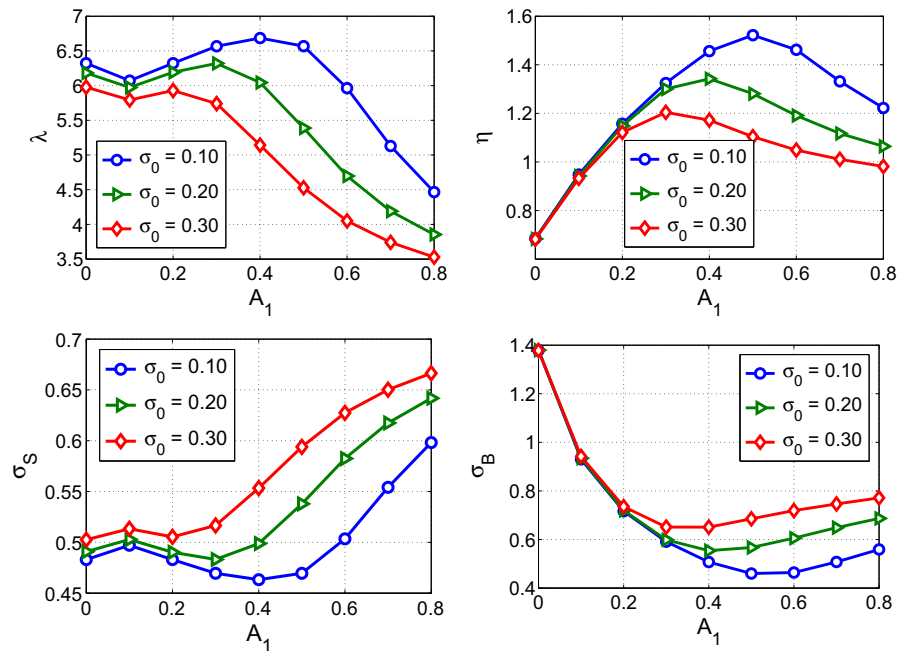


Fig. 9 The performance measures versus the external capital in partnership systems with internal investment $A_0 = 0.6$ and risk level $\sigma_0 = 0.1, 0.2$ and 0.3 , and with fixed external risk level $\sigma_1 = 0.1$



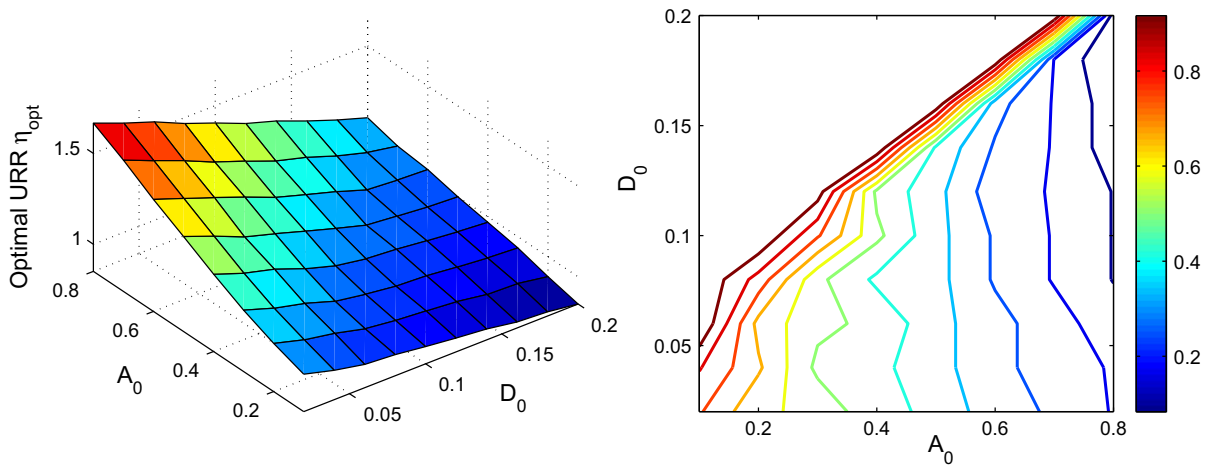
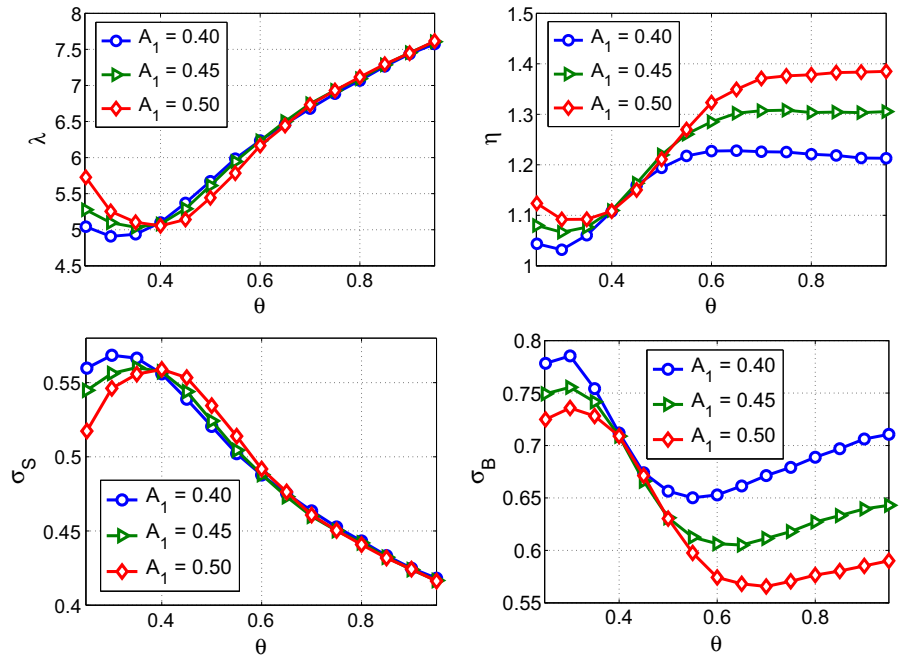


Fig. 10 The optimal URR η_{opt} and related introduced venture capital $(A_1)_{opt}$ with unit risk level $\sigma_1 = 0.3$, in partnership system with internal investment $A_0 \in [0.2, 0.8]$ and risk level $\sigma_0 \in [0.02, 0.2]$

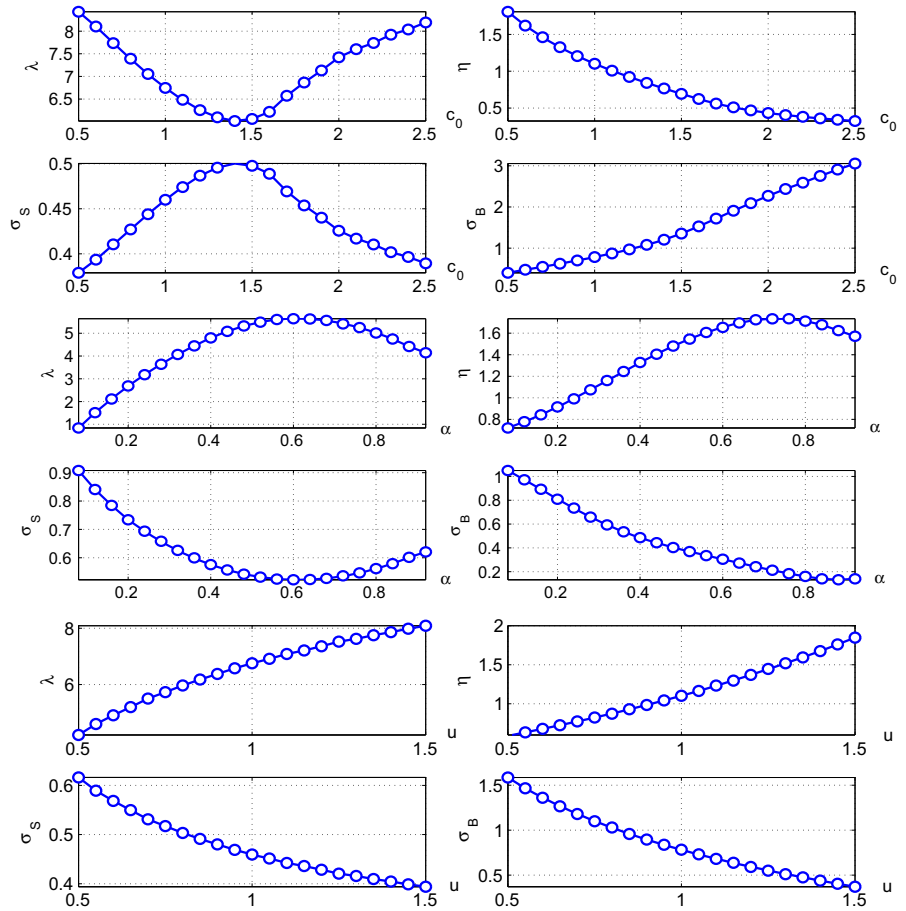
Fig. 11 The performance measures versus the technology-side share in partnership systems with internal investment $A_0 = 0.45$ and risk level $\sigma_0 = 0.2$, and with different introduced capital $A_1 = 0.40, 0.45$ and 0.50 , which involve the fixed risk level $\sigma_1 = 0.1$



$\theta < 0.4$, and the value of reverse SR valley increases with the increase of A_1 , and the position shifts obviously from the small value of θ to the large value, but the inverse resonance behavior tends to fade and disappear with the decreasing A_1 . Conversely, the resonance occurs at $\theta > 0.5$, and the value of the peak increases with increasing A_1 , and the position shifts obviously from the small value of θ to the large value, while the resonance behavior tends to fade and disappear.

Another interesting note can be observed in Fig. 11, as to the initial share $\theta = 0.6$, the technology-side may proactively transfer a certain share to marketing-side in order to maintain a good cooperative relationship, but it could backfire, even the bilateral risk resonance may even occur, especially in the system with insufficient and less $A_0 + A_1$. It is mainly because of the fact that this practice makes the technology-side unwilling to apply the effort he deserves, thus the rupture risk of bilateral cooperation, measured by σ_B , shows SR phe-

Fig. 12 The performance measures versus the parameters u , α and c_0 in partnership systems with internal state $A_0 = 0.4$, $\sigma_0 = 0.1$ and external state $A_1 = 0.4$, $\sigma_1 = 0.1$



nomena at $\theta = 0.3$. Therefore, the wise decision for lowering rupture risk is not to blindly sacrifice self-benefit and transfer the share to the other side, but to search the position of reverse SR of bilateral risk, which may occur at $\theta < 0.6$, i.e., $\theta = 0.55$ for $A_1 = 0.40$, and may occur at $\theta > 0.6$, i.e., $\theta = 0.70$ for $A_1 = 0.50$. Due to the mechanism, we can also naturally comprehend non-monotonic behaviors of SR and reverse SR.

Then, we analyze the effect of other adjustable parameters, including constrained wealth c_0 , return elasticity α and level factor u . It is seen from Fig. 12 that λ and η , respectively, show reverse SR and monotonic decrease, mainly caused by the system risk resonance of σ_S and increasing σ_S , which plays a predominant role. As α increases, σ_S decreases firstly and then increases, while σ_B monotonically decreases, thus we observe the SR peaks of λ and η , respectively, at $\alpha = 0.6$ and 0.72 . Actually, combining Fig. 2, it is not difficult to comprehend the SR behaviors in Fig. 12 from the perspective of parameter influence on the bias

and asymmetry of mono-stable CDU potential. Therefore, decision makers can be guided with the results to quantitatively adjust the parameters to control the risk and maintain productive partnership.

Furthermore, we investigate the effect of venture capital portfolio from two aspects. First, respectively, considering the venture capital $A_1 = 0.2, 0.3$ and 0.4 , which are equally introduced from N independent resources with unit risk level $\sigma_{1j} = 0.3$, the performance measures, in Fig. 13, are plotted as a function of N in partnership system with internal investment $A_0 = 0.6$ and risk level $\sigma_0 = 0.1$. As N increases, two types of risks monotonically decrease and asymptotically tend to the constant, and more A_1 corresponds to higher σ_S and lower σ_B . Thus we observe that λ increases with increasing N or decreasing A_1 , while more A_1 corresponds to higher η . Then, for non-fixed external capital, A_1 is considered to be the portfolio of N independent capital resources $c_j = 0.06, 0.10$ and 0.15 , and each one is characterized by the unit risk

Fig. 13 The performance measures versus the number of venture capital resources in partnership systems with internal investment $A_0 = 0.6$ and risk level $\sigma_0 = 0.1$, and with the total external capital $A_1 = 0.2, 0.3$ and 0.4 , which are equally introduced from N independent resources with unit risk level $\sigma_{1j} = 0.3$

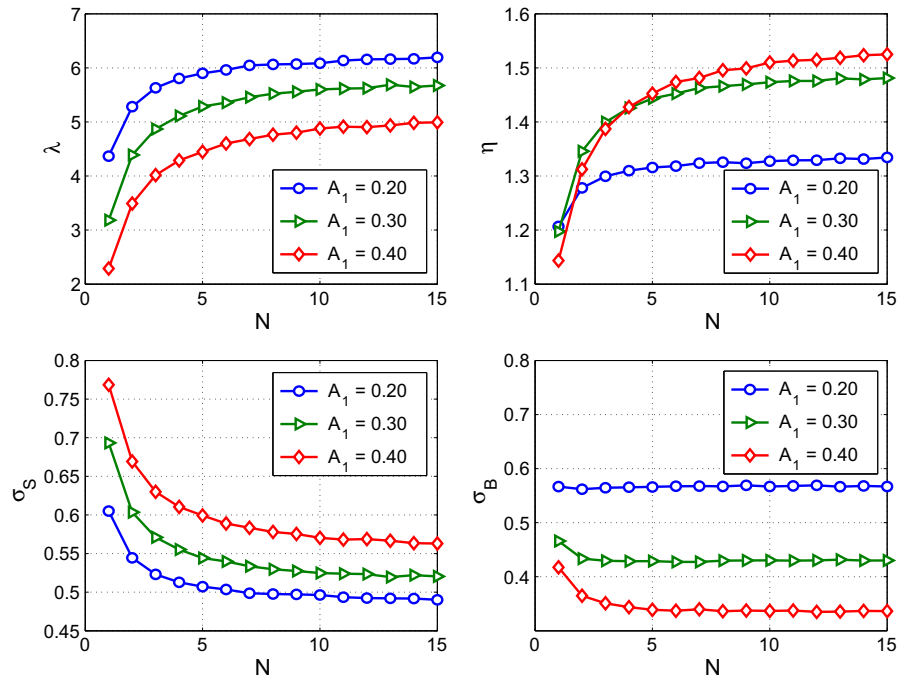


Fig. 14 The performance measures versus the number of venture capital resources in partnership systems with internal investment $A_0 = 0.6$ and risk level $\sigma_0 = 0.1$, and with external capital A_1 , which is equally introduced from N independent resources with equal capital $c_j = 0.06, 0.10$ and 0.15 and unit risk level $\sigma_{1j} = 0.3$

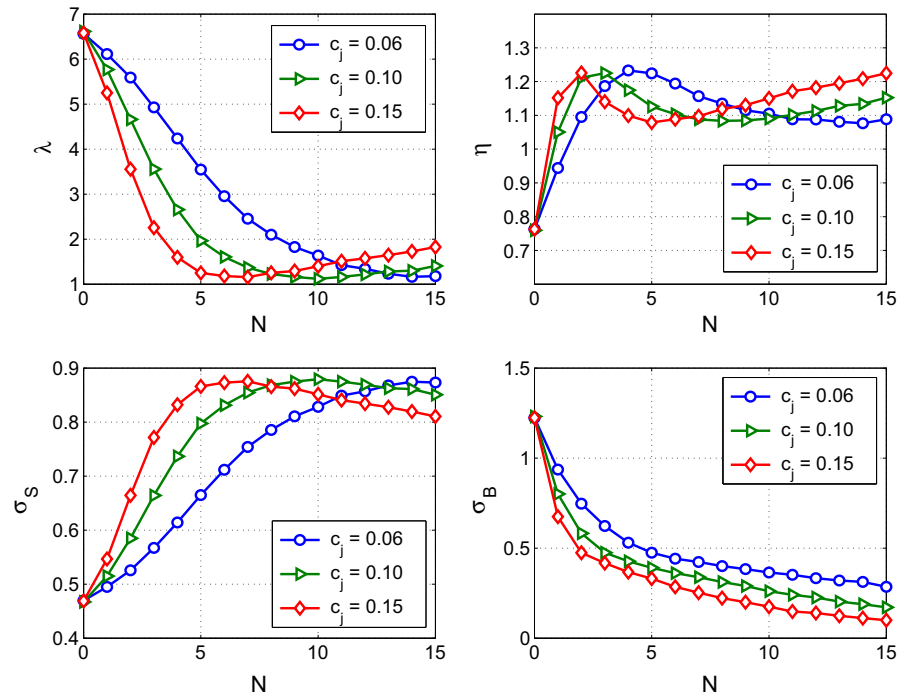


Table 1 Statistical results for SME samples (manufacturing industry)

Partnership structure			Internal investment ^a			Venture capital ^b		
<i>M</i> -lateral	Count	Prob.	<i>A</i> ₀	Count	Prob.	<i>c</i> _{<i>j</i>}	Count	Prob.
1	5968	0.1717	0.0–0.1	2735	0.1583	0.0–0.1	30,919	0.4857
2	17,276	0.4971	0.1–0.2	3062	0.1772	0.1–0.2	17,022	0.2674
3	5463	0.1572	0.2–0.3	3478	0.2013	0.2–0.3	8397	0.1319
4	2643	0.0760	0.3–0.4	722	0.0418	0.3–0.4	4001	0.0629
5	1314	0.0378	0.4–0.5	3047	0.1764	0.4–0.5	1686	0.0265
6	803	0.0231	0.5–0.6	1827	0.1058	0.5–0.6	698	0.0110
7	417	0.0120	0.6–0.7	332	0.0192	0.6–0.7	317	0.0050
8	309	0.0089	0.7–0.8	722	0.0418	0.7–0.8	299	0.0047
9	166	0.0048	0.8–0.9	369	0.0214	0.8–0.9	208	0.0033
≥ 10	396	0.0114	0.9–1.0	982	0.0568	≥ 1.0	113	0.0018

^aSample average $\bar{A}_0 = 0.3904$

^bSample average $\bar{c}_j = 0.1546$, and it satisfies the power-law distribution $P(c_j) = k \times c_j^{-\hat{\phi}}$ with estimated parameters $\hat{\phi} = 3.2368$, $\hat{k} = 4.6727 \times 10^{-4}$

level $\sigma_{1j} = 0.3, j = 1, 2, \dots, N$. In Fig. 14, one can clearly observe SR and reverse SR behaviors of λ and η . It is explained as follows, on the one hand, σ_S firstly increases and then decreases with increasing N . The value of SR peak remains unchanged with the increase of A_1 , and the position shifts obviously from the large value of N to the small value. On the other hand, in partnership system with insufficient $A_0 = 0.6$, the increase of N leads to decrease in probability that the enterprise is in the local optimum of the technology-side, that is, the introduced venture capital decreases the unfair competition of constrained resource, and this has made the marketing-side willing to apply more effort, thus the bilateral risk σ_B decreases, and it is lower with more A_1 . Obviously, it is important for enterprises to determine the optimal number N of venture capital resources to avoid the reverse SR of λ , or to pursuit the SR of η .

For potential applications, we finally extend the simulations to some actual scenes, and illustrate actual phenomenon observed from the practice [8]. Based on the help of HIGGS, a big data sharing platform developed by BBD Inc., (<http://www.bbdservice.com>), we collected 34,755 small and micro enterprise (SME) samples (Jan 2015–Dec 2017) from the sector of manufacturing industry in China.

Here SMEs are defined as the registered capital less than ¥1,000,000, and it is used to normalize the internal investment and external venture capital. The statistical results are listed in Table 1. One can clearly

observe that the bilateral partnership structure with $M = 2$ accounts for 49.71% (17,276 samples), whose introduced venture capital are from 63,660 resources and satisfies the power-law distribution (as shown in Fig. 15a) with the mean $\bar{c}_j = 0.1546$ and estimated parameters $\hat{\phi} = 3.2368, \hat{k} = 4.6727 \times 10^{-4}$. The average number of external capital resources is $\bar{N} = 3.6849$ based on 17,276 bilateral samples ($M = 2$), thus $\bar{A}_1 = 0.5697$. The basic internal and external profiles are listed and selected to set the system parameters. In order to make an objective evaluation of the development state, the information about bank loans and repayments is extracted from 17,276 bilateral samples, in which 9989 samples have the bank records, and there had been 1132 repayment-default samples, as shown in Fig. 15b. Analyzing the default rate, an interesting result is revealed in the subgraph, that is, as N increases from 0 to 6, the rate decreases firstly and then increases, and the similar reverse SR behavior occurs at $N = 3$. It means that more external cooperators are not better, which is against the intuition and traditional understanding.

However, the result can be explained by our proposed bilateral partnership system with mono-stable CDU potential. Combining with the actual scene, the internal investment A_0 is set to the sample average 0.3904, and the external capital $A_1 = \sum_{j=1}^N c_j$ is supposed to be introduced from N independent resources, c_1, c_2, \dots, c_N , which are randomly generated by the

Fig. 15 **a** The estimated power-law distribution of introduced venture capital c_j in SME samples; **b** the default statistics for bank loans and repayments of SME samples

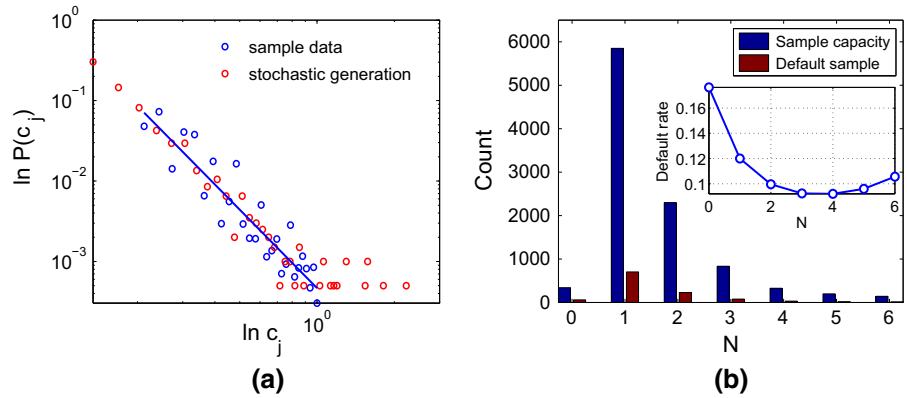
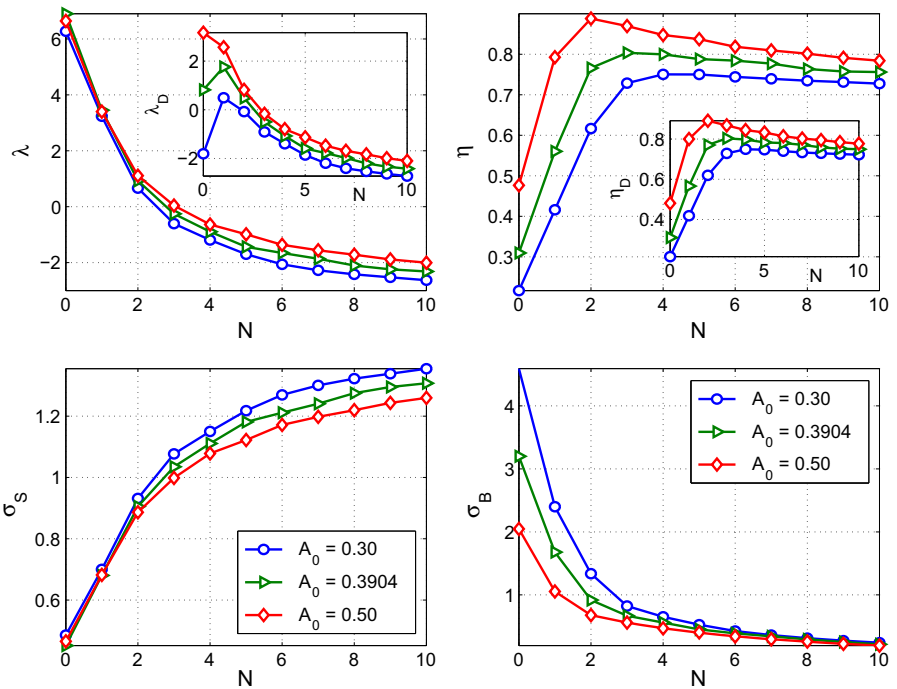


Fig. 16 The performance measures versus the number of venture capital resources in partnership systems with estimated parameters



estimated power-law distribution $P(c_j) = 4.6727 \times 10^{-4} \times c_j^{-3.2368}$, and each capital resource is characterized by unit risk level $\sigma_{1j} = 0.3$. In Fig. 16, the performance indexes are plotted as a function of N . Due to the SR mechanism, it is easy to interpret that the global optimum, in the statistical sense, occurs at the peaks, and the SR peak of stationary URR η for $A_0 = 0.3904$ occurs at $N = 3$, which perfectly matches the sample result shown in Fig. 15b. Besides, analyzing the partnership system using the concepts of the time value, here the discounted performance indexes, λ_D and η_D , from process $x_D(t)$ with rate $r = 0.1$ are considered and the results are included in the subgraph. Moreover,

for any internal state $A_0 \in [0.2, 0.5]$, $\sigma_0 \in [0.05, 0.2]$, the optimal URR η_{opt} partnership system can achieve, and related number N_{opt} is shown in Fig. 17.

6 Conclusion

In summary, we establish a nonlinear stochastic dynamical equation to describe the partnership system and analyze the performance based on linear response theory. A dynamical method and the associated prototype system are developed to beg the questions of how the external venture capital incents the partners especially associated with partnership success and what

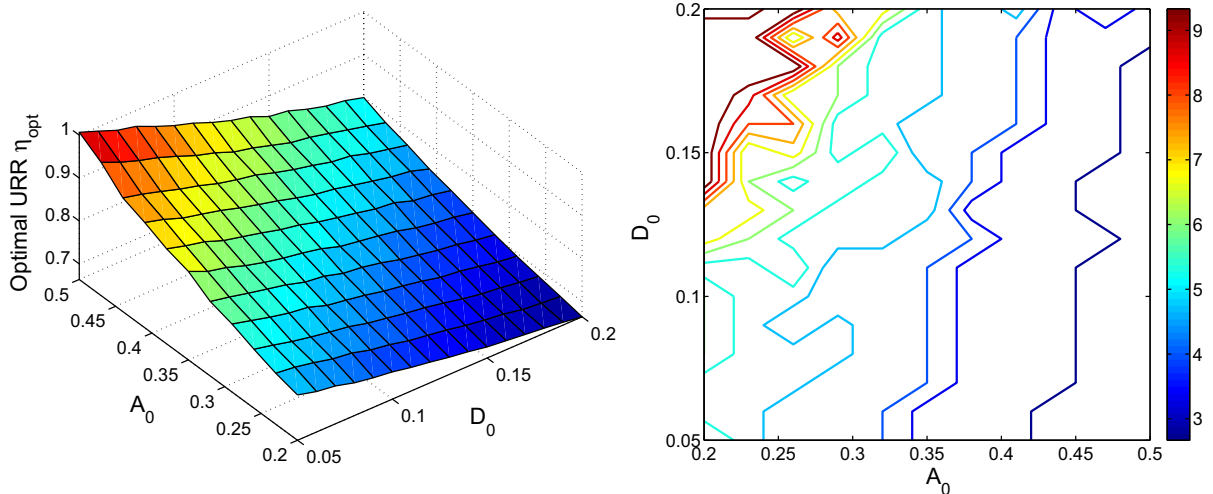


Fig. 17 The optimal URR η_{opt} and related number N_{opt} of venture capital resources in partnership systems with estimated parameters

roles the internal and external risks play, respectively, and two types of risks are proposed to comprehend the monotonic or non-monotonic behaviors, which are all observed in the simulations. We believe that these results can not only supply the theoretical investigations of a new bias mono-stable system, but also be instructive for enterprises to create portfolio strategies of introducing venture capital and optimizing portfolio risk, based on the internal investment and risk profile.

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