Nonlinear Dyn (2018) 91:2541–2550
https://doi.org/10.1007/s11071-017-4030-z

ORIGINAL PAPER

A new model of dry friction oscillator colliding with a rigid obstacle

Madeleine Pascal

Received: 7 September 2017 / Accepted: 21 December 2017 / Published online: 6 February 2018

Abstract We consider a system of two masses connected by linear springs and in contact with a belt moving at a constant velocity. One of the masses can collide with a fixed rigid obstacle. The contact forces between the masses and the belt are given by Coulomb's laws. Moreover, when the colliding mass is in contact with the obstacle, we assume that a perfect elastic impact occurs. Several periodic orbits including contact against the fixed obstacle followed by slip and stick phases are obtained in analytical form.

Keywords Coupled oscillators · Dry friction · Impact · Periodic motions · Stick–slip motions

1 Introduction

Non-smooth dynamical systems are related to force or motion characteristics which are non-continuous. One example of them is dry friction oscillators. Another kind of non-smooth systems is vibrating systems with clearance between the moving parts. In many industrial applications like brake systems, machine tools or turbo machines, the combined actions of dry friction and impact induce some undesirable effects. Non-smooth systems are very complex and they are usually modeled as spring mass oscillators. In the past, such systems have been the subject of several investigations, mainly in the case of one-degree-of-freedom systems [1–8]. For multidegrees-of-freedom systems, very often, only numerical methods have been used [9–12]. However, in [13], a two-degree-of-freedom oscillator with a colliding component is considered and several results about the existence of periodic motions are obtained in analytical form. On the other hand, double dry friction oscillators have been considered in [14,15]. Assuming that the friction forces are modeled by Coulomb's laws, closed-form solutions including stick–slip phases are presented. More recently, in [16], a two-degree-of-freedom vibro-impact system with multiple constraints has been investigated by using the flow switch ability theory of discontinuous systems, while in [17,18], mathematical investigation of a dry friction oscillator in contact with a speed-varying traveling belt or submitted to a switching control law has been performed.

In this paper, a two-degree-of-freedom oscillator excited by dry friction and in the presence of a rigid obstacle is considered. The existence of periodic orbits including an impact with the fixed obstacle and several phases of stick and slip motions is proved.

2 Description of the model

The system (Fig. 1) consists of two masses m_1, m_2 connected by linear springs of stiffness k_1, k_2. The two masses are in contact with a driving belt moving at a constant velocity v_0. Friction forces F_1, F_2 act between

M. Pascal (✉)
Laboratoire IBISC, Universite d'Evry Val d'Essonne, Évry, France
e-mail: madeleine.pascal3@wanadoo.fr

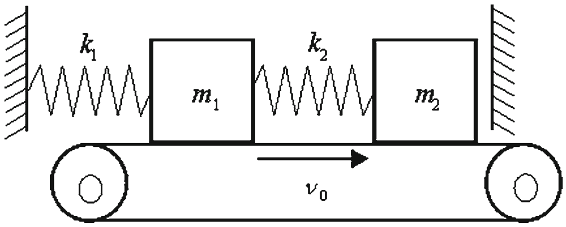

Fig. 1 Description of the model

the masses and the belt. Moreover, the second mass can collide with a fixed rigid obstacle.

Two different cases will be considered:

2.1 Motion impact-less case

This case has been investigated in [14]. The motion equations are given by:

$$M\ddot{y} + Ky = F,\ \ y = (y_1, y_2)^t\ ,\ \ F = (F_1, F_2)^t$$
$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix},\quad K = \begin{pmatrix} k_1+k_2 & -k_2 \\ -k_2 & -k_2 \end{pmatrix},$$
$$y_2 < e,\quad \ddot{y} = \frac{d^2y}{dt'^2} \tag{1}$$

Here (y_1, y_2) are the displacements of the masses, e is the clearance ,and (F_1, F_2) are the contact friction forces obtained from Coulomb's laws:

- $v_0 - \dot{y}_i \neq 0, \quad F_i = F_{si}\text{sign}(v_0 - \dot{y}_i)\ (i = 1, 2)$
- $v_0 - \dot{y}_1 = 0$

$$F_1 = \begin{cases} (k_1+k_2)y_1 - k_2y_2 \text{ if } |(k_1+k_2)y_1 - k_2y_2| < F_{r1} \\ \varepsilon F_{s1} \text{ if } \varepsilon[(k_1+k_2)y_1 - k_2y_2] > F_{r1}(\varepsilon = \pm 1) \end{cases} \tag{2}$$

- $v_0 - \dot{y}_2 = 0$

$$F_2 = \begin{cases} k_2(y_2 - y_1) & \text{if } |k_2(y_2 - y_1)| < F_{r2} \\ \varepsilon F_{s2} & \text{if } \varepsilon k_2(y_2 - y_1) > F_{r2}(\varepsilon = \pm 1) \end{cases}$$

F_{s1}, F_{s2} are the friction forces when slip motion occurs, while F_{r1}, F_{r2} are the static friction forces $(F_{si} < F_{ri})$.

The systems (1), (2) are normalized using:

$$t = \omega_3 t', \omega_3 = \sqrt{\frac{k_1+k_2}{m_1}},\quad (o') = \frac{d(o)}{dt},$$
$$x_i = \frac{y_i}{e}, (i = 1, 2), V = \frac{v_0}{e\omega_3} \tag{3}$$

From (1), it follows:

$$x'' + \tilde{K}x = R, x = (x_1, x_2)^t\ , x_2 < 1$$
$$\tilde{K} = \begin{pmatrix} 1 & -\chi \\ -\chi\eta & \chi\eta \end{pmatrix},\quad R = (u_1, \eta u_2)^t$$
$$\chi = \frac{k_2}{k_1+k_2},\quad \eta = \frac{m_1}{m_2},$$
$$u_i = \frac{F_i}{(k_1+k_2)e}(i = 1, 2) \tag{4}$$

For each mass, three kinds of motions occur: slip motion with a velocity less than the belt velocity, overshoot motion with a velocity greater than the belt velocity and stick motion with a velocity equal to the belt one. For each kind of motion, the closed-form solution is available [14]. In the following, instead of the parameters (x, x'), a new set of variables is introduced:

$$Z = \begin{pmatrix} z \\ z' \end{pmatrix},\quad z = x - d_0,\quad d_0 = (d_{01}, d_{02})^t$$
$$z' = x',\quad d_{01} = \frac{u_{s1} + u_{s2}}{1-\chi},$$
$$d_{02} = \frac{\chi u_{s1} + u_{s2}}{\chi(1-\chi)}. \tag{5}$$

2.2 Rigid impact

In the case of a contact of the second mass with the stop at $t = \tau$, i.e., $x_2(\tau) = 1$, the positions and the velocities $x^-(\tau)$, $x'^-(\tau)$ of the system before the impact and the positions and the velocities $x^+(\tau)$, $x'^+(\tau)$ after the impact are related by:

$$x^+(\tau) = x^-(\tau),\ \ x'^+(\tau) = \text{E}x'^-(\tau),\ \ E = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

Several transitions between all these kinds of regimes (i.e., slip motion, overshooting motion, stick motion and contact) can occur. In [7,8], a more simple system, with only one-degree-of-freedom has been investigated. Several periodic motions including a shock have been obtained. In the following, we show that a similar investigation can be performed for this more complex system.

3 First example of periodic orbits with impact (symmetric solution)

Let us assume the following initial conditions:

$$x_{10}, x_{20} = 1, x'_{10}, 0 < x'_{20} < V \tag{7}$$

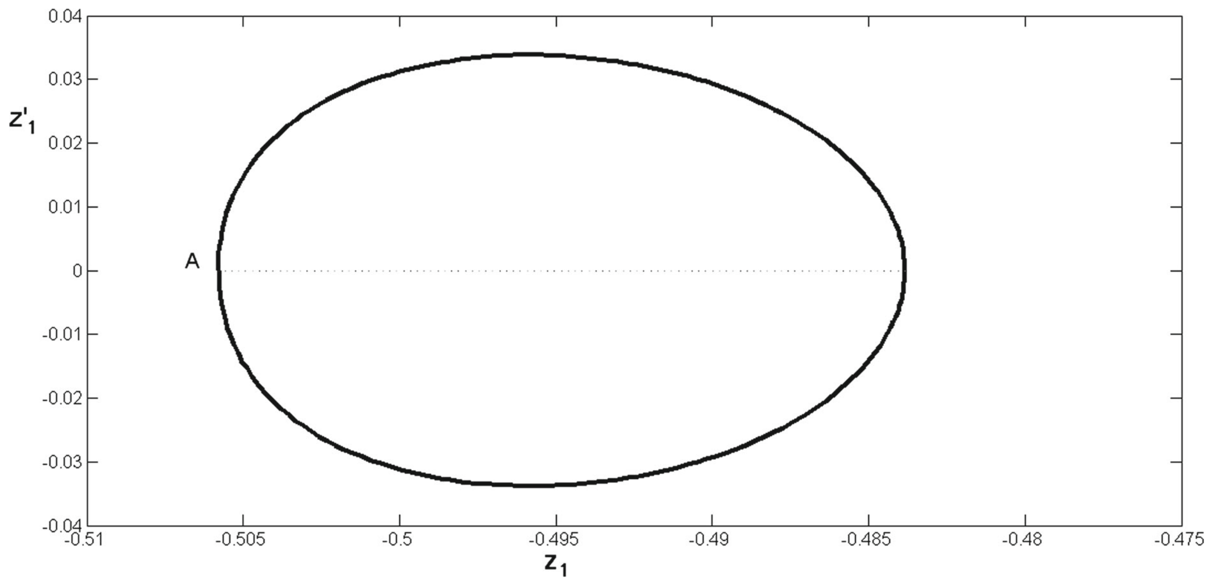

Fig. 2 Phase portrait of the non-colliding mass

At $t = 0$ an impact occurs. After the impact, the positions and the velocities of the system are obtained by the formula:

$$Z_0^+ = H_0 Z_0,\ \ H_0 = \begin{pmatrix} I & 0 \\ 0 & E \end{pmatrix},$$
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \ Z_0 = Z(0) \tag{8}$$

For $0 < t < T$, the system undergoes a slip–slip motion given by

$$Z(t) = H(t) Z_0^+,\ \ H(t) = \begin{pmatrix} H_1(t) & H_2(t) \\ H_3(t) & H_1(t) \end{pmatrix} \tag{9}$$

The two-by-two matrices $H_i(t), (i = 1, 2, 3)$ are obtained [14] from a modal analysis of system (4). A periodic solution of period T is obtained if

$$Z_0 = H(T) Z_0^+ = H(T) H_0 Z_0 \tag{10}$$

From (10), we deduce:

$$(H_1 - I) z_0 - H_2 z_0' = 0,\ \ -H_3 z_0 + (H_1 - E) z_0' = 0$$
$$H_i = H_i(T), (i = 1, 2, 3) \tag{11}$$

Taking into account the following properties [13] of the H_i matrices:

$$H_1^2 - H_2 H_3 = I,\ \ H_i H_j = H_j H_i,\ (i, j = 1, 2, 3) \tag{12}$$

(11) gives the relation:

$$H_2 (E + I) z_0' = 0,\quad \text{if } \det(H_2) \neq 0, (E + I) z_0' = 0 \tag{13}$$

From (13), as in Ref. [13], we deduce the initial conditions for this orbit:

$$z_{10}' = 0, z_0 = (H_1 - I)^{-1} H_2 z_0' \tag{14}$$

This periodic orbit depends on 3 parameters (z_{10}, z_{20}', T) defined by 2 scalar equations deduced from (14). As in Ref. [13], the period T can be chosen, and the two parameters (z_{10}, z_{20}') are obtained from (14) in term of the period. Moreover, as in [14], an interesting property of symmetry for the phase portraits of the system is obtained (see "Appendix").

Example: For

$$\eta = 4,\ \chi = .3,\ \ T = 2, u_{s1} = 0.5,\ \ u_{s2} = 0.3,$$
$$V = 2,\ \ u_{r1} = 0.6,\ \ u_{r2} = 0.4 \tag{15}$$

we obtain:

$$z_{10} = -0.5058,\ \ z_{20} = -1.1429,\ \ z_{20}' = 1.3837 \tag{16}$$

The phase portraits of (m_1, m_2) are shown in Figs. 2 and 3.

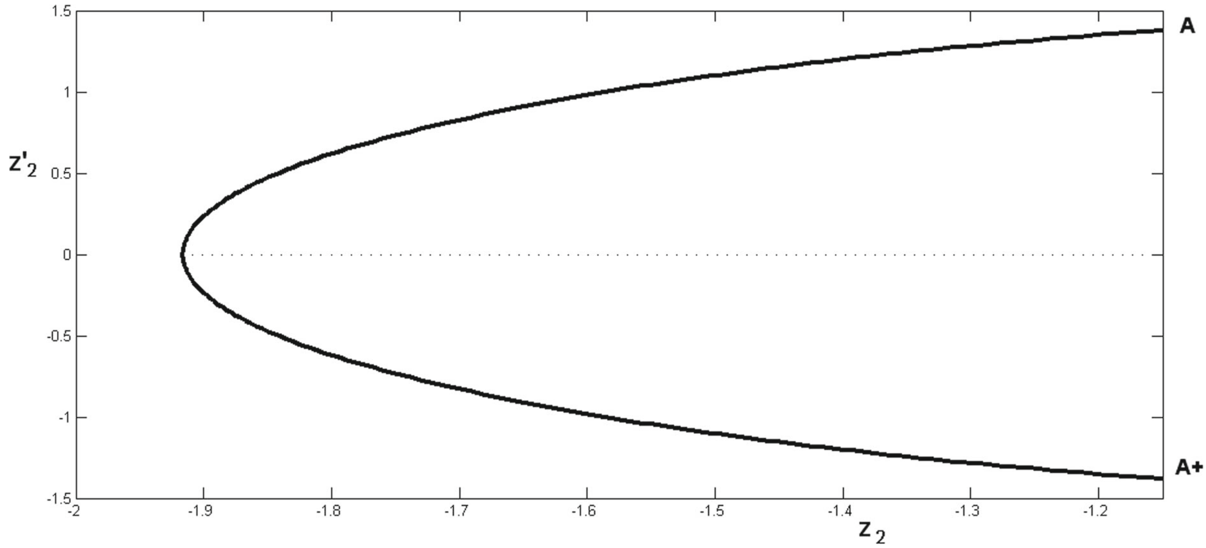

Fig. 3 Phase portrait of the colliding mass

4 Second kind of periodic solutions with impact

Let us consider the following initial conditions:

$$x_{10}, x_{20} < 1,\ x'_{10} < V,\ x'_{20} = V, |\chi(x_{20} - x_{10}| < u_{r2} \tag{17}$$

For $0 < t < \tau$, the system performs a slip–stick motion [14] given by

$$Z(t) = \Gamma(t) Z_0, \Gamma(t) = \begin{pmatrix} \Gamma_1(t) & \Gamma_2(t) \\ \Gamma_3(t) & \Gamma_1(t) \end{pmatrix} \tag{18}$$

This motion ends at $t = \tau$ if at this time

$$x_{2B} \equiv x_2(\tau) = 1 \tag{19}$$

At $t = \tau$, an impact occurs and we have:

$$Z_B^+ = H_0 Z_B, Z_B \equiv Z(\tau) = \Gamma(\tau) Z_0 \tag{20}$$

For $0 < t - \tau < \tau_1$, the system undergoes a slip–slip motion. A periodic motion of period

$$T = \tau + \tau_1 \quad \text{is obtained if } Z_0 = H(\tau_1) Z_B^+ \\ = H(\tau_1) H_0 \Gamma(\tau) Z_0 \tag{21}$$

This motion depends on 5 parameters ($z_{10}, z_{20}, z'_{10}, \tau, \tau_1$). From (19), (21), we obtain 5 scalar equations for the determination of these parameters.

Example: For

$$\eta = 4,\ \chi = 0.3,\ V = 1,\ u_{r1} = 0.5, \\ u_{s1} = 0.1,\ u_{r2} = 0.4,\ u_{s2} = 0.0225 \tag{22}$$

we obtain:

$$\tau = 2, \tau_1 = 2.5075,\ z_{10} = 0.1015, z_{20} \\ = -1.2502,\ z'_{10} = -0.2532 \tag{23}$$

The phase portraits of m_1 and m_2 are shown in Figs. 4 and 5, and the curves

$$f_1 = \chi(z_2 - z_1) + u_{s2} - u_{r2}, \\ f_2 = \chi(z_2 - z_1) + u_{s2} + u_{r2} \tag{24}$$

connected to the constraints ($f_1 \leq 0,\ f_2 > 0$) during the sticking motion of m_2 are shown in Fig. 6.

5 Third kind of periodic solutions with impact

Let us assume the following initial conditions:

$$x'_{10} = x'_{20} = V,\ \ x_{10} - \chi x_{20} = u_{r1}, \\ \chi(x_{20} - x_{10}) < -u_{r2} \tag{25}$$

For $0 < t < \tau$, the system performs a slip–overshoot motion:

($x'_1 < V, x'_2 > V,\ u_1 = u_{s1},\ u_2 = -u_{s2}$) given by [14]:

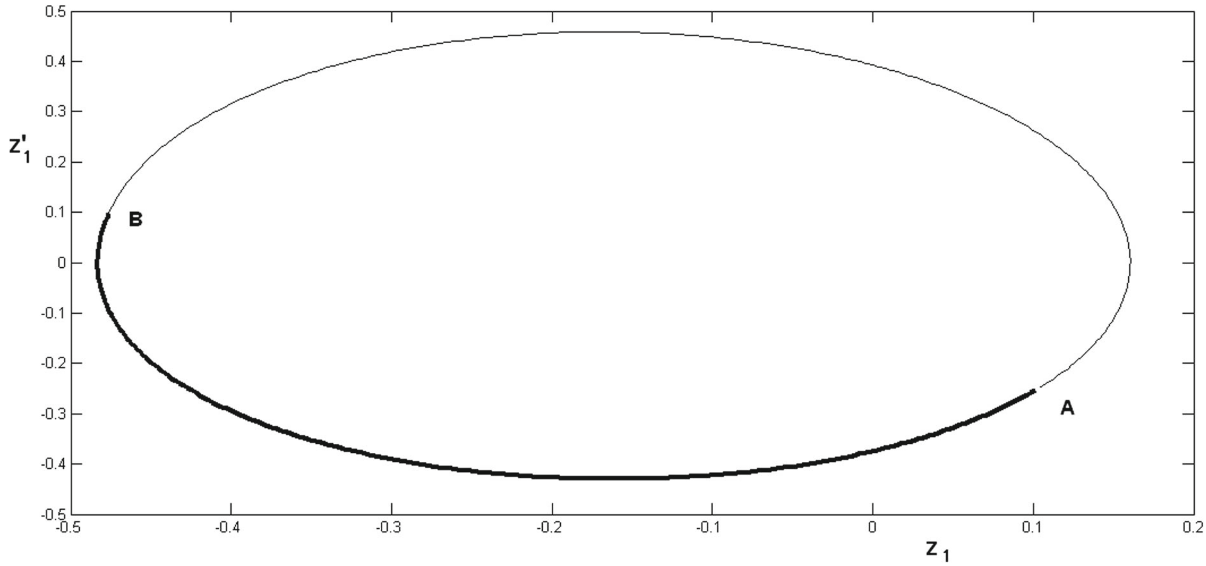

Fig. 4 Phase portrait of the non-colliding mass

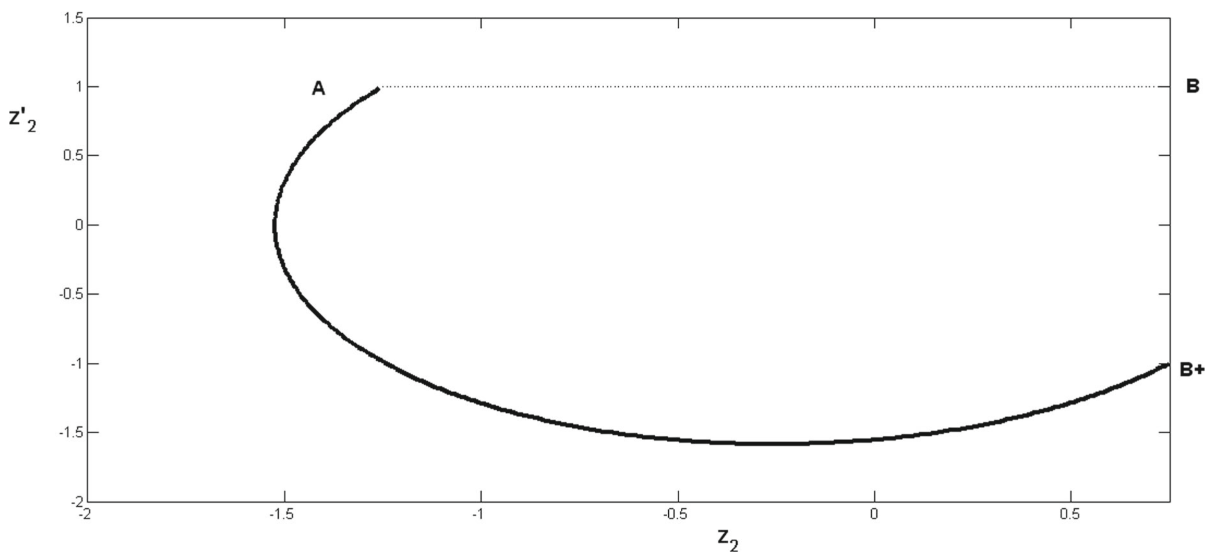

Fig. 5 Phase portrait of the colliding mass

$$Z(t) = H(t)Z_0 + 2u_{s2}(H(t) - I_4)A_0$$
$$A_0 = \begin{pmatrix} \alpha_0 \\ 0 \end{pmatrix}, \alpha_0 = \begin{pmatrix} 1/(1-\chi) \\ 1/\chi(1-\chi) \end{pmatrix}, I_4 = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{26}$$

This motion ends at $t = \tau$, if

$$x'_{1B} < V,\ x'_{2B} = V,\ -u_{r2} < \chi(x_{2B} - x_{1B}) < u_{r2},$$
$$x_{iB} \equiv x_i(\tau), x'_{iB} \equiv x'_i(\tau), (i = 1, 2) \tag{27}$$

At this time, we have:

$$Z_B = HZ_0 + 2u_{s2}(H - I_4)A_0,\ Z_B \equiv Z(\tau),\ H \equiv H(\tau) \tag{28}$$

For $0 < t - \tau < \tau_1$, the system performs a slip–stick motion ($x'_1 < V, x'_2 = V, u_1 = u_{s1}$).

This motion ends at $t = \tau + \tau_1$ if at this time:

$$x_{2C} \equiv x_2(\tau + \tau_1) = 1,\ \chi\,|x_{2C} - x_{1C}| < u_{r2} \tag{29}$$

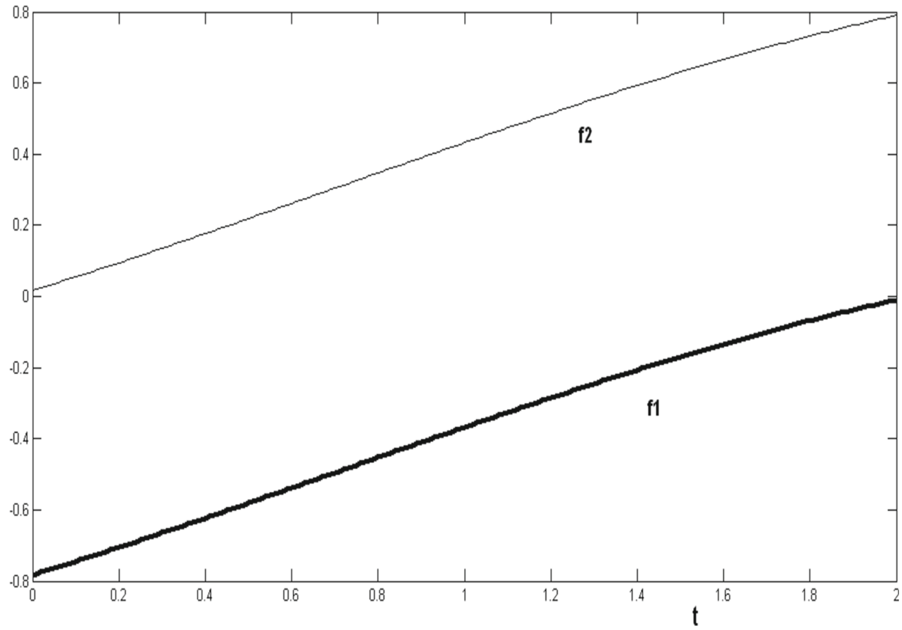

Fig. 6 Constraints during the sticking motion of m_2

After the impact, the positions and the velocities of the system are obtained from:

$$Z_C^+ = H_0 Z_C, Z_C = \Gamma Z_B, \Gamma \equiv \Gamma(\tau_1) \tag{30}$$

For $0 < t-\tau-\tau_1 < \tau_2$, the system undergoes a slip–slip motion ($x_1' < V, x_2' < V, u_1 = u_{s1}, u_2 = u_{s2}$)

This motion ends at $t = \tau + \tau_1 + \tau_2$ if at this time we have:

$$x_{1D}' \equiv x_1'(\tau + \tau_1 + \tau_2) = V, |x_{1D} - \chi x_{2D}| < u_{r1}$$
$$Z_D \equiv Z(\tau + \tau_1 + \tau_2) = hZ_C^+, h \equiv H(\tau_2) \tag{31}$$

For $0 < t - \tau - \tau_1 - \tau_2 < \tau_3$, the system performs a stick–slip motion ($x_1' = V, x_2' < V, u_2 = u_{s2}$) given by [14]:

$$Z(t - \tau - \tau_1 - \tau_2) = C(t)Z_D,$$
$$C(t) = \begin{pmatrix} C_1(t) & C_2(t) \\ C_3(t) & C_1(t) \end{pmatrix} \tag{32}$$

A periodic solution of period $T = \tau + \tau_1 + \tau_2 + \tau_3$ is obtained if

$$Z_D = C(-\tau_3)Z_0, Z_D = QZ_0 + 2u_{s2}(Q - hH_0)A_0$$
$$Q = hH_0\Gamma H \tag{33}$$

This solution depends on 6 parameters ($x_{10}, x_{20}, \tau, \tau_1, \tau_2, \tau_3$). Taking into account that the condition (31) is included in the periodicity conditions (33), these parameters are linked by 7 scalar equations deduced from (25), (27), (29), (33).

We solve a semi-inverse problem, assuming that u_{r1} is defined by (25). The solution is obtained by solving the 6 scalar equations deduced from (27), (29), (33) with respect to ($x_{10}, x_{20}, \tau, \tau_1, \tau_2, \tau_3$).

Example: For

$$\eta = 1,\ \chi = 0.2,\ V = 0.556,\ u_{s1} = 0.0535,$$
$$u_{r1} = 1.2018,\ u_{s2} = 0.2161,\ u_{r2} = 0.28 \tag{34}$$

we obtain:

$$\tau = 1.5775,\ \tau_1 = 1,\ \tau_2 = 1.5,\ \tau_3 = 3.281,$$
$$z_{10} = 0.7653,\ z_{20} = -1.9152 \tag{35}$$

The phase portraits of m_1 and m_2 are shown in Figs. 7 and 8. Moreover, the constraints during the sticking motion of $m_2 (0 < t - \tau < \tau_1)$ and the constraints during the sticking motion of $m_1 (0 < t-\tau-\tau_1-\tau_2 < \tau_3)$ are fulfilled.

6 Fourth kind of periodic solutions with impact

Let us consider the following initial conditions:

$$x_{10}' = x_{20}' = V, x_{20} = 1, x_{10} - \chi x_{20}$$
$$< -u_{r1}, \chi\,|x_{20} - x_{10}| < u_{r2} \tag{36}$$

At $t = 0$, an impact of the second mass occurs with the post-impact rule:

$$Z_0^+ = H_0 Z_0 \tag{37}$$

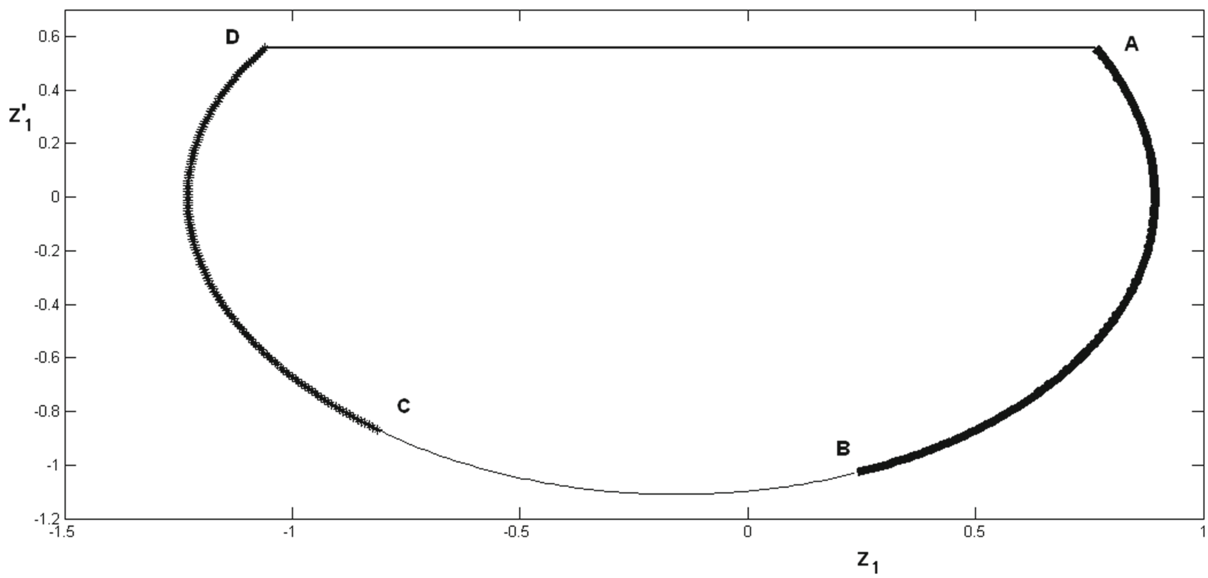

Fig. 7 Phase portrait of the non-colliding mass

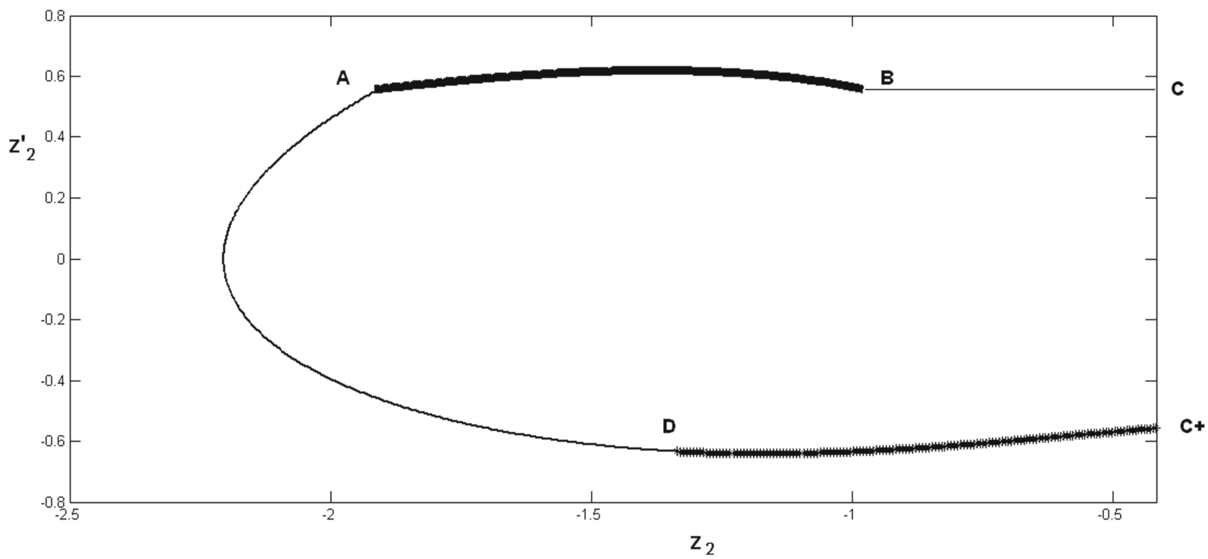

Fig. 8 Phase portrait of the colliding mass

For $0 < t < \tau$, the system performs an overshoot–slip motion $(x_1' > V, x_2' < V)$. This motion ends at $t = \tau$ if at this time, we have:

$$x_{1B}' \equiv x_1'(\tau) = V \tag{38}$$

The positions and the velocities at $t = \tau$ are obtained from the formula [14]:

$$Z_B \equiv Z(\tau) = H(\tau)Z_0^+ + 2u_{s1}(H(\tau) - I_4)B_0$$

$$B_0 = \begin{pmatrix} \beta_0 \\ 0 \end{pmatrix}, \beta_0 = \begin{pmatrix} 1/(1-\chi) \\ 1/(1-\chi) \end{pmatrix} \tag{39}$$

Let us assume the following properties:

$$|x_{1B} - \chi x_{2B}| < u_{r1}, x_{2B}' < V \tag{40}$$

For $0 < t - \tau < \tau_1$, the system undergoes a stick–slip motion $(x_1' = V, x_2' < V)$

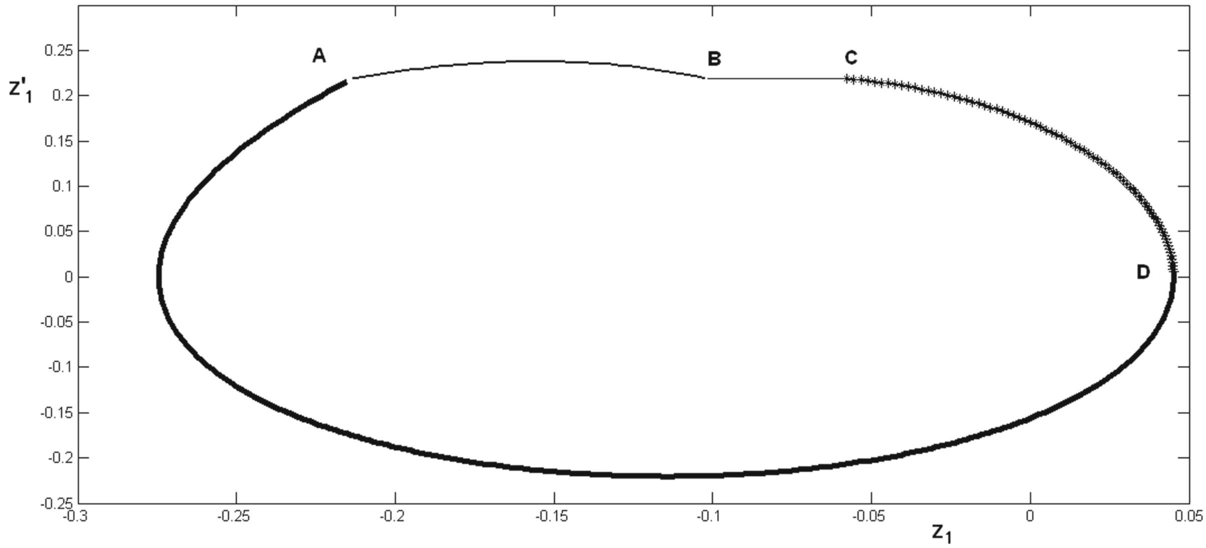

Fig. 9 Phase portrait of the non-colliding mass

This motion ends at $t = \tau + \tau_1$ if at this time, we have:

$$x_{1C} - \chi x_{2C} = u_{r1} \text{ where } Z_C \equiv Z(\tau+\tau_1) = C(\tau_1)Z_B \tag{41}$$

For $0 < t - \tau - \tau_1 < \tau_2$, the system performs a slip–slip motion ($x'_1 < V, x'_2 < V$). This motion ends at $t = \tau + \tau_1 + \tau_2$ if at this time, we have:

$$x'_{2D} = V \text{ where } Z_D \equiv Z(\tau + \tau_1 + \tau_2) = H(\tau_2)Z_C \tag{42}$$

Let us assume that at this time, we have:

$$x'_{1D} < V, \chi\,|x_{2D} - x_{1D}| < u_{r2} \tag{43}$$

For $0 < t - \tau - \tau_1 - \tau_2 < \tau_3$, the system undergoes a slip–stick motion ($x'_1 < V, x'_2 = V$). A periodic motion of period $T = \tau + \tau_1 + \tau_2 + \tau_3$ is obtained if we have:

$$Z_0 = \Gamma(\tau_3)Z_D \tag{44}$$

This motion depends on 5 parameters ($x_{10}, \tau, \tau_1, \tau_2, \tau_3$). Assuming that u_{r1} is defined by (41), these parameters are subjected to 5 scalar conditions related to (38) and (44), taking into account that (42) is included in (44).

Example: For

$$\eta = 4.2, \chi = 0.8,\ V = 0.2186, u_{s1} = 0.1278,\ u_{s2} = 0.0219,\ u_{r2} = 0.4 \tag{45}$$

we obtain:

$$\tau = 0.4845,\ \tau_1 = 0.2,\ \tau_2 = 0.78,\ \tau_3 = 2.809,\ u_{r1} = 0.1921, z_{10} = -0.2134,\ z_{20} = 0.2243 \tag{46}$$

The phase portraits of the system are shown in Figs. 9 and 10.

Moreover, the constraints during the sticking motion of $m_1(0 < t - \tau < \tau_1)$ and the constraints during the sticking motion of $m_2(0 < t - \tau - \tau_1 - \tau_2 < \tau_3)$ are fulfilled.

7 Concluding remarks

In this work, a two-degree-of-freedom oscillator excited by a moving base with constant velocity and colliding with a rigid obstacle is considered. This system is strongly nonlinear; however, assuming that the dry friction forces are given by the Coulomb's laws and assuming perfect elastic impact when a collision with the obstacle occurs, several sets of periodic motions including some phases of slip motion, overshooting motion, sticking motion and impact are found in analytic form.

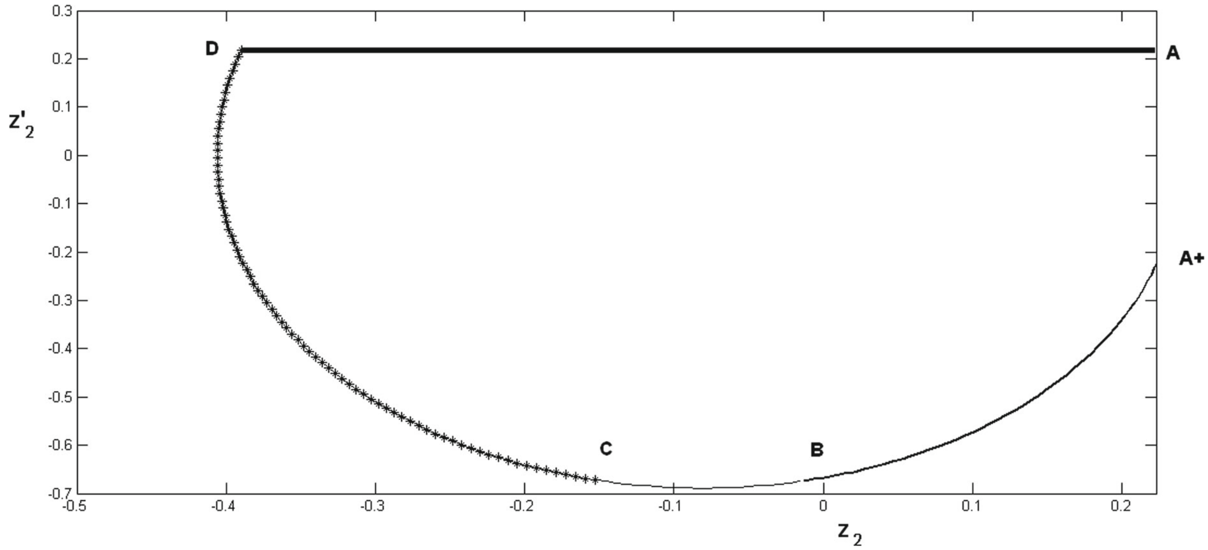

Fig. 10 Phase portrait of the colliding mass

Appendix: Symmetry of the solution

For $T/2 < t < T$, the motion is defined by

$$Z(t) = H(t)H_0 Z_0,\ Z_0 = \begin{pmatrix} z_0 \\ z_0' \end{pmatrix}$$

$$z_0' = \begin{pmatrix} 0 \\ z_{20}' \end{pmatrix},\ Z(T) = H(T)H_0 Z_0 = Z_0 \tag{47}$$

From (47) it results

$$Z(T-t) = H(T-t)H_0 Z_0 \tag{48}$$

From the following properties [13] of the matrix $H(t)$:

$$H(T-t) = H(-t)H(T)$$

$$H(-t) = FH(t)F,\ F = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{49}$$

we deduce

$$Z(T-t) = FH(t)FZ_0 \tag{50}$$

Taking into account that $z_{10}' = 0$, $FZ_0 = H_0 Z_0$. It results:

$$Z(T-t) = FZ(t) \tag{51}$$

The phase portrait of the first mass is symmetric with respect to the line

$z_1' = 0$ and the phase portrait of the second mass is symmetric with the line $z_2' = 0$.

References

1. Shaw, S.W., Holmes, P.J.: A periodically forced piecewise linear oscillator. J. Sound Vib. **90**(1), 129–155 (1983)
2. Hindmarsh, M.B., Jeffries, D.J.: On the motions of the impact oscillator. J. Phys. A **17**, 1791–1803 (1984)
3. Hong, H.K., Liu, C.S.: Non-sticking formulae for Coulomb friction under harmonic loading. J. Sound Vib. **244**(5), 883–898 (2001)
4. Csernak, G., Stepan, G.: On the periodic response of a harmonically excited dry friction oscillator. J. Sound Vib. **295**(4), 649–658 (2006)
5. Andreaus, U.: Sliding-uplifting response of rigid blocks to base excitation. Earthq. Eng. Struct. Dyn. **19**(8), 1181–1196 (1990)
6. Andreaus U., Casini P.: Forced response of a SDOF friction oscillator colliding with a hysteretic obstacle. In: Proceedings of DETC'01 ASME 2001 Design Engineering Technical Conferences and Computers and Information in Engineering Conference. Pittsburgh, Pennsylvania, September 9–12 (2001)
7. Andreaus, U., Casini, P.: Forced motion of friction oscillators limited by a rigid or deformable obstacle. Mech. Struct. Mach. **29**(2), 177–198 (2001)
8. Andreaus, U., Casini, P.: Friction oscillator excited by moving base and colliding with a rigid or deformable obstacle. Int. J. Non Linear Mech. **37**, 117–133 (2002)
9. Aidanpan, J.O., Gupta, R.D.: Periodic and chaotic behavior of a threshold-limited two degree of freedom system. J. Sound Vib. **165**(2), 305–327 (1993)

10. Valente, A.X., McClamroch, N.H., Mezie, I.: Hybrid impact of two coupled oscillators that can impact a fixed stop. Int. J. Non Linear Mech. **38**, 677–689 (2003)
11. Galvanetto, U., Bishop, S.R.: Stick–slip vibrations of a 2-degree-of-freedom geophysical fault model. Int. J. Mech. Sci. **36**(8), 683–698 (1994)
12. Khizgiyayev, S.V.: Self-excited oscillations of a two-mass oscillator with dry stick–slip friction. J. Appl. Math. Mech. **71**, 905–913 (2007)
13. Pascal, M.: Dynamics and stability of a two degree of freedom oscillator with an elastic stop. J. Comput. Nonlinear Dyn. **1**(1), 94–102 (2006)
14. Pascal, M.: Dynamics of coupled oscillators excited by dry friction. ASME J. Comput. Nonlinear Dyn. **3**(3), 20–26 (2008)
15. Pascal, M.: New events in stick–slip oscillators behaviour. J. Appl. Math. Mech. **75**(3), 402–409 (2011)
16. Xue, S., Fan, J.: Discontinuous dynamical behaviors in a vibro-impact system with multiple constraints. Int. J. Non Linear Mech. **98**, 75–101 (2018)
17. Fan, J., Li, S., Chen, G.: On dynamical behavior of a friction-induced oscillator with 2-DOF on a speed-varying traveling belt. In: Mathematical Problems in Engineering, 2017, Article ID 1208563 (2017)
18. Fan, J., Xue, S., Li, S.: Analysis of dynamical behaviors of a friction-induced oscillator with switching control law. Chaos Solitons Fractals **103**, 513–531 (2017)