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CORRECTION

Correction to: Stabilization of a class of fractional-order chaotic systems using a non-smooth control methodology

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Abstract This note provides a corrigendum to the paper "Stabilization of a class of fractional-order chaotic systems using a non-smooth control methodology" [Nonlinear Dynamics, 89 (2017) 1357–1370]. It is pointed out that we have relied on a wrong formula in [2] to compute the upper bounds of the finite settling times of the proposed control methods in [1]. Fortunately, the minor errors appeared in [1] are readily corrected via the Mean Value Theorem for definite integrals. It is proved that the mentioned minor errors do not affect the main results and designs of the control methods in [1].

Keywords Sliding mode control · Fractional-order system · Mean value theorem

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1 Main discussion

In the recently published paper [1], the author obtained the following inequality for the Lyapunov function of the proposed control method (see Eq. (24) in [1]):

$$V_1(t) - V_1(0) \leq -\frac{\beta\eta}{\sqrt{0.5}\Gamma(q)} \int_0^t (V_1(\tau))^{0.5} (t-\tau)^{q-1} d\tau \qquad (1)$$

Then, the author relied on a formula published in [2] (the formula appears in the last line of page 489 in [2]) to compute a bound for the finite settling time of the proposed control approach (see Eq. (25) in [1]). Letting $V_1(t) - V_1(0) = \int_0^t \dot{V}_1(\tau) d\tau$ yields

$$\int_0^t \frac{\dot{V}_1(\tau)}{(V_1(\tau))^{0.5}} d\tau \leq -\frac{\beta\eta}{\sqrt{0.5}\Gamma(q)} \int_0^t (t-\tau)^{q-1} d\tau \qquad (2)$$

However, since $(V_1(\tau))^{0.5}$ is part of the integrand function, it is not possible to eliminate it by division. Here, it is shown that although inequality (2) cannot be obtained from (1), the main results obtained in [1] are still correct and the finite-time stability is guaranteed.

In this note, we use the well-known first Mean Value Theorem (MVT) for definite integrals to achieve an upper bound for the settling time of the system. First, the MVT is restated below.

Theorem 1 [3]. *Consider a continuous function $f(t)$ on the closed interval $[a, b]$. There is at least one number $c \in (a, b)$, such that the following equality holds:*

$$\int_a^b f(\tau) d\tau = f(c)(b-a) \qquad (3)$$

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Now, in order to obtain the finite settling time, replacing t by T_1 in (1) to obtain a definite integral and applying MVT to the right-hand side of Eq. (1) (Eq. (24) in [1]), one obtains

$$V_1(T_1) - V_1(0) \leq -\frac{\beta\eta}{\sqrt{0.5}\Gamma(q)} \int_0^{T_1} (V_1(\tau))^{0.5} (t-\tau)^{q-1} \, d\tau$$
$$= -\frac{\beta\eta T_1}{\sqrt{0.5}\Gamma(q)} (V_1(c_1))^{0.5} (T_1 - c_1)^{q-1} \quad (4)$$

where $c_1 \in (0, t)$ is a finite number.

Setting $V_1(T_1) \equiv 0$ for $0 < T_1 \leq t \leq \infty$, we have

$$\frac{\beta\eta T_1}{\sqrt{0.5}\Gamma(q)} (V_1(c_1))^{0.5} (T_1 - c_1)^{q-1} \leq V_1(0)$$
$$\rightarrow T_1 (T_1 - c_1)^{q-1} \leq \frac{\sqrt{0.5}\Gamma(q) V_1(0)}{\beta\eta (V_1(c_1))^{0.5}} \quad (5)$$

Owing to $q - 1 < 0$ and $(T_1 - c_1) > 0$, the inequality $T_1 (T_1 - c_1)^{q-1} \geq (T_1 - c_1)(T_1 - c_1)^{q-1} = (T_1 - c_1)^q$ is always satisfied. According to this fact and using (5), one obtains

$$(T_1 - c_1)^q \leq \frac{\sqrt{0.5}\Gamma(q) V_1(0)}{\beta\eta (V_1(c_1))^{0.5}} \quad (6)$$

Therefore, the finite settling time for the sliding motion of the method proposed in [1] is achieved as follows:

$$T_1 \leq \sqrt[q]{\frac{\sqrt{0.5}\Gamma(q) V_1(0)}{\beta\eta (V_1(c_1))^{0.5}}} + c_1 \quad (7)$$

Thus, the origin of the sliding mode dynamics (15) in [1] is finite-time stable with the settling time (7) instead of that given in (27) of [1].

Similarly, the settling times T_2, T_3, T_4 and T_5 expressed in Theorems 3, 4 and 5 in [1], respectively, are modified as follows:

$$T_2 \leq \sqrt[q]{\frac{\sqrt{0.5}\Gamma(q) V_2(0)}{k (V_2(c_2))^{0.5}}} + c_2 \quad (8)$$

$$T_3 \leq \sqrt[q]{\frac{\sqrt{0.5}\Gamma(q) V_3(0)}{(K - \rho - \theta)(V_3(c_3))^{0.5}}} + c_3 \quad (9)$$

$$T_4 \leq \sqrt[q]{\frac{\sqrt{0.5}\Gamma(q) V_4(0)}{\beta\eta (V_4(c_4))^{0.5}}} + c_4 \quad (10)$$

$$T_5 \leq \sqrt[q]{\frac{\sqrt{0.5}\Gamma(q) V_5(0)}{(L - \rho - \theta - \gamma)(V_5(c_5))^{0.5}}} + c_5 \quad (11)$$

where $c_2, c_3, c_4, c_5 \in (0, t)$ are finite numbers.

Remark 1 Based on the above discussions and corrections, it is clear that the mentioned minor mistakes do not influence the main results of the original work [1] and the finite-time stability of the proposed control methodologies in [1] is still ensured.

2 Conclusion

In this note, some minor errors appeared in the computation the settling times in [1] were pointed out. The errors were due to the fact that some wrong results of [2] were exploited. Accordingly, we readily modified the finite settling times in [1] using the well-known Mean Value Theorem for definite integrals. We proved that the introduced minor errors do not have serious consequences on the main results of the original paper [1].

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Compliance with Ethical Standards

Conflict of Interest The author declares that he has no conflict of interest.

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