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ORIGINAL PAPER

Sliding mode control with an extended disturbance observer for a class of underactuated system in cascaded form

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Abstract A sliding mode controller based on an extended disturbance observer is investigated to control a class of underactuated system in this paper. By using strict feedback technique, the underactuated system is presented as a special cascade form. First, an extended disturbance observer is designed to estimate the unknown external disturbances and model uncertainties of the underactuated system. Furthermore, a sliding mode control strategy is proposed to stabilize the underactuated part directly and drive the variables to the sliding mode surface. Finally, combining the sliding mode controller with the extended disturbance observer, a sliding mode controller with disturbance observer is designed. The stability of the overall system is proved and a numerical example is presented to illustrate the effectiveness of the proposed disturbance observer and controller.

Keywords Underactuated system · Cascaded form · Sliding mode control · Disturbance observer

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1 Introduction

Underactuated systems are mechanical control systems with greater number of degrees of freedom than the controls, and the actuated degrees of freedom and the underactuated degrees of freedom are usually nonlinear coupling. Some degrees of freedom cannot be controlled but must be self-stabilized when the controlled state variables converge to the desired value. In resent years, there has been extensive interest among the researchers in controlling underactuated systems due to their extensive applications. Many control systems including the mobile robots [1,2], the acrobot [3], the underactuated surface vessel [4,5] and the crane systems [6,7] are underactuated. The study of underactuated systems is meaningful and challenging in light of these considerations.

Robust control has been widely used in nonlinear systems [8,9], underactuated systems [11–17] and time-delay systems [18,19]. So far, there is no general control method for all underactuated systems except feedback linearization methods, while work has been done on a certain class or kind of underactuated systems. Fuzzy control [10], sliding mode control [2,11] and intelligent backstepping control [12] are proposed to control the wheeled inverted pendulum. Planning-based adaptive control [6], adaptive fuzzy sliding mode control [13], integral sliding mode control method [14] and robust control [15] are applied to the crane system. Since the sliding mode control (SMC) is robust to the model uncertainties and less sensitive to distur-

bances, it is widely applied to nonlinear underactuated systems.

It has been proved that the SMC algorithm can robustly stabilize a class of underactuated nonlinear system such as a mobile robot and an underactuated underwater vehicle [20]. Huang [21] proposed a velocity controller for the mobile wheeled inverted pendulum based on sliding mode. Tuan [7] proposed a control strategy which combines the SMC and the partial feedback linearization for 3D overhead cranes. Much work has been done based on benchmark cascade form for a class of underactuated system by using the SMC. Saleh [22] proposed a sliding mode controller for the control of a class of underactuated systems which are described in cascaded form with external disturbances. Mcninch [23] proposed an optimizing sliding mode cascade control structure for the underactuated unmanned surface vessel systems which look upon the physical limits of the underactuated system as input saturations.

The conventional SMC loses the property of invariance in the presence of unmatched uncertainties and disturbances. The robustness of SMC controller can be obtained by increasing the switch gain k of SMC to suppress the disturbances and model uncertainties. In some cases, it is difficult to obtain or estimate the upper bounds of the unknown disturbances, or the estimated upper bounds are too conservative. In order to stabilize the system, the parameter of sliding mode control k in the item $k\mathrm{sgn}(s)$ has to be significantly increased. Unfortunately, with the increasing k, the chattering phenomenon also becomes serious accordingly. It is well known that the "chattering" is the main drawback of the SMC. Combining the SMC with other approaches which estimate the disturbances is an attractive proposition to overcome this deficiency. The disturbance observer (DO) technology might be a candidate solution for this problem. The DO is originally proposed by Chen [24] to deal with the disturbances of nonlinear system. Recently, several results using the disturbance observer combining with the SMC approach for mismatched nonlinear system have been reported in the researches [25–28]. It turns out that the DO-based SMC strategies can enhance the control accuracies as well as reduce the "chattering" in the above researches.

It should be noted that most of the DOs are based on the assumption that the disturbance is bounded [29] and its first-order derivative dies away gradually [24,30] or is bounded [31–33], or the disturbance itself is of vanishing type in the steady states [25,26]. Whereas this assumption is too restrictive for many practical systems. For example, this assumption cannot describe the cases in which the control system is affected by external disturbance which is a time varying such as polynomials with respect to time t [34]. In these cases, the derivative of disturbance does not converge to zero in the steady states.

In this paper, we propose an extended disturbance observer (EDO) which relaxes the aforementioned assumption. To the best of our knowledge, this study might be the first attempt to discuss the high-order DO-based control design method for "general" underactuated systems. The main contribution of our work includes: (1) the extension of DO technology to enable the estimations of both the disturbance and its derivatives for a class of underactuated systems; (2) a sliding mode controller based on the extended disturbance observer (SMCDO) which can both alleviate the chattering and enhance the control accuracy for the underactuated systems; (3) the stability analysis for both the proposed EDO and SMCDO approaches.

The rest of this paper is organized as follows. The EDO for a class of underactuated system is proposed in Sect. 2. Based on the EDO, a sliding mode controller based on a cascade form of a general underactuated system is derived in Sect. 3. An example is presented to demonstrate the theoretical analysis in Sect. 4, followed by a conclusion in Sect. 5.

2 Extended disturbance observer design for underactuated systems

In this section, we propose an extended disturbance observer (EDO) for a general second-order underactuated system. The proposed EDO relaxes the conventional assumption that the first-order derivatives of disturbances go to zero or they are bounded.

2.1 Problem statement

A general second-order underactuated system is considered. The Lagrange equations of motion for the underactuated system with disturbances can be written as:

$$\begin{aligned} &m_{11}(q)\ddot{q}_1 + m_{12}(q)\ddot{q}_2 + h_1(q,\dot{q}) = d_1 \\ &m_{21}(q)\ddot{q}_1 + m_{22}(q)\ddot{q}_2 + h_2(q,\dot{q}) = u + d_2 \end{aligned} \tag{1}$$

where h_1 and h_2 contain the Coriolis, the centrifugal and the gravity terms. d_1 and d_2 are lumped disturbance terms containing the model uncertainties and disturbances of the system.

Consider the underactuated system described by (1) with two degrees of freedom (q_1, q_2) and symmetry property $M(q) = M(q_2)$. Obviously, the degree of freedom q_2 is actuated. First we introduce the following global transform of coordinate [35]

$$\begin{aligned} x_1 &= q_1 + \beta(q_2), \\ x_2 &= m_{11}\dot{q}_1 + m_{12}\dot{q}_2 = \frac{\partial L}{\partial \dot{q}_1}, \\ x_3 &= q_2, \\ x_4 &= \dot{q}_2, \end{aligned} \tag{2}$$

where $\beta(q_2) = \int_0^{q_2} m_{11}^{-1}(s)m_{12}(s)\mathrm{d}s$. $L = E_\mathrm{k} - Ep = \frac{1}{2}\dot{q}^\mathrm{T}M(q)\dot{q} - E_\mathrm{p}(q)$. E_k is the kinetic energy. E_p is the potential energy.

Based on the global transform of coordinate (2), the dynamic model (1) can be further transformed into a special cascade nonlinear system in feedback form:

$$\begin{aligned} \dot{x}_1 &= \dot{q}_1 + m_{11}^{-1}m_{12}\dot{q}_2 = m_{11}^{-1}x_2, \\ \dot{x}_2 &= \frac{\mathrm{d}(m_{11}\dot{q}_1 + m_{12}\dot{q}_2)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_1} = f_1 + d_1, \\ \dot{x}_3 &= \dot{q}_2, \\ \dot{x}_4 &= f_2 + bu + d_{12}, \end{aligned} \tag{3}$$

where

$$mm = m_{11}m_{22} - m_{12}m_{21}, \quad f_1 = \frac{\partial L}{\partial q_1},$$

$$f_2 = \frac{m_{21}h_1 - m_{11}h_2}{mm},$$

$$b = \frac{m_{11}}{mm}, \quad d_{12} = \frac{-m_{21}d_1 + m_{11}d_2}{mm}.$$

Remark 1 The dynamic model (1) is derived by Lagrangian method, where $f_1 = \partial L/\partial q_1$, $L = E_\mathrm{k} - Ep = \frac{1}{2}\dot{q}^\mathrm{T}M(q)\dot{q} - E_\mathrm{p}(q)$. Since q_1 is the underactuated joint, we have $M(q) = M(q_2)$ and $f_1 = -\partial E_\mathrm{p}/\partial q_1$, which is related to the first-order partial derivative of potential energy of the system with respect to q_1. In most cases, the potential energy E_p is only related to the position variables (q_1, q_2) and has nothing to do with the velocity variables $(\dot{q}_1, \dot{q}_2)$. That is, $\partial f_1/\partial x_2$ and $\partial f_1/\partial x_4$ are usually equal to zero, and $\partial f_1/\partial x_3$ is invertible and bounded. In this paper, only this situation is considered.

For simplicity, we rewrite (3) as the following vector form:

$$\dot{X} = F_1(X) + G_1(X)u + G_2 d, \tag{4}$$

where

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4]^\mathrm{T}$$

$$F_1(X) = \begin{bmatrix} m_{11}^{-1}x_2 \\ f_1 \\ x_4 \\ f_2 \end{bmatrix}, \; G_1(X) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \; d = \begin{bmatrix} d_1 \\ d_{12} \end{bmatrix}.$$

2.2 A second-order disturbance observer

Assumption 1 The disturbance d is continuous, and its first-order derivative $\dot{d}$ and second-order derivative $\ddot{d}$ exist.

Assumption 2 $\ddot{d}$ is bounded and satisfies $||\ddot{d}|| \le \mu_1$. Here $\mu_1 > 0$ is a constant.

The second-order disturbance observer for the underactuated system (4) is proposed as

$$\begin{aligned} \hat{d} &= z_1 + p_1(X), \\ \dot{z}_1 &= L_1\left(-F_1(X) - G_1(X)u - G_2\hat{d}\right) + \hat{\dot{d}}, \\ \hat{\dot{d}} &= z_2 + p_2(X), \\ \dot{z}_2 &= L_2\left(-F_1(X) - G_1(X)u - G_2\hat{d}\right), \end{aligned} \tag{5}$$

where $\hat{d}$ and $\hat{\dot{d}}$ are the estimates of d and $\dot{d}$, respectively. The disturbance observer constant gain matrix L_i is defined as

$$L_i = \partial p_i(X)/\partial X, \quad i = 1, 2, \tag{6}$$

where

$$L_i = \begin{bmatrix} k_{2i-1,1} & k_{2i-1,2} & k_{2i-1,3} & k_{2i-1,4} \\ k_{2i,1} & k_{2i,2} & k_{2i,3} & k_{2i,4} \end{bmatrix}.$$

Theorem 1 *Considering the underactuated system (4), assume that the disturbance d and its first-order derivative $\dot{d}$ exist, and its second-order derivative can*

be negligible (i.e., $\ddot{d} = 0$). Choosing proper L_i satisfying that A_1 is negative definite, the disturbance observer given in (5) with the auxiliary vector L_i defined in (6) ensures that the disturbance tracking error converges exponentially to zero. If the disturbance d and its first-order derivative $\dot{d}$ exist, and its second-order derivative $\ddot{d}$ is bounded (i.e., $||\ddot{d}|| \leq \mu_1$), choosing proper L_i satisfying that A_1 is negative definite, then the disturbance observer given in (5) with the auxiliary vector L_i defined in (6) ensures that the disturbance tracking error is globally uniformly ultimately bounded.
Here

$$A_1 = \begin{bmatrix} -L_1G_2 & \mathbf{I} \\ -L_2G_2 & \mathbf{0} \end{bmatrix},$$

where $\mathbf{I}$ *is a* 2×2 *identity matrix.* $\mathbf{0}$ *is a* 2×2 *zero matrix.*

Proof Let the estimation errors to be defined as

$$\begin{aligned} \tilde{d} &= d - \hat{d}, \\ \dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}}, \end{aligned} \tag{7}$$

where $\tilde{d}$ is the estimation error between d and its estimate $\hat{d}$, and $\dot{\tilde{d}}$ is the estimation error between $\dot{d}$ and its estimate $\dot{\hat{d}}$.

Differentiating the first equation of (7), we can get:

$$\begin{aligned} \dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}} \\ &= \dot{d} - L_1\left(-F_1 - G_1u - G_2\hat{d}\right) - \dot{\hat{d}} - \dot{p}_1 \\ &= \dot{d} - L_1G_2\tilde{d} - \dot{\hat{d}} = \dot{\tilde{d}} - L_1G_2\tilde{d}. \end{aligned} \tag{8}$$

According to the second equation of (7), we have:

$$\begin{aligned} \ddot{\tilde{d}} &= \ddot{d} - \ddot{\hat{d}} \\ &= \ddot{d} - L_2\left(-F_1 - G_1u - G_2\hat{d}\right) - L_2\dot{X} \\ &= \ddot{d} - L_2G_2\tilde{d}. \end{aligned} \tag{9}$$

According to (8) and (9), the error vector $D_1 = \begin{bmatrix} \tilde{d} & \dot{\tilde{d}} \end{bmatrix}^{\mathrm{T}}$ can be expressed as

$$\dot{D}_1 = A_1D_1 + B_1\ddot{d}, \tag{10}$$

where $B_1 = [\mathbf{0_1} \quad \mathbf{I_1}]^{\mathrm{T}}$. $\mathbf{0_1}$ is a 2×2 zero matrix. $\mathbf{I_1}$ is a 2×2 identity matrix.

Since A_1 is a negative definite matrix, the eigenvalues of A_1 are negative. There exists a positive defined matrix Q_1 satisfying $A_1^{\mathrm{T}}P_1 + P_1A_1^{\mathrm{T}} = -Q_1$.

Choose the following Lyapunov function

$$V_1 = D_1^{\mathrm{T}}P_1D_1,$$

where P_1 is a positive defined matrix.

By differentiating V_1, we have

$$\begin{aligned} \dot{V}_1 &= D_1^{\mathrm{T}}(A_1^{\mathrm{T}}P_1 + P_1A_1^{\mathrm{T}})D_1 + 2{D_1}^{\mathrm{T}}P_1B_1\ddot{d} \\ &= -{D_1}^{\mathrm{T}}Q_1D_1 + 2{D_1}^{\mathrm{T}}P_1B_1\ddot{d} \\ &\leq -\lambda_{1\min}\|D_1\|^2 + 2\|D_1^{\mathrm{T}}P_1B_1\| \cdot \mu_1 \\ &\leq -\|D_1\|\left(\lambda_{1\min}\|D_1\| - 2\|P_1B_1\| \cdot \mu_1\right), \end{aligned}$$

where λ_{1min} is the minimum eigenvalue of Q_1.

$$\dot{V}_1 \leq -\|D_1\|(\lambda_{1\min}\|D_1\| - 2\|P_1B_1\| \cdot \mu_1).$$

That is, after a sufficiently long time, the norm of the estimation error is bounded by

$$\|D_1\| \leq \frac{2\|P_1B_1\| \cdot \mu_1}{\lambda_{1\min}}.$$

Thus, if the disturbance d and its first-order derivative $\dot{d}$ exist, and its second-order derivative satisfies $\ddot{d} = 0$, the disturbance tracking error converges exponentially to zero. If the disturbance d and its first-order derivative $\dot{d}$ exist, and its second-order derivative $\ddot{d}$ is bounded, the tracking error is globally uniformly ultimately bounded. □

2.3 Extension of disturbance observer to nth-order cases

Assumption 3 The disturbance d is continuous, and its nth-order derivative exists.

Assumption 4 $d^{(n)}$ is bounded and satisfies $\|d^{(n)}\| \leq \mu$. Here $\mu > 0$ is a constant.

The extended disturbance observer for the underactuated system (4) is proposed as:

$$\begin{aligned} \hat{d} &= z_1 + p_1(X) \\ \dot{z}_1 &= L_1(-F_1(X) - G_1(X)u - G_2\hat{d}) + \dot{\hat{d}} \\ &\vdots \\ \hat{d}^{(i-1)} &= z_i + p_i(X) \\ \dot{z}_i &= L_i(-F_1(X) - G_1(X)u - G_2\hat{d}) + \hat{d}^{(i)} \\ &\vdots \\ \hat{d}^{(n-1)} &= z_n + p_n(X) \\ \dot{z}_n &= L_n(-F_1(X) - G_1(X)u - G_2\hat{d}), \end{aligned} \tag{11}$$

where $\hat{d}^{(i)}$ is the estimate of $d^{(i)}$. The disturbance observer gain matrix L_i is defined as:

$$L_i = \partial p_i(X)/\partial X, \quad i = 1, 2, \ldots, n. \tag{12}$$

Theorem 2 *Considering the underactuated system (4), assume that the disturbance d and its derivatives $d^{(i)}$ exist, and its nth derivative can be negligible (i.e., $d^{(n)} = 0$). Choosing proper L_i satisfying that A is negative definite, the extended disturbance observer given in (11) with the auxiliary vector L_i defined in (12) ensures that the disturbance tracking error converges exponentially to zero. If the disturbance d and its derivatives $d^{(i)}$ exist, and its n-th derivative $d^{(n)}$ is bounded (i.e., $||d^{(n)}|| \leq \mu$), choosing proper L_i satisfying that A is negative definite, the disturbance observer given in (11) with the auxiliary vector L_i defined in (12) ensures that the disturbance tracking error is globally uniformly ultimately bounded.*
Here

$$A = \begin{bmatrix} -L_1G_2 & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ -L_2G_2 & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -L_{n-1}G_2 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \\ -L_nG_2 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix},$$

where $i = 1, 2 \ldots n$. $\mathbf{I}$ is 2×2 identity matrix. $\mathbf{0}$ is 2×2 zero matrix.

Proof Let the estimation errors to be defined as

$$\begin{aligned} \tilde{d} &= d - \hat{d} \\ &\vdots \\ \tilde{d}^{(i)} &= d^{(i)} - \hat{d}^{(i)} \\ &\vdots \\ \tilde{d}^{(n-1)} &= d^{n-1} - \hat{d}^{(n-1)}, \end{aligned} \tag{13}$$

where $i = 1, 2, \ldots, n-1$. $\tilde{d}^{(i)}$ is the estimation error between $d^{(i)}$ and its estimate $\hat{d}^{(i)}$.

Differentiate the first $n-1$ equations of (13), and substitute (11) into them. It follows that

$$\begin{aligned} \dot{\tilde{d}}^{(i)} &= d^{(i+1)} - \dot{\hat{d}}^{(i)} \\ &= d^{(i+1)} - L_{i+1}(-F_1 - G_1u - G_2\hat{d}) - \hat{d}^{(i+1)} - \dot{p}_{i+1} \\ &= d^{(i+1)} - L_{i+1}G_2\tilde{d} - \hat{d}^{(i+1)} = \tilde{d}^{(i+1)} - L_{i+1}G_2\tilde{d}, \end{aligned} \tag{14}$$

where $i = 0, 1, \ldots, n-2$.

According to the last two equations of (11), we obtain

$$\begin{aligned} \dot{\hat{d}}^{(n-1)} &= \dot{z}_n + \dot{p}_n(X) \\ &= L_n(-F_1 - G_1u - G_2\hat{d}) + L_n\dot{X}. \end{aligned} \tag{15}$$

Differentiating the last equation of (13) and substituting it to (15), we have

$$\dot{\tilde{d}}^{(n-1)} = d^n - L_n\left(-F_1 - G_1u - G_2\hat{d}\right) - L_n\dot{X}.$$

That is, we have:

$$\dot{\tilde{d}}^{(n-1)} = d^{(n)} - L_nG_2\tilde{d}. \tag{16}$$

According to (14) and (16), the error vector $D = \left[\tilde{d} \quad \dot{\tilde{d}}, \ldots, \tilde{d}^{(n-1)}\right]^{\mathrm{T}}$ can be expressed as

$$\dot{D} = AD + Bd^{(n)}, \tag{17}$$

where $B = [\mathbf{0_{(n-1)}} \quad \mathbf{I_1}]^{\mathrm{T}}$. $\mathbf{0_{(n-1)}}$ is a $2(n-1)\times 2$ zero matrix. $\mathbf{I_1}$ is a 2×2 identity matrix.

Since A is a negative definite matrix, the eigenvalues of A are negative. There exists a positive defined matrix Q satisfying $A^{\mathrm{T}}P + PA^{\mathrm{T}} = -Q$.

Choose Lyapunov function

$$V = D^{\mathrm{T}}PD,$$

where P is a positive defined matrix.

In the light of the mathematical derivation in Sect. 2.2, it is easy to obtain

$$\dot{V} \leq -\|D\|(\lambda_{\min}\|D\| - 2\|PB\|\cdot\mu),$$

where λ_{min} is the minimum eigenvalue of Q. That is, after a sufficiently long time, the norm of the estimation error is bounded by

$$\|D\| \leq \frac{2\|PB\|\cdot\mu}{\lambda_{\min}}.$$

Based on the above inequality, we can easily draw the conclusion of this theorem. □

3 Sliding mode control with an extended disturbance observer

Based on the obtained EDO in Sect. 2, we propose a sliding mode controller with disturbance observer

(SMCDO) for the general second-order underactuated system (3) in this section.

For simplicity, we rewrite (3) as

$$\begin{aligned}\dot{x}_1 &= m_{11}^{-1}x_2,\\ \dot{x}_2 &= f_1(x_1,x_2,x_3,x_4)+d_1,\\ \dot{x}_3 &= x_4,\\ \dot{x}_4 &= f_2(x_1,x_2,x_3,x_4)+b(x_1,x_2,x_3,x_4)u+d_{12}.\end{aligned} \tag{18}$$

The sliding surface is chosen as:

$$s = c_1e_1 + c_2e_2 + c_3e_3 + e_4, \tag{19}$$

where $e_1 = x_1 - x_{1d}$, $e_2 = x_2 - x_{2d}$, $e_3 = f_1 + d_1 - f_{1d} - \hat{d}_1$, $e_4 = \frac{\mathrm{d}f_1}{\mathrm{d}t} + d_{12} - \hat{d}_{12} - \dot{f}_{1d}$. Here x_{1d}, x_{2d}, f_{1d}, $\dot{f}_{1d}$ are the desired values.

Remark 2 Since we have $\dot{x}_3 = x_4$, and $\dot{x}_4$ is related to input u, then $\dot{s}$ should include $\dot{x}_4$. To meet the requirement, if f_1 is related to x_4, the sliding mode surface is selected as $s = c_1e_1 + c_2e_2 + e_3$. If f_1 only has a relationship with x_3, the sliding mode surface is then chosen as $s = c_1e_1 + c_2e_2 + c_3e_3 + e_4$. As analyzed in *Remark* 1, we have $\partial f_1/\partial x_4 = 0$. Then the sliding mode surface is chosen as (19).

Differentiating the errors e_1, e_2, e_3, it follows that

$$\begin{aligned}\dot{e}_1 &= \dot{x}_1 - \dot{x}_{1d} = m_{11}^{-1}x_2 - m_{11}^{-1}x_{2d} = m_{11}^{-1}(x_2 - x_{2d})\\ &= m_{11}^{-1}e_2,\\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_{2d} = f_1 + d_1 - (f_{1d} + \hat{d}_1) = e_3,\\ \dot{e}_3 &= \frac{\mathrm{d}f_1}{\mathrm{d}t} - \dot{f}_{1d} + \dot{d}_1 - \dot{\hat{d}}_1 = e_4 - d_{12} + \hat{d}_{12} + \dot{d}_1 - \dot{\hat{d}}_1\\ &= e_4 - \tilde{d}_{12} + \dot{\tilde{d}}_1.\end{aligned} \tag{20}$$

When the system state approaches the sliding mode surface (19), it satisfies $s = \gamma$ where $|\gamma| \le \kappa$, i.e.,

$$e_4 = -c_1e_1 - c_2e_2 - c_3e_3 + \gamma. \tag{21}$$

According to (20) and (21), the error $E = [e_1 \quad e_2 \quad e_3]^{\mathrm{T}}$ can be expressed as

$$\dot{E} = ME + ND_1 + \Upsilon, \tag{22}$$

where

$$M = \begin{bmatrix} 0 & m_{11}^{-1} & 0\\ 0 & 0 & 1\\ -c_1 & -c_2 & -c_3\end{bmatrix}, N = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & -1 & 1 & 0\end{bmatrix}, \Upsilon = \begin{bmatrix} 0\\ 0\\ \gamma\end{bmatrix}.$$

Proper parameters c_1, c_2, c_3 can be selected satisfying the condition that the real parts of eigenvalues of M are negative.

Considering the sliding surface (19), the system dynamics on the sliding surface is asymptotically stable if D_1 and γ vanish as time goes by. That is, the underactuated part of the system can be self-stabilized on the sliding mode surface. And the system dynamics is uniformly ultimately bounded on the sliding surface if D_1 and γ exist all the time.

Theorem 3 *Considering a general nonlinear underactuated system* (18)*, the state trajectories will be driven to the sliding mode surface* (19) *and uniformly ultimately bounded when the SMCDO* (23) *is applied:*

$$u = -\left(\frac{\partial f_1}{\partial x_3}b\right)^{-1}(K + ksgn(s) + \lambda s), \tag{23}$$

where

$$\begin{aligned}K &= c_1m_{11}^{-1}x_2 + c_2f_1 + c_3\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}x_2 + \frac{\partial f_1}{\partial x_3}x_4\right)\\ &\quad + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}f_1 + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)x_2 + \frac{d}{dt}\frac{\partial f_1}{\partial x_3}x_4\\ &\quad + \frac{\partial f_1}{\partial x_3}f_2 - c_1\dot{x}_{1d}\\ &\quad - c_2\dot{x}_{2d} - c_3\dot{f}_{1d} - \ddot{f}_{1d} + \left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)\hat{d}_1\\ &\quad + \frac{\partial f_1}{\partial x_3}\hat{d}_{12},\ k > \left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)\tilde{d}_1 + \frac{\partial f_1}{\partial x_3}\tilde{d}_{12}\\ &\quad + c_3\dot{\tilde{d}}_1 + \dot{\tilde{d}}_{12},\ \lambda > 0.\end{aligned}$$

Proof Choose the Lyapunov function as

$$V = \frac{1}{2}s^{\mathrm{T}}s.$$

Differentiating both sides of V and substituting (23) into it, we have

$$\dot{V} = \dot{s}s$$
$$= s\left(\underbrace{c_1 m_{11}^{-1}x_2 - c_1\dot{x}_{1d}}_{c_1\dot{e}_1} + \underbrace{c_2 f_1 + c_2 d_1 - c_2\dot{x}_{2d}}_{c_2\dot{e}_2} + \underbrace{c_3\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}x_2 + \frac{\partial f_1}{\partial x_3}x_4 - \dot{f}_{1d} + \dot{\tilde{d}}_1\right)}_{c_3\dot{e}_3} + \dot{e}_4\right).$$

The derivative of e_4 can be calculated as follows:

$$e_4 = \frac{\mathrm{d}f_1}{\mathrm{d}t} + d_{12} - \widehat{d}_{12} - \dot{f}_{1d} \Rightarrow$$
$$\dot{e}_4 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}f_1}{\mathrm{d}t}\right) + \dot{\tilde{d}}_{12} - \ddot{f}_{1d}.$$

As analyzed in *Remark* 1 , we have $\partial f_1/\partial x_2 = 0$. It follows that

$$\dot{e}_4 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1}\dot{x}_1 + \frac{\partial f_1}{\partial x_3}\dot{x}_3\right) + \dot{\tilde{d}}_{12} - \ddot{f}_{1d}$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}x_2 + \frac{\partial f_1}{\partial x_3}x_4\right) + \dot{\tilde{d}}_{12} - \ddot{f}_{1d}.$$

Substituting (18) into it, we have

$$\dot{e}_4 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)x_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}\dot{x}_2 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3}\right)x_4$$
$$+ \frac{\partial f_1}{\partial x_3}\dot{x}_4 + \dot{\tilde{d}}_{12} - \ddot{f}_{1d}$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)x_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}(f_1 + d_1)$$
$$+ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3}\right)x_4 + \frac{\partial f_1}{\partial x_3}(f_2 + bu + d_{12}) + \dot{\tilde{d}}_{12} - \ddot{f}_{1d}.$$

Thus, we have

$$\dot{V} = s\left(c_1 m_{11}^{-1}x_2 + c_2 f_1 + c_3\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}x_2 + \frac{\partial f_1}{\partial x_3}x_4\right)\right.$$
$$+ \frac{\partial f_1}{\partial x_1}m_{11}^{-1}f_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)x_2 + \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial f_1}{\partial x_3}x_4$$
$$+ \frac{\partial f_1}{\partial x_3}f_2 - c_1\dot{x}_{1d} - c_2\dot{x}_{2d} - c_3\dot{f}_{1d} - \ddot{f}_{1d}$$
$$+ \left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)d_1 + \frac{\partial f_1}{\partial x_3}d_{12}$$
$$\left. + c_3\dot{\tilde{d}}_1 + \dot{\tilde{d}}_{12} + \frac{\partial f_1}{\partial x_3}bu\right)$$
$$= s\left(\left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1}\right)\tilde{d}_1 + \frac{\partial f_1}{\partial x_3}\tilde{d}_{12} + c_3\dot{\tilde{d}}_1 + \dot{\tilde{d}}_{12}\right)$$
$$- \lambda s^2 - ks\cdot\mathrm{sgn}(s)$$
$$\leq |s|\left(\left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1} + \frac{\partial f_1}{\partial x_3} + c_3 + 1\right)\|D_1\| - \lambda|s| - k\right)$$
$$\leq |s|\left(\left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1} + \frac{\partial f_1}{\partial x_3} + c_3 + 1\right)\right.$$
$$\left.\frac{2\|PB\|\cdot\mu}{\lambda_{\min}} - \lambda|s| - k\right).$$

It is easy to obtain that after a sufficiently long time, $|s|$ is bounded by

$$|s| \leq \frac{2\left(c_2 + \frac{\partial f_1}{\partial x_1}m_{11}^{-1} + \frac{\partial f_1}{\partial x_3} + c_3 + 1\right)\|PB\|\mu - k\lambda_{\min}}{\lambda\lambda_{\min}}.$$

It is worth to notice that $|s|$ can be lowered by increasing λ or k.

If the nth-order derivative of d can be negligible (i.e., $d^{(n)} = 0$, $\mu = 0$), then the Lyapunov function V is globally asymptotic convergent to 0, and the actuated states are driven to the sliding mode surface. The effect of the disturbance can also be eliminated completely.

On the other hand, $\|D\|$ is uniformly ultimately bounded if the nth-order derivative $d^{(n)}$ is bounded ($||d^{(n)}|| \leq \mu$). In this case, the magnitude of sliding variable $|s|$ and Lyapunov function V are uniformly ultimately bounded. The actuated states move around the sliding mode surface, and the bounds can be lowered by selecting proper control parameter k, λ. □

4 Simulation results

In order to test and verify the proposed EDO and SMCDO, we provide an example to illustrate the theory analysis in this section.

4.1 The model of acrobot system

The acrobot is a two-link planar robot with two joints and the elbow is the actuator, which is an inherent unstable, underactuated system, as shown in Fig. 1.

The dynamic model of acrobot is given by [36]

$$m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + h_1 + \phi_1 = d_1,$$
$$m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + h_2 + \phi_2 = \tau + d_2, \quad (24)$$

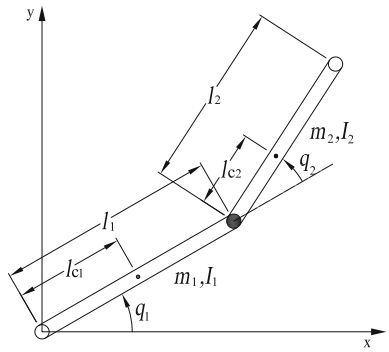

Fig. 1 The mechanical structure of acrobot

where

$$
\begin{aligned}
&m_{11} = a + b\cos(q_2), \quad m_{12} = c + 0.5b\cos(q_2),\\
&m_{12} = m_{21}, m_{22} = c\\
&\quad a = m_1 l_{c1}^2 + m_2\left(l_1^2 + l_{c2}^2\right) + I_1 + I_2,\\
&\quad c = m_2 l_{c2}^2, b = 2m_2 l_1 l_{c2}\\
&\quad h_1 = -m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2^2 - 2m_2 l_2 l_{c2}\sin(q_2)\dot{q}_1\dot{q}_2,\\
&\quad h_2 = m_2 l_1 l_{c2}\sin(q_2)\dot{q}_1^2,\\
&\quad \phi_1 = (m_1 l_{c1} + m_2 l_1)\, g\cos(q_1) + m_2 l_{c2} g\cos(q_1+q_2),\\
&\quad \phi_2 = m_2 l_{c2} g\cos(q_1+q_2).
\end{aligned}
$$

d_1, d_2 are the disturbances including model uncertainties and external disturbance.

Obviously, it satisfies that $M(q) = M(q_2)$ and q_2 is actuated. Choose new states and the global change of the coordinate

$$
\begin{aligned}
x_1 &= q_1 + \beta(q_2),\\
x_2 &= m_{11}\dot{q}_1 + m_{12}\dot{q}_2 = \partial L/\partial\dot{q}_1,\\
x_3 &= q_2,\\
x_4 &= \dot{q}_2,
\end{aligned} \tag{25}
$$

where $\beta(q_2) = \frac{q_2}{2} + \frac{2c-a}{\sqrt{a^2-b^2}}\arctan(\sqrt{\frac{a-b}{a+b}}\tan(\frac{q_2}{2}))$.

The dynamic model (24) is then transformed into a cascade nonlinear system in strict feedback form:

$$
\begin{aligned}
\dot{x}_1 &= m_{11}^{-1}x_2,\\
\dot{x}_2 &= f_1 + d_1,\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= f_2 + bu + d_{12},
\end{aligned} \tag{26}
$$

Table 1 The parameters of an acrobot

m_1	1	kg	l_{c2}	1	m
m_2	1	kg	I_1	0.083	kg m
l_1	1	m	I_2	0.33	kg m
l_2	2	m	g	9.8	m/s^2
l_{c1}	0.5	m			

where

$$
\begin{aligned}
f_1 &= -\,(m_1 l_{c1} + m_2 l_1)\, g\cos(q_1) - m_2 l_{c2} g\cos(q_1+q_2)\\
&= -\,(m_1 l_{c1} + m_2 l_1)\, g\cos(x_1 - \beta)\\
&\quad - m_2 l_{c2} g\cos(x_1 - \beta + x_3)\\
f_2 &= \frac{m_{21}(h_1 + \phi_1) - m_{11}(h_2 + \phi_1)}{m_{11}m_{22} - m_{12}m_{21}},\\
b &= \frac{m_{11}}{m_{11}m_{22} - m_{12}m_{21}},\\
d_{12} &= \frac{(-m_{12}d_1 + m_{11}d_2)}{m_{11}m_{22} - m_{12}m_{21}}.
\end{aligned}
$$

An obvious equilibrium of the acrobot system can be easily obtained

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4]^{\mathrm{T}} = [\pi/2 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}.$$

4.2 The simulation results of acrobot controlled by the proposed method

In the simulation, the parameters of the acrobot are given in Table 1. The initial conditions are chosen as:

$$q_1 = \pi/2 + 0.2, \quad q_2 = -0.5, \quad \dot{q}_1 = 0, \quad \dot{q}_2 = 0.$$

The friction and model uncertainties will vanish as the system gradually approaches the equilibrium. In order to test the EDO and SMCDO, the external disturbance is also considered. Without loss of the generality, the first-order, second-order, third-order and fourth-order disturbance observers are discussed, respectively, in the following. The equilibrium control of acrobot will be discussed in the following cases.

Case 1 The situation that there is no disturbance is considered.

$$d_1 = 0, \quad d_2 = 0.$$

The simulation of the acrobot system without disturbances controlled by sliding mode controller is shown in Fig. 2. It is shown that the sliding mode controller can drive the inherent unstable system to the sliding surface, and it takes about 25 s.

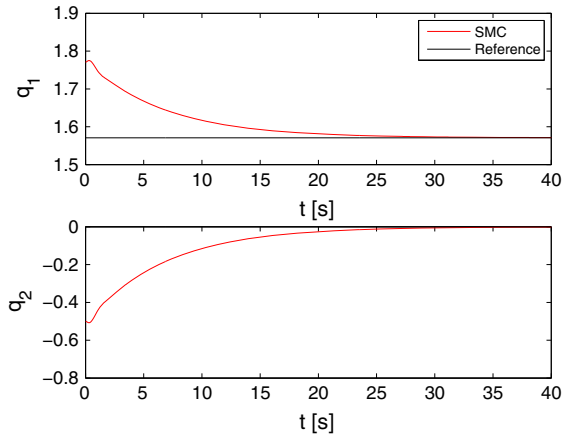

Fig. 2 The simulation result of sliding mode control for acrobot with $d_1 = 0$, $d_2 = 0$

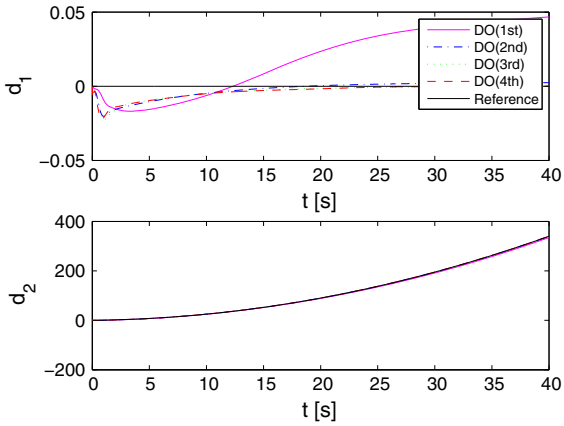

Fig. 3 The simulation result of EDOs with $d_1 = 0.2(q_1 - \pi/2) + 0.1q_2 + 0.3\dot{q}_1$ and $d_2 = 0.2(q_1 - \pi/2) + 0.5q_2 + 0.3\dot{q}_1 + 0.5\dot{q}_2 + 0.5t + 0.2t^2$

Case 2 In this case, the disturbances are assumed to be

$$d_1 = 0.2(q_1 - \pi/2) + 0.1q_2 + 0.3\dot{q}_1,$$
$$d_2 = 0.2(q_1 - \pi/2) + 0.5q_2 + 0.3\dot{q}_1 + 0.5\dot{q}_2 + 0.5t + 0.2t^2,$$

where d_1 dies away as the states tend to converge to zero. d_2 satisfies $d_2 \to \infty$, $\dot{d}_2 \to \infty$, $|\ddot{d}_2| \leq \mu$ and $d_2^{(3)} = d_2^{(4)} = 0$.

The simulation results of EDOs are shown in Figs. 3 and 4. From the figures, it can be seen that the first-order disturbance observer (DO(1st)) cannot estimate the disturbance. The second-order disturbance observer (DO(2nd)) tracks the disturbance with a steady-state

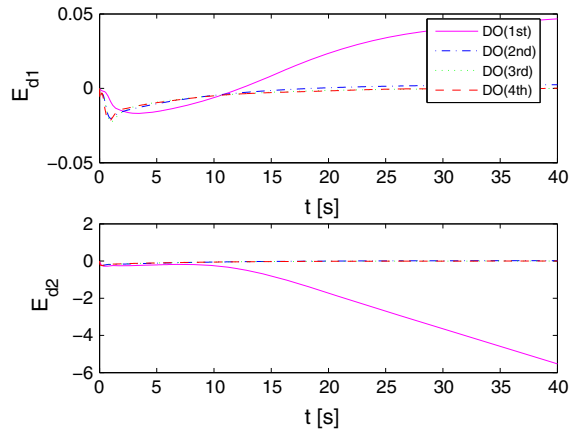

Fig. 4 The errors between disturbance and its estimations with $d_1 = 0.2(q_1 - \pi/2) + 0.1q_2 + 0.3\dot{q}_1$, $d_2 = 0.2(q_1 - \pi/2) + 0.5q_2 + 0.3\dot{q}_1 + 0.5\dot{q}_2 + 0.5t + 0.2t^2$

error. The third-order disturbance observer (DO(3rd)) and the fourth-order disturbance observer (DO(4th)) are able to track the disturbance accurately.

According to (17), for the DO(1st), the estimation error satisfies $\dot{D} = AD + B\dot{d}$. Since we have $\dot{d} \to \infty$, the stability of estimation error cannot be guaranteed. This is why the DO(1st) cannot estimate the disturbance. For the DO(2nd), the estimation error satisfies $\dot{D} = AD + B\ddot{d}$. Since we have $\|\dot{d}\| \leq \mu$, the DO(2nd) estimates the disturbance with a steady error. For the DO(3rd) and DO(4th), the estimation errors satisfy $\dot{D} = AD + Bd^{(3)}$ and $\dot{D} = AD + Bd^{(4)}$, respectively. Since we have $d^{(3)} = d^{(4)} = 0$, these DOs can estimate the disturbance accurately, as shown in Fig. 3

The simulation results of the acrobot controlled by SMCDO (23) are shown in Fig. 5. Obviously, the sliding mode control with first-order disturbance observer (SMCDO(1st)) cannot drive the states to the equilibrium. The sliding mode control with second-order disturbance observer (SMCDO(2nd)) drives the states to the equilibrium with a steady error. The sliding mode control with third-order disturbance observer (SMCDO(3rd)) and fourth-order disturbance observer (SMCDO(4th)) drives the states to the equilibrium.

Case 3 In this case, more complex disturbance which is superimposed on types of bounded disturbance is considered to test the EDO and the SMCDOs. Choose d_1 and d_2 as

$$d_1 = 0.1\sin(t + 1),$$
$$d_2 = \sin(0.5t),$$

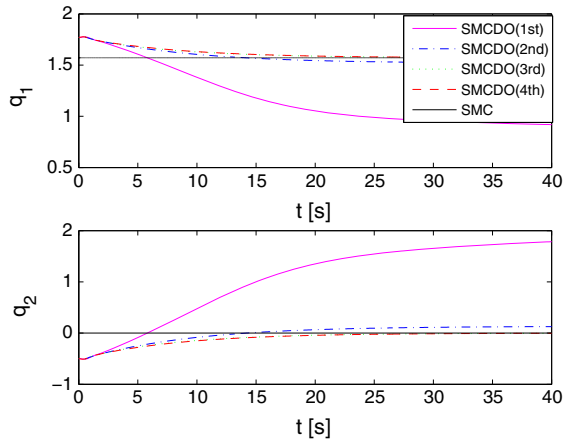

Fig. 5 The simulation result of SMCDO for acrobot with $d_1 = 0.2(q_1 - \pi/2) + 0.1q_2 + 0.3\dot{q}_1$, $d_2 = 0.2(q_1 - \pi/2) + 0.5q_2 + 0.3\dot{q}_1 + 0.5\dot{q}_2 + 0.5t + 0.2t^2$

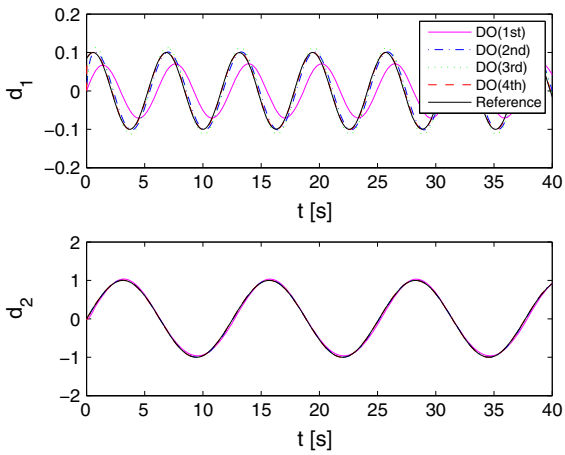

Fig. 6 The simulation results of EDOs with $d_1 = 0.1\sin(t + 1)$, $d_2 = \sin(0.5t)$

where d_1, d_2 include the friction, model uncertainties and external disturbance. Their nth-order derivatives exist and are bounded ($\|d^{(n)}\| \leq \mu$).

The simulation results of the EDOs are shown in Figs. 6 and 7. The tracking errors between system states and the reference are globally uniformly ultimately bounded. Because of the slow change of disturbance($f_1 = 1/2\pi$, $f_2 = 1/4\pi$), the higher order of the derivative of d_2, the smaller the upper bounded of $d^{(n)}$. As analyzed in Theorem 2, we can get smaller tracking error by using the disturbance observer of higher order.

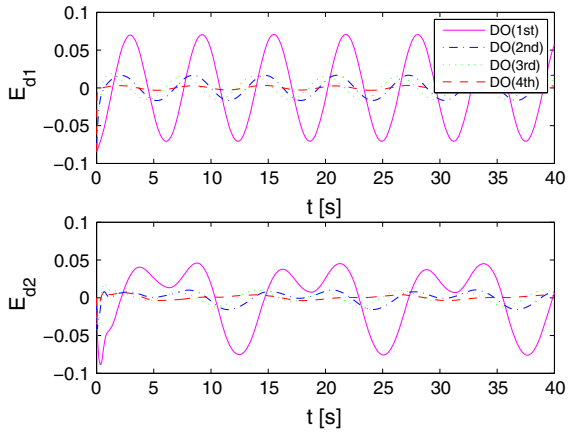

Fig. 7 The errors between disturbance and its estimations with $d_1 = 0.1\sin(t + 1)$, $d_2 = \sin(0.5t)$

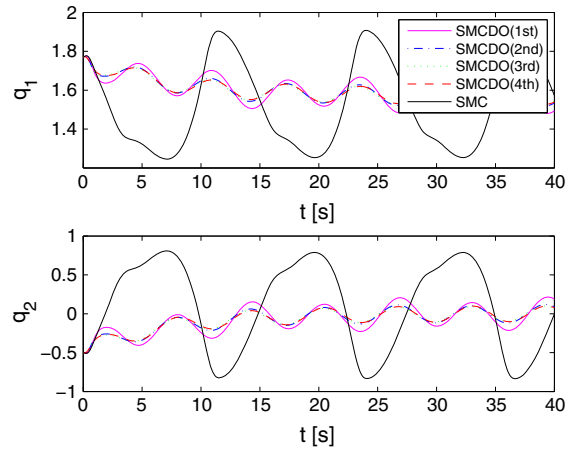

Fig. 8 The simulation results of SMC and SMCDOs for acrobot with $d_1 = 0.1\sin(t + 1)$, $d_2 = \sin(0.5t)$

The simulation results of the acrobot controlled by the SMC and the SMCDOs are shown in Fig. 8. Both the SMC and the SMCDOs cannot guarantee that the states converge to the desired value when $d^{(n)}$ does not vanish. As analyzed in Theorem 3, the effect of disturbance cannot be eliminated completely. As shown in Fig. 8, compared to the SMC, the proposed SMCDOs are more effective. Using the EDO of higher order, better control performance can be obtained.

4.3 Comparison results

In order to verify the superiority of the proposed method, a comparison study is carried out in this subsection.

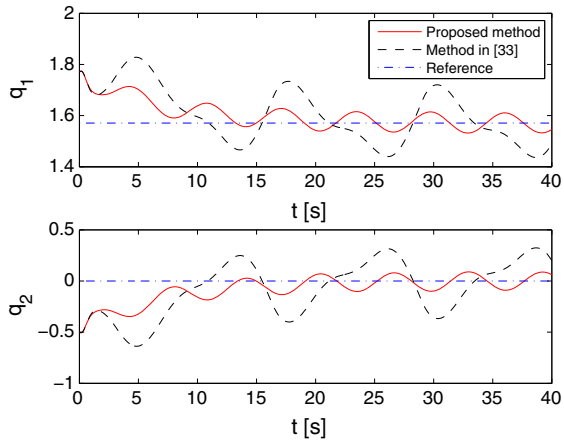

Fig. 9 The results of SMCDO(4th) compared with the method in Ref. [33] for acrobot with $d_1 = 0.1\sin(t+1)$, $d_2 = \sin(0.5t)$

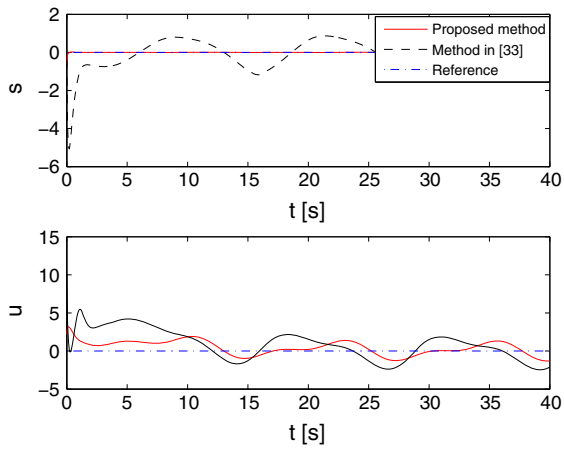

Fig. 10 The results of SMCDO(4th) compared with the method in Ref. [33] for acrobot with $d_1 = 0.1\sin(t+1)$, $d_2 = \sin(0.5t)$

The simulation results of proposed SMCDO controller are compared with those of method in Ref. [33]. In Ref. [33], a global sliding mode control is presented to improve the robustness and stability of the underactuated system with external disturbances, and the conditions of asymptotic stability are presented by linear matrix inequalities. Based on the controller, states of the system converge to the sliding mode surface exponentially.

Considering the method in Ref. [33] applied to a class of underactuated systems with bounded external disturbances, the disturbance is chosen as given in Case 3 in Sect. 4.2. In the comparison, the proposed SMC with fourth-order DO is used. The system trajectories with initial states $q_1 = \pi/2 + 0.2$, $q_2 = -0.5$ are illustrated in Fig. 9. The trajectories of sliding mode motion and control signal are shown in Fig. 10. These results verify that the proposed control approach has better performance in comparison with the controller in Ref. [33].

5 Conclusion

In this paper, we proposed an extended disturbance observer which relaxes the assumption that the first-order derivatives of disturbances go to zero or they are bounded in steady states. The sliding mode surface control is also generalized for a class of underactuated systems based on the nth-order disturbance observer. It is proved that the ultimate boundedness of the sliding variable and disturbance estimation error can be guaranteed. Compared to the conventional SMC, the switching gain of proposed SMCDO is only required to be designed greater than the bound of the disturbance estimation error rather than that of the disturbance. The SMCDO has the ability to compensate the disturbances and obtain more satisfactory control performance. An example of acrobot is proposed to verify the effectiveness of proposed method in comparison with the results of the controller proposed in Ref. [33]. By using the proposed method, both the chattering and the tracking error are significantly reduced even there exist various disturbances. The proposed technology also yields better robust performance compared to the results presented in Ref. [33]. In the future, experiments will be conducted to further validate the theory analysis.

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