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ORIGINAL PAPER

New periodic solitary wave solutions for the (3+1)-dimensional generalized shallow water equation

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Abstract Many important physical situations such as fluid flows, plasma physics and solid-state physics have been described by (3+1)-dimensional generalized shallow water equation. In this article, we construct new periodic solitary wave solutions for the (3+1)-dimensional generalized shallow water equation by using the auto-Bäcklund transformation and a direct test function. These obtained new periodic solitary wave solutions enrich the solution structure. Evidently, with the help of symbolic computation, the physical structure for these periodic solitary wave solutions is presented with some figures. The direct test function approach can be also applied to solve other nonlinear differential equations.

Keywords Auto-Bäcklund transformation · Periodic solitary wave solutions · Generalized shallow water equation · Symbolic computation

Mathematics Subject Classification 35C08 · 68M07 · 33F10

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1 Introduction

Nonlinear evolution equations (NLEEs) are widely used to describe complex sciences phenomena such as the marine engineering, fluid dynamics, plasma physics, chemistry and physics and many other applications [1–15]. During the past several decades, many efficient methods have been proposed to obtain the exact solutions of NLEEs such as inverse scattering method [16], the homotopy perturbation method [17], Hirota direct method [18–26], hyperbolic function method [27], homogeneous balance method [28–30], F-expansion method [31], exp function method [32–34], the extended mapping method [35], the (G′/G)-expansion method [36–38] and three-wave approach [39–44].

In this paper, we will research the following (3+1)-dimensional generalized shallow water equation:

$$u_{xxxy} - 3\,u_x\,u_{xy} - 3\,u_y\,u_{xx} + u_{yt} - u_{xz} = 0. \qquad (1)$$

Equation (1) has been used in weather simulations, tidal waves, river and irrigation ows, tsunami prediction and researched in different ways. Tian [45] obtained the soliton-type solutions of Eq. (1) by using the generalized tan$h$ algorithm method. Zayed [46] got the traveling wave solutions of Eq. (1) by using the (G′/G)-expansion method. Tang [47] presented the Grammian and Pfaffian solutions of Eq. (1) by the Hirota bilinear form. Multiple soliton solutions of Eq. (1) are discussed by Zeng [1]. Next, we will discuss the new periodic solitary wave solutions for Eq. (1) by using the direct test function method. The direct test function is used instead of text function in the original Hirota bilinear method. If the bilinear equation of nonlinear evolution equations is available, then rich variety of exact solutions can be presented by using the direct test function method. These exact solutions are found to possess dynamic characteristics. This used method being simple and straightforward than the method in Refs. [45–47].

The paper is organized as follows: in Sect. 2, by using the auto-Bäcklund transformation and a direct test function, new periodic solitary wave solutions for the (3+1)-dimensional generalized shallow water equation are obtained. In Sect. 3, the conclusions are presented.

2 New periodic solitary wave solutions for the (3+1)-dimensional generalized shallow water equation

According to the idea of the extended variable-coefficient homogeneous balance method (EvcHB) [48], the solutions of Eq. (1) can be supposed as follows:

$$u(x,y,z,t)=[-2\ln(\xi)+\delta(\eta)+\xi\,\sigma(\eta)]_x+u_0(x,y,z,t), \tag{2}$$

where $\xi=\xi(x,y,z,t)$, $\eta=\eta(y,z,t)$ and $u_0(x,y,z,t)$ is a special solution of Eq. (1). Substituting Eq. (2) into (1), we have the following auto-Bäcklund transformation:

$$\xi\,\sigma(\eta)\,\xi_x^2\,(\xi_x\,\xi_{xy}+\xi_y\,\xi_{xx})=0, \tag{3}$$

$$\begin{aligned}&-2\,\xi_z\,\xi_x^2+2\,\xi_t\,\xi_y\,\xi_x+[3\,\xi\,\sigma(\eta)+2]\,\xi_y\,\xi_{xxx}\,\xi_x\\&-6\,u_{0y}\,\xi_x^3+3\,\xi_y\,[\xi\,\sigma(\eta)\,\xi_{xx}^2-2\,\xi_x^2\,u_{0x}]\\&+3\{[5\,\xi\,\sigma(\eta)-2]\,\xi_{xy}\,\xi_{xx}\\&+[\xi\,\sigma(\eta)+2]\,\xi_x\,\xi_{xxy}\}\,\xi_x=0, \quad -3u_{0xy}\,\xi_x^2 \qquad (4)\\&+\xi_{yt}\,\xi_x-(2\,\xi_{xz}+6\,u_{0x}\,\xi_{xy}+9\,u_{0y}\,\xi_{xx})\,\xi_x\\&+4\,\xi_{xxxy}\,\xi_x-\xi_z\,\xi_{xx}+\xi_{xy}\,(\xi_t-2\,\xi_{xxx})\\&-\frac{3}{2}\,\xi\,\sigma(\eta)\,[\xi\,\sigma(\eta)-4]\,(\xi_{xx}\,\xi_{xxy}+\xi_{xy}\,\xi_{xxx})\\&+\xi_y\,(\xi_{xt}-3\,u_{0x}\,\xi_{xx}-3\,\xi_x\,u_{0xx}+\xi_{xxxx})=0,\end{aligned} \tag{5}$$

$$\begin{aligned}&\xi_{xyt}-3\,u_{0xy}\,\xi_{xx}-3\,\xi_{xy}\,u_{0xx}-\xi_{xxz}-3\,u_{0x}\,\xi_{xxy}\\&-3\,u_{0y}\,\xi_{xxx}+\xi_{xxxxy}=0,\end{aligned} \tag{6}$$

$$\sigma'(\eta)\,(\eta_t\,\xi_{xy}-\eta_z\,\xi_{xx})=0, \tag{7}$$

$$u_{0xxxy}-3\,u_{0x}\,u_{0xy}-3\,u_{0y}\,u_{0xx}+u_{0yt}-u_{0xz}=0. \tag{8}$$

Aiming at the new periodic solitary wave solutions, we suppose that $\sigma(\eta)=0$, $u_0(x,y,z,t)=0$ and a direct test function

$$\xi(x,y,z,t)=k_1\,e^{\theta_1}+e^{-\theta_1}+k_2\,\tan(\theta_2)+k_3\,\tan h\,(\theta_3)\,, \tag{9}$$

where $\theta_i=\alpha_i\,x+\beta_i\,y+\gamma_i\,z+\delta_i\,t$, $i=1,2,3,4$ and α_i, β_i, γ_i, δ_i are constants to be determined later. Substituting Eq. (9) into (3)–(8) and equating all the coefficients of different powers of e^{θ_1}, $e^{-\theta_1}$, $\tan(\theta_2)$, $\tan h\,(\theta_3)$ and constant term to zero, we can obtain a set of algebraic equations for α_i, β_i, γ_i, δ_i, $k_i (i=1,2,3,4)$. Solving the system with the aid of Mathematical, we obtain the following results:

Case(1)

$$k_2=\beta_1=\gamma_1=\alpha_3=\delta_3=0,\ \gamma_3=\frac{\beta_3\alpha_1^3+\beta_3\delta_1}{\alpha_1}, \tag{10}$$

where α_1, δ_1, β_3, k_1 and k_3 are arbitrary constants. Substituting Eq. (10) into (9), we have

$$\xi(x,y,z,t)=e^{x\alpha_1+t\delta_1}k_1+e^{-x\alpha_1-t\delta_1}+k_3\tan h[y\beta_3+\frac{z\left(\beta_3\alpha_1^3+\beta_3\delta_1\right)}{\alpha_1}]. \tag{11}$$

Therefore, we obtain the first new periodic solitary wave solution for Eq. (1):

$$u_1=-\frac{2\left(e^{x\alpha_1+t\delta_1}k_1\alpha_1-e^{-x\alpha_1-t\delta_1}\alpha_1\right)}{e^{x\alpha_1+t\delta_1}k_1+e^{-x\alpha_1-t\delta_1}+k_3\tan h\left[y\beta_3+\frac{z(\beta_3\alpha_1^3+\beta_3\delta_1)}{\alpha_1}\right]}. \tag{12}$$

The evolution and mechanical feature of Eq. (12) are shown in Figs. 1, 2.

Case(2)

$$k_3=\beta_1=\gamma_1=\alpha_2=\delta_2=0,\ \gamma_2=\frac{\beta_2\alpha_1^3+\beta_2\delta_1}{\alpha_1}, \tag{13}$$

where α_1, δ_1, β_2, k_1 and k_2 are arbitrary constants. Substituting Eq. (13) into (9), we have

$$\xi(x,y,z,t)=e^{x\alpha_1+t\delta_1}k_1+e^{-x\alpha_1-t\delta_1}+k_2\tan\left[y\beta_2+\frac{z\left(\beta_2\alpha_1^3+\beta_2\delta_1\right)}{\alpha_1}\right]. \tag{14}$$

Therefore, we obtain the second new periodic solitary wave solutions for Eq. (1):

$$u_2 = -\frac{2\left(e^{x\alpha_1+t\delta_1}k_1\alpha_1 - e^{-x\alpha_1-t\delta_1}\alpha_1\right)}{e^{x\alpha_1+t\delta_1}k_1 + e^{-x\alpha_1-t\delta_1} + k_2 \tan\left[y\beta_2 + \frac{z(\beta_2\alpha_1^3+\beta_2\delta_1)}{\alpha_1}\right]}. \tag{15}$$

The evolution and mechanical feature of Eq. (15) are shown in Figs. 3, 4 and 5.

Case(3)

$$k_1 = \alpha_2 = \delta_2 = \alpha_3 = \delta_3 = 0,\ \gamma_1 = \frac{\beta_1\alpha_1^3 + \beta_1\delta_1}{\alpha_1},$$
$$\gamma_2 = \frac{\beta_2\alpha_1^3 + \beta_2\delta_1}{\alpha_1},\ \gamma_3 = \frac{\beta_3\alpha_1^3 + \beta_3\delta_1}{\alpha_1}, \tag{16}$$

where α_1, δ_1, β_1, β_2, β_3, k_2 and k_3 are arbitrary constants. Substituting Eq. (16) into (9), we have

$$\xi(x, y, z, t) = k_2 \tan\left[y\beta_2 + \frac{z\left(\beta_2\alpha_1^3 + \beta_2\delta_1\right)}{\alpha_1}\right] + e^{-x\alpha_1 - y\beta_1 - t\delta_1 - \frac{z(\beta_1\alpha_1^3+\beta_1\delta_1)}{\alpha_1}} + k_3 \tan h\left[y\beta_3 + \frac{z\left(\beta_3\alpha_1^3 + \beta_3\delta_1\right)}{\alpha_1}\right]. \tag{17}$$

Therefore, we obtain the third new periodic solitary wave solutions for Eq. (1):

$$u_3 = 2e^{-x\alpha_1 - y\beta_1 - t\delta_1 - \frac{z(\beta_1\alpha_1^3+\beta_1\delta_1)}{\alpha_1}}\alpha_1 \Big/ \left[k_2 \tan\left[y\beta_2 + \frac{z\left(\beta_2\alpha_1^3 + \beta_2\delta_1\right)}{\alpha_1}\right] + e^{-x\alpha_1 - y\beta_1 - t\delta_1 - \frac{z(\beta_1\alpha_1^3+\beta_1\delta_1)}{\alpha_1}} + k_3 \tan h\left[y\beta_3 + \frac{z\left(\beta_3\alpha_1^3 + \beta_3\delta_1\right)}{\alpha_1}\right]\right]. \tag{18}$$

The evolution and mechanical feature of Eq. (18) are shown in Figs. 6, 7.

Case(4)

$$k_1 = \alpha_2 = \beta_2 = \alpha_3 = \beta_3 = 0,\ \gamma_1 = \frac{\beta_1\alpha_1^3 + \beta_1\delta_1}{\alpha_1},\ \gamma_2 = \frac{\beta_1\delta_2}{\alpha_1},\ \gamma_3 = \frac{\beta_1\delta_3}{\alpha_1}, \tag{19}$$

where α_1, δ_1, β_1, δ_2, δ_3, k_2 and k_3 are arbitrary constants. Substituting Eq. (19) into (9), we have

$$\xi(x, y, z, t) = k_2 \tan\left(t\delta_2 + \frac{z\beta_1\delta_2}{\alpha_1}\right) + k_3 \tan h\left(t\delta_3 + \frac{z\beta_1\delta_3}{\alpha_1}\right) + e^{-x\alpha_1 - y\beta_1 - t\delta_1 - \frac{z(\beta_1\alpha_1^3+\beta_1\delta_1)}{\alpha_1}}. \tag{20}$$

Therefore, we obtain the fourth new periodic solitary wave solutions for Eq.(1):

$$u_4 = 2e^{-x\alpha_1 - y\beta_1 - t\delta_1 - \frac{z(\beta_1\alpha_1^3+\beta_1\delta_1)}{\alpha_1}}\alpha_1 \Big/ \left[k_2 \tan\left(t\delta_2 + \frac{z\beta_1\delta_2}{\alpha_1}\right) + k_3 \tan h\left(t\delta_3 + \frac{z\beta_1\delta_3}{\alpha_1}\right) + e^{-x\alpha_1 - y\beta_1 - t\delta_1 - \frac{z(\beta_1\alpha_1^3+\beta_1\delta_1)}{\alpha_1}}\right]. \tag{21}$$

The evolution and mechanical feature of Eq. (21) are shown in Figs. 8, 9.

3 Conclusion

By using the auto-Bäcklund transformation and a direct test function, we obtain new periodic solitary wave solutions of the (3+1)-dimensional generalized shallow

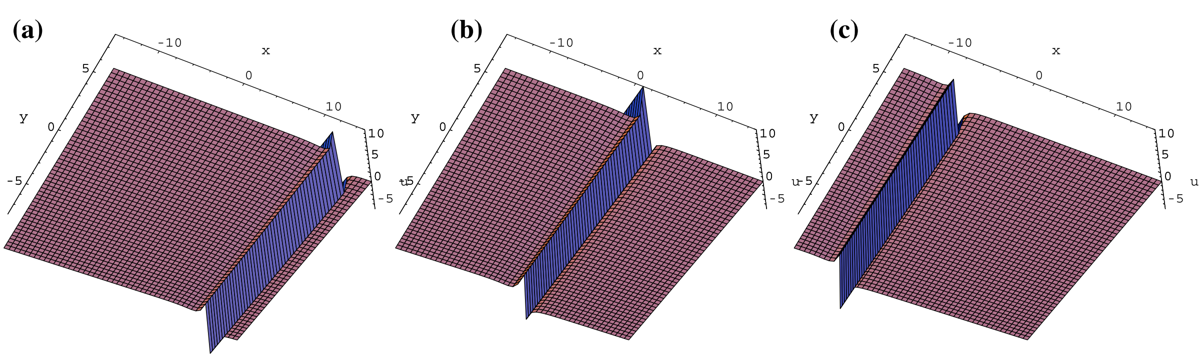

Fig. 1 The solitary wave solution (12) at $k_1 = k_3 = \delta_1 = -2$, $\alpha_1 = -1$, $\beta_3 = 5$, $z = 10$, **a** $t = -5$, **b** $t = 0$, **c** $t = 5$

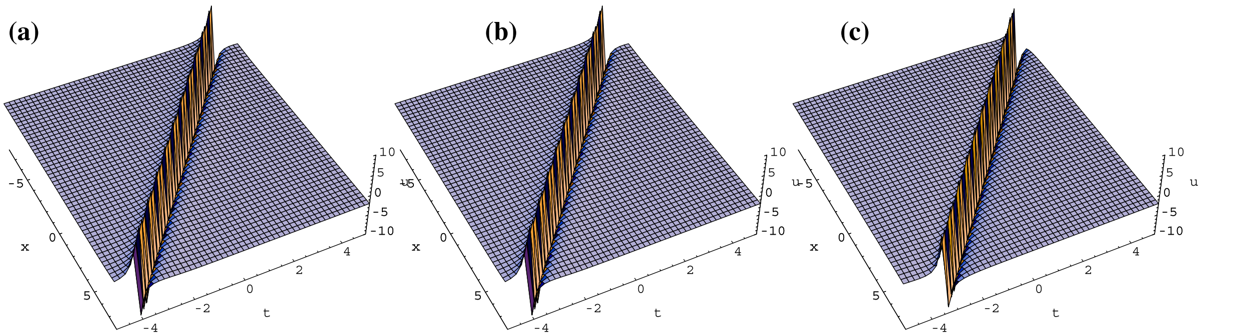

Fig. 2 The solitary wave solution (12) at $k_1 = k_3 = \delta_1 = -2$, $\alpha_1 = -1$, $\beta_3 = 5$, $y = -1$, **a** $z = -20$, **b** $z = 0$, **c** $z = 20$

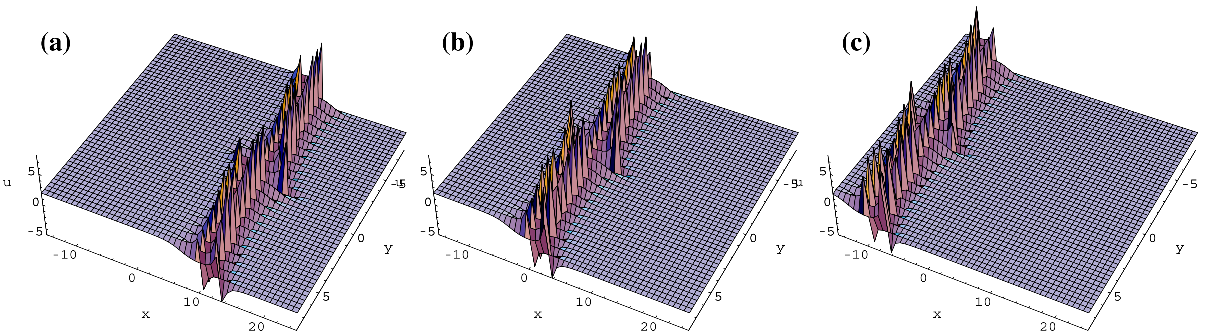

Fig. 3 The solitary wave solution (15) at $k_1 = k_2 = \delta_1 = -2$, $\alpha_1 = -1$, $\beta_2 = 5$, $z = 10$, **a** $t = -5$, **b** $t = 0$, **c** $t = 5$

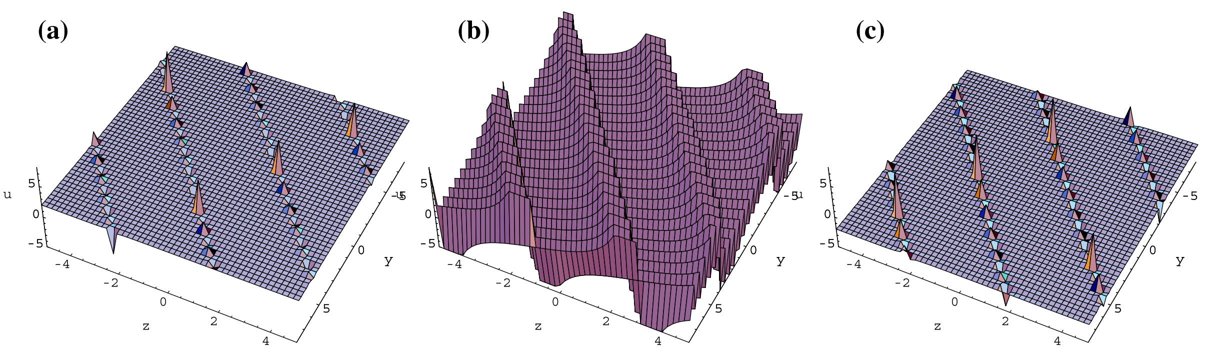

Fig. 4 The solitary wave solution (15) at $k_1 = k_2 = \delta_1 = -2$, $\alpha_1 = -1$, $\beta_2 = 5$, $t = -5$, **a** $x = 5$, **b** $x = 10$, **c** $x = 15$

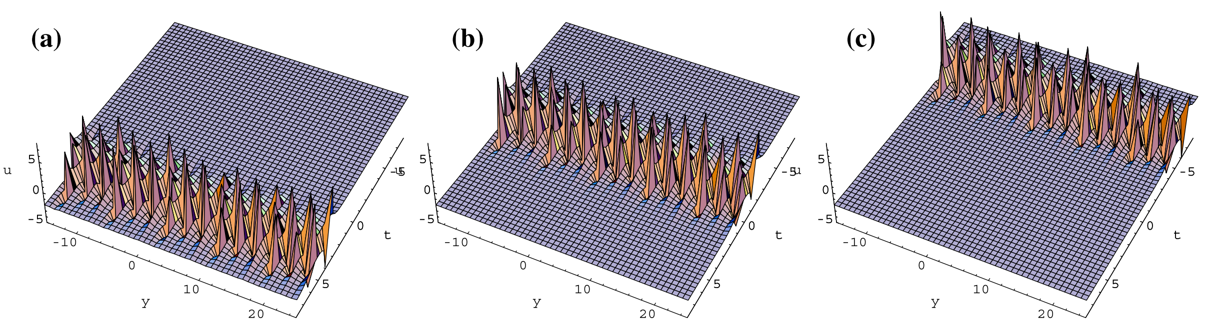

Fig. 5 The solitary wave solution (15) at $k_1 = k_2 = \delta_1 = -2$, $\alpha_1 = -1$, $\beta_2 = 5$, $z = 10$, **a** $x = -10$, **b** $x = 0$, **c** $x = 10$

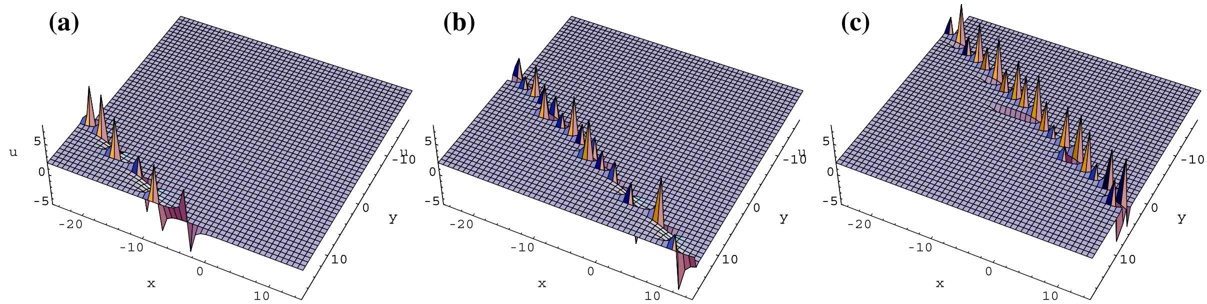

Fig. 6 The solitary wave solution (18) at $k_3 = k_2 = \beta_1 = \delta_1 = -2$, $\alpha_1 = 1$, $\beta_2 = \beta_3 = -5$, $z = 10$, **a** $t = -10$, **b** $t = 0$, **c** $t = 10$

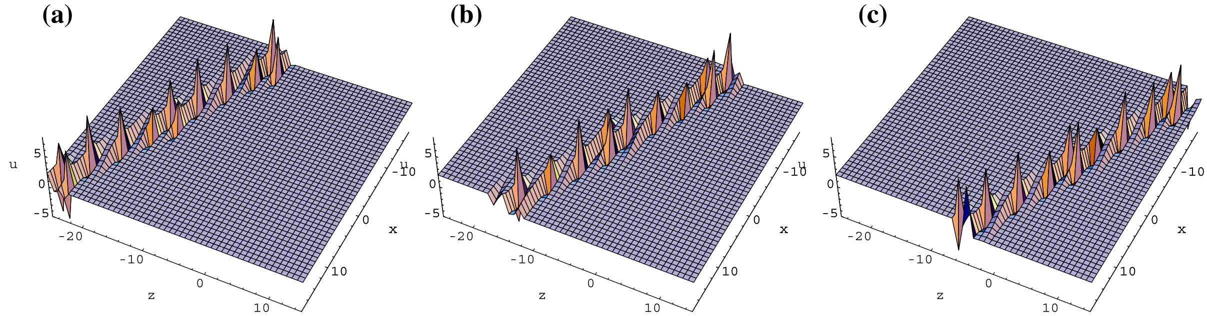

Fig. 7 The solitary wave solution (18) at $k_2 = \beta_1 = \delta_1 = -2$, $k_3 = 0$, $\alpha_1 = 1$, $\beta_2 = \beta_3 = -5$, $t = -5$, **a** $y = -10$, **b** $y = 0$, **c** $y = 10$

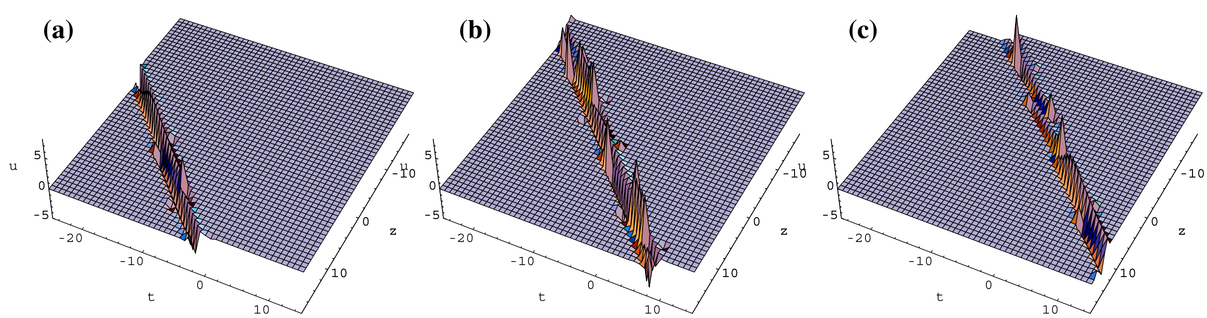

Fig. 8 The solitary wave solution (21) at $k_3 = k_2 = \beta_1 = \delta_1 = -2$, $\alpha_1 = 1$, $\delta_2 = \delta_3 = -5$, $y = 10$, **a** $x = -20$, **b** $x = 0$, **c** $x = 20$

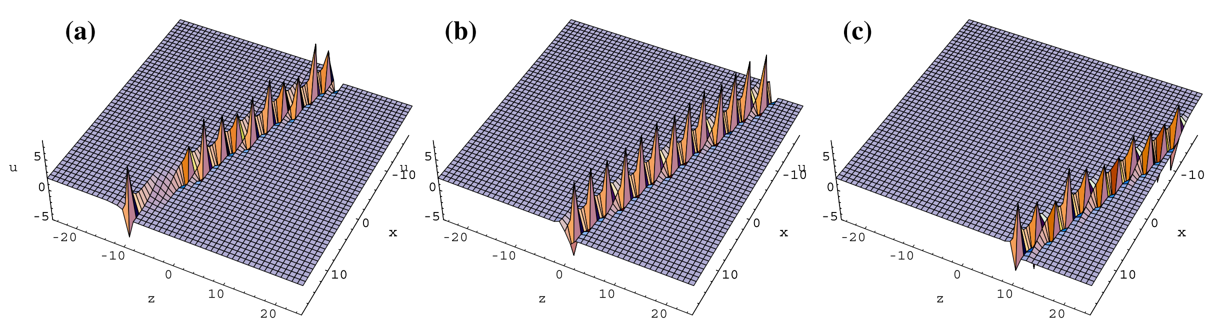

Fig. 9 The solitary wave solution (21) at $k_3 = k_2 = \beta_1 = \delta_1 = -2$, $\alpha_1 = 1$, $\delta_2 = \delta_3 = -5$, $y = 10$, **a** $t = -10$, **b** $t = 0$, **c** $t = 10$

water equation. Moreover, the evolution and mechanical feature of solutions (12), (15), (18) and (21) are clearly presented in Figs. 1, 2, 3, 4, 5, 6, 7 , 8 and 9.

The direct test function method is reliable and effective and obtains many new periodic solitary wave solutions. The applied method will be used in further works to seek more entirely periodic solitary wave solutions of higher dimensional nonlinear evolution equations.

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