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REVIEW

A review for dynamics in neuron and neuronal network

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Abstract The biological Hodgkin–Huxley model and its simplified versions have confirmed its effectiveness for recognizing and understanding the electrical activities in neurons, and bifurcation analysis is often used to detect the mode transition in neuronal activities. Within the collective behaviors of neurons, neuronal network with different topology is designed to study the synchronization behavior and spatial pattern formation. In this review, the authors give careful comments for the presented neuron models and present some open problems in this field, nonlinear analysis could be effective to further discuss these problems and some results could be helpful to give possible guidance in the field of neurodynamics.

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1 Introduction

Neuron is thought as the basic unit in neuronal system and the electrical activities show distinct nonlinear properties. For example, external forcing can induce mode transition in electrical activities from quiescent state to spiking, bursting and even chaotic states in neurons. The Hodgkin–Huxley neuron [1] model and its many developed versions [2–6] have been available for bifurcation analysis and understanding the dynamical response to external stimuli, synchronization stability and evolution of collective behaviors under coupling. Based on these neuron models, dynamical analysis is carried out on the isolate neuron model, particularly, coherence resonance and stochastic resonance are induced by imposing appropriate noise, as a result, distinct regularity can be found in the sampled time series for membrane potentials. Furthermore, collective behaviors are investigated on the neuronal network connected with different topological connection, such as synchronization transition, pattern selection in the network. And these results are important and helpful to understand potential mechanism for occurrence of neuronal disease, so reliable schemes can be presented to prevent the breakdown of neuronal systems. Brain is a complex neuronal system, which contains a large number of neurons, and excitation–inhibition balance between neurons is important to keep normal and healthy brain. As a result, it is interesting to detect and analyze the collective behaviors in brain by using nonlinear dynamics and model setting from the sampled time series for oscillating behaviors [7–10]. In the following section, relevant topics will be summarized with short comments.

The first topic: Functional connection and anatomic connection to neuron In the last decades, eyes are often emphasized on the electrical activities of neurons and many neuron models and relevant nonlinear circuits are used to produce the complex sampled time series that are consistent with biological experimental data by setting appropriate parameters and external forcing. Extensive evidences confirmed that astrocyte could also be helpful to regulate the electrical activities of neuron beside the protection from injury for neurons. As a result, neuron-coupled astrocyte network [11–19] is designed to investigate the dynamical response and changes of calcium ions, and it is believed to explain the occurrence mechanism of seizure [20]. Indeed, some intermediary neurons have autapse connection [21–26], which the synapse connects to neuron or soma via a close loop, and the modulation of autapse driving is described by a time-delayed feedback on the membrane potential [27]. Some evidence have confirmed that autapse connection can enhance the self-adaption of neuron to external stimuli and appropriate distribution of autapse in the network can induce continuous waves to regulate the collective behaviors of neuronal network. For example, autapse driving in neuron and network can induce coherence resonance [28]. A various spatial pattern can be formed and selected by applying appropriate time delay and feedback gain in the autapse in the neuronal network [29]. Autapse is classified as chemical autapse and electric autapse, it is found that electric autapse can give quick response to external stimuli while chemical can modulate the electrical activities slowly. Within the network, negative feedback in autapse can generate defects to block the wave propagation while positive feedback can induce stable pulses, fronts of target wave to regulate the collective behavior of the neuronal network [30,31]. That is to say, autapse connection can be of importance for signal transmission and self-adaption to external stimuli. Particularly, Wang et al. argued that the formation mechanism of autapse could be associated with the injury of neuron, and the development of autapse can be helpful to propagate the blocked signal via an auxiliary loop [32]. As a result, Guo et al. [33] suggested that electrical field can be imposed on the injured area of the axon so that blocked signal can be transmitted. Readers can find the original illustration for the autapse function on injured axon of neuron, it is also shown in Fig. 1 as follows.

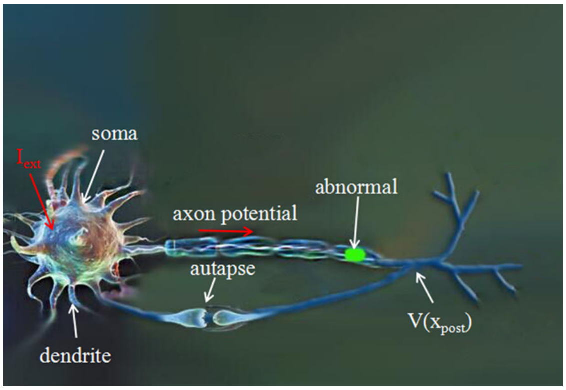

Fig. 1 Autapse formation on a cable neuron model, it is shown in Fig. 1 in Ref. [32]

The autapse current for electric type is often described by

$$I_{\text{auta}} = g(V(t-\tau) - V(t)) \tag{1}$$

where τ is the time delay in the autapse and g is the feedback gain, V is the membrane potential of neuron. A switch between negative feedback and positive feedback in autapse can be realized by setting appropriate gains in autapse, and a time-varying feedback gain can enhance the self-adaption of neuron to external forcing and electrical stimuli. Furthermore, autapse connection to neuron can enhance the robustness to electromagnetic radiation, and the disturbance resulting from electromagnetic radiation on neurons can be suppressed by autaptic modulation [34]. For biological neuron, described by Hodgkin–Huxley model [1], channel noise [35] and temperature are important factors, which can change the excitability and modes in electrical activities, it often reads as follows

$$\begin{aligned}
C_m \mathrm{d}V/\mathrm{d}t &= \tilde{g}_{\mathrm{K}} n^4 (V_{\mathrm{K}} - V) + \tilde{g}_{\mathrm{Na}} m^3 h (V_{\mathrm{Na}} - V) \\
&\quad + \tilde{g}_L (V_L - V) + I_{\text{ext}}; \\
\mathrm{d}m/\mathrm{d}t &= a_m(V)(1-m) - \beta_m(V)m + \xi_m(t); \\
\mathrm{d}h/\mathrm{d}t &= a_h(V)(1-h) - \beta_h(V)h + \xi_h(t); \\
\mathrm{d}n/\mathrm{d}t &= a_n(V)(1-n) - \beta_n(V)n + \xi_n(t); \\
a_m &= 0.1(V+40)\phi(T)/[1-\exp(-(V+40)/10)];
\end{aligned}$$

$$\beta_m = 4\phi(T)\exp(-(V+65)/18);$$
$$a_h = 0.07\phi(T)\exp(-(V+65)/20);$$
$$\beta_h = \phi(T)/[1+\exp(-(V+35)/10)];$$
$$a_n = 0.01(V+55)\phi(T)/[1-\exp(-(V+55)/10)];$$
$$\beta_n = 0.125\phi(T)\exp(-(V+65)/80);$$
$$\phi(T) = 3^{(T-6.3\,^{\circ}\mathrm{C})/10\,^{\circ}\mathrm{C}}; \tag{2}$$

where the variable V describes the membrane potential of the neurons, m , n and h are parameters for gate channel, T is temperature of membrane, I_{ext} is external stimuli and the capacitance of membrane is $C_m = 1\,\mu\text{F/cm}^2$. The maximal conductance of potassium is $\tilde{g}_{\text{K}} = 36\,\text{mS/cm}^2$, the maximal conductance of sodium is $\tilde{g}_{\text{Na}} = 120\,\text{mS/cm}^2$, the conductance of leakage current is $\tilde{g}_{\text{L}} = 0.3\,\text{mS/cm}^2$. The reversal potential $V_{\text{K}} = -77\,\text{mV}$, $V_{Na} = 50\,\text{mV}$ and $V_{\text{L}} = -54.4\,\text{mV}$. $\xi_m(t)$, $\xi_h(t)$, $\xi_n(t)$, are independent Gaussian white noise [36] and the statistic properties [37,38] of the channel noise are defined by

$$\langle \xi_m(t) \rangle = 0;$$
$$\left\langle \xi_m(t)\xi_m(t^{'}) \right\rangle = \frac{2\alpha_m\beta_m\delta(t-t^{'})}{N_{\text{Na}}(\alpha_m+\beta_m)} = D_m\delta(t-t^{'});$$
$$\langle \xi_n(t) \rangle = 0;$$
$$\left\langle \xi_n(t)\xi_n(t^{'}) \right\rangle = \frac{2\alpha_n\beta_n\delta(t-t^{'})}{N_{\text{K}}(\alpha_n+\beta_n)} = D_n\delta(t-t^{'});$$
$$\langle \xi_h(t) \rangle = 0;$$
$$\left\langle \xi_h(t)\xi_h(t^{'}) \right\rangle = \frac{2\alpha_h\beta_h\delta(t-t^{'})}{N_{\text{Na}}(\alpha_h+\beta_h)} = D_h\delta(t-t^{'}); \tag{3}$$

where D_m, D_n, and D_h describe the intensity of noise, and $\delta(t-t^{'}) = 1$ is Dirac-δ function, N_{Na} and N_{K} are the total numbers of sodium and potassium channels present in a given patch of the membrane, respectively. In the case of homogeneous ion channel density, $\rho_{\text{Na}} = 60\,\mu\text{m}^{-2}$ and $\rho_{\text{K}} = 18\,\mu\text{m}^{-2}$, the total channels number is decided by $N_{\text{Na}} = \rho_{\text{Na}}s$ and $N_{\text{K}} = \rho_{\text{K}}s$, and s describes the membrane patch. In case of channel blocking and poisoning, the conductance is modulated as follows

$$g_{\text{K}}(n) = \tilde{g}_K\chi_k = g_K^{\max}\chi_k n^4; \quad g_{\text{Na}}(m,h)$$
$$= \tilde{g}_{\text{Na}}\chi_{\text{Na}} = g_{\text{Na}}^{\max}\chi_{\text{Na}}m^3h; \tag{4}$$

where χ_k, χ_{Na} is the fractions of working, i.e., non-blocked ion channels, to the overall number of potassium, N_{K}, or sodium, N_{Na}, ion channels, respectively. When the effect of channel blocking is considered, the statistical properties of channel noise is described by

$$\langle \xi_m(t) \rangle = 0;$$
$$\langle \xi_m(t)\xi_m(t') \rangle = \frac{2\alpha_m\beta_m\delta(t-t^{'})}{\chi_{\text{Na}}N_{\text{Na}}(\alpha_m+\beta_m)};$$
$$\langle \xi_n(t) \rangle = 0;$$
$$\langle \xi_n(t)\xi_n(t') \rangle = \frac{2\alpha_n\beta_n\delta(t-t^{'})}{\chi_k N_{\text{K}}(\alpha_n+\beta_n)};$$
$$\langle \xi_h(t) \rangle = 0;$$
$$\langle \xi_h(t)\xi_h(t') \rangle = \frac{2\alpha_h\beta_h\delta(t-t^{'})}{\chi_{\text{Na}}N_{\text{Na}}(\alpha_h+\beta_h)}; \tag{5}$$

Compared the description in Eq. (3) with the definition in Eq. (5), it is found that the noise intensity is enhanced when partial ion channels are blocked. Furthermore, according to the definition in Eq. (2), the excitability of the media will be changed by the temperature, as a result, the electrical activities of isolate neuron and even collective behaviors of neuronal network can be modulated by changing the temperature of the cell. However, when autapse connection is considered, the self-adaption can be enhanced to keep robust the effect of temperature fluctuation on membrane potential and also the mode transition in electrical activities. That is, readers can introduce autaptic current into Eq. (2) to investigate the modulation and self-adaption from the autapse driving when temperature is fluctuated.

The second topic: pattern formation and synchronization in neuronal network In the case of network, statistical function or auto-correlation function is often defined to calculate the phase transition induced by bifurcation parameter and topology connection. Based on the mean field theory, a statistical variable [29] (factor of synchronization) is defined to study the collective behaviors and statistic property

$$F = \sum_{j=1}^{N}\sum_{i=1}^{N} V_{ij}/N^2 \tag{6}$$

$$R = \frac{\langle F^2 \rangle - \langle F \rangle^2}{\sum_{j=1}^{N}\sum_{i=1}^{N}\left(\left\langle V_{ij}^2 \right\rangle - \langle V_{ij} \rangle^2\right)/N^2} \tag{7}$$

where R is factor of synchronization, the number of neurons is N^2 and the variable V_{ij} is the membrane potential of neuron. The symbol $\langle . \rangle$ represents the average of variable over time. It is found that perfect synchronization can be reached when the factor of synchronization R is close to 1 while regular spatial patterns such as target wave or spiral waves can be formed to occupy the network when the synchronization factor is very low. That is, non-perfect synchronization is reached when R is very close to zero. More often, regular connection and small-world connection are used to design network, time delay, noise and pacemaker (periodical forcing) are used to induce spatial coherence resonance, spiral wave [39–43] can be formed in the two-dimensional space which the collective behavior can be controlled completely. And it is believed that the emergence of spiral wave in neuronal network can explain the biological function in neocortex. In the nerve system, most of the researchers believed that small-world connection could be more reliable than regular network connection, in fact, the small-world network can be approached by using a local regular connection (nearest-neighbor connection) and long-range connection with certain probability. It is also found that local regular connection can enhance the formation of regular pattern while long-range connection can destroy pattern formation. In numerical studies, no-flux or periodical boundary condition is often used, respectively. In the chain or ring network, spatiotemporal development is often detected for dynamical analysis. As it is well known, strong enough coupling can enhance the synchronization stability and pattern formation, however, collapse-induced instability of network can be triggered by external violent attack or intrinsic destroy in the network [44,45], for example, parameter shift or switch can cause breakdown and instability of spiral waves in the neuronal network, and nonlinear analysis from the sampled time series are helpful to predict the occurrence of breakdown in the network. Synapse plays important role in signal exchange and mode transition in electrical activities of neurons. The normal function of neural networks depends on a delicate balance between excitatory and inhibitory synaptic inputs [46,47]. It is thought that excitatory synaptic inputs are helpful to trigger the electrical activities of neurons, while inhibitory synaptic inputs can calm down the firing in electrical activities. Indeed, many evidences confirmed that inhibitory synapse can enhance neural firing pattern or enhance synchronous degree of coupled neurons and neuronal network [48–51]. Therefore, with the view of pattern selection and dynamics, neuronal network composed of excitatory and inhibitory neurons could be detected to understand the cooperation and self-organization in neuronal network, and further feasible schemes can be used to select appropriate spatial patterns [52].

The third topic: Model setting under electromagnetic induction and radiation The motion of charge particle can be controlled by electromagnetic field and the spatial distribution of charge particles become complex when these charge particles are exposed to external electromagnetic field. For the neuron and biological cell, the electrical activities can be changed due to the electromagnetic induction during the exchange of ion currents and fluctuation of ion concentrations. As reviewed in Ref. [53], the effects of electromagnetic radiation on neuronal electrical activity, energy metabolism, genomic responses, neurotransmitter balance, bloodCbrain barrier permeability, cognitive function, sleep, and various brain diseases including brain tumors should be considered with the increasing use of mobile communication. Lisi et al. [54] investigated the effect of electromagnetic radiations (EMF) at a frequency of 50 Hz on the development of cerebellar granule neurons (CGN). References [55,56] presented experiments to study the oxidative damage to mitochondrial DNA in primary cultured neurons exposed to 1800 MHz radio frequency radiation. Masuda et al. [57] presented experimental verification and discussion about effects of 915 MHz electromagnetic field radiation in TEM cell on the blood–brain barrier and neurons in the rat brain. Xu et al. [58] discussed the effects of microwave exposure on the function of cultured hippocampal neurons of rats using whole cell patch-clamp analysis combined with immunocytochemistry. They found that chronic exposure (15 min per day for 8 days) to global system for mobile communication (GSM) 1800-MHz microwaves at specific absorption rate (SAR) of 2.4 W/kg induced a selective decrease in the amplitude of -amino-3-hydroxy-5-methyl-4-soxazole propionic acid (AMPA) miniature excitatory postsynaptic currents (mEPSCs), whereas the frequency of AMPA mEPSCs and the amplitude of N-methyl-D-aspartate (NMDA) mEPSCs did not change. According to the physical law of electromagnetic induction, the distribution and density of magnetic flux across membrane can be changed when the cell is exposed to electromagnetic field. As mentioned in the effect of autapse, time delay, e.g., response time delay can be used to describe the effect of memory in neuron. However, in physical view, magnetic field can be effective to describe the effect of memory of neuron by generating appropriate spatial distribution and the fluctuation of electromagnetic field will make effective information exchange. On the other hand, neuron is thought as an intelligent circuit to process complex signals in nerve system, and memristor [59] is proposed to design reliable neuronal circuit due to its memory effect because the memductance is dependent on the external stimuli. It reads as follows

$$\rho(\varphi) = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi} = \alpha + 3\beta\varphi^2 \tag{8}$$

where α, β are parameters to be dependent on the memristor. As reported in Refs. [60,61], time-varying change of intercellular and extracellular ion concentration can induce electromagnetic induction, and this effect can be described by using magnetic flux according to the law of electromagnetic induction, the induced current from electromagnetic induction can modulate the membrane potential via feedback by using memristor[59]. The induced current and electromagnetic induction can be described by

$$i^{'} = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}t} = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi}\frac{\mathrm{d}\varphi}{dt} = \rho(\varphi)\frac{\mathrm{d}\varphi}{\mathrm{d}t};$$
$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = kV \tag{9}$$

where the induction coefficient k is dependent on the media and V is the membrane potential. We found that multiple modes of electrical activities [61] can be induced by electromagnetic radiation and these results are consistent with biological experiments. Furthermore, we developed a new cardiac tissue model [62] and explained the potential mechanism for heart diseased induced by electromagnetic radiation. Continuous wave emitting from the sino-atrial node of heart can form stable target waves to regulate the electrical activities and shrinkage of heart, strong electromagnetic radiation and induction can cause breakdown of heartbeat and functional role by breaking and suppressing the propagation of target waves. Furthermore, the synchronization behavior of electrical activities of neurons are discussed when neurons are exposed to noise-like electromagnetic radiation [63], and double coherence resonance behavior is observed.

The fourth topic: neuronal circuit Since the original biological Hodgkin–Huxley neuron model setting, which can describe the electrical activities on large axon of squid and the effect of ion channel is also considered, many oscillator-like neuron models have been designed for dynamical analysis thus the outputs can produce spiking, bursting and even chaotic behaviors to consistent with the sampled series from biological experiments. On the other hand, these neuronal model can be realized in nonlinear circuits by using appropriate nonlinear devices. Besides the well-known nonlinear resistor, capacitor, two important sensitive electric devices should be mentioned. One is the Josephson junction [64–67], which shows superconductivity and quantum characteristics, it is found that Josephson junction coupled resonator can produce main properties of electrical activities in neurons. Another important device is memristor [68–74], which its memductance is dependent on the external stimuli thus memory effect is approached, as a result, nonlinear circuits and systems [75] composed of memristor can be initial-dependent to trigger different attractors in integer and even fractional-order dynamical systems. In circuit realization, mapped from the theoretical neuronal model, parameter can be carefully modulated to trigger spiking, bursting states, PSpice [76,77] and FPGA can be used to verify the reliability of neuronal circuits. In the circuit setting, memristor and Josephson junction can use to generate nonlinear response thus high nonlinearity is formed in the dynamical models and systems, furthermore, memristor can be used to bridge coupling between nonlinear circuits so that both of the coupled circuits can be suppressed by the induced currents.

Furthermore, noise and autapse distribution can be considered in the improved network, the synchronization transition and pattern stability could be more attractive for investigation. The neuronal system is made of different types of functional neurons such as excitatory and inhibitory neurons, and neurons show distinct diversity, readers can set multilayer network to investigate the collective behaviors in networks.

2 Open problems and discussion

The nerve system contains a large number of neurons with complex connection type. Within the collective behavior and synchronization problems, chemical and electric synapse are connected between neurons, neuronal networks are designed via synapse connection and coupling, as a result, bifurcation parameters and noise can be considered to investigate the pattern stability [78–82], synchronization transition [83–87], spatial coherence resonance and stochastic resonance [88–93]. In our view, field coupling could be another effective way to regulate the collective behaviors of neurons and networks. The electrical activities in neurons show certain diversity in amplitude, and the rhythm can carry more important information because phase synchronization can be associated with memory [94]. In the complex physical and biological condition, e.g., noise driving and electromagnetic radiation, complete synchronization between neurons become more difficult while the phase synchronization or rhythm becomes available [95,96]. In Ref. [96], we argued that magnetic coupling could be another effective way to realize phase synchronization though most of the researchers believed that synapse coupling is the most important bridge to exchange signals between neurons. In our view, in the neuronal system, each neuron can set isolate electromagnetic field during the fluctuation of ion concentration and exchange of ion currents via the channels embedded into the membrane, further more, each neuron is also exposed to the integrated electromagnetic field contributed by other neurons according to superposition principle of field. As a result, for two isolate neurons, the field coupling can be reduced by magnetic coupling as follows

$$\begin{aligned}
\frac{\mathrm{d}V_1}{\mathrm{d}t} &= f(V_1, p);\\
\frac{\mathrm{d}\varphi_1}{\mathrm{d}t} &= kV_1 + g(\varphi_2 - \varphi_1);\\
\frac{\mathrm{d}V_2}{\mathrm{d}t} &= f(V_2, p);\\
\frac{\mathrm{d}\varphi_2}{\mathrm{d}t} &= kV_2 + g(\varphi_1 - \varphi_2);
\end{aligned} \tag{10}$$

where $V_1, V_2, \varphi_1, \varphi_2$, denotes the membrane potential, magnetic flux for the two neurons, g is the coupling intensity and p is parameter for the neuron, k is the induction coefficient associated with the media, in $f(V, p)$ represents the local kinetics of neuron model. As confirmed in Ref. [96], phase synchronization can be reached by setting appropriate field coupling intensity. In the previous works, collective behavior and consensus of electrical activities are discussed on regular, small-world, scale-free network, and the topology connection is often considered as bifurcation parameter to study the stability of synchronization behaviors. However, in the case of field coupling, signal exchange can also be realized even neurons are not connected via synapse. The dynamical equations for the network can be described by field coupling as follows

$$\begin{aligned}
\frac{\mathrm{d}V_i}{\mathrm{d}t} &= f(V_i, p);\\
\frac{\mathrm{d}\varphi_i}{\mathrm{d}t} &= kV_i + g\left(\sum_{j\neq i}^{N} \varphi_j - \varphi_i\right);
\end{aligned} \tag{11}$$

where the subscript i is used to discern the neuron in the network without synapse connection and position setting. $\sum_{j\neq i}^{N} \varphi_j$ represents the field and magnetic contribution of other neurons to the ith neuron. For example, a chain distribution for Hindmarsh–Rose neuron can be described by

$$\begin{aligned}
\frac{\mathrm{d}x_i}{\mathrm{d}t} &= y_i - ax_i^3 + bx_i^2 - z_i + I_{\mathrm{ext}} - k\rho(\varphi_i)x_i;\\
\frac{\mathrm{d}y_i}{\mathrm{d}t} &= c - \mathrm{d}x_i^2 - y_i;\\
\frac{\mathrm{d}z_i}{\mathrm{d}t} &= r[s(x_i + 1.6) - z_i];\\
\frac{\mathrm{d}\varphi_i}{\mathrm{d}t} &= kx_i + g\left(\sum_{j\neq i}^{N} \varphi_j - \varphi_i\right);
\end{aligned} \tag{12}$$

where the memductance $\rho(\varphi_i)$ is calculated according to Eq. (8), x, y, z is membrane potential, slow current associated with recovery variable and adaption current, respectively. Furthermore, this problem can also be discussed in the two-dimensional space such as square array, and the dynamical equations can be described by

$$\begin{aligned}
\frac{\mathrm{d}x_{ij}}{dt} &= y_{ij} - ax_{ij}^3 + bx_{ij}^2 - z_{ij} + I_{\mathrm{ext}} - k\rho(\varphi_{ij})x_{ij};\\
\frac{\mathrm{d}y_{ij}}{\mathrm{d}t} &= c - dx_{ij}^2 - y_{ij};\\
\frac{\mathrm{d}z_{ij}}{\mathrm{d}t} &= r[s(x_{ij} + 1.6) - z_{ij}];\\
\frac{\mathrm{d}\varphi_{ij}}{\mathrm{d}t} &= kx_{ij} + g\left(\sum_{l\neq i, m\neq j}^{N} \varphi_{lm} - \varphi_{ij}\right);
\end{aligned} \tag{13}$$

where the subscript ij, l, m is used to mark the neurons in the network, statistical function such as factor of synchronization can be used to detect the stability of synchronization, pattern selection in the network by setting different parameters. Differs from the well-known synapse coupling, maybe, the field coupling between neurons can give new sight to understand the collective behaviors in neuronal networks. This problem can also be carried out on the biological Hodgkin–Huxley neuron model, channel noise, additive noise can also be further considered to investigate the synchronization transition and pattern control for a large number of neurons, it could be helpful to understand the occurrence mechanism of neuronal systems induced by electromagnetic radiation. Finally, it is important to clarify concepts of nonlinear dynamics and their possible function in the real nervous system. Biological experiments can provide enough data for nonlinear analysis and further investigation on health. For example, biological data are helpful to set more reliable neuron models to understand the mode transition in electrical activities of neurons, and parameter regions can also be verified to be consistent with the biological experiments [97,98]. Furthermore, electroencephalograph (EEG) [99–101] and functional magnetic resonance imaging (FMRI) [102–104] can give enough information to understand the activities and connection in brain network, further nonlinear analysis can be helpful to understand emergence mechanism of some neuronal diseases such as Parkinson's disease and epilepsy. That is, nonlinear analysis and dynamical transition based on these biological data could be helpful to understand the biological function and give guidance to prevent the occurrence of neuronal disease.

3 Conclusions

In this review, some important results on dynamics of neuronal electrical activities are introduced for readers in the field of dynamics control. Physical and biological factors should be considered for model setting for complex dynamical systems. Reliable neuron models are important for dynamical analysis and understanding the synchronization behavior, occurrence mechanism of neuronal diseases. As a result, the anatomical structure and functional connection such as autapse connection, physical effect such as electromagnetic induction and radiation should be considered for model setting. In the end, some open problems are presented and argued that field coupling between neurons could be another effective way to exchange signals and information encoding. These discussion could be helpful for further investigation on neurodynamics and relevant dynamical problems. We wish the prediction about field coupling between neurons can be helpful to understand the neuronal disease induced by electromagnetic radiation from the view of dynamical control and physical principle. Readers in this field can present extensive further investigation on this network model for dynamical discussion.

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