

Finite-time synchronization analysis for general complex dynamical networks with hybrid couplings and time-varying delays

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Abstract This paper focuses on the finite-time synchronization problem for a kind of general complex networks with intrinsic time-varying delays and hybrid couplings (i.e., containing current-state couplings and time-varying delay couplings). By designing a simple discontinuous state feedback controller and using strict analytical techniques, several synchronization criteria are proposed to guarantee that the complex dynamical networks can be synchronized onto an isolated chaotic system in finite time. Besides, the upper bound of the synchronization time could be estimated, which is dependent on the initial values of the system as well as the delays. Then, some finite-time synchronization criteria about special cases of the complex networks are also obtained. Here, the coupling configuration matrices are not required to be symmetric or irreducible in all cases. Finally, numerical examples are provided to demonstrate the correctness of our theoretical results.

Keywords Complex dynamical network · Finite-time synchronization · Hybrid coupling · Time-varying delay

1 Introduction

The past decades have witnessed an increasing interest in complex networks because an ocean of natural and man-made systems can be exceedingly described as complex networks, such as the World Wide Web (i.e., search engine optimization), electrical power grids, communication networks, biological networks (i.e., transcriptional regulatory network, virus–host network) and so on [1–5]. Synchronization, which means the dynamical behaviors of coupled nodes in a network achieve the same state both in time and spatial, has become one of the hottest topics in complex networks. Up to date, various types of synchronizations have been considered: complete synchronization [6, 7], exponential synchronization [8, 9], cluster synchronization [10, 11], lag synchronization [12, 13], generalized synchronization [14, 15] and so on.

However, most of existing results concerning synchronization including those mentioned above are actually asymptotic results, which can only be guaranteed when time goes to infinity. Recently, another kind of synchronization, called finite-time synchronization, has attracted more attentions. Finite-time synchronization means that the synchronization goal can be achieved in a setting time, which has many applications, especially in engineering fields. For example, recovering the transmitted signals in a setting time in secure communication is requested to improve the efficiency and confidentiality; therefore, we applied

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finite-time synchronization in it, which has not only the optimality in convergence time, but also better disturbance rejection properties and robustness against uncertainties [16, 17]. Because of various advantages, an increasing number of researchers focused on the finite-time synchronization by using many kinds of control approaches. In [18], the authors designed finite-time synchronization controllers for a class of coupled neural networks by using switching approach. Subsequently, Mei et al. investigated finite-time synchronization of complex dynamical networks with time delay through impulsive and intermittent control schemes simultaneously in [19]. Additionally, [20] discussed finite-time synchronization for a class of chaotic and hyperchaotic systems via adaptive control method. Aghababa et al. studied finite-time synchronization of two different chaotic systems via slid mode control in [21]. Besides, there were also some literatures about the finite-time synchronization of drive-response systems [24, 25]. Here we would focus on the finite-time synchronization of general complex dynamical networks.

However, time delays are unavoidable in practice due to the finite speed of transmission and conjunction [22, 23]. We do not want to see that synchronization of systems with time delays can not be achieved. Consequently, it is indispensable to consider the time delays in studies of synchronization of complex network. As we all know, there generally exist two types of time delays in dynamical networks in practice. One is internal delay occurring inside the dynamical nodes. For instance, the effect of autapse on neuronal activity is often illustrated by adding an additive forcing current along a close loop, which is described by a time-delayed feedback on the membrane potential. In neural processing and signal transmission, axonal signal transmission delays often occur. The other is coupling delay caused by the exchange of information between dynamical nodes. Taking the communication network as an example, we contact our friends by telephone, and the information received would be affected by time delays. Many papers have considered the finite-time synchronization of complex networks without time delays or only with internal delay or coupling delay [18–21, 27–30]. Fortunately, there are a few works about finite-time synchronization with two kinds of time delays in recent years. Such as in [31], finite-time synchronization for complex dynamical networks with hybrid coupling and time-varying delay was developed, but the internal time-varying delays and coupled

time-varying delays are supposed to be the same, which does not match the realities. Very Recently, Li and Cao worked on the finite-time synchronization of coupled networks with one single time-varying delay coupling in [24]. The controller in [24] included the term like “ $(\int_{t-\tau(t)}^t e_i^T(s)e_i(s)ds)^{\frac{1+\eta}{2}} \frac{e_i(t)}{\|e_i(t)\|^2}$,” which are nonlinear and extremely complicated to implement in real applications. Compared to them, the controller proposed in [25, 26] is easier to be applied to the real world, but it only analyzed the neural networks with delays, which is one-dimension networks. To the best of our knowledge, there are few results concerning finite-time synchronization (or finite-time boundedness) for general complex networks with both internal time-varying delays and coupled time-varying delays. In this paper, two types time delay will be considered in the network systems and they are supposed to be time-varying.

Motivated by the above discussions, in this paper, the finite-time synchronization issue for multi-dimension complex dynamical networks with internal time-varying delays and coupled time-varying delays is investigated by applying a novel control scheme. We introduce the 1-norm of each error's vector which can be described the finite-time synchronization rather than 2-norm which have been used for all most existing related literatures. By employing the Lyapunov method combined with rigorous mathematical analysis, a set of simple and easily verifiable sufficient conditions are derived to guarantee finite-time synchronization for the general network system. Meanwhile, the upper bound of the synchronization time is estimated, which is dependent on both initial values of the interest system and the delays. Additionally, some numerical examples are given to verify the effectiveness of the theoretical results.

The main contributions of this paper can be summarized as follows. The first one is that both internal delays and coupling delays are modeled in this paper, which are all time-varying and may be more consistent with the real-world case. In fact, internal time delay can facilitate synchronization. The second one is that a novel controller for achieving finite-time synchronization is designed here, which is discontinuous but easy to be applied in the real world. Moreover, some sufficient conditions guaranteeing the finite-time synchronization are derived through designing a new Lyapunov functional and the proof is rigorous rather than using the well-known finite-time stability theorem, which avoids designing complex controller for meeting the condi-

tions of finite-time stability theorem in most existing literatures. Moreover, some corollaries are presented which show our main results are less conservative and more general than previous ones.

The rest of this paper is organized as follows: Section 2 presents some preliminaries consisting of assumptions and definitions. Section 3 addresses the main results about finite-time synchronization for the present model and its relevant corollaries. Simulation results are given in Sect. 4 to show the effectiveness of the theoretical results. We make a conclusion in the last section.

2 Preliminaries

Consider the following complex dynamical network with time-varying delays

$$\begin{aligned} \dot{x}_i(t) = & f(t, x_i(t), x_i(t - \tau(t))) + c_1 \sum_{j=1}^N a_{ij} \Gamma x_j(t) \\ & + c_2 \sum_{j=1}^N b_{ij} \Gamma x_j(t - \tau_{ij}(t)), i = 1, 2, \dots, N \end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the node i , $f(t, x_i(t), x_i(t - \tau(t))) = (f_1(t, x_i(t), x_i(t - \tau(t))), f_2(t, x_i(t), x_i(t - \tau(t))), \dots, f_n(t, x_i(t), x_i(t - \tau(t))))^T \in \mathbb{R}^n$ represents the dynamical behavior of the i th node, which is a continuous vector-valued function. The constants $c_1 > 0, c_2 > 0$ denote the non-delayed and the delayed coupling strength, respectively. $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ means the inner coupling matrix, and $\gamma_j > 0, j = 1, \dots, n$. $\tau(t)$ is the internal time delay, and $\tau_{ij}(t)$ are called coupling delays. Let $A = (a_{ij}), B = (b_{ij}) \in \mathbb{R}^{N \times N}$ represent the non-delayed and the delayed coupling matrices, respectively, which satisfy $a_{ij} > 0, b_{ij} > 0$ if for all node j receives information from node i and otherwise $a_{ij} = 0, b_{ij} = 0$. The diagonal elements of A and B can be defined as $a_{ii} = -\sum_{j=1, j \neq i}^N a_{ij}, b_{ii} = -\sum_{j=1, j \neq i}^N b_{ij}, i = 1, 2, \dots, N$.

The initial states of the network (1) are $x_i(t) = \varphi_i(t) \in \mathbb{C}([-\tau, 0], \mathbb{R}^n), i = 1, 2, \dots, N$, where $\tau = \max\{\bar{\tau}_{ij}, \bar{\tau}\}, \bar{\tau}_{ij}$ and $\bar{\tau}$ are positive constants satisfying $\tau_{ij}(t) \leq \bar{\tau}_{ij}, \tau(t) \leq \bar{\tau}$, for all $i = 1, 2, \dots, N, j =$

$1, 2, \dots, n. \mathbb{C}([-\tau, 0], \mathbb{R}^n)$ means the set of continuous functions mapping the interval $[-\tau, 0]$ into \mathbb{R}^n . In addition, suppose that $\dot{\tau}_{ij}(t) \leq \mu_{ij} < 1, \dot{\tau}(t) \leq \mu < 1$, for $\forall i = 1, 2, \dots, N, j = 1, 2, \dots, n$.

Let $s(t)$ be a solution of an isolated node described by

$$\dot{s}(t) = f(t, s(t), s(t - \tau(t))) \tag{2}$$

with initial condition $s(t) = \varphi(t) \in \mathbb{C}[-\bar{\tau}, 0]$. In this paper, our objective is to force the network (1) to the desired trajectory $s(t)$ with suitable control strategies in finite time.

Denote $e_i(t) = x_i(t) - s(t)$. Subtracting Eq. (2) from (1) and noting the property of matrices A and B , we have

$$\begin{aligned} \dot{e}_i(t) = & f(t, x_i(t), x_i(t - \tau(t))) - f(t, s(t), s(t - \tau(t))) \\ & + c_1 \sum_{j=1}^N a_{ij} \Gamma e_j(t) + c_2 \sum_{j=1}^N b_{ij} \Gamma e_j(t - \tau_{ij}(t)), \\ & i = 1, 2, \dots, N. \end{aligned} \tag{3}$$

In order to achieve the synchronization of complex network (1), some controllers $u_i(t)$ are needed to be introduced and the controlled error system can be described as

$$\begin{aligned} \dot{e}_i(t) = & f(t, x_i(t), x_i(t - \tau(t))) - f(t, s(t), s(t - \tau(t))) \\ & + c_1 \sum_{j=1}^N a_{ij} \Gamma e_j(t) + c_2 \sum_{j=1}^N b_{ij} \Gamma e_j(t - \tau_{ij}(t)) \\ & + u_i(t), i = 1, 2, \dots, N. \end{aligned} \tag{4}$$

To proceed our study, the following assumption conditions are needed.

Assumption 1 [33] There exist two constant matrices $\Theta = (\theta_{ij})_{n \times n}$ and $\Phi = (\varphi_{ij})_{n \times n}$ in which $\varphi_{ij} \geq 0, \theta_{ij} \geq 0$, such that

$$\begin{aligned} & |f_i(t, x(t), x(t - \tau(t))) - f_i(t, y(t), y(t - \tau(t)))| \\ & \leq \sum_{j=1}^n (\theta_{ij} |x_j(t) - y_j(t)| + \varphi_{ij} |x_j(t - \tau(t)) - y_j(t - \tau(t))|), \\ & \forall x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n, y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n, \\ & i = 1, 2, \dots, n. \end{aligned} \tag{5}$$

Remark 1 When $\tau(t) = 0$, then inequality (5) in assumption 1 is changed to

$$|f_i(t, x(t)) - f_i(t, y(t))| \leq \sum_{j=1}^n \theta_{ij} |x_j(t) - y_j(t)|$$

Remark 2 The reason why we use Assumption 1 is to deal with the nonlinear dynamical function of every node, generally speaking, the QUAD condition and the Lipschitz condition are usually used for 2-norm to manage nonlinear term, while the Lyapunov functional is based on 1-norm in this paper instead of 2-norm.

Definition 1 [25] The complex networks (1) are said to be synchronized with (2) in finite time if, for a suitable designed feedback controller, there exists a constant $t_1 > 0$, such that $\|e_i(t_1)\|_1 = 0$ and $\|e_i(t)\|_1 \equiv 0$ for $t > t_1$ and $i = 1, 2, \dots, N$, where $\|e_i(t)\|_1 = \sum_{j=1}^n |e_{ij}(t)|$, for $e_i(t) = (e_{i1}(t), e_{i2}(t), \dots, e_{in}(t))^T$, t_1 is called the setting time.

3 Main results

In this section, we will investigate the finite-time synchronization of complex network (1), which is equivalent to study the stability of error dynamical system (3) at the origin in a setting time. some sufficient criteria to guarantee the origin stab are given.

Design the controller as

$$u_i(t) = -\xi_i e_i(t) - \delta_i \text{sign}(e_i(t)), \quad i = 1, 2, \dots, N, \tag{6}$$

where $\text{sign}(\cdot)$ is the sign function, $\text{sign}(e_i(t)) = (\text{sign}(e_{i1}(t)), \text{sign}(e_{i2}(t)), \dots, \text{sign}(e_{in}(t)))^T$, and $\xi_i > 0, \delta_i > 0$ are the tunable constants for $i = 1, 2, \dots, N$.

Remark 3 The traditional controllers contain such as “ $u_i(t) = -\delta_i \text{sign}(e_i(t))|e_i(t)|^\beta, 0 < \beta < 1$ ” in [18] and most existing literature, if the considered system contains time delay, the designed controller is more complicated [24]. It is clear that the controller we designed here is simpler than previous one and more easier to implement in reality.

In the following, we will present the main results of this paper, which ensure the achievement of the synchronization for the general complex networks in finite time.

Theorem 1 Suppose that Assumption 1 holds. If there exist $\delta_i > 0$ and $\xi_i > 0$ such that

$$\begin{aligned} & \sum_{j=1}^n \theta_{jv} + \sum_{j=1}^n \frac{1}{1-\mu} \varphi_{jv} + c_1 \sum_{j=1}^N |a_{ji}| \gamma_v \\ & + c_2 \sum_{j=1}^N \frac{1}{1-\mu_{ji}} \\ & \times |b_{ji}| \gamma_v - \xi_i \leq 0, \quad i = 1, \dots, N, \quad v = 1, \dots, n, \end{aligned} \tag{7}$$

then error network (4) is stable in finite time, which means synchronization between complex dynamical network (1) and (2) is achieved under controller (6). Furthermore, for the given initial value $e_i(0)$, the setting time is estimated as

$$\begin{aligned} \Upsilon \leq & (1/\delta) \left[\sum_{i=1}^N \text{sign}^T(e_i(0))e_i(0) \right. \\ & + \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \frac{1}{1-\mu} \varphi_{jv} \times \int_{-\bar{\tau}}^0 |e_{iv}(s)| ds \\ & \left. + c_2 \sum_{i=1}^N \sum_{j=1}^N \frac{1}{1-\mu_{ij}} |b_{ij}| \int_{-\bar{\tau}_{ij}}^0 \text{sign}^T(e_j(s)) \Gamma e_j(s) ds \right] - \tau, \end{aligned}$$

where $\underline{\delta} = \min\{\delta_i, i = 1, 2, \dots, N\}$.

Proof Consider the Lyapunov–Krasovskii functional candidate as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{8}$$

where

$$V_1(t) = \sum_{i=1}^N \text{sign}^T(e_i(t))e_i(t), \tag{9}$$

$$V_2(t) = \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \frac{1}{1-\mu} \varphi_{jv} \int_{t-\tau}^t |e_{iv}(s)| ds, \tag{10}$$

and

$$\begin{aligned} V_3(t) = & c_2 \sum_{i=1}^N \sum_{j=1}^N \frac{1}{1-\mu_{ij}} |b_{ij}| \\ & \times \int_{t-\tau_{ij}(t)}^t \text{sign}^T(e_j(s)) \Gamma e_j(s) ds. \end{aligned} \tag{11}$$

Calculating the time derivative of $V_1(t)$, $V_2(t)$ and $V_3(t)$ along controlled error system (4), it is followed from inequation (5) that

$$\begin{aligned} \frac{dV_1(t)}{dt} &= \sum_{i=1}^N \text{sign}^T(e_i(t))[f(t, x_i(t), x_i(t - \tau(t))) \\ &\quad - f(t, s(t), s(t - \tau(t)))] + c_1 \sum_{j=1}^N a_{ij} \Gamma e_j(t) \\ &\quad + c_2 \sum_{j=1}^N b_{ij} \Gamma e_j(t - \tau_{ij}(t)) - \xi_i e_i(t) - \delta_i \text{sign}(e_i(t))] \\ &\leq \sum_{i=1}^N \sum_{j=1}^n |f_j(t, x_i(t), x_i(t - \tau(t))) - f_j(t, s(t), s(t - \tau(t)))| \\ &\quad + c_1 \sum_{i=1}^N \sum_{j=1}^N a_{ij} \text{sign}^T(e_i(t)) \Gamma e_j(t) \\ &\quad + c_2 \sum_{i=1}^N \sum_{j=1}^N b_{ij} \text{sign}^T(e_i(t)) \Gamma e_j(t - \tau_{ij}(t)) \\ &\quad - \sum_{i=1}^N \xi_i \text{sign}^T(e_i(t)) e_i(t) - \sum_{i=1}^N \delta_i \text{sign}^T(e_i(t)) \text{sign}(e_i(t)) \\ &\leq \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \theta_{jv} |e_{iv}(t)| + \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \varphi_{jv} |e_{iv}(t - \tau(t))| \\ &\quad + c_1 \sum_{i=1}^N \sum_{j=1}^N \sum_{v=1}^n |a_{ij}| \gamma_v |e_{jv}(t)| \\ &\quad + c_2 \sum_{i=1}^N \sum_{j=1}^N \sum_{v=1}^n |b_{ij}| \gamma_v |e_{jv}(t - \tau_{ij}(t))| \\ &\quad - \sum_{i=1}^N \sum_{j=1}^n \xi_i |e_{ij}(t)| - \sum_{i=1}^N \delta_i \text{sign}^T(e_i(t)) \text{sign}(e_i(t)), \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{dV_2(t)}{dt} &\leq \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \frac{1}{1 - \mu} \varphi_{jv} |e_{iv}(t)| \\ &\quad - \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \varphi_{jv} |e_{iv}(t - \tau(t))|, \end{aligned} \tag{13}$$

and

$$\begin{aligned} \frac{dV_3(t)}{dt} &\leq c_2 \sum_{i=1}^N \sum_{j=1}^N \sum_{v=1}^n \frac{1}{1 - \mu_{ij}} |b_{ij}| \gamma_v |e_{jv}(t)| \\ &\quad - c_2 \sum_{i=1}^N \sum_{j=1}^N \sum_{v=1}^n |b_{ij}| \gamma_v |e_{jv}(t - \tau_{ij}(t))|. \end{aligned} \tag{14}$$

According to Eqs. (12), (13) and (14), we get

$$\begin{aligned} \frac{dV(t)}{dt} &\leq \sum_{i=1}^N \sum_{v=1}^n (\sum_{j=1}^n \theta_{jv} + \sum_{j=1}^n \frac{1}{1 - \mu} \varphi_{jv} + c_1 \sum_{j=1}^N |a_{ji}| \gamma_v \\ &\quad + c_2 \sum_{j=1}^N \frac{1}{1 - \mu_{ji}} |b_{ji}| \gamma_v - \xi_i) |e_{iv}(t)| - \sum_{i=1}^N \delta_i \lambda_i, \end{aligned} \tag{15}$$

where $\lambda_i = \text{sign}^T(e_i(t)) \text{sign}(e_i(t))$. It can be easily found that $\lambda_i \geq 1$ if there exists $j = 1, 2, \dots, n$ such that $e_{ij}(t) \neq 0$, otherwise $\lambda_i = 0$.

Noticing condition (7), (15) yields the following inequality

$$\frac{dV(t)}{dt} \leq - \sum_{i=1}^N \delta_i \lambda_i. \tag{16}$$

Since $V(t)$ is positive and non-increasing, with considering the above analysis, there must exist nonnegative constant V^* such that

$$\lim_{t \rightarrow +\infty} V(t) = V^* \quad \text{and} \quad V(t) \geq V^*, \quad \forall t \geq 0. \tag{17}$$

On the other hand, taking the integration both sides of inequation (16) from 0 to t , we can obtain that

$$V(t) - V(0) \leq - \sum_{i=1}^N \delta_i \lambda_i t. \tag{18}$$

If there exists $i = 1, 2, \dots, N$ such that $\|e_i(t)\|_1 > 0$ hold for all $t \in [0, +\infty)$, then $-\sum_{i=1}^N \delta_i \lambda_i < 0$. From (18), we can get $\lim_{t \rightarrow +\infty} V(t) = -\infty$, which contradicts to (17). Therefore, there exists $t_1 \in [0, +\infty)$ such that

$$\lim_{t \rightarrow t_1} V(t) = V^* \quad \text{and} \quad V(t) \equiv V^*, \quad \forall t \geq t_1. \tag{19}$$

Next, we will prove that

$$\|e_i(t_1)\|_1 = 0 \quad \text{and} \quad \|e_i(t)\|_1 \equiv 0, \quad \forall t \geq t_1, \quad i = 1, \dots, N. \tag{20}$$

First of all, we prove that $\forall i = 1, \dots, N$, $\|e_i(t_1)\|_1 = 0$ holds. For the contradiction, that $\exists i = 1, \dots, N$ such that $\|e_i(t_1)\|_1 > 0$, then there exists a small positive constant ϵ such that $\|e_i(t)\|_1 > 0$ for all $t \in [t_1, t_1 + \epsilon]$. Thus, there exists at least one $j_0 \in \{1, 2, \dots, n\}$ such that $|e_{ij_0}(t)| > 0$ for any instant $t \in [t_1, t_1 + \epsilon]$. Based on the previous analysis, it is easy to know that $\dot{V}(t) \leq -\delta_i < 0$ holds for the instant

$t \in [t_1, t_1 + \epsilon]$, which is contradicted to (19). Therefore, $\|e_i(t_1)\|_1 = 0$ holds for $i = 1, 2, \dots, N$.

Then, we would prove that $\forall i = 1, 2, \dots, N, \|e_i(t)\|_1 \equiv 0$ for $\forall t \geq t_1$. Otherwise, $\exists i = 1, 2, \dots, N$ and $t_2 > t_1$ such that $\|e_i(t_2)\|_1 > 0$. Let $t_s = \sup\{t \in [t_1, t_2] : \|e_i(t)\|_1 = 0\}$. We have $t_s < t_2, \|e_i(t_s)\|_1 = 0$ and $\|e_i(t)\|_1 > 0, \forall t \in (t_s, t_2]$. Moreover, there exists $t_3 \in (t_s, t_2]$ such that $\|e_i(t)\|_1$ is monotonously increasing on the interval $(t_s, t_3]$. From the definition of $\|e_i(t)\|_1$, we know that $V(t)$ is monotonously increasing on the interval $(t_s, t_3]$, too. Hence, $\dot{V}(t) > 0, \forall t \in (t_s, t_3]$. On the other hand, by $\|e_i(t)\|_1 > 0, \forall t \in (t_s, t_3]$, there exists at least one $j_0 \in \{1, 2, \dots, n\}$ such that $|e_{ij_0}| > 0$ at any instant $t \in (t_s, t_3]$. By the same argument as above, it follows that $\dot{V}(t) \leq -\underline{\delta} < 0$ holds for $t \in (t_s, t_3]$, which is a contradiction with $\dot{V}(t) > 0, \forall t \in (t_s, t_3]$. Therefore, $\|e_i(t)\|_1 \equiv 0$ for $\forall t \geq t_1, i = 1, 2, \dots, N$.

Next, we would prove $V^* = 0$. Note that $\lim_{t \rightarrow t_1} V_1(t) = 0$ and $\exists t_2 = t_1 + \bar{\tau}$ such that $\lim_{t \rightarrow t_2} V_2(t) = \lim_{t \rightarrow t_2} \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \frac{1}{1-\mu} \varphi_{jv} \int_{t-\tau(t)}^t |e_{iv}(s)| ds = 0$. Moreover, there exists $t_3 = t_1 + \bar{\tau}_{ij}$ to ensure that $\lim_{t \rightarrow t_3} V_3(t) = \lim_{t \rightarrow t_3} c_2 \sum_{i=1}^N \sum_{j=1}^n \frac{1}{1-\mu_{ij}} |b_{ij}| \int_{t-\tau_{ij}(t)}^t \text{sign}^T(e_j(s)) \times \Gamma e_j(s) ds = 0$. Because $V(t) = V_1(t) + V_2(t) + V_3(t)$, we have $\lim_{t \rightarrow t_4} V(t) = \lim_{t \rightarrow t_4} V_1(t) + \lim_{t \rightarrow t_4} V_2(t) + \lim_{t \rightarrow t_4} V_3(t) = 0$, where $t_4 = \max\{t_2, t_3\}$. Therefore, $V^* = 0$ is proved.

Last, the synchronization time Υ could be estimated. We can get $\dot{V}(t) \leq -\underline{\delta}$ when $\|e_i(t_1)\|_1 \neq 0$ from inequation (16), then $t_1 \leq \frac{V(0)}{\underline{\delta}}$. Thus, $\Upsilon = t_1 - \tau$.

Therefore, the upper bound of the synchronization time is estimated as $\Upsilon \leq \frac{V(0)}{\underline{\delta}} - \tau$. This completes the proof.

Remark 4 From the criteria of Theorem 1, there are totally $n \times N$ inequalities to be satisfied. The reason is that the Lyapunov–Krasovskii functional we designed here is 1-norm, and there are so many components to multi-dimension situation for the considered system. Different from using linear matrix inequality method in previous relative literature, our conditions here are easier to be realized if the value of control strength ξ_i is large enough. Moreover, note that it is crucial to select the variable λ_i in (15) according to the value of $e_i(t)$ which makes our proposed method valid for this finite-time problem. We can see λ_i plays a key role in analyzing $\|e_i(t)\|_1 = 0$ from the Theorem 1 in the end.

Remark 5 From above one can see that ξ_i in controller (6) decide whether the system achieves synchronization, and δ_i can regulate the synchronization time, later mentioned in the simulation. Besides, δ_i is tunable and generally the time delay is relatively small; thus, we can choose proper δ_i such that the setting time $\Upsilon > 0$.

Remark 6 Most of the existing results on finite-time synchronization of coupled networks were discussed by using the finite-time stability theory and did not consider time delays (see [18] and reference therein). Although both intrinsic time delay and coupled time delay were considered in [24], they assume that there is only one single time-varying delay coupling, which is not comprehensive in a practical network because there are different time delays during communication between different two nodes. In this paper, taking into account the fact that time delay is inevitable ubiquity in coupled network, we study the global synchronization in finite time of general coupled complex network with both intrinsic time-varying delay and different coupled time-varying delay. Different from the existing works, we obtain simpler criteria for achieving finite-time synchronization for considered systems by constructing proper Lyapunov–Krasovskii functional and employing the property of sign function and strict analytical techniques. The controller designed here is very simple and can be easily implemented in practical applications. Recently, in [34] we designed the same controller like (6) to achieve the finite-time synchronization for some networks that not considering the coupling delay, in a sense, our work here generalizes the previous works.

Theorem 1 is very general and its conclusion can be applicable to complex networks with or without delays. In order to make Theorem 1 more applicable, we give some corollaries as follows.

First of all, we consider the complex network without internal delay and coupling delays. Thus, complex dynamical network (1) can be described as

$$\dot{x}_i(t) = f(t, x_i(t)) + c_1 \sum_{j=1}^N a_{ij} \Gamma x_j(t), \quad i = 1, 2, \dots, N. \tag{21}$$

Correspondingly, $s(t)$ is a solution of the isolated node of network (21) given by

$$\dot{s}(t) = f(t, s(t)). \tag{22}$$

Our control objective is to synchronize complex network (21) to homogenous trajectory (22) within the setting time.

Therefore, the controlled error dynamical system is

$$\begin{aligned} \dot{e}_i(t) &= f(t, x_i(t)) - f(t, s(t)) \\ &+ c_1 \sum_{j=1}^N a_{ij} \Gamma e_j(t) + u_i(t), \\ i &= 1, 2, \dots, N. \end{aligned} \tag{23}$$

It is easy to derive the following result from Theorem 1.

Corollary 1 *Suppose that Assumption 1 holds. If there exist $\delta_i > 0$ and $\xi_i > 0$ such that*

$$\begin{aligned} \sum_{j=1}^n \theta_{jv} + c_1 \sum_{j=1}^N |a_{ji}| \gamma_v - \xi_i &\leq 0, \quad i = 1, \dots, N, \\ v &= 1, \dots, n. \end{aligned} \tag{24}$$

Then network (21) synchronized to (22) in finite-time under controller (6). Moreover, for the given initial value $e_i(0)$, the upper bound of the synchronization time is $t_1 \leq (\frac{1}{\delta})[\sum_{i=1}^N \text{sign}^T(e_i(0))e_i(0)]$.

When $\tau_{ij}(t) \neq 0$ and $\tau(t) \equiv 0, \forall t > 0$, i.e., the network only contain coupling delay, then the dynamical system of the complex network can be described as:

$$\begin{aligned} \dot{x}_i(t) &= f(t, x_i(t)) + c_1 \sum_{j=1}^N a_{ij} \Gamma x_j(t) \\ &+ c_2 \sum_{j=1}^N b_{ij} \Gamma x_j(t - \tau_{ij}(t)), \quad i = 1, 2, \dots, N. \end{aligned} \tag{25}$$

We synchronize complex network (25) to homogenous trajectory (22) in finite-time.

From Theorem 1, one can obtain the following result:

Corollary 2 *Suppose Assumption 1 holds, If there exist $\delta_i > 0$ and $\xi_i > 0$ such that*

$$\begin{aligned} \sum_{j=1}^n \theta_{jv} + c_1 \sum_{j=1}^N |a_{ji}| \gamma_v + c_2 \sum_{j=1}^N \frac{1}{1 - \mu_{ji}} |b_{ji}| \gamma_v - \xi_i &\leq 0, \\ i &= 1, \dots, N, \quad v = 1, \dots, n. \end{aligned} \tag{26}$$

Then network (25) finite-timely synchronized to (22) under the same controller designed as (6). Furthermore, for the given initial value $e_i(0)$, the upper bound of the synchronization time is estimated as

$$\begin{aligned} \Upsilon &\leq (1/\delta) \left[\sum_{i=1}^N \text{sign}^T(e_i(0))e_i(0) \right. \\ &+ c_2 \sum_{i=1}^N \sum_{j=1}^N \frac{1}{1 - \mu_{ij}} |b_{ij}| \int_{-\bar{\tau}_{ij}}^0 \text{sign}^T(e_j(s)) \Gamma e_j(s) ds \left. \right] \\ &- \bar{\tau}_{ij} \end{aligned}$$

Remark 7 Mei et al. studied the similar model with (25) in [19]. However, not only the time delay considered in [19] was assumed to be a constant, but also the designed complex controllers which contain integral term and impulsive control (or intermittent control) were used simultaneously for synchronization system. Compared with [19], the controllers we design here are much simpler and the conditions we obtained are more concise. Furthermore, our results show another kind of effective control strategy for finite-time synchronization of complex dynamic network, which is easier to implement. In this sense, our work improve greatly the relative existing ones.

When there are not coupling delays in network model (1), i.e., $\tau(t) > 0$, and $\tau_{ij}(t) \equiv 0, \forall t > 0$. In this case, complex network (1) can be turned to

$$\begin{aligned} \dot{x}_i(t) &= f(t, x_i(t), x_i(t - \tau(t))) + c_1 \sum_{j=1}^N a_{ij} \Gamma x_j(t), \\ i &= 1, 2, \dots, N. \end{aligned} \tag{27}$$

The corresponding result for this case is as follows:

Corollary 3 *Suppose Assumption 1 holds, if there exist $\delta_i > 0$ and $\xi_i > 0$ such that*

$$\begin{aligned} \sum_{j=1}^n \theta_{jv} + \sum_{j=1}^n \frac{1}{1 - \mu} \varphi_{jv} + c_1 \sum_{j=1}^N |a_{ji}| \gamma_v - \xi_i &\leq 0, \\ i &= 1, \dots, N, \quad v = 1, \dots, n. \end{aligned} \tag{28}$$

Then network (28) can be finite-timely synchronized onto (2) under controller (6). Moreover, for the given initial value $e_i(0)$, we estimate

$$\begin{aligned} \Upsilon &\leq (1/\delta) \left[\sum_{i=1}^N \text{sign}^T(e_i(0))e_i(0) \right. \\ &+ \sum_{i=1}^N \sum_{j=1}^n \sum_{v=1}^n \frac{1}{1 - \mu} \varphi_{jv} \int_{-\bar{\tau}}^0 |e_{iv}(s)| ds \left. \right] - \bar{\tau} \end{aligned}$$

as the upper bound of the synchronization time.

Remark 8 The above three corollaries are direct applications of Theorem 1. Their proofs are similar to the proof of Theorem 1 with only a little difference in constructing the Lyapunov–Krasovskii functional. we can consider the $V_1(t)$ as the Lyapunov functional in Corollary 1, since $V_1(t) = 0$ is equivalent to $\|e_i(t)\|_1 = 0, i = 1, 2, \dots, N$. Correspondingly, we can construct the Lyapunov function of corollary 2 and 3, respectively. Here we omit it.

Remark 9 Some sufficient conditions of finite-time synchronization for several kinds of coupled network are given by the above three corollaries, respectively. In a manner, those conclusions improve the existing relative works. For example, in [30], the authors also study the finite-time synchronization control for delayed network with intrinsic delay only, the sufficient conditions of finite-time synchronization in [30] are more complex than the conditions given by Corollary 3, which is simpler to be verified and be implemented in practical applications. Furthermore, the isolate node dynamics here are more general than [30], which was supposed to be a kind of special neural network. On the other hand, in our previous work [34], the finite-time synchronization of system (27) was studied also, the result of Corollary 3 here is the same as Theorem 1 in [34]. In a sense, the conclusion there is a special case of our work. In other words, the current work improves the previous one.

4 Numerical simulation

In this section, we present some numerical simulations to illustrate the effectiveness of the Theorem and Corollaries in the previous section. Examples 1 and 2 verify the correction of Theorem 1 and Corollary 2, respectively. In the following examples, we make the following regulations: $x_i(t) = (x_{i1}(t), x_{i2}(t))^T$, in other words, the nodes in the system are two-dimension and the complex networks have 6 nodes in total. Besides, $\Gamma = I_{2 \times 2}, c_1 = c_2 = 1$, initial condition is chosen as $x_1(0) = [1.5, -2.7]^T, x_2(0) = [1.6, 4.8]^T, x_3(0) = [-1.3, -4.4]^T, x_4(0) = [0.8, 2.1]^T, x_5(0) = [-4.3, 1.4]^T, x_6(0) = [1.3, -1.7]^T$, and the initial value of $s(t)$ is $s(0) = [0.4, 0.6]^T$. Coupling matrices A and B are defined as:

$$A = \begin{pmatrix} -1.4 & 0.4 & 0 & 1 & 0 & 0 \\ 0.4 & -0.7 & 0.3 & 0 & 0 & 0 \\ 0 & 0.3 & -0.3 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1.6 & 0.6 & 0 \\ 0 & 0 & 0 & 0.6 & -1.1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & -0.5 \end{pmatrix},$$

$$B = \begin{pmatrix} -0.75 & 0.5 & 0.25 & 0 & 0 & 0 \\ 0.5 & -2.5 & 0 & 0.5 & 0 & 1.5 \\ 0.25 & 0 & -0.65 & 0 & 0.4 & 0 \\ 0 & 0.5 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & -0.4 & 0 \\ 0 & 1.5 & 0 & 0 & 0 & -1.5 \end{pmatrix}$$

Example 1 The effectiveness of Theorem 1 is verified in this section, which consider both the coupling delays and the internal delay. Without loss of generality, we introduce $\tau(t) = \tau_{ij}(t) = 1$. Therefore, $\mu = \mu_{ij} = 0$, and we take $f(t, x_i(t), x_i(t - \tau(t)))$ in complex network (1) as follows [32]:

$$f(t, x_i(t), x_i(t - \tau(t))) = -Cx_i(t) + Mg(x_i(t)) + Ng(x_i(t - \tau(t))),$$

in which $g(x_i(t)) = 0.5(|x_{i1}(t) + 1| - |x_{i1}(t) - 1|, |x_{i2}(t) + 1| - |x_{i2}(t) - 1|)^T$, and

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} 1 + \frac{\pi}{4} & 20 \\ 0.1 & 1 + \frac{\pi}{4} \end{pmatrix},$$

$$N = \begin{pmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{pmatrix}$$

From the definition of $f(t, x_i(t), x_i(t - \tau(t)))$, we have

$$\begin{aligned} & |f_1(t, x_1(t), x_1(t - 1)) - f_1(t, s(t), s(t - 1))| \\ & \leq (2 + \pi/4)|e_{11}(t)| + 20|e_{12}(t)| \\ & \quad + 1.3\sqrt{2}\pi/4|e_{11}(t - 1)| + 0.2|e_{12}(t - 1)|, \end{aligned} \tag{29}$$

and according to assumption 1, $\theta_{11} = 2 + \pi/4, \theta_{12} = 20, \varphi_{11} = 1.3\sqrt{2}\pi/4, \varphi_{12} = 0.1$, using the same methods, we possess $\theta_{21} = 1.1, \theta_{22} = 1 + \pi/4, \varphi_{21} = 0.1, \varphi_{22} = 1.3\sqrt{2}\pi/4$. In other words,

$$\Theta = \begin{pmatrix} 2 + \pi/4 & 20 \\ 1.1 & 1.3\sqrt{2}\pi/4 \end{pmatrix},$$

$$\Phi = \begin{pmatrix} 1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & 1.3\sqrt{2}\pi/4 \end{pmatrix}.$$

Therefore, we should take $\max \sum_{j=1}^2 \theta_{jv} = 21.7875$, and $\max \sum_{j=1}^2 \varphi_{jv} = 1.5478$. Then from the criteria

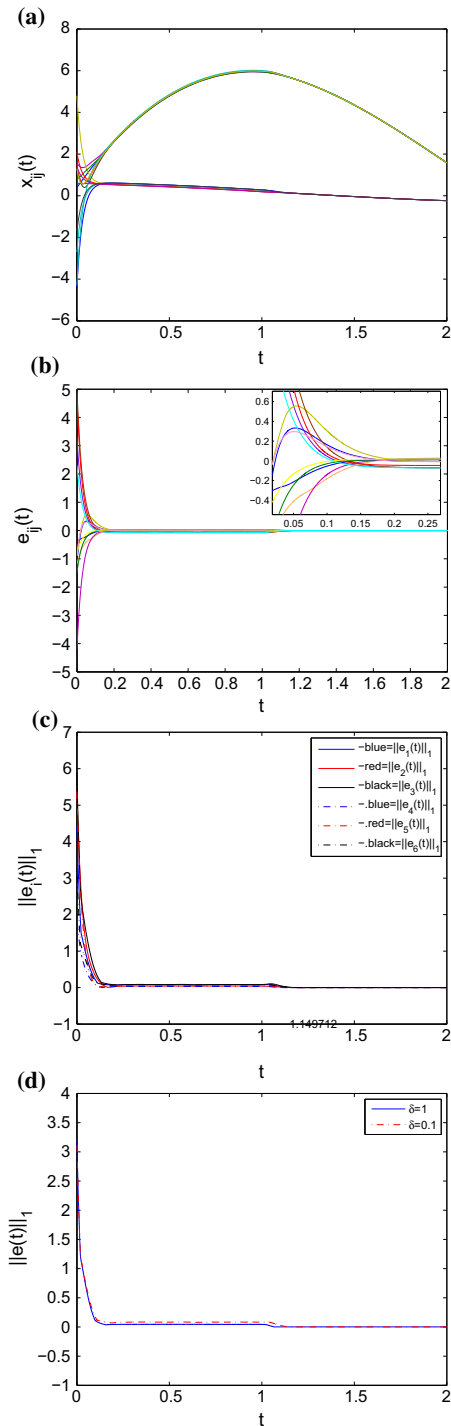


Fig. 1 **a** Time response of the state variables when $\delta_i = 0.1$; **b** time response of the error variables when $\delta_i = 0.1$; **c** time response of the error variables' 1-norm for each node when $\delta_i = 0.1$; **d** time evolution of $\|e(t)\|_1$ with $\delta_i = 1$ and $\delta_i = 0.1, i = 1, \dots, 6$

in Theorem 1, we take $\xi_i = 31.5353, i = 1, \dots, 6$, which satisfy the condition.

Figure 1 describes the time evolution of $e_i(t)$ and $\|e_i(t)\|_1, i = 1, \dots, 6$ when $\delta_i = 0.1, i = 1, \dots, 6$ and $\|e(t)\|_1$ with different values of $\delta_i = \delta_j, i, j = 1, \dots, 6$. When $\delta_i = 1, i = 1, \dots, 6$ we can see from Fig. 1 (d) clearly that $\|e(t)\|_1$ reaches zero earlier than when $\delta_i = 0.1, i = 1, \dots, 6$, and they all reach zero at almost $t = 1.149712s$. Consequently, Theorem 1 is verified from Fig. 1. At the same time, through the simple computation, $V(0) = 61.9343$, then the estimated upper bound is $t = \frac{1}{\delta}V(0) - 1 = 618.343s$ when $\delta_i = \delta_j = 0.1, i, j = 1, \dots, 6$, meeting the estimated upper bound we proposed. Besides, we can see the synchronization is realized quickly from above figures.

Example 2 The next step is to demonstrate the validity of Corollary 2, which is the complex network only with coupling time delays. In this part, we let $\tau_{ij}(t) = 1$, then we can get $\mu_{ij} = 0, \forall i = 1, \dots, 6, j = 1, 2$. Here, $f(x)$ displayed as follows: $f(t, x_i(t), x_i(t - \tau(t))) = -Cx_i + Mg(x_i(t))$, in which $g(x_i)$ is taken as the same as that in Example 1. According to Remark 1, obviously, we take $\xi_i = 29.9875$ to satisfy the criteria in the Corollary 2.

Therefore, Corollary 2 is verified. Evidently, Fig. 2 describes the time evolution of $e_i(t)$ and $\|e_i(t)\|_1, i = 1, \dots, 6$ when $\delta_i = 0.1, i = 1, \dots, 6$ and $\|e(t)\|_1$ with different values of $\delta_i = \delta_j, i, j = 1, \dots, 6$. When $\delta_i = 1, i = 1, \dots, 6$, we can see from Fig. 2 (d) that $\|e(t)\|_1$ reaches zero earlier than when $\delta_i = 0.1, i = 1, \dots, 6$, and they all reach zero at almost $t = 0.254319s$. At the same time, through the simple computation, $V(0) = 33.3$, then the estimated upper bound is $t = \frac{1}{\delta}V(0) - 1 = 332s$ when $\delta_i = \delta_j = 0.1, i, j = 1, \dots, 6$, which meets the estimated upper bound we proposed.

From the above two examples, we can conclude that the larger $\delta_i = \delta_j, i, j = 1, \dots, 6$ can shorten the synchronization time when the initial value is fixed. Evidently, the synchronization time we estimated is much larger than the real synchronization time. However, when δ is increasing, the gap between the real synchronization time and the setting time will decrease. Choosing $\delta_i = 10, \forall i = 1, \dots, 6$ and keeping the other parameters same as those in example 1, we have Fig. 3 in which the synchronization time is almost $t = 0.1s$, and the gap between the synchronization time and the setting time $t = 5.19343s$ decrease.

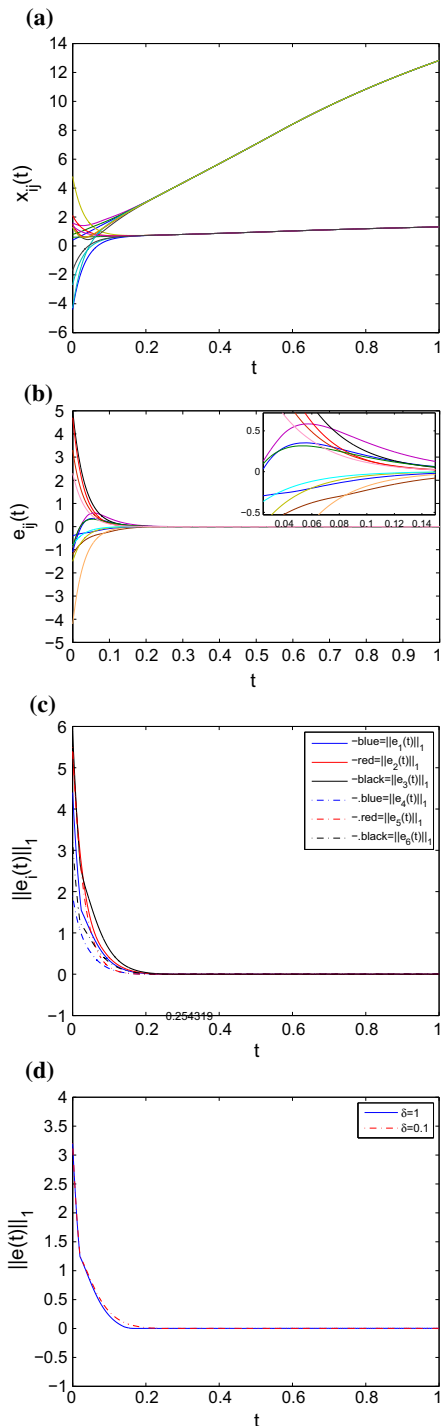


Fig. 2 **a** Time response of the state variables when $\delta_i = 0.1$; **b** time response of the error variables when $\delta_i = 0.1$; **c** Time response of the error variables' 1-norm for each node when $\delta_i = 0.1$; **d** Time evolution of $\|e(t)\|_1$ with $\delta_i = 1$ and $\delta_i = 0.1, i = 1, \dots, 6$

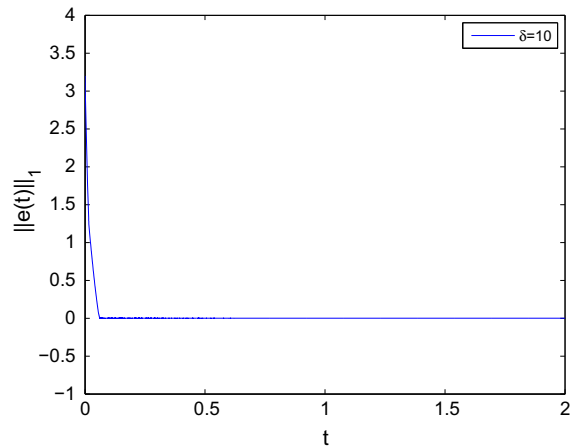


Fig. 3 Time evolution of $\|e(t)\|_1$ with $\delta_i = 10, i = 1, \dots, 6$

5 Conclusion

This paper has investigated the finite-time synchronization issues for general complex network. The proposed network model may shed some new lights on the synchronization without or with intrinsic time-varying delay and different coupled time-varying delays. From our perspective, the controller we used reap huge fruits, which is discontinuous but simple to handle in the real situations. Some results are obtained to guarantee synchronization of the coupled systems by constructing a proper Lyapunov–Krasovskii functional and strict analytical techniques. Moreover, the upper bound of the synchronization time is estimated for general complex dynamical network with bounded delay and without delay. The control law and the synchronization criteria are very simple and can be easily extended to other complex dynamical networks. Several numerical examples have been presented to demonstrate the effectiveness of our theoretical results. In the future, we will consider the fixed-time synchronization problem for general complex network with time delay, whose estimated synchronization time is independent of initial value of the state variable, in other words, the finite time we estimated in this paper exists a upper bound independent to the initial value of Lyapunov functional.

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