Nonlinear Dyn (2017) 88:529–546
DOI 10.1007/s11071-016-3258-3

ORIGINAL PAPER

Distributed attitude control for multiple flexible spacecraft under actuator failures and saturation

Di Huang · Qishao Wang · Zhisheng Duan

Received: 29 June 2016 / Accepted: 2 December 2016 / Published online: 27 December 2016

Abstract This paper solves the problem of attitude consensus for flexible spacecraft formation under actuator failures and saturation constraints. Three insightful distributed consensus control laws are designed based on the Lyapunov's stability theory and graph theory. The induced oscillations of the spacecraft's flexible appendages are compensated online with adaptive update parameters. Attitude consensus for the multiple spacecraft system can be achieved with limited information transfer. The modal variables of the flexible appendages are avoided in the distributed controllers in order to reduce the payload of the spacecraft. In addition, the issue of actuator saturation is rejected by applying a switching control scheme. Numerical simulations are performed to demonstrate that the proposed controller can guarantee attitude consensus despite the presence of modeling uncertainties, external disturbances, and simultaneous loss of actuator effectiveness faults and additive faults.

Keywords Attitude consensus · Distributed control · Flexible spacecraft · Spacecraft formation

D. Huang · Q. Wang · Z. Duan (✉)
State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Engineering Science, College of Engineering, Peking University, Beijing 100871, China
e-mail: duanzs@pku.edu.cn

1 Introduction

Spacecraft cooperative control has drawn extensive attention recently because of potential advantages over a single spacecraft, including lower cost and greater flexibility [1,2]. It has broad potential applications in a wide range of fields, including interferometry, synthetic aperture imaging, autonomous in-orbit assembly of large real structures, etc. Spacecraft attitude consensus, which aims to drive multiple spacecraft to achieve prescribed consensus on their states, is one of the most intensively studied topics within the realm of spacecraft cooperative control. The design of a distributed controller for attitude consensus is a challenging task because of the highly nonlinear dynamics of spacecraft [3–6].

Several results have been given on attitude consensus for a group of rigid spacecraft. Following a decentralized coordination architecture, Ren and Beard [7] introduced decentralized formation control strategies by using the virtual structure approach. Yu et al. [8] developed distributed adaptive controllers for synchronization which adaptively tune the coupling weights of the network, and Yang et al. [9] further studied the adaptive synchronization problem by using the Lagrangian formulations of the dynamics of spacecraft. Despite the presence of unknown disturbances, distributed attitude coordinated control techniques for spacecraft formation were proposed by applying adaptive control [10] and sliding mode control [11–13].

For the circumstance that angular velocity measurement is unavailable, Abdessameud and Tayebi [14] presented a velocity-free attitude coordination control law for a group of spacecraft. To achieve faster convergence rate, authors in Refs. [15–20] analyzed the finite-time attitude tracking control problem for a rigid body with external disturbances and inertia uncertainties. Du et al., Zhou et al., and Meng et al. [21–23] proposed a finite-time attitude synchronization algorithm for a group of spacecraft. The relative position and relative attitude control are also studied in the spacecraft rendezvous and docking. In addition, consensus algorithms which are proposed in [24] have applications in rendezvous. Many insightful control techniques [25,26] have been investigated to deal with the position tracking and attitude synchronization problem for spacecraft rendezvous. Recently, flexible spacecraft, which carry some flexible appendages, such as solar arrays and manipulators, have received considerable attention. The vibration induced by the flexible appendages makes the dynamics of the spacecraft more complicated. Based on the backstepping design, Du and Li [27] presented a distributed attitude synchronization control law for a group of flexible spacecraft. Du and Li [28] extended the result to the case with communication delay. When the spacecraft formation is assumed to contain two different kinds of spacecraft, Du et al. [29] proposed a distributed attitude control algorithm to achieve attitude synchronization.

One common assumption in the aforementioned references is that there are no failures in the actuators. For a multiple spacecraft system in a practical situation, the actuator in every spacecraft may not work in the ideal way due to the presence of unexpected failures. In such circumstances, actuator failures cannot be ignored in achieving attitude consensus of multiple spacecraft since a single undetected failure in one spacecraft can cause a severe impact on the overall system performance, such as performance degradation or even system divergence. Realizing this, researchers began to take the actuator failures problems into consideration during the consensus controller design procedure. Based on the adaptive control law, Zou and Kumar [30] presented a distributed attitude coordination controller for spacecraft formation under actuator failures. Wu et al. [31] analyzed a similar problem and proposed a distributed control law without using angular accelerations. Zhou and Xia [32] investigated distributed fault-tolerant control design for spacecraft finite-time attitude synchronization. Note that the above results are mainly concerned with rigid spacecraft. To the best knowledge of the authors, there are no results about flexible spacecraft consensus under actuator failures.

Another important problem encountered in practical situation is that of actuator saturation. It is known that the available torque amplitude is limited in the actual spacecraft. If the command input signals exceed the input bounds of the actuators, the desired performance of the closed-loop system cannot be reached. Besides, it may also lead to instability of the closed-loop system. In this paper, the problem of attitude consensus for flexible spacecraft formation under actuator failures and saturation constraints is investigated. The desired attitude is allowed to be available to only a small subset of the spacecraft. Based on the Lyapunov's stability theory and graph theory, a distributed consensus control law for flexible spacecraft in the presence of loss of actuator effectiveness is presented. An adaptive parameter is utilized to damp out the induced oscillations of the spacecraft's flexible appendages. In practical situations, the mass, damping, and stiffness properties of the flexible spacecraft may be uncertain and external disturbances always exist. Thus, the case with model uncertainties, external disturbances, and simultaneous loss of actuator effectiveness faults as well as additive faults is also considered. Finally, a switching control scheme is established to reject the issue of actuator saturation. In contrast with the available literature on attitude consensus control for spacecraft formation, the contributions of this paper are as follows:

1. A new kind of distributed attitude consensus controllers for multiple flexible spacecraft formation under actuator failures and saturation constraints is proposed for the first time, while the existing results [13,19] either consider rigid spacecraft or lack analysis of actuator failures and saturation affecting the model.
2. The proposed first controller (6) only needs the states of its neighbors and modal variables of its own, thus less information is needed to be exchanged and the communication burden is lightened compared with the methods in [27–29]. An adaptive parameter is utilized to deal with the oscillations induced by modal variables. In addition, controller (6) is more effective although less communication load is required, which can be verified in the simulations.

3. The second controller (16) considers the case where the inertia matrix is not exactly known, and damping stiffness matrices are completely unknown. The designed controller is robust against modeling uncertainties, external disturbances, and actuator failures. Moreover, the modal variables are avoided in the controller, which reduces the burden of measurement and the payload of the spacecraft.
4. In order to overcome the defect that the control torques are overlarge at the beginning of the control task by using controllers (6) and (16), a simple switching control scheme (21) is presented to reject the issue of actuator saturation which will be encountered in practical situation.

The rest of the paper is organized as follows. In Sect. 2, the flexible spacecraft attitude dynamics and algebraic graph theory are briefly described. In Sect. 3, distributed consensus control laws for flexible spacecraft under actuator failures and saturation constraints are presented. Section 4 gives the simulation examples, and concluding remarks are finally given in Sect. 5.

The following notations will be used throughout this paper. For a matrix A, symbol A^{T} denotes its transpose and A^{-1} represents its inverse if it exists. Suppose that A is a square matrix with real eigenvalues, $\lambda_{\max}(A)$ denotes the largest eigenvalue of matrix A and $\lambda_{\min}(A)$ represents the smallest eigenvalue accordingly. $\mathbf{1}$ denotes the column vector with all entries equal to one. $A \otimes B$ denotes the Kronecker product of matrices A and B. For any given $n-$dimensional real vector $x = [x_1, \ldots, x_n]^{\mathrm{T}}$, let $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_n)]^{\mathrm{T}}$ where $\mathrm{sgn}(\cdot)$ denotes the sign function.

2 Preliminaries and problem formulation

2.1 Kinematics and dynamics of flexible spacecraft attitude

The attitude of flexible spacecraft can be described by two sets of equations: the kinematic equation and the dynamic equation. Adopting the unit quaternion, then the kinematic equation of the ith flexible spacecraft can be given by Shuster [33]

$$\dot{q}_i = \frac{1}{2}(-s(q_i) + q_{i,0}I_3)\omega_i,$$
$$\dot{q}_{i,0} = -\frac{1}{2}q_i^{\mathrm{T}}\omega_i, \quad i \in \varOmega = 1, \ldots, n, \tag{1}$$

where $\bar{q}_i = [q_{i,0}, q_{i,1}, q_{i,2}, q_{i,3},]^{\mathrm{T}} = [q_{i,0}, q_i^{\mathrm{T}}]^{\mathrm{T}}$ is the unit quaternion which satisfies $q_{i,0}^2 + q_i^{\mathrm{T}} q_i = 1$, and $\omega_i = [\omega_{i,1},\ \omega_{i,2},\ \omega_{i,3}]^{\mathrm{T}}$ is the angular velocity. The matrix $s(x)$ for a vector $x = [x_1,\ x_2,\ x_3]^{\mathrm{T}}$ is used to denote the skew-symmetric matrix $s(x) = \begin{bmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{bmatrix}$. The dynamic equation of the ith flexible spacecraft can be given by Gennaro [34]

$$J_i\dot{\omega}_i + \delta_i^{\mathrm{T}}\ddot{\eta}_i = s(\omega_i)(J_i\omega_i + \delta_i^{\mathrm{T}}\dot{\eta}_i) + D_i\tau_i + d_i,$$
$$\ddot{\eta}_i + C_i\dot{\eta}_i + K_i\eta_i = -\delta_i\dot{\omega}_i, \quad i \in \varOmega, \tag{2}$$

where $J_i = J_i^{\mathrm{T}}$ is the positive definite inertia matrix, $\tau_i = [\tau_{i,1}, \tau_{i,2}, \tau_{i,3}]^{\mathrm{T}}$ is the control torque, δ_i is the coupling matrix between the rigid body and the flexible attachments, η_i is the modal coordinate vector, d_i is the external disturbance, $C_i = \mathrm{diag}\{2\xi_{i,j}\omega_{i,nj}, j = 1, \ldots, N_i\}$ is the damping matrix, $K_i = \mathrm{diag}\{\omega_{i,nj}, j = 1, \ldots, N_i\}$ is the stiffness matrix, N_i is the number of flexible attachments for ith spacecraft, $\omega_{i,nj}$ is the natural frequencies, $\xi_{i,j}$ is the associated damping, $D_i = \mathrm{diag}\{D_{i,j}, j = 1, \ldots, m\}$ is the control actuator distribution matrix, m is the number of actuators.

Consider the situation in which each of the actuators partially loses its actuation effectiveness, and additive faults also exist. Denote $\psi_i = \dot{\eta}_i + \delta_i\omega_i$ and $J_{m,i} = J_i - \delta_i^{\mathrm{T}}\delta_i$. The spacecraft attitude kinematic and dynamic equations can be rewritten as

$$\dot{q}_i = \frac{1}{2}(-s(q_i) + q_{i,0}I_3)\omega_i,$$
$$\dot{q}_{i,0} = -\frac{1}{2}q_i^{\mathrm{T}}\omega_i,$$
$$\dot{\eta}_i = \psi_i - \delta_i\omega_i,$$
$$\dot{\psi}_i = -(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i), \quad i \in \varOmega, \tag{3}$$

and

$$J_{m,i}\dot{\omega}_i = s(\omega_i)(J_{m,i}\omega_i + \delta_i^{\mathrm{T}}\psi_i)$$
$$+ \delta_i^{\mathrm{T}}(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) + D_i\varGamma_i\tau_i$$
$$+ D_i f_i + d_i, \quad i \in \varOmega, \tag{4}$$

where $\varGamma_i = \mathrm{diag}\{\varGamma_{i,j}, j = 1, \ldots, m\}$ is the time-varying and uncertain bounded actuation effectiveness matrix with $0 < \varGamma_{i,j} \le 1$. The case in which $\varGamma_{i,j} = 1$ denotes that the actuator is healthy, $0 < \varGamma_{i,j} < 1$ is the case in which the jth actuator partially loses its actuating power. f_i denotes the additive faults term of the ith spacecraft.

The following assumptions are made about the above systems:

Assumption 1 Let $J_i = \bar{J}_i + \Delta J_i$, where $\bar{J}_i, \Delta J_i$ are the nominal and uncertain part of the inertia matrix. The uncertain inertia matrix ΔJ_i satisfies $\|\Delta J_i\| \le \mu_{J,i}$.

Assumption 2 The additive faults term f_i satisfies $\|f_i\| \le \mu_{f,i}$.

Assumption 3 The disturbances term d_i satisfies that $\|d_i\| \le \mu_{d,i}$.

In the above assumptions, $\mu_{J,i},\ \mu_{f,i},\ \mu_{d,i},\ i \in \Omega$ are unknown nonnegative constants.

2.2 Algebraic graph theory

The topology of the information flow among n flexible spacecraft is described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, in which an edge is represented by a pair of distinct nodes. $N_i = \{j \in \mathcal{V} : \{i, j\} \in \mathcal{E}\}$ is the neighborhood set of i. In the undirected graph, $(v_i, v_j) \in \mathcal{E}$ means $(v_j, v_i) \in \mathcal{E}$, and the node $v_i(v_j)$ is called the neighbor of $v_j(v_i)$. A path from node v_{i_1} to node v_{i_j} is a sequence of ordered edges of the form $(v_{i_k}, v_{i_{k+1}}),\ k = 1, \ldots, j$. An undirected graph is connected if there exists a path between every pair of distinct nodes. Otherwise, it is disconnected. The associated adjacency matrix of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is denoted by $\mathcal{A} = (a_{ij}) \in \mathbf{R}^{N\times N}$. $a_{ii} = 0$, and $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$. The Laplacian matrix of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, associated with the adjacency matrix $\mathcal{A}$, is denoted by $\mathcal{L} = (l_{ij}) \in \mathbf{R}^{N\times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij},\ l_{ij} = -a_{ij}$ for $i \ne j$. For an undirected graph, both its adjacency matrix and its Laplacian matrix are symmetric. The desired attitude is represented by a virtual leader. The connection weight between the leader and the spacecraft is denoted by $B = \operatorname{diag}\{b_1, \ldots, b_n\}$. If the ith spacecraft can obtain the information of the virtual leader, $b_i > 0$; otherwise, $b_i = 0$.

Assumption 4 The communication topology graph for n flexible spacecraft is connected, and there is at least one spacecraft that can directly access to the information of the virtual leader.

Lemma 1 *If Assumption 4 holds, the matrix* $\mathcal{L}+B > 0$.

2.3 Control objective

The goal of this paper is to design a distributed attitude control law τ_i for n flexible spacecraft such that all the attitudes can reach consensus and the induced oscillations of the spacecraft's flexible appendages are damped out, which can be expressed as

$$\lim_{t\to\infty} q_i = q_d,\ \lim_{t\to\infty} \omega_i = 0,\ \lim_{t\to\infty} \eta_i = 0,\ \lim_{t\to\infty} \dot{\eta}_i = 0,$$

where q_d is a constant reference attitude.

3 Attitude consensus control law design

3.1 Consensus control law design for spacecraft under loss of actuator effectiveness

In order to make the design process clear, a simple case with $f_i = 0, d_i = 0, \Delta J_i = 0$ is first considered. That is to say, in this case, additive faults, disturbances and inertia matrix uncertainty are not considered in the dynamic equation of the spacecraft, which can be written as

$$J_{m,i}\dot{\omega}_i = s(\omega_i)(J_{m,i}\omega_i + \delta_i^{\mathrm{T}}\psi_i) + \delta_i^{\mathrm{T}}(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) + D_i\Gamma_i\tau_i, \quad i \in \Omega, \tag{5}$$

Assume that δ_i, C_i, K_i are known, the following theorem can be established to achieve attitude consensus.

Theorem 1 *For the multiple flexible spacecraft systems (3) and (5), if Assumption 4 holds and the control torque* τ_i *is designed as*

$$\tau_i = -\frac{1}{\lambda_{\tau_i}} D_i^{\mathrm{T}} J_{m,i} Q_i^{-1}\Big\{k_1(v_i - v_i^*) + \Big[\|\dot{Q}_i\omega_i + Q_i J_{m,i}^{-1}\cdot s(\omega_i)(J_{m,i}\omega_i + \delta_i^{\mathrm{T}}\psi_i) + Q_i J_{m,i}^{-1}\delta_i^{\mathrm{T}}(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i)\| + k_2^2(\|\psi_i\| + \|\eta_i\|)\hat{\theta}_i\Big]\operatorname{sgn}(v_i - v_i^*)\Big\},$$
$$i \in \Omega, \tag{6}$$

where $Q_i = \frac{1}{2}(-s(q_i) + q_{i,0}I_3)$, $v_i = Q_i\omega_i$, *and* $v_i^* = -k_2[\sum_{j\in N_i} a_{ij}(q_i - q_j) + b_i(q_i - q_d)]$, *the parameters* $\lambda_{\tau_i}, k_1, k_2$ *satisfy* $0 < \lambda_{\tau_i} \le \lambda_{\min}\{D_i\Gamma_i D_i^{\mathrm{T}}\}$, $k_1 \ge k_2^2[0.5(\mu_1 + n\mu_a + 1) + \frac{1}{k_2}(\mu_l + n\mu_a) + k_3]$, *and* $k_2 \ge 0.5 + 0.5(\mu_l + n\mu_a) + k_3$, *where* $k_3 > 0$, $\mu_l = \max_{i\in\Omega}\{l_{ii} + b_i\}$, $\mu_a = \max_{i\in\Omega, j\in N_i}\{a_{ij}\}$, *and the adaptive law for parameter* $\hat{\theta}_i$ *is set as*

$$\dot{\hat{\theta}}_i = -k_{\hat{\theta}_i}\hat{\theta}_i + k_{\theta_i}(\|\psi_i\| + \|\eta_i\|)\sum_{w=1}^{3}|v_{iw} - v_{iw}^*|, \tag{7}$$

where $k_{\theta_i} > 0, k_{\hat{\theta}_i} > 0$, w subscript denotes the wth element of the corresponding vector, i.e., v_{iw}, v_{iw}^* denote the wth elements of vectors v_i, v_i^*, respectively. Then, attitude consensus can be achieved asymptotically and the induced oscillations of the spacecraft's flexible appendages are damped out.

Proof Consider the following candidate Lyapunov function

$$V = V_0 + \sum_{i=1}^{n}(W_{i,1} + W_{i,2} + W_{i,3}), \tag{8}$$

with

$$V_0 = \frac{1}{4}\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_i - q_j)^2 + \frac{1}{2}\sum_{i=1}^{n} b_i(q_i - q_d)^2,$$

$$W_{i,1} = \frac{1}{k_2^2}\sum_{w=1}^{3}\int_{v_{iw}^*}^{v_{iw}}(s - v_{iw}^*)\mathrm{d}s,$$

$$W_{i,2} = \frac{1}{2}\psi_i^{\mathrm{T}}\psi_i + \eta_i^{\mathrm{T}}K_i\eta_i + \frac{1}{2}(\psi_i + C_i\eta_i)^{\mathrm{T}}(\psi_i + C_i\eta_i),$$

$$W_{i,3} = \frac{1}{2k_{\theta_i}}(\theta_i - \hat{\theta}_i)^2.$$

Since $\dot{q}_i = Q_i\omega_i$ and $v_i = Q_i\omega_i$, it can be obtained that

$$\frac{\mathrm{d}\left[\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_i - q_j)^2\right]}{\mathrm{d}t} = 2\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_i - q_j)^{\mathrm{T}}(v_i - v_j)$$
$$= 2\sum_{i=1}^{n}\sum_{j\in N_i}\left[a_{ij}(q_i - q_j)^{\mathrm{T}}v_i + a_{ij}(q_j - q_i)^{\mathrm{T}}v_j\right]$$
$$= 4\sum_{i=1}^{n}\sum_{j\in N_i} a_{ij}(q_i - q_j)^{\mathrm{T}}v_i.$$

Then the time derivative of V_0 is

$$\dot{V}_0 = \sum_{i=1}^{n}\left[\sum_{j\in N_i} a_{ij}(q_i - q_j)^{\mathrm{T}} + b_i(q_i - q_d)^{\mathrm{T}}\right]v_i$$
$$= \sum_{i=1}^{n}\left[\sum_{j\in N_i} a_{ij}(q_i - q_j)^{\mathrm{T}} + b_i(q_i - q_d)^{\mathrm{T}}\right]v_i^*$$
$$+ \sum_{i=1}^{n}\left[\sum_{j\in N_i} a_{ij}(q_i - q_j)^{\mathrm{T}} + b_i(q_i - q_d)^{\mathrm{T}}\right]\times(v_i - v_i^*)$$
$$\leq -\left(k_2 - \frac{1}{2}\right)\sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i} a_{ij}(q_{iw} - q_{jw}) + b_i(q_{iw} - q_{dw})\right]^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2, \tag{9}$$

where q_{iw}, q_{jw} and q_{dw} denote the wth element of vectors q_i, q_j and q_d.

The time derivative of $W_{i,1}$ is

$$\dot{W}_{i,1} = \frac{1}{k_2^2}\sum_{w=1}^{3}\left[-\frac{\partial v_{iw}^*}{\partial t}\int_{v_{iw}^*}^{v_{iw}} 1\mathrm{d}s + (v_{iw} - v_{iw}^*)\frac{\partial v_{iw}}{\partial t}\right]. \tag{10}$$

According to the definition of v_{iw}^*, one can obtain

$$\frac{\partial v_{iw}^*}{\partial t} = -k_2\left[\sum_{j\in N_i} a_{ij}(v_{iw} - v_{jw}) + b_i v_{iw}\right] \leq k_2\left(\mu_l|v_{iw}| + \mu_a\sum_{m=1}^{n}|v_{mw}|\right),$$

and

$$|v_{mw}||v_{iw} - v_{iw}^*| \leq (|v_{mw} - v_{mw}^*| + |v_{mw}^*|)\times|v_{iw} - v_{iw}^*|$$
$$\leq \left(\frac{1}{2} + \frac{k_2}{2}\right)|v_{iw} - v_{iw}^*|^2 + \frac{1}{2}|v_{mw} - v_{mw}^*|^2 + \frac{k_2}{2}\left|\sum_{j\in N_m} a_{mj}(q_{mw} - q_{jw}) + b_m(q_{mw} - q_{dw})\right|^2.$$

Then, Eq. (10) can be rewritten as

$$\dot{W}_{i,1} \leq \sum_{w=1}^{3}\frac{\mu_l}{k_2}\left[\frac{1+k_2}{2}|v_{iw} - v_{iw}^*|^2 + \frac{1}{2}|v_{iw} - v_{iw}^*|^2 + \frac{k_2}{2}\left|\sum_{j\in N_i} a_{ij}(q_{iw} - q_{jw}) + b_i(q_{iw} - q_{dw})\right|^2\right]$$
$$+ \sum_{w=1}^{3}\frac{\mu_a}{k_2}\sum_{m=1}^{n}\left[\frac{1+k_2}{2}|v_{iw} - v_{iw}^*|^2 + \frac{1}{2}|v_{mw} - v_{mw}^*|^2 + \frac{k_2}{2}\left|\sum_{j\in N_m} a_{mj}(q_{mw} - q_{jw}) + b_m(q_{mw}\right.\right.$$

$$
\begin{aligned}
&-q_{dw})|^2\Bigg]+\sum_{w=1}^{3}\frac{1}{k_2^2}(v_{iw}-v_{iw}^*)\frac{\partial v_{iw}}{\partial t}\\
&\le \sum_{w=1}^{3}\Bigg\{\frac{1}{k_2}\left[(\mu_l+n\mu_a)\frac{k_2+1}{2}+\frac{\mu_l}{2}\right]|v_{iw}-v_{iw}^*|^2\\
&+\frac{\mu_l}{2}\left|\sum_{j\in N_i}a_{ij}(q_{iw}-q_{jw})+b_i(q_{iw}-q_{dw})\right|^2\\
&+\frac{1}{k_2^2}(v_{iw}-v_{iw}^*)\frac{\partial v_{iw}}{\partial t}+\frac{\mu_a}{2k_2}\sum_{m=1}^{n}|v_{mw}-v_{mw}^*|^2\\
&+\frac{\mu_a}{2}\sum_{m=1}^{n}\left|\sum_{j\in N_m}a_{mj}(q_{mw}-q_{jw})+b_m(q_{mw}-q_{dw})\right|^2\Bigg\}.
\end{aligned}
\tag{11}
$$

The time derivative of $W_{i,2}$ is

$$
\begin{aligned}
\dot{W}_{i,2} &= \psi_i^{\mathrm{T}}\dot{\psi}_i+2\eta_i^{\mathrm{T}}K_i\dot{\eta}_i+(\psi_i+C_i\eta_i)^{\mathrm{T}}\left(\dot{\psi}_i+C_i\dot{\eta}_i\right)\\
&= -\psi_i^{\mathrm{T}}C_i\psi_i-\eta_i^{\mathrm{T}}C_iK_i\eta_i\\
&\quad +(\psi_i^{\mathrm{T}}C_i\delta_i-2\eta_i^{\mathrm{T}}K_i\delta_i)\omega_i\\
&= -\psi_i^{\mathrm{T}}C_i\psi_i-\eta_i^{\mathrm{T}}C_iK_i\eta_i\\
&\quad +\sum_{w=1}^{3}(\psi_i^{\mathrm{T}}C_i\delta_i-2\eta_i^{\mathrm{T}}K_i\delta_i)\Big|_w\omega_{iw}\\
&\le -\psi_i^{\mathrm{T}}C_i\psi_i-\eta_i^{\mathrm{T}}C_iK_i\eta_i\\
&\quad +\left(\theta_{0,\psi_i}\|\psi_i\|+\theta_{0,\eta_i}\|\eta_i\|\right)\sum_{w=1}^{3}|\omega_{iw}|\\
&\le -\psi_i^{\mathrm{T}}C_i\psi_i-\eta_i^{\mathrm{T}}C_iK_i\eta_i\\
&\quad +\theta_{0,i}(\|\psi_i\|+\|\eta_i\|)\sum_{w=1}^{3}|\omega_{iw}|,
\end{aligned}
\tag{12}
$$

where $\theta_{0,i},\theta_{0,\psi_i},\theta_{0,\eta_i}$ are positive constants that satisfy $\theta_{0,i}=\max\{\theta_{0,\psi_i},\theta_{0,\eta_i}\}$, and $(\cdot)\Big|_w$ denotes the wth element of vector $(\cdot)$. On the other hand, based on the definition of v_{iw}, v_{iw}^*, and $q_{iw}\le 1$, one can get

$$
\begin{aligned}
\theta_i\sum_{w=1}^{3}|v_{iw}-v_{iw}^*| &= \theta_i\sum_{w=1}^{3}\Bigg|(Q_i\omega_i)\Big|_w-k_2\sum_{j\in N_i}a_{ij}(q_{iw}\\
&\quad -q_{jw})+b_i(q_{iw}-q_{dw})\Bigg|\\
&\ge \theta_i\sum_{w=1}^{3}\theta_{1,iw}|\omega_{iw}|\\
&\ge \theta_i\theta_{1,i}\sum_{w=1}^{3}|\omega_{iw}|,
\end{aligned}
$$

where $\theta_i,\theta_{1,i}$, and $\theta_{1,iw}$ are positive time-varying parameters since Q_i, q_{iw}, q_{jw} are time varying, $\theta_{1,i}$ satisfy $\theta_{1,i}=\min_{w\in\{1,2,3\}}\{\theta_{1,iw}\}$. Let $\theta_i\theta_{1,i}\ge\theta_{0,i}$, then

$$
\begin{aligned}
\dot{W}_{i,2} &\le -\psi_i^{\mathrm{T}}C_i\psi_i-\eta_i^{\mathrm{T}}C_iK_i\eta_i\\
&\quad +\theta_i(\|\psi_i\|+\|\eta_i\|)\sum_{w=1}^{3}|v_{iw}-v_{iw}^*|.
\end{aligned}
\tag{13}
$$

From the definition of $(\cdot)\Big|_w$, it can be obtained that $(\cdot)\Big|_w\le\|(\cdot)\|$ for a given vector $(\cdot)$. Substituting the control torque (6), one has

$$
\begin{aligned}
&(v_{iw}-v_{iw}^*)\frac{\partial v_{iw}}{\partial t}\\
&= (v_{iw}-v_{iw}^*)\left(\frac{\partial v_i}{\partial t}\right)\Big|_w\\
&= (v_{iw}-v_{iw}^*)(\dot{Q}_i\omega_i+Q_i\dot{\omega}_i)\Big|_w\\
&= (v_{iw}-v_{iw}^*)(\dot{Q}_i\omega_i)\Big|_w+(v_{iw}-v_{iw}^*)\\
&\quad \cdot\Big\{Q_iJ_{m,i}^{-1}\big[s(\omega_i)(J_{m,i}\omega_i+\delta_i^{\mathrm{T}}\psi_i)+\delta_i^{\mathrm{T}}\\
&\quad \cdot(C_i\psi_i+K_i\eta_i-C_i\delta_i\omega_i)+D_i\Gamma_i\tau_i\big]\Big\}\Big|_w\\
&\le |v_{iw}-v_{iw}^*|\,\Big\|(\dot{Q}_i\omega_i)+Q_iJ_{m,i}^{-1}\\
&\quad \cdot[s(\omega_i)(J_{m,i}\omega_i+\delta_i^{\mathrm{T}}\psi_i)+\delta_i^{\mathrm{T}}\\
&\quad \cdot(C_i\psi_i+K_i\eta_i-C_i\delta_i\omega_i)]\Big\|\\
&\quad +(v_{iw}-v_{iw}^*)\left(Q_iJ_{m,i}^{-1}D_i\Gamma_i\tau_i\right)\Big|_w\\
&\le -k_2^2(\|\psi_i\|+\|\eta_i\|)\hat{\theta}_i|v_{iw}-v_{iw}^*|\\
&\quad -k_1(v_{iw}-v_{iw}^*)^2,
\end{aligned}
\tag{14}
$$

where $\{\cdot\}\Big|_w$ also denotes the wth element of vector $\{\cdot\}$. Applying the adaptive law (7), the time derivative of Lyapunov function V is

$$
\begin{aligned}
\dot{V} &= \dot{V}_0+\sum_{i=1}^{n}(\dot{W}_{i,1}+\dot{W}_{i,2}+\dot{W}_{i,3})\\
&\le -\left(k_2-\frac{1}{2}-\frac{\mu_l+n\mu_a}{2}\right)\sum_{i=1}^{n}\sum_{w=1}^{3}\Bigg[\sum_{j\in N_i}a_{ij}(q_{iw}\\
&\quad -q_{jw})+b_i(q_{iw}-q_{dw})\Bigg]^2\\
&\quad +\left[\frac{1}{2}(\mu_l+n\mu_a+1)+\frac{1}{k_2}(\mu_l+n\mu_a)\right]\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw}\\
&\quad -v_{iw}^*)^2+\frac{1}{k_2^2}\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw}-v_{iw}^*)\frac{\partial v_{iw}}{\partial t}
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{i=1}^{n}(\psi_i^{\mathrm{T}}C_i\psi_i+\eta_i^{\mathrm{T}}C_iK_i\eta_i)+\sum_{i=1}^{n}\theta_i(\|\psi_i\|+\|\eta_i\|)\\
&\cdot\sum_{w=1}^{3}|v_{iw}-v_{iw}^{*}|-\sum_{i=1}^{n}\frac{1}{k_{\theta_i}}(\theta_i-\hat{\theta}_i)\dot{\hat{\theta}}_i\\
\le&-k_3\sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i}a_{ij}(q_{iw}-q_{jw})+b_i(q_{iw}-q_{dw})\right]^2\\
&-k_3\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw}-v_{iw}^{*})^2\\
&-\sum_{i=1}^{n}(\psi_i^{\mathrm{T}}C_i\psi_i+\eta_i^{\mathrm{T}}C_iK_i\eta_i)+\sum_{i=1}^{n}\frac{k_{\hat{\theta}_i}}{k_{\theta_i}}(\theta_i-\hat{\theta}_i)\hat{\theta}_i.
\end{aligned}
\tag{15}
$$

where the parameter k_3 satisfies the following inequalities simultaneously

$$
\begin{aligned}
&k_3\le\frac{k_1}{k_2^2}-0.5(\mu_1+n\mu_a+1)-\frac{1}{k_2}(\mu_l+n\mu_a),\\
&k_3\le k_2-0.5-0.5(\mu_l+n\mu_a).
\end{aligned}
$$

Note that

$$
\begin{aligned}
V_0&=\frac{1}{2}(q-\mathbf{1}_n\otimes q_d)^{\mathrm{T}}(\mathcal{L}+B)(q-\mathbf{1}_n\otimes q_d)\\
&\le\frac{1}{2\lambda_{LB}}\sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i}a_{ij}(q_{iw}-q_{jw})\right.\\
&\qquad\left.+b_i(q_{iw}-q_{dw})\right]^2,
\end{aligned}
$$

$$
\sum_{i=1}^{n}W_{i,1}\le\frac{1}{k_2^2}\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw}-v_{iw}^{*})^2,
$$

$$
\begin{aligned}
\sum_{i=1}^{n}W_{i,2}\le{}&\lambda_{CK}\sum_{i=1}^{n}(\psi_i^{\mathrm{T}}C_i\psi_i+\eta_i^{\mathrm{T}}C_iK_i\eta_i)\\
&+\sum_{i=1}^{n}\eta_i^{\mathrm{T}}C_i^{\mathrm{T}}\psi_i,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n}W_{i,3}\le{}&\frac{1}{2\lambda_{k_{\hat{\theta}}}}\sum_{i=1}^{n}\left[-\frac{k_{\hat{\theta}_i}}{k_{\theta_i}}(\theta_i-\hat{\theta}_i)\hat{\theta}_i\right.\\
&\left.+\frac{k_{\hat{\theta}_i}}{k_{\theta_i}}\theta_i\left(\theta_i-\hat{\theta}_i\right)\right],
\end{aligned}
$$

where $\lambda_{LB}=\lambda_{\min}\{\mathcal{L}+B\}$, $\lambda_{k_{\hat{\theta}}}=\min_{i\in\Omega}\{k_{\hat{\theta}_i}\}$, $q=[q_1^{\mathrm{T}},\ldots,q_n^{\mathrm{T}}]^{\mathrm{T}}$, and

$$
\lambda_{CK}=\frac{1}{\min_{i\in\Omega}\{\lambda_{\min}\{C_i\}\}}+\frac{\max_{i\in\Omega}\{\lambda_{\max}\{C_i\}\}}{2\min_{i\in\Omega}\{\lambda_{\min}\{K_i\}\}}.
$$

Then, $\dot{V}\le-\lambda_V V+\varepsilon_V$, where

$$
\lambda_V=\min\left\{2\lambda_{LB}k_3,\ k_2^2k_3,\ \frac{1}{\lambda_{CK}},\ 2\lambda_{k_{\hat{\theta}}}\right\},
$$

and

$$
\varepsilon_V=\sum_{i=1}^{n}\left[\frac{1}{\lambda_{CK}}\eta_i^{\mathrm{T}}C_i^{\mathrm{T}}\psi_i+\frac{k_{\hat{\theta}_i}}{k_{\theta_i}}\theta_i(\theta_i-\hat{\theta}_i)\right].
$$

Hence,

$$
V(t)\le e^{-\lambda_V(t-t_0)}V(t_0)+\int_{t_0}^{\mathrm{T}}e^{-\lambda_V(t-s)}\varepsilon_V\mathrm{d}s,
$$

where t_0 denotes the initial time. Applying Gronwall–Bellman inequality [35], $\dot{V}\le\varepsilon_V e^{-\lambda_V(t-t_0)}$. Then, one can obtain $\dot{V}\le 0$ if $V\ge\|\frac{\varepsilon_V}{\lambda_V}\|$. As a result, it can be concluded that the states $(q_i-q_d,\ \omega_i,\eta_i,\dot{\eta}_i)$ are globally bounded. That is to say, attitude consensus is achieved asymptotically and the induced oscillations are damped out. □

Remark 1 The above theorem proposes the control technique for attitude consensus of flexible spacecraft under loss of actuator effectiveness. The idea of constructing the candidate Lyapunov function is partly borrowed from the results of [22]. In the work of [22], all the spacecraft are assumed to be rigid. A more complicated issue with flexible spacecraft is considered in the above theorem. In addition, this paper analyzes flexible spacecraft with faults and adopts an adaptive law to deal with the oscillations induced by the flexible appendages.

Remark 2 In the work of [29], the controller not only depends on the states q,ω of its neighbors but also requires the modal variables η,ψ of its neighbors. However, the proposed controller in this paper only needs the states of its neighbors and modal information of its own, so less information is needed to be exchanged. In this sense, the proposed control technique can reduce the communication burden. From a theoretical point of view, it is only required to stabilize the modal variables. Therefore, the proposed controller in this paper is more reasonable. Moreover, compared with the method in [29], controller (6) can achieve better performance while requiring less communication load. This is verified in the simulations mentioned afterward.

3.2 Consensus control law design for spacecraft under modeling uncertainties, disturbances and actuator failures

Note that during operation the mass, damping and stiffness properties of the flexible spacecraft may be uncertain or may change due to onboard payload motion, rotation and fuel consumption. This makes J_i, δ_i, C_i, K_i time varying and uncertain. On the other hand, it is of theoretical and practical importance to consider the factors of additive faults and disturbances.

In such circumstances, it is assumed that $\Delta J_i, \delta_i, C_i, K_i$ are unknown. Then, the following theorem can be established to achieve attitude consensus for spacecraft with modeling uncertainties, disturbances and actuator failures.

Theorem 2 *For the multiple flexible spacecraft systems (3) and (4), if Assumptions 1–4 hold and the control torque τ_i is designed as*

$$\tau_i = -\frac{1}{\lambda_{\tau_i}} D_i^{\mathrm{T}} \bar{J}_i Q_i^{-1} \Big[k_1(v_i - v_i^*) + \Big(\|\dot{Q}_i\omega_i + Q_i\bar{J}_i^{-1} \cdot s(\omega_i)\bar{J}_i\omega_i\| + k_2^2\hat{e}_i\Big)\,\mathrm{sgn}(v_i - v_i^*)\Big], \quad i \in \Omega. \tag{16}$$

Parameters $\lambda_{\tau_i}, k_1, k_2$ satisfy $0 < \lambda_{\tau_i} \le \lambda_{\min}\{D_i\Gamma_i D_i^{\mathrm{T}}\}$, $k_1 \ge k_2^2[0.5(\mu_1 + n\mu_a + 1) + \frac{1}{k_2}(\mu_l + n\mu_a) + k_3]$, and $k_2 \ge 0.5 + 0.5(\mu_l + n\mu_a) + k_3$, where $k_3 > 0$, $\mu_l = \max_{i\in\Omega}\{l_{ii} + b_i\}$, $\mu_a = \max_{i\in\Omega, j\in N_i}\{a_{ij}\}$, and the adaptive law for parameter $\hat{e}_i$ is set as

$$\dot{\hat{e}}_i = -k_{\hat{e}_i}\hat{e}_i + k_{e_i}\sum_{w=1}^{3}|v_{iw} - v_{iw}^*|, \tag{17}$$

where $k_{e_i} > 0, k_{\hat{e}_i} > 0$, then the attitude consensus can be achieved asymptotically and the induced oscillations of the spacecraft's flexible appendages are damped out.

Proof The candidate Lyapunov function is constructed as the same as Theorem 1 except for

$$W_{i,3} = \frac{1}{2k_{e_i}}(e_i - \hat{e}_i)^2.$$

From (13), one can further obtain

$$\dot{W}_{i,2} \le -\psi_i^{\mathrm{T}} C_i\psi_i - \eta_i^{\mathrm{T}} C_i K_i\eta_i + e_{1,i}\sum_{w=1}^{3}|v_{iw} - v_{iw}^*|, \tag{18}$$

where $e_{1,i}$ is a positive time-varying parameter that depends on θ_i, ψ_i, η_i. From Assumptions 1–3,

$$\Big\|Q_i\bar{J}_i^{-1}\Big[s(\omega_i)(\Delta J_i - \delta_i^{\mathrm{T}}\delta_i)\omega_i + s(\omega_i)\delta_i^{\mathrm{T}}\psi_i + \delta_i^{\mathrm{T}}(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) - (\Delta J_i - \delta_i^{\mathrm{T}}\delta_i)\dot{\omega}_i + D_i f_i + d_i\Big]\Big\| \le k_2^2 e_{2,i},$$

where $e_{2,i}$ is a positive time-varying parameter that depends on $\omega_i, \dot{\omega}_i, \psi_i, \eta_i$. Substituting the control torque (16), one has

$$\begin{aligned}(v_{iw} - v_{iw}^*)\frac{\partial v_{iw}}{\partial t} &\le |v_{iw} - v_{iw}^*|\|\dot{Q}_i\omega_i + Q_i\bar{J}_i^{-1}s(\omega_i)\bar{J}_i\omega_i\| + k_2^2 e_{2,i}|v_{iw} - v_{iw}^*| \\ &\quad + (v_{iw} - v_{iw}^*)\Big(Q_i\bar{J}_i^{-1}D_i\Gamma_i\tau_i\Big)\Big|_w \\ &\le k_2^2(e_{2,i} - \hat{e}_i)|v_{iw} - v_{iw}^*| - k_1(v_{iw} - v_{iw}^*)^2.\end{aligned} \tag{19}$$

Denote $e_i = e_{1,i} + e_{2,i}$. Applying the adaptive law (17), the time derivative of Lyapunov function V is

$$\begin{aligned}\dot{V} &= \dot{V}_0 + \sum_{i=1}^{n}(\dot{W}_{i,1} + \dot{W}_{i,2} + \dot{W}_{i,3}) \\ &\le -k_3\sum_{i=1}^{n}\sum_{w=1}^{3}\Bigg[\sum_{j\in N_i}a_{ij}(q_{iw} - q_{jw}) + b_i(q_{iw} - q_{dw})\Bigg]^2 - k_3\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2 \\ &\quad - \sum_{i=1}^{n}(\psi_i^{\mathrm{T}} C_i\psi_i + \eta_i^{\mathrm{T}} C_i K_i\eta_i) \\ &\quad + \sum_{i=1}^{n}\frac{k_{\hat{e}_i}}{k_{e_i}}(e_i - \hat{e}_i)\hat{e}_i.\end{aligned} \tag{20}$$

Following the same line of Theorem 1, it can be concluded that the states $(q_i - q_d,\ \omega_i, \eta_i, \dot{\eta}_i)$ are globally bounded. Thus, the attitude consensus is achieved and the induced oscillations are damped out, which completes the proof. □

Remark 3 The above theorem proposes the control technique for attitude consensus of flexible spacecraft with uncertainty, disturbances and faults. Since information on $\Delta J_i, \delta_i, C_i, K_i$ is not used in the control torque, the proposed method does not highly depend on the accuracy of the system model which cannot usually be guaranteed in practice. In this sense, the proposed method has the advantage over the method in [29] which needs the precise information of the system model.

Remark 4 The modal variables η_i, ψ_i, which are implied in ω_i and $\dot{\omega}_i$, are not directly used in the feedback controller. They are damped out along with the convergence of ω_i and $\dot{\omega}_i$ indirectly. Hence, it is not necessary to add extra sensors to measure the modal variables, which can lighten the payload of the spacecraft.

Remark 5 Compared with controller (6), less information is used in the control law (16) since the information of modal variables is excluded. In addition, the adaptive parameter $\hat{e}_i$ in controller (16) is utilized for handling the modal variables, modeling uncertainties, disturbances and additive faults, while the adaptive parameter in controller (6) is only used to deal with the oscillations induced by modal variables. However, the system performance under controller (6) is better than that under control law (16). More specifically, by applying controller (6), the convergence time of the adaptive parameter is shorter, and the control torque is smaller. The numerical simulations in Sect. 4 will illustrate this statement.

3.3 Consensus control law design for spacecraft under actuator failures and saturations

It can be seen from the control laws (6) and (16) that the terms $\frac{D_i^{\mathrm{T}}}{\lambda_{\tau_i}} J_{m,i} Q_i^{-1} k_1(v_i - v_i^*)$ and $\frac{D_i^{\mathrm{T}}}{\lambda_{\tau_i}} \bar{J}_i Q_i^{-1} k_1(v_i - v_i^*)$ will lead to large control torques at the beginning of the control task which can be verified by the simulations. Hence, it is urgently necessary that the consensus control law can reject the issue of actuator saturation.

The switching control scheme ensuring attitude consensus for spacecraft under the conditions as mentioned in Sect. 3.2 and actuator saturation is given by

$$\tau_i = -\frac{D_i^{\mathrm{T}} \bar{J}_i Q_i^{-1}}{\|D_i\| \|\bar{J}_i Q_i^{-1}\|} \mathrm{sat}(\tau_i,\ v_i,\ v_i^*), \quad i \in \Omega \tag{21}$$

with

$$\mathrm{sat}(\tau_i,\ v_i,\ v_i^*) = \begin{cases} \frac{1}{\sqrt{3}} \tau_i^{\max} \mathrm{sgn}(v_i - v_i^*), & \text{if } \Upsilon_i > \tau_i^{\max}, \\ \frac{1}{\lambda_{\tau_i}} \left[k_1(v_i - v_i^*) + k_2^2 \hat{\epsilon}_i \mathrm{sgn}(v_i - v_i^*) \right], & \text{if } \Upsilon_i \le \tau_i^{\max}, \end{cases}$$

where $\Upsilon_i = \frac{1}{\lambda_{\tau_i}}(k_1\|v_i - v_i^*\| + \sqrt{3} k_2^2 \hat{\epsilon}_i)$, and $\tau_i^{\max} = \min\{\tau_{i,1}^{\max}, \tau_{i,2}^{\max}, \ldots, \tau_{i,m}^{\max}\}$ where $\tau_{i,j}^{\max}(j \in \{1, \ldots, m\})$ denotes the maximum allowable torque of the jth actuator of the ith spacecraft.

Theorem 3 *For the multiple flexible spacecraft systems (3) and (4) under thrust limits* $\tau_{i,j} \le \tau_{i,j}^{\max}(i \in \Omega, j \in \{1, \ldots, m\})$, *if Assumptions 1–4 hold and*

$$k_1 \max_{w \in \{1,2,3\}} \{|v_{iw} - v_{iw}^*|\} + \epsilon_i k_2^2 \|D_i\| \|\bar{J}_i Q_i^{-1}\| \le \frac{\lambda_{\tau_i} \tau_i^{\max}}{\sqrt{3}}, \tag{22}$$

and the control scheme (21) is applied, the parameters $\lambda_{\tau_i}, k_1, k_2$ *satisfy* $0 < \lambda_{\tau_i} \le \lambda_{\min}\{D_i \Gamma_i D_i^{\mathrm{T}}\}$, $k_1 \ge k_2^2 \|D_i\| \|\bar{J}_i Q_i^{-1}\| [0.5(\mu_1 + n\mu_a + 1) + \frac{1}{k_2}(\mu_l + n\mu_a) + k_3]$, *and* $k_2 \ge 0.5 + 0.5(\mu_l + n\mu_a) + k_3$, *where* $k_3 > 0$, $\mu_l = \max_{i \in \Omega}\{l_{ii} + b_i\}$, $\mu_a = \max_{i \in \Omega, j \in N_i}\{a_{ij}\}$,, *and the adaptive law for parameter* $\hat{\epsilon}_i$ *is set as*

$$\dot{\hat{\epsilon}}_i = -k_{\hat{\epsilon}_i} \hat{\epsilon}_i + k_{\epsilon_i} \sum_{w=1}^{3} |v_{iw} - v_{iw}^*|, \tag{23}$$

where $k_{\epsilon_i} > 0, k_{\hat{\epsilon}_i} > 0$, *then the attitude consensus can be achieved asymptotically and the induced oscillations of the spacecraft's flexible appendages are damped out.*

Proof To prove the theorem, two cases are addressed.

Case A: If $\frac{1}{\lambda_{\tau_i}}(k_1\|v_i - v_i^*\| + \sqrt{3} k_2^2 \hat{\epsilon}_i) > \tau_i^{\max}$ for $i \in \Omega$, the control torques are

$$\tau_i = -\frac{D_i^{\mathrm{T}} \bar{J}_i Q_i^{-1}}{\|D_i\| \|\bar{J}_i Q_i^{-1}\|} \frac{1}{\sqrt{3}} \tau_i^{\max} \mathrm{sgn}(v_i - v_i^*).$$

The candidate Lyapunov function is constructed as

$$V = V_0 + \sum_{i=1}^{n} (W_{i,1} + W_{i,2}), \tag{24}$$

where V_0, $W_{i,1}$, $W_{i,2}$ are defined as the same as Theorems 1 and 2.

Under the Assumptions 1–3, it can be verified that

$$\begin{aligned} &\Bigg\| (\psi_i^{\mathrm{T}} C_i \delta_i - 2\eta_i^{\mathrm{T}} K_i \delta_i)\omega_i + \frac{1}{k_2^2} \sum_{w=1}^{3} (v_{iw} - v_{iw}^*) \big\{ \dot{Q}_i \omega_i \\ &\quad + Q_i \bar{J}_i^{-1} \Big[s(\omega_i)(J_i - \delta_i^{\mathrm{T}} \delta_i)\omega_i + s(\omega_i)\delta_i^{\mathrm{T}} \psi_i + \delta_i^{\mathrm{T}}(C_i \psi_i \\ &\quad + K_i \eta_i - C_i \delta_i \omega_i) - (\Delta J_i - \delta_i^{\mathrm{T}} \delta_i)\dot{\omega}_i + D_i f_i + d_i \Big] \big\} \Bigg\| \\ &\le \epsilon_i \sum_{w=1}^{3} |v_{iw} - v_{iw}^*|, \end{aligned}$$

where ϵ_i is a positive time-varying parameter that depends on $\omega_i, \dot{\omega}_i, \psi_i, \eta_i$. The time derivative of Lyapunov function V is

$$
\begin{aligned}
\dot{V} &= \dot{V}_0 + \sum_{i=1}^{n}(\dot{W}_{i,1} + \dot{W}_{i,2}) \\
&\le -k_3 \sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i} a_{ij}(q_{iw} - q_{jw}) + b_i(q_{iw} - q_{dw})\right]^2 \\
&\quad + \left[\frac{1}{2}(\mu_l + n\mu_a + 1) + \frac{1}{k_2}(\mu_l + n\mu_a)\right] \\
&\quad \times \sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2 + \frac{1}{k_2^2}\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)\frac{\partial v_{iw}}{\partial t} \\
&\quad - \sum_{i=1}^{n}(\psi_i^{\mathrm{T}} C_i \psi_i + \eta_i^{\mathrm{T}} C_i K_i \eta_i) \\
&\quad + \sum_{i=1}^{n}(\psi_i^{\mathrm{T}} C_i \delta_i - 2\eta_i^{\mathrm{T}} K_i \delta_i)\omega_i,
\end{aligned}
$$

where k_3 satisfies

$$k_3 \le k_2 - 0.5 - 0.5(\mu_l + n\mu_a).$$

Substituting the aforementioned control torque, one has

$$
\begin{aligned}
\dot{V} &\le -k_3 \sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i} a_{ij}(q_{iw} - q_{jw}) + b_i(q_{iw} - q_{dw})\right]^2 \\
&\quad + \left[\frac{1}{2}(\mu_l + n\mu_a + 1) + \frac{1}{k_2}(\mu_l + n\mu_a)\right] \\
&\quad \times \sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2 - \sum_{i=1}^{n}(\psi_i^{\mathrm{T}} C_i \psi_i + \eta_i^{\mathrm{T}} C_i K_i \eta_i) \\
&\quad + \sum_{i=1}^{n}\left(\epsilon_i - \frac{\lambda_{\tau_i}\tau_i^{\max}}{\sqrt{3}k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|}\right)\sum_{w=1}^{3}|v_{iw} - v_{iw}^*|.
\end{aligned} \tag{25}
$$

According to inequality (22), it can be obtained that

$$
\begin{aligned}
&\frac{k_1 \max_{w\in\{1,2,3\}}|v_{iw} - v_{iw}^*|}{k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|} + \epsilon_i \\
&\quad \le \frac{\lambda_{\tau_i}\tau_i^{\max}}{\sqrt{3}k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|},
\end{aligned}
$$

then

$$
\begin{aligned}
&\left(\epsilon_i - \frac{\lambda_{\tau_i}\tau_i^{\max}}{\sqrt{3}k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|}\right)\sum_{w=1}^{3}|v_{iw} - v_{iw}^*| \\
&\quad \le -\frac{k_1 \max_{w\in\{1,2,3\}}|v_{iw} - v_{iw}^*|}{k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|}\sum_{w=1}^{3}|v_{iw} - v_{iw}^*| \\
&\quad \le -\frac{k_1}{k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\dot{V} &\le -k_3 \sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i} a_{ij}(q_{iw} - q_{jw})\right. \\
&\quad \left. + b_i(q_{iw} - q_{dw})\right]^2 - k_3\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2 \\
&\quad - \sum_{i=1}^{n}(\psi_i^{\mathrm{T}} C_i \psi_i + \eta_i^{\mathrm{T}} C_i K_i \eta_i) \le 0,
\end{aligned} \tag{26}
$$

where k_3 also satisfies the following inequality

$$
\begin{aligned}
k_3 &\le \frac{k_1}{k_2^2\|D_i\|\|\bar{J}_i Q_i^{-1}\|} \\
&\quad - \left[0.5(\mu_1 + n\mu_a + 1) + \frac{1}{k_2}(\mu_l + n\mu_a)\right].
\end{aligned}
$$

If $\frac{1}{\lambda_{\tau_i}}(k_1\|v_i - v_i^*\| + \sqrt{3}k_2^2\hat{\epsilon}_i) > \tau_i^{\max}$, one has $\dot{V} \le 0$. Thus, the states of the systems (3) and (4) will converge to the region $\frac{1}{\lambda_{\tau_i}}(k_1\|v_i - v_i^*\| + \sqrt{3}k_2^2\hat{\epsilon}_i) \le \tau_i^{\max}$.

Case B: If $\frac{1}{\lambda_{\tau_i}}(k_1\|v_i - v_i^*\| + \sqrt{3}k_2^2\hat{\epsilon}_i) \le \tau_i^{\max}$ for $i \in \Omega$, the control torques are

$$
\begin{aligned}
\tau_i &= -\frac{D_i^{\mathrm{T}}\bar{J}_i Q_i^{-1}}{\|D_i\|\|\bar{J}_i Q_i^{-1}\|}\frac{1}{\lambda_{\tau_i}}\left[k_1(v_i - v_i^*)\right. \\
&\quad \left. + k_2^2\hat{\epsilon}_i \operatorname{sgn}(v_i - v_i^*)\right].
\end{aligned}
$$

The candidate Lyapunov function is constructed as the same as Theorem 1 except for

$$W_{i,3} = \frac{\|D_i\|\|\bar{J}_i Q_i^{-1}\|}{2k_{\epsilon_i}}\left(\epsilon_i - \frac{1}{\|D_i\|\|\bar{J}_i Q_i^{-1}\|}\hat{\epsilon}_i\right)^2.$$

Following the same steps in Case A and applying the adaptive law (23), the time derivative of Lyapunov function V is

$$
\begin{aligned}
\dot{V} &= \dot{V}_0 + \sum_{i=1}^{n}(\dot{W}_{i,1} + \dot{W}_{i,2} + \dot{W}_{i,3}) \\
&\le -k_3 \sum_{i=1}^{n}\sum_{w=1}^{3}\left[\sum_{j\in N_i} a_{ij}(q_{iw} - q_{jw})\right. \\
&\quad \left. + b_i(q_{iw} - q_{dw})\right]^2 - k_3\sum_{i=1}^{n}\sum_{w=1}^{3}(v_{iw} - v_{iw}^*)^2
\end{aligned}
$$

$$- \sum_{i=1}^{n} (\psi_i^{\mathrm{T}} C_i \psi_i + \eta_i^{\mathrm{T}} C_i K_i \eta_i) + \sum_{i=1}^{n} \frac{k_{\hat{\epsilon}_i}}{k_{\epsilon_i}} (\epsilon_i - \frac{1}{\|D_i\| \|\bar{J}_i Q_i^{-1}\|} \hat{\epsilon}_i) \hat{\epsilon}_i. \quad (27)$$

The result is established using the same argument as in Theorem 1. It can be concluded that the states $(q_i - q_d,\ \omega_i, \eta_i, \dot{\eta}_i)$ are globally bounded. Therefore, the attitude consensus is achieved and the induced oscillations are damped out, which completes the proof. □

Remark 6 Inequality (22) implies that the actuators can produce torques sufficient to allow the multiple flexible spacecraft to achieve the attitude consensus under actuator failures. Similar assumptions were proposed in [31] and references therein.

Remark 7 The comparisons of the proposed three control laws are given in this remark. Firstly, controller (6) can only deal with the situation where an actuator partially loses its actuation effectiveness, while controllers (16) and (21) can handle the conditions under modeling uncertainties, disturbances and actuator failures. Besides, controller (6) needs the modal information and has the most complex form, while controllers (16) and (21) do not require the modal information. Thus, using controllers (16) and (21) can lighten the payload of the spacecraft and reduce the computation burden. However, the system performance under controller (6) is better than that under control laws (16) and (21), since more information and more complex schemes are utilized in controller (6). The adaptive parameters in controllers (16) and (21) are utilized for handling the modal variables, modeling uncertainties, disturbances and additive faults, while the adaptive parameter in controller (6) is only used to deal with the oscillations induced by modal variables. Thus, the convergence time of the adaptive parameter is shortened by applying controller (6). Finally, only controller (21) has the anti-saturation ability, while controllers (6) and (16) cannot reject the issue of actuator saturation.

4 Simulations

Simulation results are presented in this section to illustrate the effectiveness of the proposed consensus control law. Consider a scenario with four flexible spacecraft described by (1) and (2). The communication topology graph is described by Fig. 1,

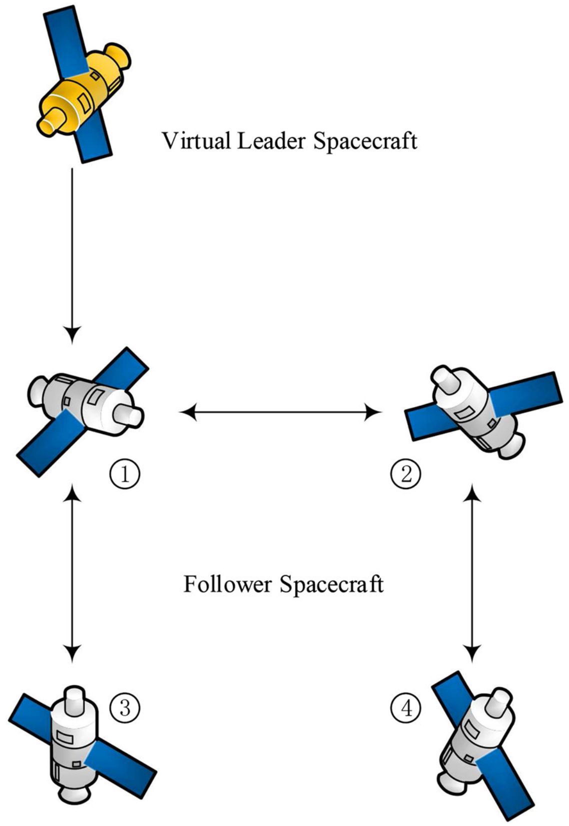

Fig. 1 Communication topology graph

where $a_{12} = a_{21} = a_{13} = a_{31} = a_{24} = a_{42} = 0.5$. Assume that only the first follower spacecraft can obtain the information of the virtual leader, i.e., $b_1 = 0.5, b_2 = b_3 = b_4 = 0$. The leader's attitude is $q_d = [\sin(\pi/12)/\sqrt{2},\ \sin(\pi/12)/\sqrt{2},\ 0]^{\mathrm{T}}$, which implies that the reference attitude is a 30° rotation around the axis $[\sqrt{2}/2, \sqrt{2}/2, 0]^{\mathrm{T}}$.

The nominal part of the inertia matrix of each spacecraft is chosen as [29]: $\bar{J}_1 = \mathrm{diag}(18, 12, 10)\,\mathrm{kg\,m}^2$, $\bar{J}_2 = \mathrm{diag}(22, 16, 12)\,\mathrm{kg\,m}^2$, $\bar{J}_3 = \mathrm{diag}(17, 14, 12)\,\mathrm{kg\,m}^2$, and parameter $\bar{J}_4 = \mathrm{diag}(19, 13, 15)\,\mathrm{kg\,m}^2$. The coupling matrix, the natural frequency, the damping for flexible attachments are given as

$$\delta_i = \begin{bmatrix} 1.3523 & 1.2784 & 2.153 \\ -1.1519 & 1.0176 & -1.2724 \\ 2.2167 & 1.5891 & -0.8324 \\ 1.23637 & -1.6537 & 0.2251 \end{bmatrix} \mathrm{kg\,m/s}^2,$$

and parameters $\omega_{i,41} = 1.0973, \omega_{i,42} = 1.2761, \omega_{i,43} = 1.6538, \omega_{i,44} = 2.2893, \xi_{i,1} = 0.056, \xi_{i,2} = 0.086, \xi_{i,3} = 0.08, \xi_{i,4} = 0.025, i \in \Omega$. The

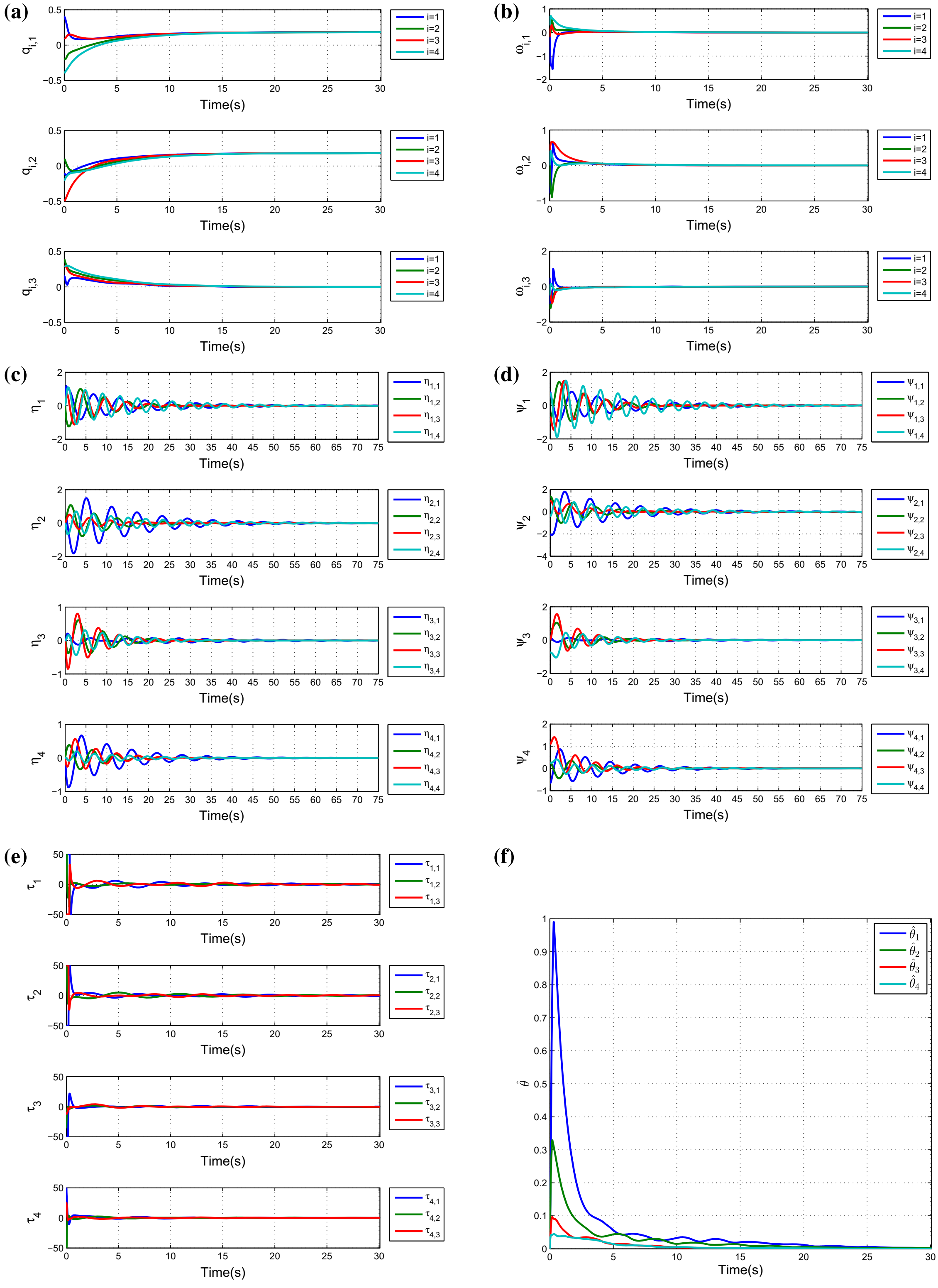

Fig. 2 Response curves under the control law (6). **a** Attitude, **b** angular velocity, **c** modal variable η, **d** modal variable ψ , **e** control torque, **f** adaptive parameters

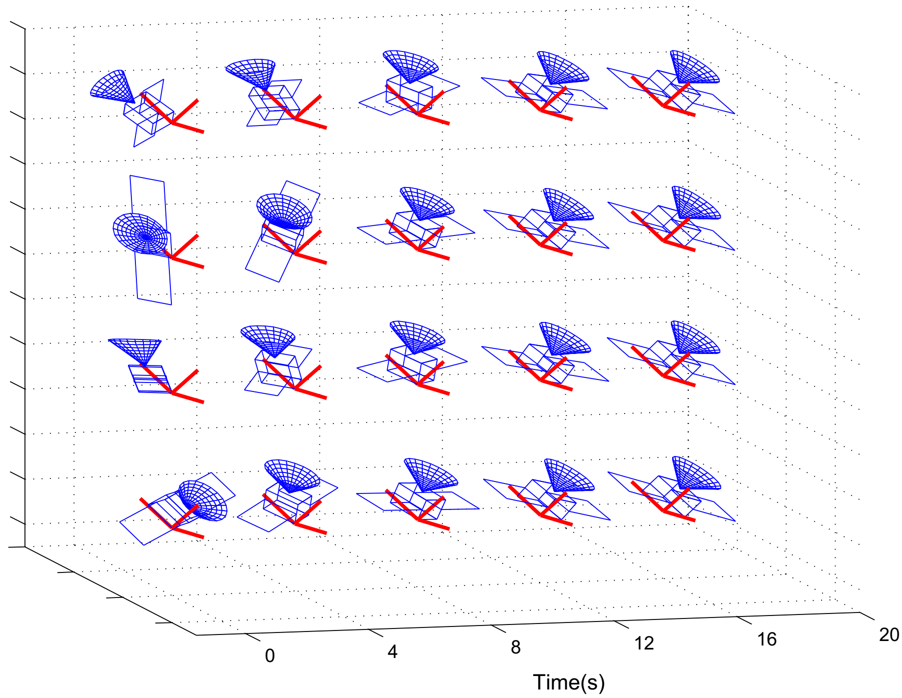

Fig. 3 Consensus procedure

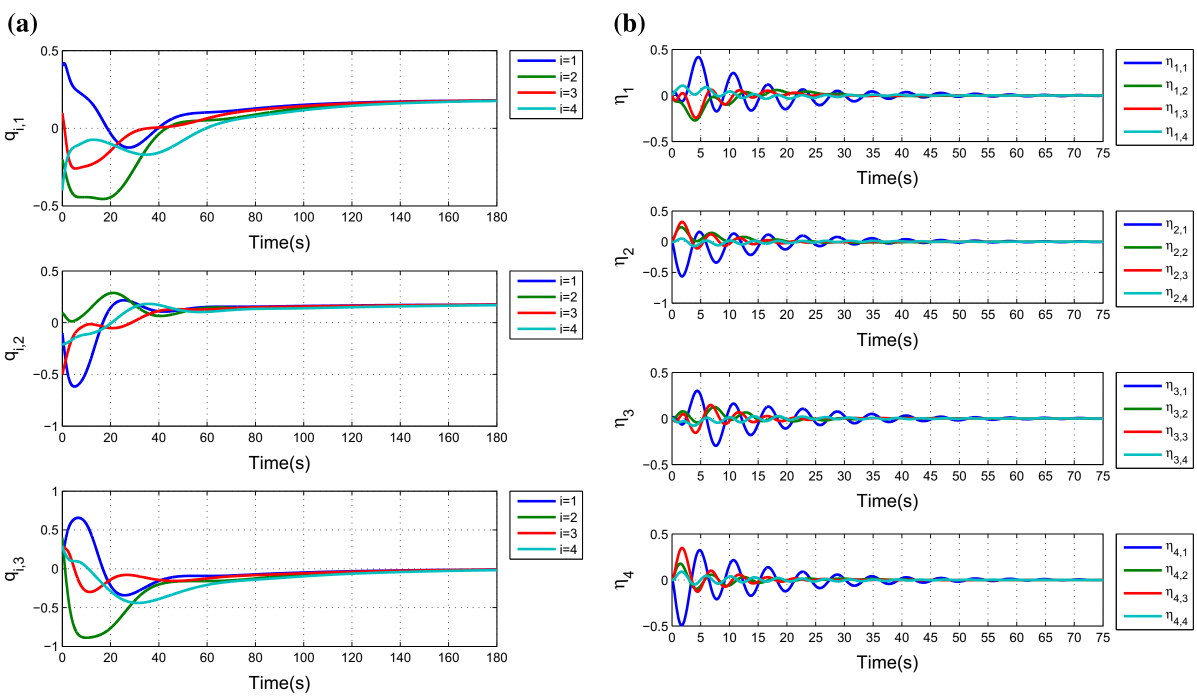

Fig. 4 Response curves under the control law proposed in [29]. **a** Attitude, **b** modal variable η

actuation effectiveness matrix is assumed as $\varGamma_i = \mathrm{diag}(0.6, 1, 0.9, 0.5)$, and the control actuator distribution matrix is

$$D_i = \begin{bmatrix} -\sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 & -\sqrt{3}/3 \end{bmatrix}.$$

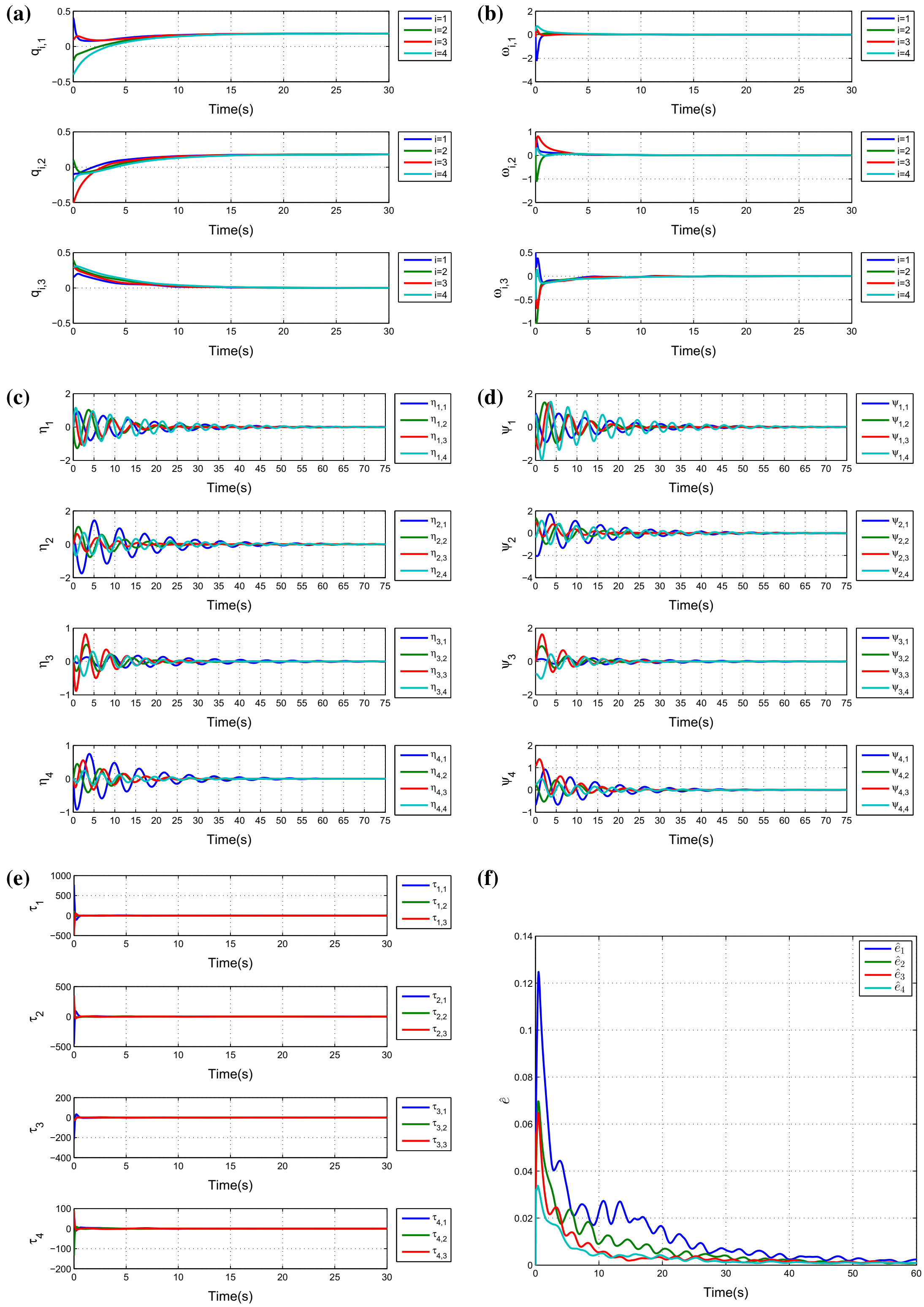

Fig. 5 Response curves under the control law (16). **a** Attitude, **b** angular velocity, **c** modal variable η, **d** modal variable ψ , **e** control torque, **f** adaptive parameters

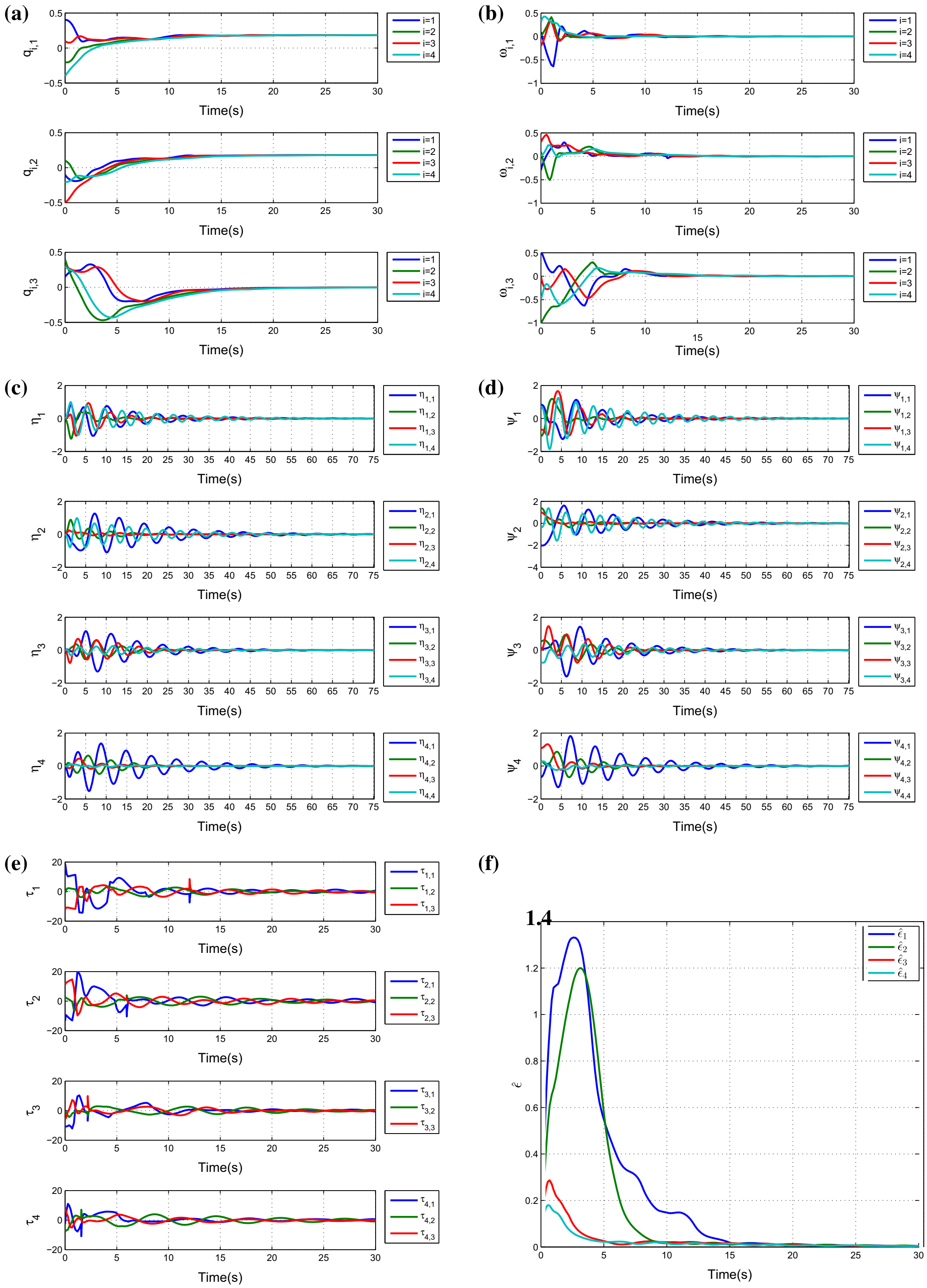

Fig. 6 Response curves under the control law (21). **a** Attitude, **b** angular velocity, **c** modal variable η, **d** modal variable ψ , **e** control torque, **f** adaptive parameters

The initial conditions of each spacecraft are chosen as $q_1(0) = [0.4, -0.1, 0.15]^{\mathrm{T}}$, $q_2(0) = [-0.2, 0.1, 0.4]^{\mathrm{T}}$, and $q_3(0) = [0.1, -0.5, 0.3]^{\mathrm{T}}$, $q_4(0) = [-0.4, -0.2, 0.3]^{\mathrm{T}}$, and $\omega_1(0) = [0.1, -0.3, 0.5]^{\mathrm{T}}$, $\omega_2(0) = [0, 0.1, -1]^{\mathrm{T}}$,and $\omega_3(0) = [-0.2, 0.3, 0]^{\mathrm{T}}$, $\omega_4(0) = [0.3, 0, -0.5]^{\mathrm{T}}$, $\eta_i(0) = [0, 0, 0, 0]^{\mathrm{T}}$, $\psi_i(0) = \delta_i \omega_i(0)$, $i \in \Omega$.

4.1 Loss of actuator effectiveness

The attitude consensus controller proposed in Theorem 1 is applied in this subsection without considering additive faults, disturbances and modeling uncertainties. Let the control gains of control law (6) be $k_1 = 20$, $k_2 = 2.5$ and the gains of adaptive law (7) be $k_{\theta_i} = 1$, $k_{\hat{\theta}_i} = 1$. The control torques are limited to within 50 Nm via saturation blocks in Simulink since the controller (6) has no anti-saturation ability. The response curves of the closed-loop system are given in Fig. 2. From Fig. 2a, b, it can be seen that the attitudes of each spacecraft can reach consensus within 20 s and the final attitude is identical with the reference attitude. Meanwhile, the modal variables (η_i, ψ_i) for each flexible spacecraft converge to zero as shown in Fig. 2c, d, which means that the vibrations of flexible appendages are damped out asymptotically. The control torques and the adaptive parameters are recorded in Fig. 2e, f from which it follows that the control torques are limited and the adaptive parameters are converged. Moreover, the changing process of the spacecraft attitudes is shown in Fig. 3, where the consensus procedure can be observed clearly.

For the purpose of comparison, the attitude cooperative control strategy proposed in [27] is also simulated under the same scenario. Based on the principles for the parameter selection, the control gains are chosen as $k_1 = 0.5$ and $k_2 = 15$. Particularly, the actuation effectiveness matrix is assumed as $\Gamma_i = \mathrm{diag}(1, 1, 1)$ since actuator failure is not considered in [27]. The response curves of the closed-loop system are shown in Fig. 4. It can be found in Fig. 4a that the attitudes of spacecraft reach consensus over 100 s, while the consensus time under the controller presented in this paper is about 20 s. Hence, the control law (6) is more effective although less communication load is required. Comparing Fig. 4b with Fig. 2c, it can be observed that there is little difference between control law proposed in [27] and that proposed in (6) when considering the convergence rate of vibrations.

4.2 Modeling uncertainties, disturbances, actuator faults and saturations

In order to study the effectiveness of the consensus control law proposed in Theorems 2 and 3, the flexible spacecraft are assumed to suffer from inertia matrix uncertainties, additive faults and disturbances. For brevity, identical uncertain part of inertia matrices, disturbances and additive faults are added to each spacecraft as $\Delta J_i = \mathrm{diag}(5, 5, 5)\,\mathrm{kg\,m^2}$, and $d_i = [0.05 + 0.06\sin(t), -0.04 + 0.06\cos(t), 0.01 - 0.03\sin(2t)]^{\mathrm{T}}$ Nm, and

$$f_i = [-0.07 + 0.05\cos(0.4t), 0.05 - 0.03\cos(0.6t), 0.09 + 0.06\sin(0.5t)), -0.06 + 0.04\sin(0.5t)]^{\mathrm{T}}\,\mathrm{Nm}, \quad i \in \Omega.$$

Let the control gains of control law (16) be $k_1 = 20$, $k_2 = 2.5$ and the gains of adaptive law (17) be $k_{e_i} = 1$, $k_{\hat{e}_i} = 1$. The response curves of the closed-loop system are given in Fig. 5. Clearly, it can be observed that attitude consensus and tracking is achieved despite the presence of uncertainties, faults and disturbances. In addition, the vibrations of flexible appendages are damped out and the adaptive parameters are converged, as shown in Fig. 5c, d, f.

However, it can be observed from Fig. 5e that the control torques become overlarge when saturation blocks are not applied. This is a serious defect since the available control torque amplitude is limited in practical situations, so the investigation of anti-saturation control is crucial. Let the control gains be $k_1 = 600$, $k_2 = 2.5$, $k_{\hat{\epsilon}_i} = 1$, $k_{\epsilon_i} = 1$ in control law (21). The response curves of the closed-loop system are given in Fig. 6. From Fig. 6e, it can be seen that the control torques are limited to within 20 Nm during the consensus process under the anti-saturation controller. In the meantime, the attitudes of each spacecraft can reach consensus with slower convergence rates as shown in Fig. 6a, b. Similarly, it can be found in Fig. 6c, d that the vibrations of flexible appendages are damped out with longer settling times.

5 Conclusions

The problem of attitude consensus for flexible spacecraft formation under actuator failures and saturation constraints has been investigated in this paper. A distributed consensus control law for flexible spacecraft with loss of actuator effectiveness faults has been presented by using combined tools from the Lyapunov stability theory and graph theory. In order to handle more complicated situations, a distributed control law for multiple spacecraft systems with modeling uncertainties, external disturbances, and simultaneous loss of actuator effectiveness faults and additive faults has also been proposed. An anti-saturation control scheme has been established by further extending the proposed controllers. The performance of the proposed methods has been discussed through numerical studies. Future work includes achieving a group of flexible spacecraft simultaneously tracking a common time-varying reference attitude with actuator failures, and considering the related topics on semi-Markov jump systems [36], Takagi–Sugeno fuzzy systems [37] in order to achieve more industrial oriented results.

Acknowledgements This work is supported by the National Natural Science Foundation of China under Grants 61225013, 61673026, 61528301 and 11332001. The authors gratefully acknowledge Luke Barnwell's help in revising this paper.

References

1. Yang, D.P., Chen, Z., Liu, X.D.: Distributed adaptive attitude tracking of multiple spacecraft with a leader of bounded unknown input. Int. J. Control Autom. Syst. **11**, 1–9 (2013)
2. Chen, M.Z.Q., Zhang, L., Su, H., Chen, G.: Stabilizing solution and parameter dependence of modified algebraic Riccati equation with application to discrete-time network synchronization. IEEE Trans. Autom. Control **60**, 228–233 (2016)
3. Qian, Y.F., Wu, X.Q., Lv, J.H., Lu, J.A.: Consensus of second-order multi-agent systems with nonlinear dynamics and time delay. Nonlinear Dyn. **78**, 495–503 (2014)
4. Zhao, Y., Liu, Y., Li, Z., Duan, Z.: Distributed average tracking for multiple signals generated by linear dynamical systems: an edge-based framework. Automatica **75**, 158–166 (2017)
5. Zhao, Y., Liu, Y.F.: Distributed average computation for multiple time-varying signals with output measurements. Int. J. Robust Nonlinear Control **26**, 2899–2915 (2016)
6. Wen, G., Yu, W., Li, Z., Yu, X., Cao, J.: Neuro-adaptive consensus tracking of multiagent systems with a high-dimensional leader. IEEE Trans. Cybern. (2016). doi:10.1109/TCYB.2016.2556002
7. Ren, W., Beard, R.W.: Decentralized scheme for spacecraft formation flying via the virtual structure approach. J. Guid. Control Dyn. **27**, 73–82 (2004)
8. Yu, W.W., DeLellis, P., Chen, G., Bernardo, M., Kurths, J.: Distributed adaptive control of synchronization in complex networks. IEEE Trans. Autom. Control **57**, 2153–2158 (2012)
9. Yang, D.P., Ren, W., Liu, X.D.: Fully distributed adaptive sliding-mode controller design for containment control of multiple Lagrangian systems. Syst. Control Lett. **72**, 44–52 (2014)
10. Dong, X.G., Cao, X.B., Zhang, J.X., Shi, L.: A robust adaptive control law for satellite formation flying. Acta Autom. Sin. **39**, 128–137 (2013)
11. Zhao, L., Jia, Y.M.: Decentralized adaptive attitude synchronization control for spacecraft formation using nonsingular fast terminal sliding mode. Nonlinear Dyn. **78**, 2779–2794 (2014)
12. Wu, Y.H., Cao, X.B., Zheng, P.F., Zeng, Z.K.: Variable structure-based decentralized relative attitude-coordinated control for satellite formation. IEEE Aerosp. Electron. Syst. Mag. **27**, 18–25 (2012)
13. Wu, B.L., Wang, D.W., Poh, E.K.: Decentralized sliding-mode control for attitude synchronization in spacecraft formation. Int. J. Robust Nonlinear Control **23**, 1183–1197 (2013)
14. Abdessameud, A., Tayebi, A.: Attitude synchronization of a group of spacecraft without velocity measurements. IEEE Trans. Autom. Control **54**, 2642–2648 (2009)
15. Liu, Y.F., Geng, Z.Y.: Finite-time optimal formation tracking control of vehicles in horizontal plane. Nonlinear Dyn. **76**, 481–495 (2014)
16. Liu, Y.F., Zhao, Y., Chen, G.R.: Finite-time formation tracking control for multiple vehicles: a motion planning approach. Int. J. Robust Nonlinear Control **26**, 3130–3149 (2016)
17. Yang, X.X., Wu, Z.Y., Cao, J.D.: Finite-time synchronization of complex networks with nonidentical discontinuous nodes. Nonlinear Dyn. **73**, 2313–2327 (2013)
18. Lu, K.F., Xia, Y.Q.: Adaptive attitude tracking control for rigid spacecraft with finite-time convergence. Automatica **49**, 3591–3599 (2013)
19. Lu, K.F., Xia, Y.Q.: Finite-time attitude stabilization for rigid spacecraft. Int. J. Robust Nonlinear Control **25**, 32–51 (2015)
20. Lu, K.F., Xia, Y.Q., Fu, M.Y., Yu, C.M.: Adaptive finite-time attitude stabilization for rigid spacecraft with actuator faults and saturation constraints. Int. J. Robust Nonlinear Control **26**, 28–46 (2016)
21. Du, H.B., Li, S.H., Qian, C.J.: Finite-time attitude tracking control of spacecraft with application to attitude synchronization. IEEE Trans. Autom. Control **56**, 2711–2717 (2011)
22. Zhou, J.K., Hu, Q.L., Friswell, M.I.: Decentralized finite time attitude synchronization control of satellite formation flying. J. Guid. Control Dyn. **36**, 185–195 (2013)
23. Meng, Z.Y., Ren, W., You, Z.: Distributed finite-time attitude containment control for multiple rigid bodies. Automatica **46**, 2092–2099 (2010)
24. Ren, W., Beard, R.W., Atkins, E.M.: Information consensus in multivehicle cooperative control. IEEE Control Syst. Mag. **27**, 71–82 (2007)
25. Xia, K.W., Huo, W.: Robust adaptive backstepping neural networks control for spacecraft rendezvous and docking with uncertainties. Nonlinear Dyn. **84**, 1683–1695 (2016)
26. Sun, L., Huo, W.: Adaptive robust control with $L2$-gain performance for autonomous spacecraft proximity maneuvers. J. Spacecr. Rockets **53**, 249–257 (2016)
27. Du, H.B., Li, S.H.: Attitude synchronization control for a group of flexible spacecraft. Automatica **50**, 646–651 (2014)

28. Du, H.B., Li, S.H.: Attitude synchronization for flexible spacecraft with communication delays. IEEE Trans. Autom. Control **61**, 3625–3630 (2016)
29. Du, H.B., Chen, M.Z.Q., Wen, G.H.: Leader-following attitude consensus for spacecraft formation with rigid and flexible spacecraft. J. Guid. Control Dyn. **39**, 941–948 (2016)
30. Zou, A.M., Kumar, K.D.: Robust attitude coordination control for spacecraft formation flying under actuator failures. J. Guid. Control Dyn. **35**, 1247–1255 (2012)
31. Wu, B.L., Wang, D.W., Poh, E.K.: Decentralized sliding-mode control for spacecraft attitude synchronization under actuator failures. Acta Astronaut. **105**, 333–343 (2014)
32. Zhou, N., Xia, Y.Q.: Distributed fault-tolerant control design for spacecraft finite-time attitude synchronization. Int. J. Robust Nonlinear Control **26**, 2994–3017 (2016)
33. Shuster, M.D.: A survey of attitude representations. J. Astronaut. Sci. **41**, 439–517 (1993)
34. Gennaro, S.D.: Output attitude tracking for flexible spacecraft. Automatica **38**, 1719–1726 (2002)
35. Huang, L.: Foundamentals of Stability and Robust Control. Science Press, Beijing (2003)
36. Shen, H., Wu, Z.G., Park, J.H.: Reliable mixed passive and H_∞ filtering for semi-Markov jump systems with randomly occurring uncertainties and sensor failures. Int. J. Robust Nonlinear Control **25**, 3231–3251 (2015)
37. Shen, H., Park, J.H., Wu, Z.G.: Finite-time reliable L_2-L_∞/H_∞ control for Takagi–Sugeno fuzzy systems with actuator faults. IET Control Theory Appl. **8**, 688–696 (2014)