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ORIGINAL PAPER

Analytical soliton solutions of the (2+1)-dimensional sine-Gordon equation

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Abstract In this letter, we investigated a new (2+1)-dimensional sine-Gordon equation. By the subsidiary ordinary differential equation method, some new explicit solutions are given. These solutions include hyperbolic function solutions and trigonometric function solutions amongst others. In particular, a topological 1-soliton solution is derived.

Keywords New (2+1)-dimensional sine-Gordon equation · Sub-ODE method · The ansatz approach · Topological 1-soliton solution · Travelling wave solution

1 Introduction

It is well known that the sine-Gordon equation plays an important roles in many fields, such as nonlinear optic [1], quantum field theory [2], differential geometry [3], plasma physics [4] and relativistic field theory. There are many methods to derive the exact solutions of nonlinear partial differential equations, such as Lie symmetry method [5–15], sub-ODE method [15,16] and the ansatz approach [17–27].

Compared with the (1+1)-dimensional sine-Gordon equation, the (2+1)-dimensional sine-Gordon equation may provide greater significance. A more plausible form is the following (2+1)-dimensional sine-Gordon equation

$$u_{xx} - u_{xy} - u_{xt} + u_{yt} = \sin u, \tag{1}$$

which generates the more situation than the (1+1)-dimensional case. It is clear that this equation includes the classical (1+1)-dimensional sine-Gordon equation. One of the authors derived this equation from extended Lax pair [28]. They considered the kink waves and symmetries. However, they just give a little explicit solutions. In order to give more solutions, we reconsider this equation.

In this letter, we will consider the (2+1)-dimensional sine-Gordon equation using sub-ODE method and ansatz approach. Some new exact solutions are derived.

2 Exact solutions of new (2+1)-dimensional sine-Gordon equation using sub-ODE method

Considering the transformation $v = e^{iu}$, one can get

$$2\left(v_{xx}v - v_x^2 - v_{xt}v + v_x v_t - v_{xy}v + v_x v_y + v_{yt}v - v_y v_t\right) - v^3 + v = 0. \tag{2}$$

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Now, using travelling transformation $\xi = B_1x + B_2y - ct$ and substituting $v(\xi) = f(B_1x + B_2y - ct)$ into Eq. (2), we get the following ODE:

$$Mf''f - Mf'^2 - f^3 + f = 0, \tag{3}$$

where $M = 2B_1^2 + 2B_1v - 2B_1B_2 - 2B_2v$. Next, balancing the highest derivative term and nonlinear term, we assume Eq. (3) has the following solutions

$$f = a_0 + a_1\phi + a_2\phi^2, \tag{4}$$

where ϕ satisfies the following ODE

$$\phi' = A + B\phi + C\phi^2. \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (3) and collecting different terms of ϕ, we get

$$\begin{aligned}
&2MC^2a_2^2 - a_2^3 = 0,\\
&2MBCa_2^2 + 4MC^2a_1a_2 - 3a_1a_2^2 = 0,\\
&5MBCa_1a_2 + 6MC^2a_0a_2 + MC^2a_1^2\\
&\quad -3a_0a_2^2 - 3a_1^2a_2 = 0,\\
&\quad -2AMBa_2^2 + 2AMCa_1a_2 + MB^2a_1a_2 + 10MBCa_0a_2\\
&\quad +MBCa_1^2 + 2MC^2a_0a_1 - 6a_0a_1a_2 - a_1^3 = 0,\\
&\quad -2A^2Ma_2^2 - AMBa_1a_2 + 8AMCa_0a_2 + 4MB^2a_0a_2\\
&\quad +3MBCa_0a_1 - 3a_0^2a_2 - 3a_0a_1^2 + a_2 = 0,\\
&\quad -2A^2Ma_1a_2 + 6AMBa_0a_2 - AMBa_1^2\\
&\quad +2AMCa_0a_1 + MB^2a_0a_1 - 3a_0^2a_1 + a_1 = 0,\\
&2A^2Ma_0a_2 - A^2Ma_1^2 + AMBa_0a_1 - a_0^3 + a_0 = 0.
\end{aligned} \tag{6}$$

Solving these equations by maple, one obtains two cases

Case I:

$$\begin{aligned}
&a_0 = 2ACM - 1, \quad a_1 = 2CBM,\\
&BM = RootOf(-2Ma_0 + Z^2), \quad a_2 = 2MC^2.
\end{aligned} \tag{7}$$

Case II:

$$\begin{aligned}
&a_0 = 2ACM + 1, \quad a_1 = 2CBM,\\
&BM = RootOf(-2Ma_0 + Z^2), \quad a_2 = 2MC^2.
\end{aligned} \tag{8}$$

By virtue of solutions of Eq. (5), one can find many exact travelling wave solutions for (2) as follows.

Family 1: When $B^2 - 4AC > 0$ and $BC \neq 0$ (or $AC \neq 0$),

$$\begin{aligned}
v(x, y, t) = {} & 2ACM \pm 1\\
& - BM\left[B + \sqrt{B^2 - 4AC}\tanh\left(\frac{\sqrt{B^2 - 4AC}}{2}\xi\right)\right]\\
& - MC\left(\left[B + \sqrt{B^2 - 4AC}\tanh\right.\right.\\
& \left.\left.\times\left(\frac{\sqrt{B^2 - 4AC}}{2}\xi\right)\right]\right)^2.
\end{aligned} \tag{9}$$

$$\begin{aligned}
v(x, y, t) = {} & 2ACM \pm 1\\
& - BM\left[B + \sqrt{B^2 - 4AC}\coth\left(\frac{\sqrt{B^2 - 4AC}}{2}\xi\right)\right]\\
& - MC\left(\left[B + \sqrt{B^2 - 4AC}\coth\right.\right.\\
& \left.\left.\times\left(\frac{\sqrt{B^2 - 4AC}}{2}\xi\right)\right]\right)^2.
\end{aligned} \tag{10}$$

$$\begin{aligned}
v(x, y, t) = {} & 2ACM \pm 1\\
& - BM\left[B + \sqrt{B^2 - 4AC}\left(\tanh\left(\sqrt{B^2 - 4AC}\xi\right)\right.\right.\\
& \left.\left.\pm i\,\mathrm{sech}\left(\sqrt{B^2 - 4AC}\xi\right)\right)\right]\\
& - MC\left(\left[B + \sqrt{B^2 - 4AC}\left(\tanh\left(\sqrt{B^2 - 4AC}\xi\right)\right.\right.\right.\\
& \left.\left.\left.\pm i\,\mathrm{sech}\left(\sqrt{B^2 - 4AC}\xi\right)\right)\right]\right)^2.
\end{aligned} \tag{11}$$

$$\begin{aligned}
v(x, y, t) = {} & 2ACM \pm 1\\
& - BM\left[B + \sqrt{B^2 - 4AC}\left(\coth\left(\sqrt{B^2 - 4AC}\xi\right)\right.\right.\\
& \left.\left.\pm i\,\mathrm{csch}\left(\sqrt{B^2 - 4AC}\xi\right)\right)\right]\\
& - MC\left(\left[B + \sqrt{B^2 - 4AC}\left(\coth\left(\sqrt{B^2 - 4AC}\xi\right)\right.\right.\right.\\
& \left.\left.\left.\pm i\,\mathrm{csch}\left(\sqrt{B^2 - 4AC}\xi\right)\right)\right]\right)^2.
\end{aligned} \tag{12}$$

$$\begin{aligned}
v(x, y, t) = {} & 2ACM \pm 1\\
& - \frac{BM}{2}\left[2B + \sqrt{B^2 - 4AC}\left(\tanh\left(\frac{\sqrt{B^2 - 4AC}}{4}\xi\right)\right.\right.\\
& \left.\left.+ \coth\left(\frac{\sqrt{B^2 - 4AC}}{4}\xi\right)\right)\right]\\
& - \frac{MC}{2}\left(\left[2B + \sqrt{B^2 - 4AC}\left(\tanh\left(\frac{\sqrt{B^2 - 4AC}}{4}\xi\right)\right.\right.\right.\\
& \left.\left.\left. + \coth\left(\frac{\sqrt{B^2 - 4AC}}{4}\xi\right)\right)\right]\right)^2.
\end{aligned} \tag{13}$$

$$v(x,y,t) = 2ACM \pm 1 + BM\left[-B + \frac{\sqrt{(E^2+F^2)(B^2-4AC)} - E\sqrt{B^2-4AC}\cosh(\sqrt{B^2-4AC}\xi)}{E\sinh(\sqrt{B^2-4AC}\xi)+F}\right] + MC\left(\left[-B + \frac{\sqrt{(E^2+F^2)(B^2-4AC)} - E\sqrt{B^2-4AC}\cosh(\sqrt{B^2-4AC}\xi)}{E\sinh(\sqrt{B^2-4AC}\xi)+F}\right]\right)^2. \quad (14)$$

$$v(x,y,t) = 2ACM \pm 1 + BM\left[-B - \frac{\sqrt{(F^2-E^2)(B^2-4AC)} + E\sqrt{B^2-4AC}\sinh\left(\sqrt{B^2-4AC}\xi\right)}{E\cosh(\sqrt{B^2-4AC}\xi)+F}\right] + MC\left(\left[-B - \frac{\sqrt{(F^2-E^2)(B^2-4AC)} + E\sqrt{B^2-4AC}\sinh\left(\sqrt{B^2-4AC}\xi\right)}{E\cosh(\sqrt{B^2-4AC}\xi)+F}\right]\right)^2. \quad (15)$$

where E and F are two nonzero real constants and satisfy $F^2 - E^2 > 0$.

$$v(x,y,t) = 2ACM \pm 1 + 2CBM\left[\frac{2A\cosh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}{\sqrt{B^2-4AC}\sinh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right) - B\cosh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}\right] + 2MC^2\left(\left[\frac{2A\cosh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}{\sqrt{B^2-4AC}\sinh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right) - B\cosh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}\right]\right)^2. \quad (16)$$

$$v(x,y,t) = 2ACM \pm 1 + 2CBM\left[\frac{-2A\sinh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}{-\sqrt{B^2-4AC}\cosh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right) + B\sinh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}\right] + 2MC^2\left(\left[\frac{-2A\sinh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}{-\sqrt{B^2-4AC}\cosh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right) + B\sinh\left(\frac{\sqrt{B^2-4AC}}{2}\xi\right)}\right]\right)^2. \quad (17)$$

$$v(x,y,t) = 2ACM \pm 1 + 2CBM\left[\frac{2A\cosh(\sqrt{B^2-4AC}\xi)}{\sqrt{B^2-4AC}\sinh\left(\sqrt{B^2-4AC}\xi\right) - B\cosh(\sqrt{B^2-4AC}\xi) \pm i\sqrt{B^2-4AC}\xi}\right] + 2MC^2\left(\left[\frac{2A\cosh(\sqrt{B^2-4AC}\xi)}{\sqrt{B^2-4AC}\sinh\left(\sqrt{B^2-4AC}\xi\right) - B\cosh(\sqrt{B^2-4AC}\xi) \pm i\sqrt{B^2-4AC}\xi}\right]\right)^2. \quad (18)$$

$$v(x,y,t) = 2ACM \pm 1 + 2CBM\left[\frac{2A\sinh\left(\sqrt{B^2-4AC}\xi\right)}{\sqrt{B^2-4AC}\cosh(\sqrt{B^2-4AC}\xi) - B\sinh\left(\sqrt{B^2-4AC}\xi\right) \pm \sqrt{B^2-4AC}\xi}\right] + 2MC^2\left(\left[\frac{2A\sinh\left(\sqrt{B^2-4AC}\xi\right)}{\sqrt{B^2-4AC}\cosh(\sqrt{B^2-4AC}\xi) - B\sinh\left(\sqrt{B^2-4AC}\xi\right) \pm \sqrt{B^2-4AC}\xi}\right]\right)^2. \quad (19)$$

$$v(x,y,t) = 2ACM \pm 1 + 2CBM\left[\frac{4A\sinh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)\cosh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)}{-2B\sinh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)\cosh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)+2\sqrt{B^2-4AC}\cosh^2\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)-\sqrt{B^2-4AC}}\right] + 2MC^2\left(\left[\frac{4A\sinh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)\cosh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)}{-2B\sinh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)\cosh\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)+2\sqrt{B^2-4AC}\cosh^2\left(\frac{\sqrt{B^2-4AC}}{4}\xi\right)-\sqrt{B^2-4AC}}\right]\right)^2. \tag{20}$$

Family 2: When $B^2 - 4AC < 0$ and $BC \neq 0$ (or $AC \neq 0$),

$$v(x,y,t) = 2ACM \pm 1 + BM\left[-B+\sqrt{4AC-B^2}\tan\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)\right] + MC\left(\left[-B+\sqrt{4AC-B^2}\times\tan\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)\right]\right)^2. \tag{21}$$

$$v(x,y,t) = 2ACM \pm 1 - BM\left[B+\sqrt{4AC-B^2}\cot\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)\right] - MC\left(\left[B+\sqrt{4AC-B^2}\times\cot\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)\right]\right)^2. \tag{22}$$

$$v(x,y,t) = 2ACM \pm 1 + BM\left[-B+\sqrt{4AC-B^2}\left(\tan\left(\sqrt{4AC-B^2}\xi\right)\pm\sec\left(\sqrt{4AC-B^2}\xi\right)\right)\right] + MC\left(\left[-B+\sqrt{4AC-B^2}\left(\tan\left(\sqrt{4AC-B^2}\xi\right)\pm\sec\left(\sqrt{4AC-B^2}\xi\right)\right)\right]\right)^2. \tag{23}$$

$$v(x,y,t) = 2ACM \pm 1 - BM\left[B+\sqrt{4AC-B^2}\left(\cot\left(\sqrt{4AC-B^2}\xi\right)\pm\csc\left(\sqrt{4AC-B^2}\xi\right)\right)\right] - MC\left(\left[B+\sqrt{4AC-B^2}\left(\cot\left(\sqrt{4AC-B^2}\xi\right)\pm\csc\left(\sqrt{4AC-B^2}\xi\right)\right)\right]\right)^2. \tag{24}$$

$$v(x,y,t) = 2ACM \pm 1 - \frac{BM}{2}\left[-2B+\sqrt{4AC-B^2}\left(\tan\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)-\cot\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)\right)\right] - \frac{MC}{2}\left(\left[-2B+\sqrt{4AC-B^2}\times\left(\tan\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)-\cot\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)\right)\right]\right)^2. \tag{25}$$

$$v(x,y,t)=2ACM\pm 1+BM\left[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}-E\sqrt{4AC-B^2}\cos(\sqrt{4AC-B^2}\xi)}{E\sin(\sqrt{4AC-B^2}\xi)+F}\right]+MC\left(\left[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}-E\sqrt{4AC-B^2}\cos(\sqrt{4AC-B^2}\xi)}{E\sin(\sqrt{4AC-B^2}\xi)+F}\right]\right)^2. \quad (26)$$

$$v(x,y,t)=2ACM\pm 1+BM\left[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}+E\sqrt{4AC-B^2}\sinh\left(\sqrt{4AC-B^2}\xi\right)}{E\cos(\sqrt{4AC-B^2}\xi)+F}\right]+MC\left(\left[-B+\frac{\pm\sqrt{(F^2-E^2)(4AC-B^2)}+E\sqrt{4AC-B^2}\sinh\left(\sqrt{4AC-B^2}\xi\right)}{E\cos\left(\sqrt{4AC-B^2}\xi\right)+F}\right]\right)^2. \quad (27)$$

where E and F are two nonzero real constants and satisfy $F^2-E^2>0$.

$$v(x,y,t)=2ACM\pm 1+2CBM\left[\frac{-2A\cos\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}{\sqrt{4AC-B^2}\sin\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)+B\cos\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}\right]+2MC^2\left(\left[\frac{-2A\cos\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}{\sqrt{4AC-B^2}\sin\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)+B\cos\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}\right]\right)^2. \quad (28)$$

$$v(x,y,t)=2ACM\pm 1+2CBM\left[\frac{2A\sin\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}{\sqrt{4AC-B^2}\cos\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)-B\sin\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}\right]+2MC^2\left(\left[\frac{2A\sin\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}{\sqrt{4AC-B^2}\cos\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)-B\sin\left(\frac{\sqrt{4AC-B^2}}{2}\xi\right)}\right]\right)^2. \quad (29)$$

$$v(x,y,t)=2ACM\pm 1+2CBM\left[\frac{-2A\cos(\sqrt{4AC-B^2}\xi)}{\sqrt{4AC-B^2}\sin(\sqrt{4AC-B^2}\xi)+B\cos(\sqrt{4AC-B^2}\xi)\pm i\sqrt{4AC-B^2}\xi}\right]+2MC^2\left(\left[\frac{-2A\cos(\sqrt{4AC-B^2}\xi)}{\sqrt{4AC-B^2}\sin(\sqrt{4AC-B^2}\xi)+B\cos(\sqrt{4AC-B^2}\xi)\pm i\sqrt{4AC-B^2}\xi}\right]\right)^2. \quad (30)$$

$$v(x,y,t)=2ACM\pm 1+2CBM\left[\frac{2A\sin(\sqrt{4AC-B^2}\xi)}{\sqrt{4AC-B^2}\cos(\sqrt{4AC-B^2}\xi)-B\sin(\sqrt{4AC-B^2}\xi)\pm\sqrt{4AC-B^2}\xi}\right]+2MC^2\left(\left[\frac{2A\sin(\sqrt{4AC-B^2}\xi)}{\sqrt{4AC-B^2}\cos(\sqrt{4AC-B^2}\xi)-B\sin(\sqrt{4AC-B^2}\xi)\pm\sqrt{4AC-B^2}\xi}\right]\right)^2. \quad (31)$$

$$
\begin{aligned}
&v(x, y, t)\\
&\quad = 2ACM \pm 1\\
&\qquad + 2CBM\left[\frac{4A\sin\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)}{-2B\sin\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right) + 2\sqrt{4AC-B^2}\cos^2\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right) - \sqrt{4AC-B^2}}\right]\\
&\qquad + 2MC^2\left(\left[\frac{4A\sin\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)}{-2B\sin\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right) + 2\sqrt{4AC-B^2}\cos^2\left(\frac{\sqrt{4AC-B^2}}{4}\xi\right) - \sqrt{4AC-B^2}}\right]\right)^2.
\end{aligned}
\tag{32}
$$

Family 3: When $A = 0$ and $BC \neq 0$,

$$
\begin{aligned}
v(x, y, t) &= 2ACM \pm 1\\
&\quad + 2BM\left(\frac{-Bd}{(d + \cosh(B\xi) - \sinh(B\xi))}\right)\\
&\quad + 2MC\left(\frac{-Bd}{(d + \cosh(B\xi) - \sinh(B\xi))}\right)^2,\\
v(x, y, t) &= 2ACM \pm 1\\
&\quad + 2BM\left(-\frac{\cosh(B\xi) + \sinh(B\xi)}{(d + \cosh(B\xi) + \sinh(B\xi))}\right)\\
&\quad + 2MC\left(-\frac{\cosh(B\xi) + \sinh(B\xi)}{(d + \cosh(B\xi) + \sinh(B\xi))}\right)^2,
\end{aligned}
\tag{33}
$$

where d is an arbitrary constant.

Family 4: When $A = B = 0$ and $C \neq 0$,

$$
\begin{aligned}
v(x, y, t) &= 2ACM \pm 1 + 2CBM\left(\frac{-1}{B\xi + k}\right)\\
&\quad + 2MC^2\left(\frac{-1}{B\xi + k}\right)^2,
\end{aligned}
\tag{34}
$$

where k is an arbitrary constant. Hence, one can get new travelling wave solutions of new (2+1)-dimensional sine-Gordon equation via the following equation

$$
u = \arccos\frac{v + v^{-1}}{2}. \tag{35}
$$

3 Topological 1-soliton solution via the ansatz approach

In this section, we will study the topological solitons with the ansatz approach. The starting hypothesis is given in [17–22]

$$
v(x, y, t) = A\tanh^p \tau, \tag{36}
$$

where

$$
\tau = B_1 x + B_2 y - ct, \tag{37}
$$

and A, B_1 and B_2 are free parameters; meanwhile, c is the speed of the soliton solution. At the same time, the unknown exponent p will be fixed. Therefore, we have

$$
v_t = pcA\left(\tanh^{p+1}\tau - \tanh^{p-1}\tau\right), \tag{38}
$$

$$
v_x = pAB_1\left(\tanh^{p-1}\tau - \tanh^{p+1}\tau\right), \tag{39}
$$

$$
v_y = pAB_2\left(\tanh^{p-1}\tau - \tanh^{p+1}\tau\right), \tag{40}
$$

$$
\begin{aligned}
v_{xx} &= p(p-1)AB_1^2\left(\tanh^{p-2}\tau\right)\\
&\quad -2p^2AB_1^2\left(\tanh^p\tau\right)\\
&\quad + p(p+1)AB_1^2\left(\tanh^{p+2}\tau\right),
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
v_{yy} &= p(p-1)AB_2^2\left(\tanh^{p-2}\tau\right)\\
&\quad -2p^2AB_2^2\left(\tanh^p\tau\right)\\
&\quad + p(p+1)AB_2^2\left(\tanh^{p+2}\tau\right),
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
v_{tx} &= -p(p-1)AB_1c\left(\tanh^{p-2}\tau\right)\\
&\quad +2p^2AB_1c\left(\tanh^p\tau\right)\\
&\quad - p(p+1)AB_1c\left(\tanh^{p+2}\tau\right),
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
v_{ty} &= -p(p-1)AB_2c\left(\tanh^{p-2}\tau\right)\\
&\quad +2p^2AB_2c\left(\tanh^p\tau\right)\\
&\quad - p(p+1)AB_2c\left(\tanh^{p+2}\tau\right),
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
v_{xy} &= p(p-1)AB_2B_1\left(\tanh^{p-2}\tau\right)\\
&\quad -2p^2AB_2B_1\left(\tanh^p\tau\right)\\
&\quad + p(p+1)AB_2B_1\left(\tanh^{p+2}\tau\right),
\end{aligned}
\tag{45}
$$

Substituting (38)–(45) into (2) produces

$$\begin{aligned}&2pA^2B_1^2\left[(p-1)\tanh^{2p-2}\tau-2p\tanh^{2p}\tau\right.\\&\left.+(p+1)\tanh^{2p+2}\tau\right]\\&-2p^2A^2B_1^2\left[\tanh^{2p-2}\tau-2\tanh^{2p}\tau+\tanh^{2p+2}\tau\right]\\&+2pcA^2B_1\left[(p-1)\tanh^{2p-2}\tau-2p\tanh^{2p}\tau\right.\\&\left.+(p+1)\tanh^{2p+2}\tau\right]\\&-2p^2A^2B_1c\left[\tanh^{2p-2}\tau-2\tanh^{2p}\tau+\tanh^{2p+2}\tau\right]\\&-2pA^2B_1B_2\left[(p-1)\tanh^{2p-2}\tau-2p\tanh^{2p}\tau\right.\\&\left.+(p+1)\tanh^{2p+2}\tau\right]\\&+2p^2A^2B_1B_2\left[\tanh^{2p-2}\tau-2\tanh^{2p}\tau+\tanh^{2p+2}\tau\right]\\&-2pcA^2B_2\left[(p-1)\tanh^{2p-2}\tau-2p\tanh^{2p}\tau\right.\\&\left.+(p+1)\tanh^{2p+2}\tau\right]\\&+2p^2A^2B_2c\left[\tanh^{2p-2}\tau-2\tanh^{2p}\tau+\tanh^{2p+2}\tau\right]\\&-A^3\tanh^{3p}\tau+A\tanh^{p}\tau=0.\end{aligned}\tag{46}$$

Now, considering balancing principle from (46), one can get

$$3p=2p+2,\tag{47}$$

which leads to

$$p=2.\tag{48}$$

We can get the same results from

$$p=2p-2.\tag{49}$$

Consider the coefficients of the linearly independent functions should be equal to zero, which generates

$$A=4B_1^2+4B_1c-4B_1B_2-4B_2c,\tag{50}$$

and

$$c=\frac{4A^2B_1^2-4A^2B_1B_2-A}{4A^2B_2-4A^2B_1}.\tag{51}$$

This result implies

$$B_1-B_2\neq 0.\tag{52}$$

Thus, we get

$$v(x,y,t)=A\tanh^2(B_1x+B_2y-ct),\tag{53}$$

where c and A are determined by Eq. (50) and Eq. (51). At the end, the topological 1-soliton solution of the new (2+1)-dimensional sine-Gordon equation is as follows:

$$u=\arccos\times\frac{A\tanh^2(B_1x+B_2y-ct)+A\tanh^{-2}(B_1x+B_2y-ct)}{2}.\tag{54}$$

4 Conclusion

In the present paper, we have obtained new explicit solutions of new (2+1)-dimensional sine-Gordon equation. Based on the sub-ODE method, some new explicit solutions are investigated in detail. These solutions contain hyperbolic function solutions, trigonometric function solutions and so on. Also, topological 1-soliton solution is constructed via the ansatz approach. These results are important for studying more complied physical phenomena. In future, this equation will be investigated along with its variable terms with time.

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