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ORIGINAL PAPER

Exact and soliton solutions to nonlinear transmission line model

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Abstract A nonlinear transmission line (NLTL) is comprised of a transmission line periodically loaded with varactors, where the capacitance nonlinearity arises from the variable depletion layer width, which depends both on the DC and AC voltages of the propagating wave. An equivalent circuit model of NLTL is discussed analytically, in this article, and different type of solutions are celebrated. The improved extended tanh-function method has been applied successfully to extract the solutions. The obtained solutions are solitary wave solutions, singular periodic solutions, singular soliton solutions, Jacobi elliptic doubly periodic type solutions and Weierstrass elliptic doubly periodic type solutions. It is a very convenient tool to study the propagation of electrical solitons which propagate in the form of voltage waves in nonlinear dispersive media.

Keywords Exact solutions · Solitons · Nonlinear transmission line

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1 Introduction

Nonlinear phenomena can be observed in many areas such as physics, chemistry, biology, ocean engineering, and communication engineering. In physics precisely, nonlinearity is present in fluid dynamics, nonlinear optics, plasma physics, communication technology and so on [1–16]. In order to understand the mechanisms of those physical phenomena which can be described by nonlinear evolution equations (NLEEs), it is necessary to explore their solutions and properties. At the present time, there are many powerful methods for seeking the exact and approximated solutions of these NLEEs, such as inverse scattering method [17,18], Hirota bilinear transformation [19], the modified simple equation method [20,21], the (G'/G)—expansion method [22], the trial equation method [23] and many more.

In communication engineering, a transmission line is a specialized medium or other structure designed to carry alternating current of radio frequency, that is, currents with a frequency high enough that their wave nature must be taken into account. Transmission lines are used for purposes such as connecting radio transmitters and receivers with their antennas, distributing cable television signals, trunklines routing calls between telephone switching centers, computer network connections and high speed computer data buses.

In this paper, we apply the improved extended tanh-function method to seek the soliton wave solutions in a NLTL. The NLTLs are very convenient tools to study

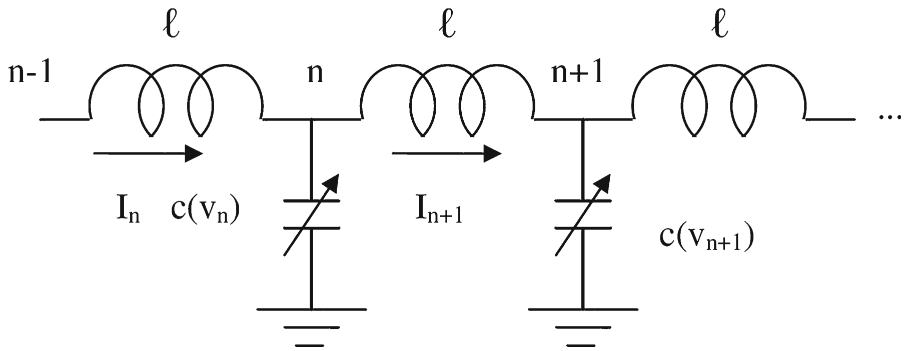

Fig. 1 A nonlinear transmission line

the propagation of electrical solitons which can propagate in the form of voltage waves in nonlinear dispersive media. The NLTL model used in this work is shown in Fig. 1 using inductors ℓ, and voltage dependent (and hence nonlinear) capacitors, $c(V)$. By applying Kirchhoff current law at node n, whose voltage with respect to ground is V_n, and applying Kirchhoff voltage law across the two inductors connected to this node, the voltages of adjacent nodes on this NLTL are related via:

$$\ell \frac{d}{\mathrm{d}t}\left[c(V_n)\frac{\mathrm{d}V_n}{\mathrm{d}t}\right] = (V_{n+1} + V_{n-1} - 2V_n). \tag{1.1}$$

The right-hand side of (1.1) can be approximated with partial derivatives with respect to distance x, from the beginning of the line, assuming that the spacing between two adjacent sections is δ (i.e., $x_n = n\delta$). An approximate continuous partial differential equation can be obtained by using the Taylor expansions of $V(x-\delta)$, $V(x)$, and $V(x+\delta)$ to evaluate the right-hand side of (1.1). Assuming a small δ, and ignoring the high order terms, we obtain:

$$L\frac{\partial}{\partial t}\left[C(V)\frac{\partial V}{\partial t}\right] = \frac{\partial^2 V}{\partial x^2} + \frac{\delta^2}{12}\frac{\partial^4 V}{\partial x^4}, \tag{1.2}$$

where C and L are the capacitance and inductance per unit length, respectively. For more detail see also [10].

2 Description of the method

In this section, we outline the main steps of the improved extended tanh-equation method as following:

Suppose that we have a nonlinear evolution equation in the form:

$$F(u, u_t, u_x, u_{xx}, u_{xt}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its various partial derivatives u_t, u_x with respect to t, x, respectively, in which the highest order derivatives and nonlinear terms are involved.

Step 1. Using the traveling wave transformation

$$u(x,t) = U(\xi), \quad \xi = k(x - vt), \tag{2.2}$$

where k, c are constant to be determined later. Then, Eq. (2.1) is reduced to a nonlinear ordinary differential equation of the form

$$P\left(U, -kvU', kU', k^2U'', \ldots\right) = 0, \tag{2.3}$$

Step 2. We assume that the solution of Eq. (2.3) can be expressed in the form

$$U(\xi) = \sum_{i=0}^{N} \alpha_i \omega^i + \sum_{i=1}^{N} \beta_i \omega^{-i}, \tag{2.4}$$

where ω satisfies

$$\omega' = \varepsilon\sqrt{a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4}, \tag{2.5}$$

where $\varepsilon = \pm 1$. This equation gives various kinds of fundamental solutions [15]. From these solutions, more new exact solutions for (2.1) can be obtained.

Step 3. Determine the positive integer number N in Eq. (2.4) by balancing the highest order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. Substitute (2.4) into (2.3) along with (2.5). As a result of this substitution, we get a polynomial of ω. In this polynomial, we gather all terms of same powers and equating them to be zero, we get an overdetermined system of algebraic equations which can be solved by the maple or mathematica to get the unknown parameters k, v, α_0, α_i and $\beta_i (i = 1, 2, \ldots)$. Consequently, we obtain the exact solutions of (2.1).

3 Exact and soliton solutions

In this section, the improved extended tanh-function method is applied to our NLTL model. To this end, we approximate the capacitor's voltage dependence using the following first-order linear relationship

$$C(V) = C_0(1 - bV), \tag{3.1}$$

where C_0 and b are arbitrary constants. In this case, Eq. (1.2) reduces to

$$\frac{\partial^2 V}{\partial t^2} - \frac{b}{2}\frac{\partial^2 V^2}{\partial t^2} = \frac{1}{LC_0}\frac{\partial^2 V}{\partial x^2} + \frac{\delta^2}{12LC_0}\frac{\partial^4 V}{\partial x^4}. \tag{3.2}$$

Introduce the voltage in the form of the traveling wave

$$V(x,t) = V(\xi), \quad \xi = x - vt \tag{3.3}$$

where v represent the velocity of propagation. Then, Eq. (3.2) reduces to the following ODE:

$$\frac{\delta^2}{12LC_0}V'''' + \left(\frac{1}{LC_0} - v^2\right)V'' + \frac{bv^2}{2}\left(V^2\right)'' = 0. \tag{3.4}$$

Integrating Eq. (3.4) twice with zero constants of integration, we obtain

$$\frac{\delta^2 v_0^2}{12}V'' + \left(v_0^2 - v^2\right)V + \frac{bv^2}{2}V^2 = 0, \tag{3.5}$$

where $v_0 = \frac{1}{\sqrt{LC_0}}$.

Balancing V'' with V^2 in Eq. (3.5), then we get $N = 2$. Then, the solution of Eq. (3.5) has the form

$$V(\xi) = \alpha_0 + \alpha_1\omega + \alpha_2\omega^2 + \beta_1\omega^{-1} + \beta_2\omega^{-2}. \tag{3.6}$$

Substituting $V(\xi)$ and its derivatives with (2.5) into (3.5) and equating all the coefficients of ω^j, $j \in [-4, 4]$ to be zero, then we obtain a system of algebraic equations. Solving this system via mathematica and consider the various kinds of fundamental solutions [15], we obtain the following cases which leads to different types of wave propagation of our model (Eq. 3.2)

Case 1: $a_0 = a_1 = a_3 = 0$. We have the following results

$$\alpha_0 = \alpha_1 = \beta_1 = \beta_2 = 0,$$
$$a_2 = \frac{3\left(v^2 - v_0^2\right)}{\delta^2 v_0^2}, \quad \alpha_2 = -\frac{a_4\delta^2 v_0^2}{v^2 b}. \tag{3.7}$$

and

$$\alpha_0 = \frac{2\left(v^2 - v_0^2\right)}{v^2 b}, \quad \alpha_1 = \beta_1 = \beta_2 = 0,$$
$$a_2 = -\frac{3\left(v^2 - v_0^2\right)}{\delta^2 v_0^2}, \quad \alpha_2 = -\frac{a_4\delta^2 v_0^2}{v^2 b}. \tag{3.8}$$

We obtain, solitary wave solutions, singular periodic solutions and rational solution

$$V(\xi) = \frac{3\left(v^2 - v_0^2\right)}{v^2 b}\,\mathrm{sec\,h}^2\left[\frac{\sqrt{3\left(v^2 - v_0^2\right)}}{\delta v_0}(x - vt)\right],$$
$$v^2 > v_0^2, \tag{3.9}$$

$$V(\xi) = \frac{3\left(v^2 - v_0^2\right)}{v^2 b}\sec^2\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}(x - vt)\right],$$
$$v^2 < v_0^2, \tag{3.10}$$

$$V(\xi) = \frac{-\delta^2}{b}\frac{1}{(x - vt)^2},$$
$$v^2 = v_0^2 \tag{3.11}$$

and

$$V(\xi) = \frac{v^2 - v_0^2}{v^2 b}\left(2 - 3\,\mathrm{sec\,h}^2\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}(x - vt)\right]\right),$$
$$v^2 < v_0^2, \tag{3.12}$$

$$V(\xi) = \frac{v^2 - v_0^2}{v^2 b}\left(2 - 3\sec^2\left[\frac{\sqrt{3\left(v^2 - v_0^2\right)}}{\delta v_0}(x - vt)\right]\right),$$
$$v^2 > v_0^2, \tag{3.13}$$

$$V(\xi) = \frac{3\left(v^2 - v_0^2\right)}{v^2 b} - \frac{\delta^2 v_0^2}{v^2 b}\frac{1}{(x - vt)^2},$$
$$v^2 = v_0^2. \tag{3.14}$$

Case 2:

(i) $a_1 = a_3 = 0$, $a_0 = \frac{a_2^2}{4a_4}$. We have

$$\alpha_1 = \alpha_2 = \beta_1 = 0, a_0 = \frac{a_2{}^2}{4a_4}, \quad \alpha_0 = \frac{3\left(v^2 - v_0^2\right)}{v^2 b},$$
$$\beta_2 = -\frac{9\left(v^2 - v_0^2\right)^2}{v^2\delta^2 a_4 b v_0^2}, \quad a_2 = \frac{6\left(-v^2 + v_0^2\right)}{\delta^2 v_0^2}. \tag{3.15}$$

We obtain, singular soliton solution and singular periodic solution

$$V(\xi)=\frac{3\left(v_0^2-v^2\right)}{v^2 b}\operatorname{csch}^2\left[\frac{\sqrt{3\left(v^2-v_0^2\right)}}{\delta v_0}(x-vt)\right],$$
$$v^2>v_0^2, \tag{3.16}$$

$$V(\xi)=\frac{3\left(v^2-v_0^2\right)}{v^2 b}\csc^2\left[\frac{\sqrt{3\left(v_0^2-v^2\right)}}{\delta v_0}(x-vt)\right],$$
$$v^2<v_0^2. \tag{3.17}$$

(ii) $a_1=a_3=0, a_0=\frac{a_2{}^2m^2(1-m^2)}{a_4(2m^2-1)^2}$. We have

$$\alpha_1=\alpha_2=\beta_1=0,\quad \alpha_0=\frac{3\left(v^2-v_0^2\right)-a_2\delta^2v_0^2}{3v^2b},$$
$$\beta_2=\frac{9m^2\left(-1+m^2\right)\left(v^2-v_0^2\right)^2}{\left(1-7m^2+7m^4\right)v^2\delta^2a_4bv_0^2},$$
$$a_2=-\frac{3\left(1-2m^2\right)\left(v^2-v_0^2\right)}{\sqrt{\left(1-7m^2+7m^4\right)}\delta^2v_0^2}, \tag{3.18}$$

and

$$\alpha_0=\frac{3\left(v^2-v_0^2\right)-a_2\delta^2v_0^2}{3v^2b},\quad \alpha_1=\beta_1=\beta_2=0,$$
$$\alpha_2=-\frac{\delta^2a_4v_0^2}{v^2b},$$
$$a_2=-\frac{3\left(1-2m^2\right)\left(v^2-v_0^2\right)}{\sqrt{\left(1-7m^2+7m^4\right)}\delta^2v_0^2}. \tag{3.19}$$

We obtain, Jacobi elliptic doubly periodic type solutions

$$V(\xi)=\frac{\left(v^2-v_0^2\right)}{v^2b}\left(1+\frac{1-2m^2}{\sqrt{1-7m^2+7m^4}}+\frac{3(1-m^2)}{\sqrt{1-7m^2+7m^4}}\operatorname{nc}^2\times\left[\xi\sqrt{\frac{3\left(v^2-v_0^2\right)}{\sqrt{1-7m^2+7m^4\delta^2v_0^2}}}\right]\right) \tag{3.20}$$

and

$$V(\xi)=\frac{\left(v^2-v_0^2\right)}{v^2b}\left(1+\frac{1-2m^2}{\sqrt{1-7m^2+7m^4}}+\frac{3m^2}{\sqrt{1-7m^2+7m^4}}\operatorname{cn}^2\times\left[\xi\sqrt{\frac{3\left(v^2-v_0^2\right)}{\sqrt{1-7m^2+7m^4\delta^2v_0^2}}}\right]\right), \tag{3.21}$$

(iii) $a_1=a_3=0, a_0=\frac{a_2{}^2(1-m^2)}{a_4(2-m^2)^2}$. We have

$$\alpha_1=\beta_1=\beta_2=0,\quad \alpha_0=\frac{3\left(v^2-v_0^2\right)-a_2\delta^2v_0^2}{3v^2b},$$
$$\alpha_2=-\frac{\delta^2a_4v_0^2}{v^2b},$$
$$\alpha_2=\frac{3\left(-2+m^2\right)\left(v^2-v_0^2\right)}{\sqrt{1-m^2+m^4}\delta^2v_0^2} \tag{3.22}$$

and

$$\alpha_0=\frac{3\left(v^2-v_0^2\right)-a_2\delta^2v_0^2}{3v^2b},\quad \alpha_1=\alpha_2=\beta_1=0,$$
$$\beta_2=\frac{9\left(-1+m^2\right)\left(v^2-v_0^2\right)^2}{\left(1-m^2+m^4\right)v^2\delta^2a_4bv_0^2},$$
$$a_2=\frac{3\left(-2+m^2\right)\left(v^2-v_0^2\right)}{\sqrt{1-m^2+m^4}\delta^2v_0^2}. \tag{3.23}$$

We obtain

$$V(\xi)=2-m^2\left(1+\frac{m^2}{2-m^2}\delta^2\operatorname{dn}^2\times\left[\xi\sqrt{\frac{3\left(v_0^2-v^2\right)}{\sqrt{1-m^2+m^4\delta^2v_0^2}}}\right]v_0^2-\frac{\left(-2+m^2\right)}{\sqrt{1-m^2+m^4}}\right) \tag{3.24}$$

and

$$V(\xi)=\frac{\left(v^2-v_0^2\right)}{v^2b}\left(1+\frac{2-m^2}{\sqrt{1-m^2+m^4}}+\frac{9\left(m^2-2\right)\left(m^2-1\right)\left(v^2-v_0^2\right)}{m^2\left(1-m^2+m^4\right)\delta^2v_0^2}\operatorname{nd}^2\times\left[\sqrt{3}\xi\sqrt{\frac{-v^2+v_0^2}{\sqrt{1-m^2+m^4\delta^2v_0^2}}}\right]\right). \tag{3.25}$$

(iv) $a_1 = a_3 = 0, a_0 = \frac{a_2{}^2 m^2}{a_4(m^2+1)^2}$. We have

$$\alpha_1 = \beta_1 = \beta_2 = 0, \quad \alpha_0 = \frac{(v^2 - v_0^2)}{v^2 b_1} - \frac{\delta^2 v_0^2 a_2}{3v^2 b},$$
$$\alpha_2 = -\frac{\delta^2 a_4 v_0^2}{v^2 b},$$
$$a_2 = \frac{3(1+m^2)(v^2 - v_0^2)}{\sqrt{1-m^2+m^4}\delta^2 v_0^2} \tag{3.26}$$

and

$$\alpha_0 = \frac{(v^2 - v_0^2)}{v^2 b} - \frac{\delta^2 v_0^2 a_2}{3v^2 b}, \quad \alpha_1 = 0, \quad \alpha_2 = 0, \quad \beta_1 = 0,$$
$$\beta_2 = -\frac{9m^2(v^2 - v_0^2)^2}{(1-m^2+m^4)v^2\delta^2 a_4 b v_0^2},$$
$$a_2 = \frac{3(v^2 - v_0^2)(1+m^2)}{\sqrt{1-m^2+m^4}\delta^2 v_0^2}. \tag{3.27}$$

We obtain

$$V(\xi) = \frac{(v^2 - v_0^2)}{v^2 b}\left(1 - \frac{1+m^2}{\sqrt{1-m^2+m^4}} + \frac{3m^2}{\sqrt{1-m^2+m^4}}\mathrm{sn}^2 \times \left[\xi\sqrt{\frac{3(v_0^2 - v^2)}{\sqrt{1-m^2+m^4}\delta^2 v_0^2}}\right]\right) \tag{3.28}$$

and

$$V(\xi) = \frac{(v^2 - v_0^2)}{v^2 b}\left(1 - \frac{1+m^2}{\sqrt{1-m^2+m^4}} + \frac{3}{\sqrt{1-m^2+m^4}}\mathrm{ns}^2 \times \left[\xi\sqrt{\frac{3(v_0^2 - v^2)}{\sqrt{1-m^2+m^4}\delta^2 v_0^2}}\right]\right). \tag{3.29}$$

Case 3: $a_2 = a_4 = 0, a_0, a_1 \neq 0$. We have

$$\alpha_0 = \frac{(v^2 - v_0^2)(7+\sqrt{21})}{7v^2 b}, \quad \alpha_1 = \alpha_2 = 0,$$
$$\beta_2 = -\sqrt{\frac{7}{48}}\frac{v^2 b\beta_1^2}{(v^2 - v_0^2)},$$
$$a_0 = \sqrt{\frac{7}{48}}\frac{v^4 b^2 \beta_1^2}{\delta^2 v_0^2 (v^2 - v_0^2)}, \quad a_1 = -\frac{2v^2 b\beta_1}{\delta^2 v_0^2},$$
$$a_3 = \frac{48(v^2 - v_0^2)^2}{7v^2\delta^2 b v_0^2 \beta_1}. \tag{3.30}$$

We obtain Weierstrass elliptic doubly periodic type solution

$$V(\xi) = \frac{(7+\sqrt{21})(v^2 - v_0^2)}{7v^2 b} + \beta_1\left(\wp\left(\frac{\sqrt{a_3}}{2}\xi; g_2, g_3\right)\right)^{-1} - \frac{\sqrt{\frac{7}{3}}v^2 b\beta_1^2}{4(v^2 - v_0^2)}\left(\wp\left(\frac{\sqrt{a_3}}{2}\xi; g_2, g_3\right)\right)^{-2}, \tag{3.31}$$

where

$$g_2 = \frac{7v^4 b^2 \beta_1^2}{6(v^2 - v_0^2)^2}, \quad g_3 = -\frac{7\sqrt{\frac{7}{3}}v^6 b^3 \beta_1^3}{48(v^2 - v_0^2)^3}. \tag{3.32}$$

Case 4: $a_3 = a_4 = 0, a_0 = \frac{a_1{}^2}{4a_2}$. We have

$$\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = 0, \beta_1 = -\frac{\delta^2 a_1 v_0^2}{2v^2 b},$$
$$\beta_2 = \frac{\delta^4 a_1^2 v_0^4}{48v^2 b(-v^2 + v_0^2)},$$
$$a_2 = -\frac{12(-v^2 + v_0^2)}{\delta^2 v_0^2} \tag{3.33}$$

and

$$\alpha_0 = \frac{2(v^2 - v_0^2)}{v^2 b}, \quad \alpha_1 = 0, \quad \alpha_2 = 0, \quad \beta_1 = -\frac{\delta^2 a_1 v_0^2}{2v^2 b},$$
$$\beta_2 = -\frac{\delta^4 a_1^2 v_0^4}{48v^2 b(-v^2 + v_0^2)}, \quad a_2 = \frac{12(-v^2 + v_0^2)}{\delta^2 v_0^2}. \tag{3.34}$$

We obtain two exponential type solutions

$$V(\xi) = -\frac{288 e^{\left(\frac{2\varepsilon\sqrt{3(v^2 - v_0^2)}}{v_0\delta}\right)\xi}\delta^2 v_0^2 a_1 (v^2 - v_0^2)^2}{v^2 b\left(24 e^{\left(\frac{2\varepsilon\sqrt{3(v^2 - v_0^2)}}{v_0\delta}\right)\xi}(v_0^2 - v^2) + \delta^2 v_0^2 a_1\right)^2} \tag{3.35}$$

and

$$V(\xi) = \frac{96\left(v^2 - v_0^2\right)}{48v^2 b} + \frac{\delta^4 a_1^2 v_0^4}{48v^2 b\left(v^2 - v_0^2\right)\left(e^{\left(\frac{2\varepsilon\sqrt{3\left(v_0^2 - v^2\right)}}{v_0 \delta}\right)\xi} + \frac{\delta^2 v_0^2 a_1}{24\left(v^2 - v_0^2\right)}\right)^2} - \frac{\delta^2 v_0^2 a_1}{2v^2 b\left(e^{\left(\frac{2\varepsilon\sqrt{3\left(v_0^2 - v^2\right)}}{v_0 \delta}\right)\xi} + \frac{\delta^2 v_0^2 a_1}{24\left(v^2 - v_0^2\right)}\right)}. \tag{3.36}$$

Case 5: (i) $a_0 = a_1 = 0$. We have

$$\alpha_0 = \frac{2\left(v^2 - v_0^2\right)}{v^2 b}, \quad \alpha_1 = 0, \ \beta_1 = 0, \quad \beta_2 = 0,$$
$$a_2 = \frac{3\left(1 - \frac{v^2}{v_0^2}\right)}{\delta^2}, \quad a_3 = 0, a_4 = -\frac{v^2 b \alpha_2}{\delta^2 v_0^2} \tag{3.37}$$

and

$$\alpha_0 = \frac{2\left(v^2 - v_0^2\right)}{v^2 b}, \quad \alpha_2 = \frac{v^2 b \alpha_1^2}{12\left(v^2 - v_0^2\right)},$$
$$\beta_1 = 0, \quad \beta_2 = 0,$$
$$a_2 = -\frac{12\left(v^2 - v_0^2\right)}{\delta^2 v_0^2},$$
$$\alpha_1 = -\frac{\delta^2 v_0^2 a_3}{2v^2 b}, \quad a_4 = -\frac{v^4 b^2 \alpha_1^2}{12\delta^2 v_0^2\left(v^2 - v_0^2\right)}. \tag{3.38}$$

We obtain singular periodic and soliton type solutions

$$V(\xi) = \frac{v^2 - v_0^2}{v^2 b}\left(2 - 3\csc^2\left[\frac{\sqrt{3\left(v^2 - v_0^2\right)}}{\delta v_0}(x - vt)\right]\right),$$
$$v^2 > v_0^2, \tag{3.39}$$
$$V(\xi) = \frac{v^2 - v_0^2}{v^2 b}\left(2 + 3\operatorname{csch}^2\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}(x - vt)\right]\right),$$
$$v_0^2 > v^2 \tag{3.40}$$

and

$$V(\xi) = \frac{\left(v^2 - v_0^2\right)}{v^2 b}\left(2 + \frac{3\operatorname{sech}\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}\xi\right]^4}{\left(1 - \varepsilon\tanh\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}\xi\right]\right)^2} - \frac{6\operatorname{sech}\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}\xi\right]^2}{1 - \varepsilon\tanh\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{\delta v_0}\xi\right]}\right) \tag{3.41}$$

(ii) $a_0 = a_1 = 0$, $a_3 = 2\varepsilon\sqrt{a_2 a_4}$. We have

$$\alpha_0 = \frac{2\left(v^2 - v_0^2\right)}{v^2 b}, \quad \alpha_1 = -\frac{\delta^2 {v_0}^2 a_3}{2v^2 b},$$
$$\alpha_2 = -\frac{\delta^2 a_4 v_0^2}{v^2 b}, \quad \beta_1 = 0, \beta_2 = 0,$$
$$a_2 = -\frac{12\left(v^2 - v_0^2\right)}{\delta^2 v_0^2}, \quad a_3 = -\frac{4\sqrt{3a_4\left(v_0^2 - v^2\right)}}{\delta v_0} \tag{3.42}$$

and

$$\alpha_0 = 0, \quad \alpha_1 = -\frac{\delta^2 {v_0}^2 a_3}{2v^2 b}, \quad \alpha_2 = -\frac{\delta^2 a_4 v_0^2}{v^2 b},$$
$$\beta_1 = 0, \quad \beta_2 = 0, \quad a_2 = \frac{12\left(v^2 - v_0^2\right)}{\delta^2 v_0^2},$$
$$a_3 = -\frac{4\sqrt{3a_4\left(v^2 - v_0^2\right)}}{\delta v_0}. \tag{3.43}$$

We obtain

$$V(\xi) = \frac{\left(v^2 - v_0^2\right)}{v^2 b}\left(2 - 6\varepsilon\left(1 + \tanh\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{v_0 \delta}\xi\right]\right) + 3\left(1 + \tanh\left[\frac{\sqrt{3\left(v_0^2 - v^2\right)}}{v_0 \delta}\xi\right]\right)^2\right) \tag{3.44}$$

and

$$V(\xi) = \frac{3\varepsilon\left(v^2 - v_0^2\right)}{v^2 b}\left(1 + \tanh\left[\frac{\sqrt{3\left(v^2 - v_0^2\right)}}{v_0 \delta}\xi\right]\right) \times\left(2 - \varepsilon\left(1 + \tanh\left[\frac{\sqrt{3\left(v^2 - v_0^2\right)}}{v_0 \delta}\xi\right]\right)\right). \tag{3.45}$$

4 Conclusions

In communication engineering, a transmission line is a specialized medium to carry the signals in wave form. It is a very convenient tool to study the propagation of electrical solitons which propagate in the form of voltage waves in nonlinear dispersive media. So, it is important to study the NLTL model analytically and discuss the type of solutions. Thus, the improved extended tanh-function method has been applied successfully to construct the solitary wave solutions, singular periodic solutions, singular soliton solutions, Jacobi elliptic doubly periodic type solutions and Weierstrass elliptic doubly periodic type solutions.

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