

Vibration control of a flexible marine riser with joint angle constraint

Shuang Zhang · Xiuyu He · Chuan Yang

Received: 2 November 2015 / Accepted: 12 August 2016 / Published online: 13 October 2016
© Springer Science+Business Media Dordrecht 2016

Abstract In this paper, the constrained problem of the joint angles for a flexible marine riser is investigated. Boundary control based on the integral-barrier Lyapunov function is achieved by three actuators equipped at the top boundary of the riser. Under the time-varying disturbances, the designed control can suppress the vibration of the riser and ensure the joint angles in the constrained ranges. The stability is proved under the designed control laws. Numerical simulations are given to illustrate the effectiveness of the designed control laws.

This work was supported by the National Natural Science Foundation of China under Grant 61403063, the National Basic Research Program of China (973 Program) under Grant 2014CB744206 and the Fundamental Research Funds for the China Central Universities of UESTC under Grant ZYGX2015J120.

S. Zhang (✉)
School of Aeronautics and Astronautic, University of Electronic Science and Technology of China,
Chengdu 611731, China
e-mail: zhangshuang.ac@gmail.com

X. He
School of Automation and Electrical Engineering,
University of Science and Technology Beijing,
Beijing 100083, China

C. Yang
School of Automation Engineering and Center for Robotics, University of Electronic Science and Technology of China, Chengdu 611731, China

Keywords Nonlinear PDE · Flexible marine riser · Boundary control · Constraints · Distributed parameter system (DPS)

List of symbols

L	Length of the riser
M	Mass of the vessel
ρ	Uniform mass per unit length of the riser
EI	Bending stiffness of the riser
EA	Axial stiffness of the riser
T	Tension of the riser
C_x, C_y, C_z	Constraints on x'_L, y'_L and z'_L
$u_x(t), u_y(t), u_z(t)$	Boundary control inputs in X, Y, Z directions
$f_x(s, t), f_y(s, t), f_z(s, t)$	Distributed disturbances of the riser in X, Y, Z directions
$d_x(t), d_y(t), d_z(t)$	Boundary disturbances of the riser in X, Y, Z directions
$x(s, t), y(s, t), z(s, t)$	Displacements in X, Y, Z directions

1 Introduction

The flexible marine riser is the connection between the well head in the sea bed and the platform/vessel on sea

surface for transporting the crude oil and natural gas. With the trend of extracting natural resources in deep ocean environment growing [1], the riser plays a crucial role. The riser is mainly subjected to environmental disturbance (waves, wind and ocean currents) and its internal forces. These may cause the vibration and deformation of the riser, which produce the premature fatigue problems. Besides, the American Petroleum Institute requires that maximum non-drilling angles should be limited to 4° [2]. Therefore, it is necessary to give an effective control to suppress vibration and prevent the violation of the angle constraints for the riser system.

The dynamics of the riser is generally described by the model of an Euler–Bernoulli beam, which is a distributed parameter system (DPS) [3–9]. The main challenge for solving the control problem of DPS is to control the infinite-dimensional state space using finite sensors and actuators [10–13, 13, 14]. Common methods to use the finite-dimensional techniques are finite element method [15, 16], Galerkin’s method [17, 18] and assumed modes method [19–21]. In these methods, only some critical modes are considered. It would happen that the controller does not stabilize the original infinite-dimensional system, which was documented by Balas in [22] and termed as the “spillover” effect.

Boundary control gains increasingly attention recently due to its practicality in implementation [23–25, 25–29]. Since actuators [30–33] and sensors [34–36] are applied at the boundary of the flexible structure, the dynamics of the system will not be influenced. Then, boundary control can be designed based on the original infinite-dimensional model, and then, the spillover problem is avoided. Lastly, in this paper, novel barrier terms are proposed in the boundary control laws. These terms can ensure the constraints of the joint angles. Without these barrier terms, the constraints would be violated. Therefore, boundary control is widely used in the control design for DPSs [37–43]. In [44], a static output feedback control is designed via the Euler–Bernoulli beam boundary and ODE measurements. Using the proposed control, the closed-loop system is proved to be exponentially stable by the Lyapunov’s direct method. In [45], the vessel dynamics is taken into consideration when modeling. In [46], an actuator at the upper riser is designed to control the transverse deflection of the riser with input saturation. The control problem for a flexible

air-breathing hypersonic vehicle (FAHV) is addressed by boundary output feedback control in [47], where a coupled system including both PDEs and ODEs is used to model the FAHV. In [48], boundary and adaptive control laws are designed for a moving beam system.

In practical application and industrial environment, constrained problems [49, 50] are commonly existing in physical stoppages, saturation and safety specifications. The barrier Lyapunov function [51–55] is an effective method to handle constraints.

In this paper, boundary control is designed to suppress the vibration of the marine riser system by three actuators at the top end. In case of larger vibrations, the control system restricts the joint angles within given ranges via the feedback signals. The stability of the riser systems is obtained by using integral-barrier Lyapunov function. The boundary control is directly designed for the original infinite-dimensional riser system, without any model discretization. Thus, the spillover problem can be eliminated entirely. The main contributions of this paper are summarized as follows:

- (i) To be more precise, the distributed disturbances along the riser are considered. Under the distributed disturbances, the governing equations of the flexible riser are described by three nonlinear non-homogeneous PDEs, making the dynamic model more complex. Therefore, the control methods based on the homogeneous PDEs cannot be applied directly.
- (ii) A novel Lyapunov function including an integral Lyapunov term and a barrier Lyapunov term is constructed to guarantee the stability of the system, and the constraints of the joint angles are not violated. The construction of the integral Lyapunov term is based on the mechanical energy $E_k(t)$ and $E_p(t)$ shown in Sect. 2.

2 Dynamics and preliminaries

Figure 1 shows a three-dimensional marine riser with its end fixed on at origin and tip clamped with a payload (vessel). The control is implemented at the tip by the actuator. In this paper, the effect of gravity is ignored. Let s and t be the independent spatial and time variables, respectively. For clarification, the notations $(*) = \partial(*)/\partial t$, $(*)' = \partial(*)/\partial s$ are used throughout this paper.

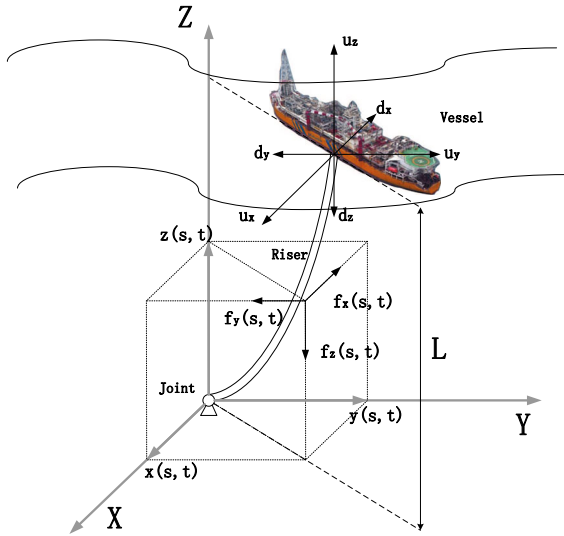


Fig. 1 A nonlinear three-dimensional marine riser

2.1 Dynamics analysis

The kinetic energy $E_k(t)$ and the potential energy $E_p(t)$ of the riser system can be represented as

$$E_k(t) = \frac{1}{2}M[(\dot{x}_L)^2 + (\dot{y}_L)^2 + (\dot{z}_L)^2] + \frac{1}{2}\rho \int_0^L [(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2]ds \tag{1}$$

$$E_p(t) = \frac{1}{2}T \int_0^L [(x')^2 + (y')^2]ds + \frac{1}{2}EI \int_0^L [(x'')^2 + (y'')^2]ds + \frac{1}{2}EA \int_0^L [z' + \frac{1}{2}(x')^2 + \frac{1}{2}(y')^2]^2 ds \tag{2}$$

where $\dot{x} = \dot{x}(s, t)$, $x' = x'(s, t)$, $x'' = x''(s, t)$ and $\dot{x}_L = \dot{x}(L, t)$ have been used, and similar abbreviations are used in the sequel.

The virtual work done by external distributed disturbances $f_x(s, t)$, $f_y(s, t)$, $f_z(s, t)$ on the riser and boundary disturbances $d_x(t)$, $d_y(t)$, $d_z(t)$ on the tip payload is given by

$$\delta W_f(t) = \int_0^L (f_x \delta x + f_y \delta y + f_z \delta z) ds + d_x \delta x_L + d_y \delta y_L + d_z \delta z_L \tag{3}$$

The virtual work done by the control force $u_x(t)$, $u_y(t)$, $u_z(t)$ to suppress vibration and prevent angle constraint violation can be represented as

$$\delta W_m(t) = u_x \delta x_L + u_y \delta y_L + u_z \delta z_L \tag{4}$$

Therefore, the virtual work δW done on the system can be represented as

$$\delta W(t) = \delta W_f(t) + \delta W_m(t) \tag{5}$$

Using the Hamilton's principle $\int_{t_1}^{t_2} \delta[E_k - E_p + W]dt = 0$ [56], we can obtain the following governing equations

$$\rho \ddot{x} = Tx'' + EA(z''x' + x''z') + \frac{3}{2}EA(x')^2x'' + \frac{1}{2}EA[x''(y')^2 + 2x'y'y''] - EIx'''' + f_x \tag{6}$$

$$\rho \ddot{y} = Ty'' + EA(z''y' + y''z') + \frac{3}{2}EA(y')^2y'' + \frac{1}{2}EA[y''(x')^2 + 2y'x'x''] - EIy'''' + f_y \tag{7}$$

$$\rho \ddot{z} = EAz'' + EAx'x'' + EAy'y'' + f_z \tag{8}$$

$\forall(s, t) \in [0, L] \times [0, \infty)$, and the boundary conditions as follows

$$x_0 = y_0 = z_0 = 0 \tag{9}$$

$$x_0'' = y_0'' = z_0'' = 0 \tag{10}$$

$$x_L'' = y_L'' = z_L'' = 0 \tag{11}$$

$$u_x + d_x = M\ddot{x}_L + Tx'_L + \frac{1}{2}EA(x'_L)^3 + EAx'_Lz'_L + \frac{1}{2}EAx'_L(y'_L)^2 - EIx_L'''' \tag{12}$$

$$u_y + d_y = M\ddot{y}_L + Ty'_L + \frac{1}{2}EA(y'_L)^3 + EAy'_Lz'_L + \frac{1}{2}EAy'_L(x'_L)^2 - EIy_L'''' \tag{13}$$

$$u_z + d_z = M\ddot{z}_L + EAz'_L + \frac{1}{2}EA(x'_L)^2 + \frac{1}{2}EA(y'_L)^2 \tag{14}$$

$\forall t \in [0, \infty)$.

Remark 1 The joint angles of the riser are given as $\arctan x'_L$, $\arctan y'_L$ and $\arctan z'_L$. Except for the vibration suppression, the control laws are designed to constrain the joint angles in given regions, i.e., $|\arctan x'_L| < A_x$, $|\arctan y'_L| < A_y$ and $|\arctan z'_L| <$

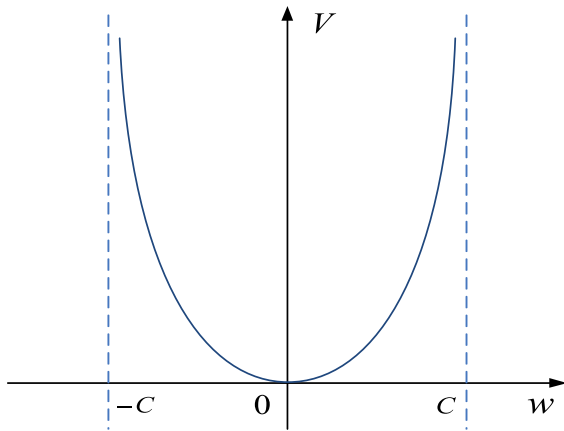


Fig. 2 Schematic illustration of a barrier Lyapunov function

A_z , where $A_x, A_y, A_z > 0$. In this paper, the constraints of the joint angles are satisfied via the constraints of the boundary slopes, i.e., the boundary slopes of the riser should satisfy $|x'_L| < C_x, |y'_L| < C_y$ and $|z'_L| < C_z$, where $C_x, C_y, C_z > 0$ and $\arctan C_x = A_x, \arctan C_y = A_y$ and $\arctan C_z = A_z$.

2.2 Preliminaries

Definition 1 [51] A barrier Lyapunov function (BLF) is a scalar function $V(x)$ defined with respect to the system $\dot{x} = f(x)$ on an open region \mathcal{D} containing the origin, which is continuous, positive definite, has continuous first-order partial derivatives at every point of \mathcal{D} , has the property $V(x) \rightarrow \infty$ as x approaches the boundary of \mathcal{D} and satisfies $V(x(t)) \leq b, \forall t \geq 0$ along the solutions of $\dot{x} = f(x)$ for $x(0) \in \mathcal{D}$ and a positive constant b .

The schematic illustration of a barrier Lyapunov function is shown in Fig. 2.

Property 1 [57] If the kinetic energy of the system (6)–(14), given by (1), is bounded, then functions $\dot{x}'(s, t), \dot{x}''(s, t), \dot{y}'(s, t), \dot{y}''(s, t), \dot{z}'(s, t)$ and $\dot{z}''(s, t)$ are bounded $\forall (s, t) \in [0, L] \times [0, \infty)$.

Property 2 [57] If the potential energy of the system (6)–(14), given by (2), is bounded, then functions $x''(s, t), x'''(s, t), x''''(s, t), y''(s, t), y'''(s, t), y''''(s, t)$ and $z''(s, t)$ are bounded $\forall (s, t) \in [0, L] \times [0, \infty)$.

Assumption 1 For the unknown distributed disturbances $f_x(s, t), f_y(s, t), f_z(s, t)$ and unknown bound-

ary disturbances $d_x(t), d_y(t), d_z(t)$, we assume that there exist constants $\bar{f}_x, \bar{f}_y, \bar{f}_z, \bar{d}_x, \bar{d}_y, \bar{d}_z \in R^+$, such that $f_x(s, t) \leq \bar{f}_x, f_y(s, t) \leq \bar{f}_y, f_z(s, t) \leq \bar{f}_z, \forall [s, t] \in [0, L] \times [0, \infty)$ and $d_x(t) \leq \bar{d}_x, d_y(t) \leq \bar{d}_y, d_z(t) \leq \bar{d}_z, \forall t \in [0, \infty)$. This is a reasonable assumption as the disturbances $f_x(s, t), f_y(s, t), f_z(s, t), d_x(t), d_y(t)$ and $d_z(t)$ have finite energy and hence are bounded, i.e., $f_x(s, t), f_y(s, t), f_z(s, t), d_x(t), d_y(t), d_z(t) \in \mathcal{L}_\infty$. The knowledge of exact values of the disturbances is not required.

3 Control design

The control objectives are to suppress the vibration of the riser in three-dimensional space and guarantee that the joint angles of the marine riser remain in constrained ranges in the presence of environment disturbances. The IBLF-based method is used to construct the control inputs u_x, u_y, u_z and analyze the stability of the closed-loop system.

For the system given by the governing Eqs. (6)–(8) and boundary conditions (9)–(14), we propose the following control laws:

$$u_x = -\text{sgn}(\dot{x}_L + \beta L x'_L) \bar{d}_x + \tau_x(t) - M\beta L \dot{x}'_L - k_{1x}(\dot{x}_L + \beta L x'_L) - (\dot{x}_L - \beta L x'_L) \times \left(k_{2x} + M \frac{x'_L \dot{x}'_L}{C^2 - x'^2_L} \right) / A - \tau_x(t) / A \quad (15)$$

$$u_y = -\text{sgn}(\dot{y}_L + \beta L y'_L) \bar{d}_y + \tau_y(t) - M\beta L \dot{y}'_L - k_{1y}(\dot{y}_L + \beta L y'_L) - (\dot{y}_L - \beta L y'_L) \times \left(k_{2y} + M \frac{y'_L \dot{y}'_L}{C^2 - y'^2_L} \right) / B - \tau_y(t) / B \quad (16)$$

$$u_z = -\text{sgn}(\dot{z}_L + \beta L z'_L) \bar{d}_z + \tau_z(t) - M\beta L \dot{z}'_L - k_{1z}(\dot{z}_L + \beta L z'_L) - (\dot{z}_L - \beta L z'_L) \times \left(k_{2z} + M \frac{z'_L \dot{z}'_L}{C^2 - z'^2_L} \right) / C - \tau_z(t) / C \quad (17)$$

where $k_{1x}, k_{2x}, k_{1y}, k_{2y}, k_{1z}, k_{2z}$ are the positive control gains, and $\text{sgn}(\cdot)$ denotes the signum function, $\tau_x(t), \tau_y(t), \tau_z(t)$ are defined as

$$\tau_x(t) = T x'_L + \frac{1}{2} E A (x'_L)^3 + E A x'_L z'_L + \frac{1}{2} E A x'_L (y'_L)^2 - E I x''_L \quad (18)$$

$$\begin{aligned} \tau_y(t) &= Ty'_L + \frac{1}{2}EA(y'_L)^3 + EAy'_Lz'_L \\ &\quad + \frac{1}{2}EAy'_L(x'_L)^2 - EIy'''_L \end{aligned} \tag{19}$$

$$\tau_z(t) = EAz'_L + \frac{1}{2}EA(x'_L)^2 + \frac{1}{2}EA(y'_L)^2 \tag{20}$$

and the barrier terms A , B and C are given as

$$\begin{aligned} A &= \ln \frac{2C_x^2}{C_x^2 - x_L^2}, \quad B = \ln \frac{2C_y^2}{C_y^2 - y_L^2}, \\ C &= \ln \frac{2C_z^2}{C_z^2 - z_L^2} \end{aligned} \tag{21}$$

Consider the Lyapunov function candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{22}$$

where

$$\begin{aligned} V_1(t) &= \frac{1}{2}\rho \int_0^L [(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2]ds \\ &\quad + \frac{1}{2}EA \int_0^L [z' + \frac{1}{2}(x')^2 + \frac{1}{2}(y')^2]^2 ds \\ &\quad + \frac{1}{2}EI \int_0^L [(x'')^2 + (y'')^2]ds \\ &\quad + \frac{1}{2}T \int_0^L [(x')^2 + (y')^2]ds \end{aligned} \tag{23}$$

$$\begin{aligned} V_2(t) &= \frac{1}{2}M(\dot{x}_L + \beta Lx'_L)^2 A + \frac{1}{2}M(\dot{y}_L \\ &\quad + \beta Ly'_L)^2 B + \frac{1}{2}M(\dot{z}_L + \beta Lz'_L)^2 C \end{aligned} \tag{24}$$

$$V_3(t) = \beta\rho \int_0^L s(\dot{x}x' + \dot{y}y' + \dot{z}z')ds \tag{25}$$

where β is a positive constant, $V_1(t)$ is designed based on the mechanical energy $E_k(t)$ and $E_p(t)$, called the energy term, the auxiliary term $V_2(t)$ is related to the payload, and the crossing term $V_3(t)$ is designed to facilitate the stability analysis.

Lemma 1 *The Lyapunov candidate function given by (22) is positive definite as*

$$\lambda_1(\zeta(t) + V_2(t)) \leq V(t) \leq \lambda_2(\zeta(t) + V_2(t)) \tag{26}$$

where λ_1 and λ_2 are two positive constants and

$$\begin{aligned} \zeta(t) &= \int_0^L [(\dot{x})^2 + (x')^2 + (\dot{y})^2 + (y')^2 + (\dot{z})^2 + (z')^2 \\ &\quad + (x')^4 + (y')^4 + (x'y')^2 + (x'')^2 + (y'')^2]ds \end{aligned} \tag{27}$$

Proof According to generalized Young's inequality [57], let δ satisfy $T - \frac{EA}{2\delta} \geq 0$ and $\frac{1}{4} - \delta \geq 0$, where δ is a positive constant, we have

$$0 \leq \eta_1\zeta(t) \leq V_1(t) \leq \eta_2\zeta(t) \tag{28}$$

where η_1 and η_2 are defined as

$$\eta_1 = \frac{1}{2} \min \left[\rho, T - \frac{EA}{2\delta}, \frac{1}{2}EA, EA \left(\frac{1}{4} - \delta \right), EI \right] \tag{29}$$

$$\eta_2 = \frac{1}{2} \max \left[\rho, T + \frac{EA}{2\delta}, \frac{1}{2}EA, EA \left(\frac{1}{4} + \delta \right), EI \right] \tag{30}$$

Similarly, we can obtain

$$\begin{aligned} |V_3(t)| &\leq \beta\rho L \int_0^L [(\dot{x})^2 + (x')^2 + (\dot{y})^2 + (y')^2 \\ &\quad + (\dot{z})^2 + (z')^2]ds \leq \beta\rho L\zeta(t) \end{aligned} \tag{31}$$

Let β satisfy $\beta\rho L < \eta_1$, we have $0 < \beta\rho L < \xi_1$. Let $\xi_1 = \eta_1 - \beta\rho L$, $\xi_2 = \eta_2 + \beta\rho L$, we further have

$$0 \leq \xi_1 V_1(t) \leq V_1(t) + V_3(t) \leq \xi_2 V_1(t). \tag{32}$$

Therefore, we have

$$0 \leq \lambda_1[(\zeta(t) + V_2(t))] \leq V(t) \leq \lambda_2(\zeta(t) + V_2(t)), \tag{33}$$

where $\lambda_1 = \min(\xi_1, 1)$ and $\lambda_2 = \max(\xi_2, 1)$ are two positive constants. \square

Lemma 2 *The time derivative of the Lyapunov candidate function in (22) is given as*

$$\dot{V}(t) \leq -\lambda V(t) + \varepsilon, \tag{34}$$

where λ and ε are two positive constants.

Proof Differentiating $V_1(t)$ by parts and applying generalized Young's inequality [57], we have

$$\begin{aligned} \dot{V}_1(t) \leq & \delta_1 \int_0^L f_x^2 ds + \delta_2 \int_0^L f_y^2 ds + \delta_3 \int_0^L f_z^2 ds \\ & + \frac{1}{\delta_1} \int_0^L (\dot{x})^2 ds + \frac{1}{\delta_2} \int_0^L (\dot{y})^2 ds \\ & + \frac{1}{\delta_3} \int_0^L (\dot{z})^2 ds + \Delta_1 \end{aligned} \quad (35)$$

where δ_1 – δ_3 are positive constants, and Δ_1 is the sum of the boundary terms in $\dot{V}_1(t)$. Substituting boundary conditions (9)–(14) to Δ_1 , we obtain

$$\Delta_1 = \tau_x(t)\dot{x}_L + \tau_y(t)\dot{y}_L + \tau_z(t)\dot{z}_L \quad (36)$$

The differentiation of $V_2(t)$ and substitution of the control laws (15)–(17) yield

$$\begin{aligned} \dot{V}_2(t) \leq & -k_{1x}(\dot{x}_L + \beta Lx'_L)^2 A - k_{2x}(\dot{x}_L)^2 \\ & + k_{2x}\beta^2 L^2 (x'_L)^2 - \tau_x(t)(\dot{x}_L + \beta Lx'_L) \\ & - k_{1y}(\dot{y}_L + \beta Ly'_L)^2 B - k_{2y}(\dot{y}_L)^2 \\ & + k_{2y}\beta^2 L^2 (y'_L)^2 - \tau_y(t)(\dot{y}_L + \beta Ly'_L) \\ & - k_{1z}(\dot{z}_L + \beta Lz'_L)^2 C - k_{2z}(\dot{z}_L)^2 \\ & + k_{2z}\beta^2 L^2 (z'_L)^2 - \tau_z(t)(\dot{z}_L + \beta Lz'_L) \end{aligned} \quad (37)$$

Time derivative of $V_3(t)$ is given as

$$\begin{aligned} \dot{V}_3(t) = & -\frac{1}{2}\beta\rho \int_0^L (\dot{x})^2 ds - \frac{1}{2}\beta\rho \int_0^L (\dot{y})^2 ds \\ & - \frac{1}{2}\beta\rho \int_0^L (\dot{z})^2 ds - \frac{1}{2}\beta EA \int_0^L (z')^2 ds \\ & - \frac{3}{8}\beta EA \int_0^L (x')^4 ds - \frac{3}{8}\beta EA \int_0^L (y')^4 ds \\ & - \beta EA \int_0^L (x')^2 z' ds - \beta EA \int_0^L (y')^2 z' ds \\ & - \frac{3}{4}\beta EA \int_0^L (x'y')^2 ds - \frac{1}{2}\beta EI \int_0^L (x'')^2 ds \\ & - \frac{1}{2}\beta EI \int_0^L (y'')^2 ds - \frac{1}{2}\beta T \int_0^L (x')^2 ds \\ & - \frac{1}{2}\beta T \int_0^L (y')^2 ds + \beta \int_0^L s(f_x x' \\ & + f_y y' + f_z z') ds + \Delta_2 \end{aligned} \quad (38)$$

where Δ_2 is the sum of the boundary terms in $\dot{V}_3(t)$. Substituting the boundary conditions (9)–(14) to Δ_2 , we have

$$\begin{aligned} \Delta_2 = & \frac{1}{2}\beta\rho L(\dot{x}_L)^2 + \frac{1}{2}\beta\rho L(\dot{y}_L)^2 + \frac{1}{2}\beta\rho L(\dot{z}_L)^2 \\ & + \tau_x(t)\beta Lx'_L + \tau_y(t)\beta Ly'_L + \tau_z(t)\beta Lz'_L \\ & - \frac{1}{2}\beta TL(x'_L)^2 - \frac{1}{2}\beta TL(y'_L)^2 \\ & - \frac{1}{2}\beta EAL \left[\frac{1}{2}(x'_L)^2 + \frac{1}{2}(y'_L)^2 + z'_L \right]^2 \end{aligned} \quad (39)$$

Then, we can further have

$$\begin{aligned} \dot{V}(t) \leq & -\left(\frac{1}{2}\beta\rho - \frac{1}{\delta_1}\right) \int_0^L (\dot{x})^2 ds \\ & - \left(\frac{1}{2}\beta\rho - \frac{1}{\delta_2}\right) \int_0^L (\dot{y})^2 ds \\ & - \left(\frac{1}{2}\beta\rho - \frac{1}{\delta_3}\right) \int_0^L (\dot{z})^2 ds \\ & - \left(\frac{1}{2}\beta EA - \beta L\delta_6\right) \int_0^L (z')^2 ds \\ & - \frac{3}{8}\beta EA \int_0^L (x')^4 ds - \frac{3}{8}\beta EA \int_0^L (y')^4 ds \\ & - \beta EA \int_0^L (x')^2 z' ds - \beta EA \int_0^L (y')^2 z' ds \\ & - \frac{3}{4}\beta EA \int_0^L (x'y')^2 ds - \frac{1}{2}\beta EI \int_0^L (x'')^2 ds \\ & - \frac{1}{2}\beta EI \int_0^L (y'')^2 ds \\ & - \left(\frac{1}{2}\beta T - \beta L\delta_4\right) \int_0^L (x')^2 ds \\ & - \left(\frac{1}{2}\beta T - \beta L\delta_5\right) \int_0^L (y')^2 ds \\ & + \left(\delta_1 + \frac{\beta L}{\delta_4}\right) \int_0^L f_x^2 ds \\ & + \left(\delta_2 + \frac{\beta L}{\delta_5}\right) \int_0^L f_y^2 ds \\ & + \left(\delta_3 + \frac{\beta L}{\delta_6}\right) \int_0^L f_z^2 ds \\ & - k_{1x}M(\dot{x}_L + \beta Lx'_L)^2 A \\ & - k_{1y}M(\dot{y}_L + \beta Ly'_L)^2 B \\ & - k_{1z}M(\dot{z}_L + \beta Lz'_L)^2 C \end{aligned}$$

$$\begin{aligned}
 & - \left(k_{2x} - \frac{1}{2}\beta\rho L\right)\dot{x}_L^2 - \left(k_{2y} - \frac{1}{2}\beta\rho L\right)\dot{y}_L^2 \\
 & - \left(k_{2z} - \frac{1}{2}\beta\rho L\right)\dot{z}_L^2 \\
 & - [2\beta TL - \beta^2 L^2(2k_{2x} + 2k_{2y} + k_{2z})]z_L^2 \\
 & - \frac{1}{2}\beta EAL \left[\frac{1}{2}(x'_L)^2 + \frac{1}{2}(y'_L)^2 + z'_L\right]^2 \\
 & \leq -\lambda_3[\zeta(t) + V_2(t)] + \varepsilon
 \end{aligned} \tag{40}$$

where δ_4 – δ_6 are positive constants, and

$$\begin{aligned}
 \lambda_3 = \min \left\{ \frac{1}{2}\beta\rho - \frac{1}{\delta_1}, \frac{1}{2}\beta\rho - \frac{1}{\delta_2}, \frac{1}{2}\beta\rho \right. \\
 \left. - \frac{1}{\delta_3}, \frac{1}{2}\beta EA - \beta L\delta_6, \frac{3\beta}{8}, \frac{1}{2}\beta T \right. \\
 \left. - \beta L\delta_4, \frac{1}{2}\beta T - \beta L\delta_5, \frac{2k_{1x}}{M}, \frac{2k_{1y}}{M}, \frac{2k_{1z}}{M} \right\}
 \end{aligned} \tag{41}$$

$$\begin{aligned}
 \varepsilon & = \left(\alpha\delta_1 + \frac{\beta L}{\delta_4}\right) \int_0^L f_x^2 ds + \left(\alpha\delta_2 + \frac{\beta L}{\delta_5}\right) \\
 & \times \int_0^L f_y^2 ds + \left(\alpha\delta_3 + \frac{\beta L}{\delta_6}\right) \int_0^L f_z^2 ds \\
 & \leq \left(\alpha\delta_1 + \frac{\beta L}{\delta_4}\right) \bar{f}_x + \left(\alpha\delta_2 + \frac{\beta L}{\delta_5}\right) \bar{f}_x \\
 & + \left(\alpha\delta_3 + \frac{\beta L}{\delta_6}\right) \bar{f}_x \in \mathcal{L}_\infty
 \end{aligned} \tag{42}$$

Choosing $2k_{2x} \geq \beta\rho L, 2k_{2y} \geq \beta\rho L, 2k_{2z} \geq \beta\rho L$, and the designed parameters β, δ_1 – δ_6 are selected to satisfy the following conditions:

$$\frac{1}{2}\beta\rho - \frac{1}{\delta_1} \geq 0 \tag{43}$$

$$\frac{1}{2}\beta\rho - \frac{1}{\delta_2} \geq 0 \tag{44}$$

$$\frac{1}{2}\beta\rho - \frac{1}{\delta_3} \geq 0 \tag{45}$$

$$EA - 2L\delta_6 \geq 0 \tag{46}$$

$$T - 2L\delta_4 \geq 0 \tag{47}$$

$$T - 2L\delta_5 \geq 0 \tag{48}$$

$$2TL - \beta L^2(2k_{2x} + 2k_{2y} + k_{2z}) \geq 0 \tag{49}$$

Combining (26) and (40), we have

$$\dot{V}(t) \leq -\lambda V(t) + \varepsilon, \tag{50}$$

where $\lambda = \lambda_3/\lambda_2$ and $\varepsilon > 0$. □

Theorem 1 For the system described by (6)–(8) and boundary conditions (9)–(14), under Assumption 1, and the control laws (15)–(17), given that the initial conditions are bounded, we can conclude that:

- (i) the states of the system $x(s, t), y(s, t)$ and $z(s, t)$ will remain in the compact set Ω_1 defined by

$$\begin{aligned}
 \Omega_1 & := \{x(s, t), y(s, t), z(s, t) \\
 & \in R \mid |x(s, t)|, |y(s, t)|, |z(s, t)| \\
 & \leq D_1, \forall (s, t) \in [0, L] \times [0, \infty)\}
 \end{aligned} \tag{51}$$

where the constant $D_1 = \sqrt{\frac{L}{\lambda_1} \left(V(0) + \frac{\varepsilon}{\lambda} \right)}$.

- (ii) the states of the system $x(s, t), y(s, t)$ and $z(s, t)$ will eventually converge to the compact Ω_2 defined by

$$\begin{aligned}
 \Omega_2 & := \{x(s, t), y(s, t), z(s, t) \in R \mid \lim_{t \rightarrow \infty} |x(s, t)|, \\
 & \lim_{t \rightarrow \infty} |y(s, t)|, \lim_{t \rightarrow \infty} |z(s, t)| \\
 & \leq D_2, \forall t \in [0, \infty)\}
 \end{aligned} \tag{52}$$

where the constant $D_2 = \sqrt{\frac{L\varepsilon}{\lambda_1\lambda}}$.

Proof Multiplying (34) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t} (V e^{\lambda t}) \leq \varepsilon e^{\lambda t} \tag{53}$$

Integration of the above inequality, we obtain

$$\begin{aligned}
 V(t) & \leq \left(V(0) - \frac{\varepsilon}{\lambda} \right) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \leq V(0) e^{-\lambda t} \\
 & + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty
 \end{aligned} \tag{54}$$

which implies $V(t)$ is bounded. Utilizing Wirtinger’s inequality (or Poincaré) [57] and (26), we obtain

$$\begin{aligned}
 \frac{1}{L} [x(s, t)]^2 & \leq \int_0^L [x'(s, t)]^2 ds \leq \zeta(t) \\
 & \leq \frac{1}{\lambda_1} V(t) \in \mathcal{L}_\infty
 \end{aligned} \tag{55}$$

$$\begin{aligned}
 \frac{1}{L} [y(s, t)]^2 & \leq \int_0^L [y'(s, t)]^2 ds \leq \zeta(t) \\
 & \leq \frac{1}{\lambda_1} V(t) \in \mathcal{L}_\infty
 \end{aligned} \tag{56}$$

$$\begin{aligned} \frac{1}{L}[z(s, t)]^2 &\leq \int_0^L [z'(s, t)]^2 ds \leq \zeta(t) \\ &\leq \frac{1}{\lambda_1} V(t) \in \mathcal{L}_\infty \end{aligned} \tag{57}$$

Rearranging the terms of the above three inequalities, we can obtain

$$\begin{aligned} |x(s, t)|, |y(s, t)|, |z(s, t)| &\leq \sqrt{\frac{L}{\lambda_1} \left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \right)} \\ &\leq \sqrt{\frac{L}{\lambda_1} \left(V(0) + \frac{\varepsilon}{\lambda} \right)}, \forall (s, t) \in [0, L] \times [0, \infty) \end{aligned} \tag{58}$$

Furthermore, from (58), we can obtain

$$\lim_{t \rightarrow \infty} |x(s, t)| \leq \sqrt{\frac{L\varepsilon}{\lambda_1\lambda}}, \forall s \in [0, L] \tag{59}$$

$$\lim_{t \rightarrow \infty} |y(s, t)| \leq \sqrt{\frac{L\varepsilon}{\lambda_1\lambda}}, \forall s \in [0, L] \tag{60}$$

$$\lim_{t \rightarrow \infty} |z(s, t)| \leq \sqrt{\frac{L\varepsilon}{\lambda_1\lambda}}, \forall s \in [0, L] \tag{61}$$

□

Remark 2 By proposing the barrier Lyapunov function bounded in a symmetrical region shown in Fig. 2, it is thus guaranteed that the barriers are not transgressed. From Ineqs. (54) and (55), we know that $V(t)$ is bounded and $\zeta(t) \leq \frac{1}{\lambda_1} V(t)$ is also bounded $\forall t \in [0, \infty)$. Furthermore, from Ineq. (26) in Lemma 1 and the fact that $V_2(t)$ and $\zeta(t)$ are positive functions, we have that $V_2(t)$ is also bounded $\forall t \in [0, \infty)$. From the definition of barrier Lyapunov function, we know that $V_2(t) \rightarrow \infty$, as $|x'_L| \rightarrow C_x, |y'_L| \rightarrow C_y, |z'_L| \rightarrow C_z$. Consequently, we know that $x'_L \neq C_x, y'_L \neq C_y$ and $z'_L \neq C_z$. Given that $|x'(L, 0)| < C_x, |y'(L, 0)| < C_y$ and $|z'(L, 0)| < C_z$, from Fig. 2, we can further infer that $|x'_L|, |y'_L|, |z'_L|$ remain in the sets $|x'_L| < C_x, |y'_L| < C_y$ and $|z'_L| < C_z, \forall t \in [0, \infty)$. Therefore, we have $|\arctan x'_L| < \arctan C_x, |\arctan y'_L| < \arctan C_y$ and $|\arctan z'_L| < \arctan C_z, \forall t \in [0, \infty)$, namely the joint angles are constrained in the given angle regions.

Remark 3 According to (28) and (55), we can state that $\zeta(t)$ and $V_1(t)$ are bounded $\forall t \in [0, \infty)$. Since $V_1(t)$ is bounded, $\dot{x}(s, t), x'(s, t), x''(s, t), \dot{y}(s, t), y'(s, t), y''(s, t), \dot{z}(s, t)$ and $z'(s, t)$ are bounded $\forall (s, t) \in$

$[0, L] \times [0, \infty)$. From (1), the kinetic energy of the system is bounded and using Property 1, we can know that $\dot{x}'(s, t)$ and $\dot{y}'(s, t)$ are bounded $\forall (s, t) \in [0, L] \times [0, \infty)$. From the boundedness of the potential energy (2), we can use Property 2 to conclude that $z''(s, t)$ is bounded, $\forall (s, t) \in [0, L] \times [0, \infty)$. Finally, using Assumption 1, (6)–(8) through (9)–(14) and the above statements, we can conclude that $\ddot{x}(s, t), \ddot{y}(s, t)$ and $\ddot{z}(s, t)$ are also bounded, $\forall (s, t) \in [0, L] \times [0, \infty)$. From Lemma 3 and the above proof, it is shown the deflection $x(s, t), y(s, t)$ and $z(s, t)$ is uniformly bounded $\forall (s, t) \in [0, L] \times [0, \infty)$. And we can conclude that the control inputs u_x, u_y and u_z are bounded $\forall (s, t) \in [0, L] \times [0, \infty)$.

Remark 4 The limitation for the proposed control is that there is no term introduced to deal with the effect of $f_x(s, t), f_y(s, t), f_z(s, t)$. Therefore, only bounded stability can be ensured due to the term ε . When the upper bounds of the distributed disturbance are large, the control performance would be affected. However, a better control performance can also be obtained by tuning the designed parameters. From Ineqs. (41), (59)–(61), by adjusting the designed parameters, such as increasing k_{1x}, k_{1y}, k_{1z} will bring a larger λ_3 . Then, the value of λ will increase, which will produce a better vibration suppression performance. However, increasing in the control gains would bring a high gain control scheme. Therefore, in practical applications, the designed parameters should be adjusted carefully for achieving suitable transient performance and control action.

4 Simulation

In this section, in order to demonstrate the effectiveness and practicability of the proposed control laws (15)–(17), we choose the infinite difference method to carry out the numerical simulation. The initial conditions of the system are given as: $x(s, t) = \dot{x}(s, t) = y(s, t) = \dot{y}(s, t) = z(s, t) = \dot{z}(s, t) = 0$.

In this paper, for simulation purposes, the external distributed disturbances are simulated with several sinusoids signals with different frequencies, which can be used to model the ocean disturbances in offshore engineering. The reader can refer to [58] for the detailed parameters of distributed disturbances on the riser. The external boundary disturbances are generated by the following equations

$$d_x(t) = d_y(t) = (3 + 0.8 \sin(0.7t) + 0.2 \sin(0.5t) + 0.2 \sin(0.9t)) \times 10^5 \tag{62}$$

$$d_z(t) = (3 + 0.2 \sin(0.5t)) \times 10^4 \tag{63}$$

From (62) and (63), the periods of boundary disturbances are $T_x = T_y = 20\pi s$, $T_z = 4\pi s$, the frequencies of boundary disturbances are $f_x = f_y = 0.016Hz$, $f_z = 0.08Hz$ and the amplitudes of boundary disturbances are $A_x = A_y = 4.2 \times 10^5$, $A_z = 3.2 \times 10^4$, respectively. The simulations for boundary disturbances are shown in Fig. 3.

Parameters of the riser system are given in Table 1. The length of simulation time is 500 s.

Table 1 Parameters of the system

Parameter	Description	Value
L	Length of riser	1000 m
ρ	Uniform mass per unit length of the riser	108 kg/m
M	Mass of the vessel	9.6×10^6 kg
EI	Bending stiffness of the riser	1.22×10^5 Nm ²
EA	Axial stiffness of the beam	3.92×10^8 Nm ²
T	Tension of the beam	1.11×10^8 N
C_x, C_y, C_z	Constraints on x'_L, y'_L and z'_L	0.03

4.1 Comparison with the PD control

In this subsection, the simulation results have been discussed thoroughly by comparing with the traditional PD control. The dynamic responses of the flexible riser are simulated in the following cases:

(i) Without control: The flexible riser is simulated without control, and the spatial time representations

are shown in Fig. 4. From the simulation results shown in Fig. 4, it can be observed that when there is no control input, there are large vibrations along the riser subjected to the external disturbances. For example, we can obtain $\max |z(x, t)| = 4.4$ m.

(ii) With the proposed boundary control force: The boundary control laws (15)–(17) developed by using integral-barrier Lyapunov function is simulated with

Fig. 3 Boundary disturbances: **(a)** $d_x(t)$ and $d_y(t)$; **(b)** $d_z(s, t)$

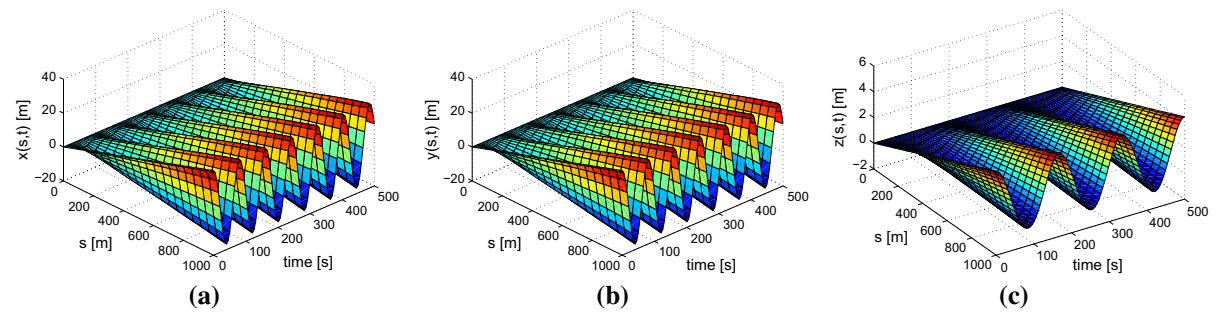
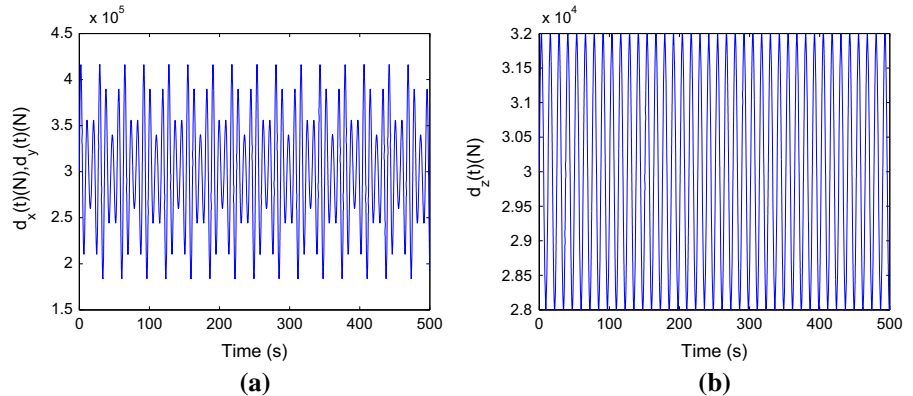


Fig. 4 Displacements of the riser without control: **a** $x(s, t)$; **b** $y(s, t)$; **c** $z(s, t)$

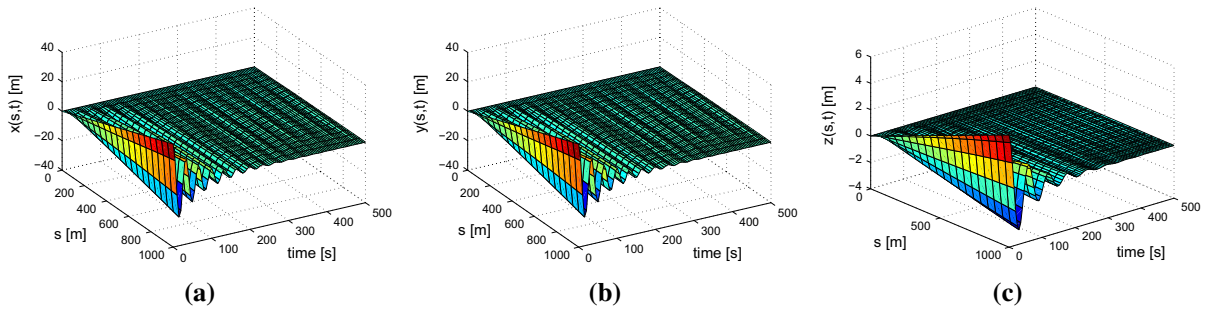


Fig. 5 Displacements of the riser with the proposed control: **a** $x(s, t)$; **b** $y(s, t)$; **c** $z(s, t)$

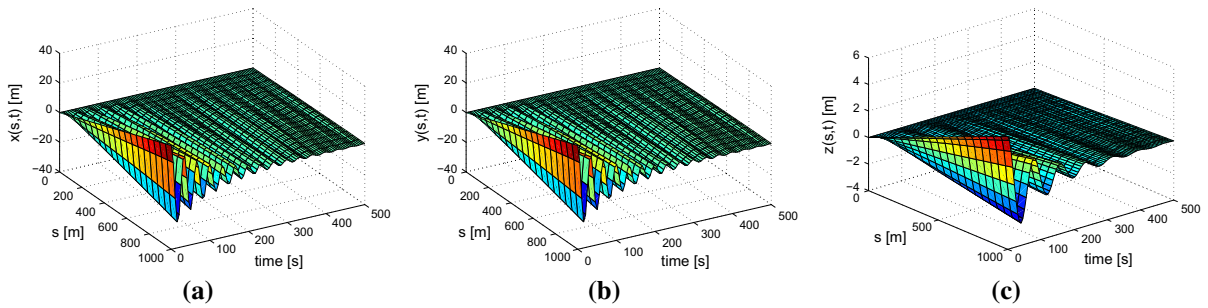


Fig. 6 Displacements of the riser with the PD control: **a** $x(s, t)$; **b** $y(s, t)$; **c** $z(s, t)$

the control parameters $k_{1x} = 2 \times 10^3$, $k_{2x} = 8 \times 10^5$, $k_{1y} = 3 \times 10^3$, $k_{2y} = 4 \times 10^5$, $k_{1z} = 5 \times 10^3$ and $k_{2z} = 10^6$. The spatial time representations are shown in Fig. 5. When the proposed control laws applied, we can see that the proposed boundary laws (15)–(17) could suppress the vibrations in all three directions. In addition, in the z direction, the displacement magnitude is regulated to around 0 at the top end of the riser, illustrating that good control performance is ensured with the proposed control.

(iii) With the PD control: The flexible riser system is analyzed with the traditional PD control $u_1(t) = -k_{p1}w(L, t) - k_{d1}\dot{w}(L, t)$, $u_2(t) = -k_{p2}y(L, t) - k_{d2}\dot{y}(L, t)$ and $u_3(t) = -k_{p3}z(L, t) - k_{d3}\dot{z}(L, t)$ by choosing $k_{p1} = k_{p2} = k_{p3} = 5 \times 10^5$ and $k_{d1} = k_{d2} = 8 \times 10^5$, $k_{d3} = 10^6$. The displacements of the riser under the PD control are shown in Fig. 6. We can observe that the vibration can also be suppressed by the PD control.

The end-point deflection has also been presented to show the advantages of the proposed method. Figures 7, 8 and 9 show the displacements of the riser at $x = L$. From Figs. 7, 9, we can conclude that both the proposed

control and the PD control are effective in regulating $w(L, t)$, $y(L, t)$ and $z(L, t)$. However, obviously, the response of the proposed control is faster than that of the PD control.

Additional, boundary slopes of the riser are simulated in Figs. 10, 11 and 12. As shown in Fig. 12, with the PD control, the constraints for boundary slopes of the riser are violated. However, the proposed control laws ensure that the boundary slopes of the riser $|x'_L| \leq C_x$, $|y'_L| \leq C_y$ and $|z'_L| \leq C_z$ are due to the existence of the barrier Lyapunov functions.

Although both the proposed control and the PD control are able to stabilize the riser at the small neighborhood of its equilibrium position. However, compared with PD control, the performance of the proposed boundary control exhibits smaller vibration and the states converge faster. And only the proposed boundary control can guarantee the joint angles that remain in the given regions. In addition, the control gains for the PD control are larger than that of the proposed control. From the comparison, we can conclude that we obtain a better control performance by using the proposed control. The control inputs are given in Fig. 13.

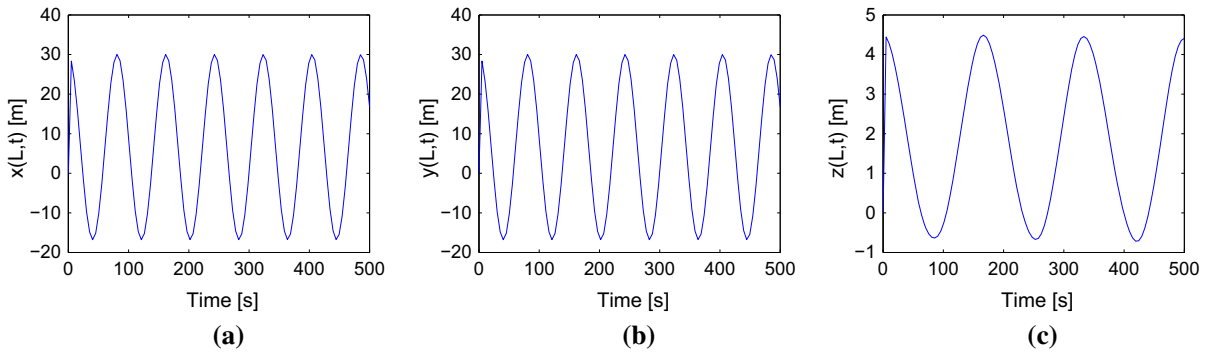


Fig. 7 Boundary displacement of the riser without control: **a** $x(L, t)$; **b** $y(L, t)$; **c** $z(L, t)$

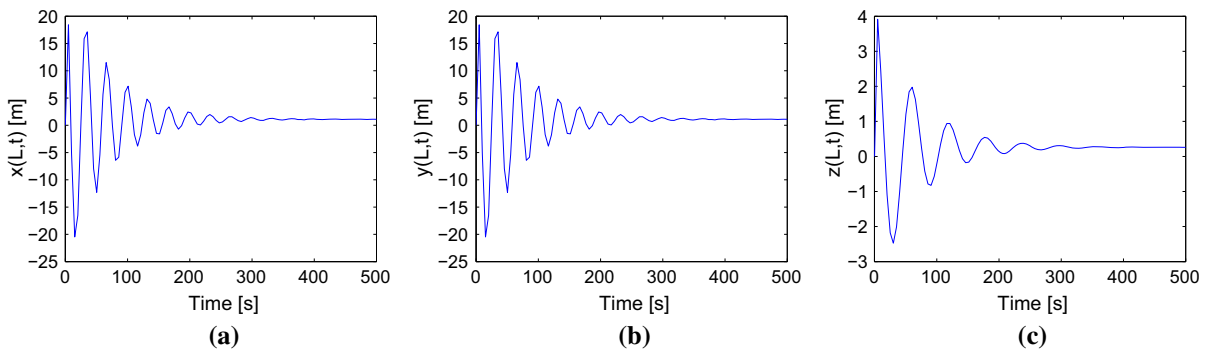


Fig. 8 Boundary displacement of the riser with the proposed control: **a** $x(L, t)$; **b** $y(L, t)$; **c** $z(L, t)$

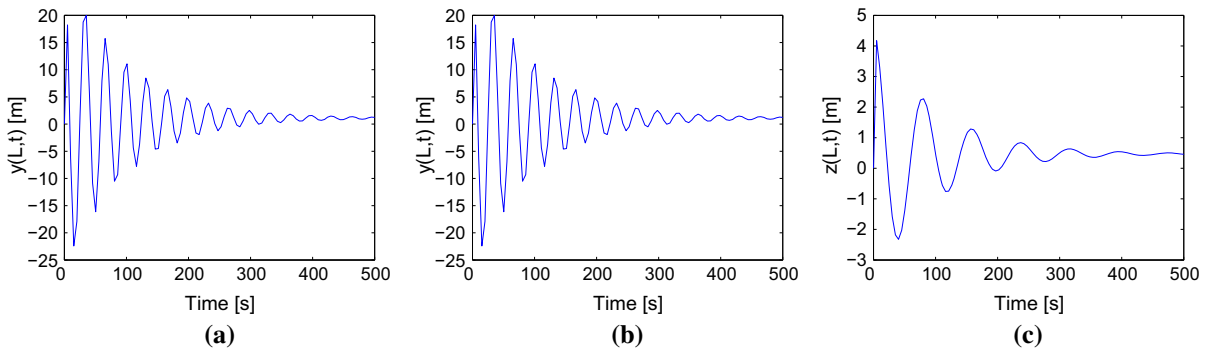


Fig. 9 Boundary displacement of the riser with the PD control: **a** $x(L, t)$; **b** $y(L, t)$; **c** $z(L, t)$

4.2 Comparison with different boundary disturbances

In order to test the effects of the frequency and amplitude of the periodic disturbances on the control performance, we have used different boundary disturbances in the simulations. The dynamic responses of the flexible riser are simulated in the following cases:

(i) Changing the frequency of the external disturbances: For this case, with the amplitude of the external disturbances unchanged, we change the frequency of the external disturbances as

$$d_{x_1}(t) = d_{y_1}(t) = (3 + 0.8 \sin(7t) + 0.2 \sin(5t) + 0.2 \sin(9t)) \times 10^5 \tag{64}$$

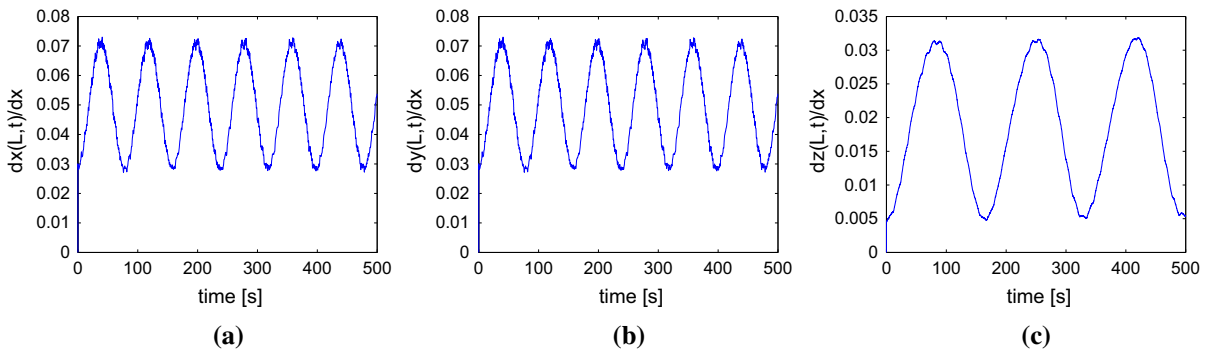


Fig. 10 Boundary slopes of the riser without control: **a** $x'(L, t)$; **b** $y'(L, t)$; **c** $z'(L, t)$

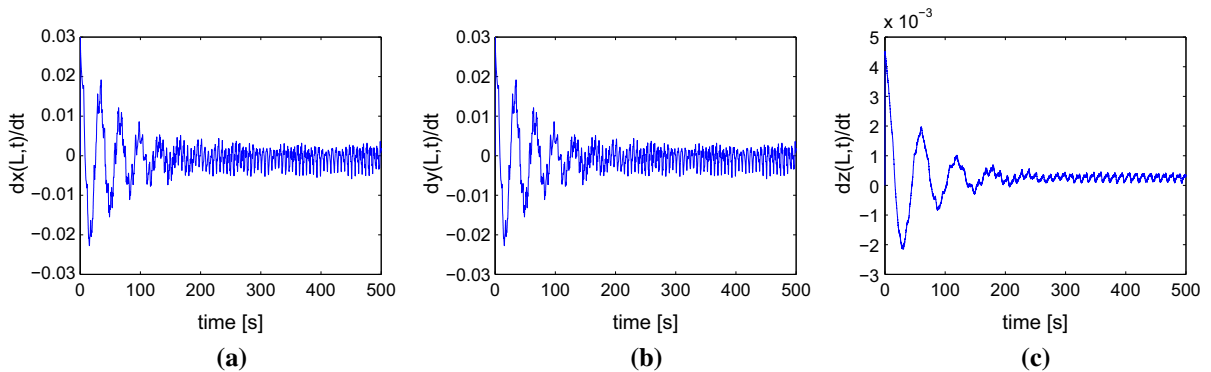


Fig. 11 Boundary slopes of the riser with the proposed control: **a** $x'(L, t)$; **b** $y'(L, t)$; **c** $z'(L, t)$

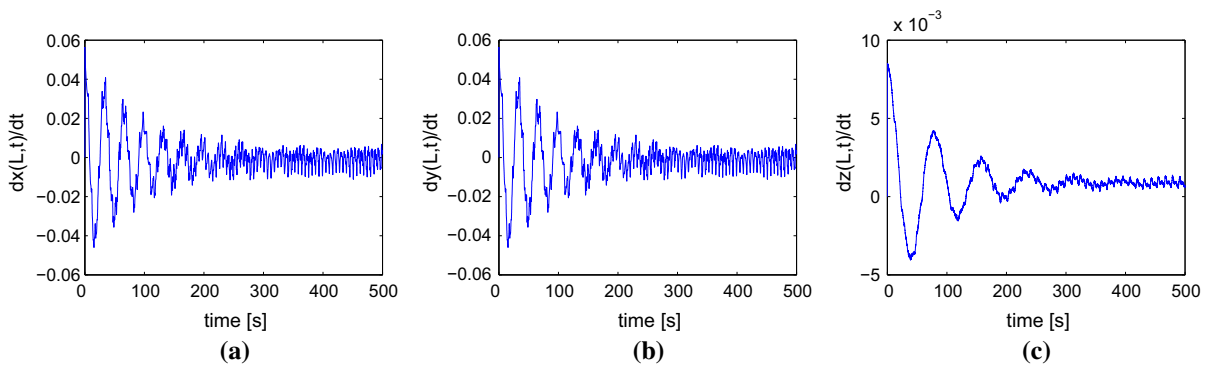


Fig. 12 Boundary slopes of the riser with the PD control: (a) $x'(L, t)$; (b) $y'(L, t)$; (c) $z'(L, t)$

$$d_{z_1}(t) = (3 + 0.2 \sin(5t)) \times 10^4 \tag{65}$$

For boundary disturbances (64) and (65), the periods are $T_{x1} = T_{y1} = 2\pi$ s, $T_{z1} = 0.4\pi$ s, and the frequencies are $f_{x1} = f_{y1} = 0.16$ Hz, $f_{z1} = 0.8$ Hz.

The simulation results are exactly the same with Figs. 5, 8 and 11 by using the same control gains,

i.e., $k_{1x} = 2 \times 10^3$, $k_{2x} = 8 \times 10^5$, $k_{1y} = 3 \times 10^3$, $k_{2y} = 4 \times 10^5$, $k_{1z} = 5 \times 10^3$ and $k_{2z} = 10^6$. From the simulation results, we can observe that changing the frequencies of the disturbances will not affect the control performance.

(ii) Changing the amplitude of the external disturbances: For this case, with the frequency of the exter-

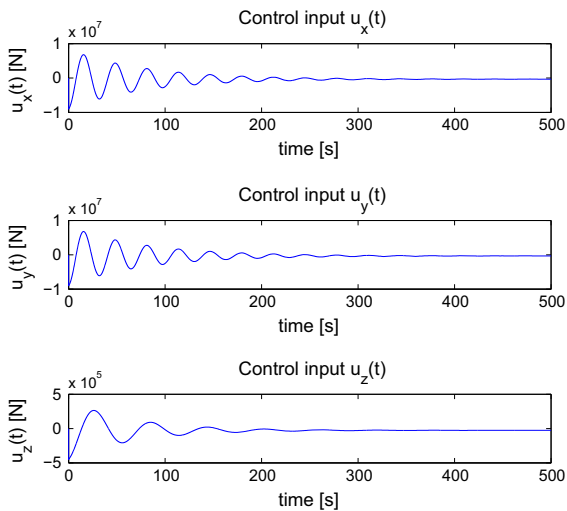


Fig. 13 Boundary control inputs $u_x(t)$, $u_y(t)$ and $u_z(t)$

nal disturbances unchanged, we change the amplitude of the external disturbances as

$$d_{x_2}(t) = d_{y_2}(t) = (5 + 0.8 \sin(0.7t) + 0.2 \sin(0.5t) + 0.2 \sin(0.9t)) \times 10^6 \tag{66}$$

$$d_{z_2}(t) = (5 + 0.2 \sin(0.5t)) \times 10^5 \tag{67}$$

The amplitudes of boundary disturbances (66) and (67) are $A_{x_2} = A_{y_2} = 6.2 \times 10^6$, $A_{z_2} = 5.2 \times 10^5$.

The simulation results are shown in Figs. 14, 15 and 16. The control gains are chosen as $k_{1x} = 8 \times 10^3$, $k_{2x} = 8 \times 10^5$, $k_{1y} = 8 \times 10^3$, $k_{2y} = 10^6$, $k_{1z} = 3 \times 10^5$ and $k_{2z} = 2 \times 10^6$.

From the simulation results, it can be observed that when we increase the amplitudes of the external disturbances, the proposed control can still suppress the vibration of the riser in three directions. In addition,

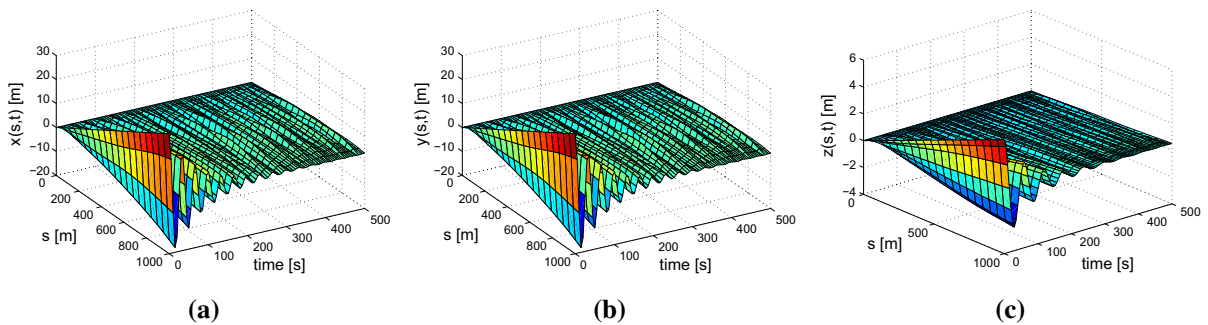


Fig. 14 Displacements of the riser with the proposed control under new disturbances (68) and (69): **a** $x(s, t)$; **b** $y(s, t)$; **c** $z(s, t)$

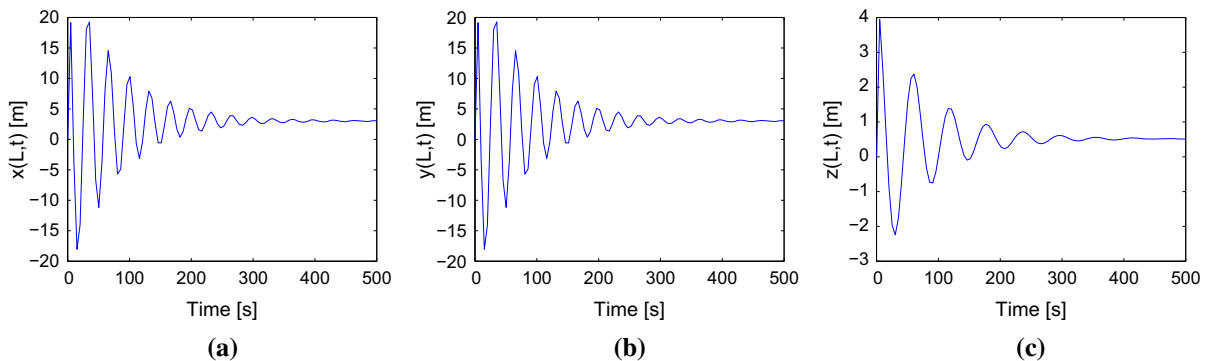


Fig. 15 Boundary displacement of the riser with the proposed control under new disturbances (68) and (69): **a** $x(L, t)$; **b** $y(L, t)$; **c** $z(L, t)$

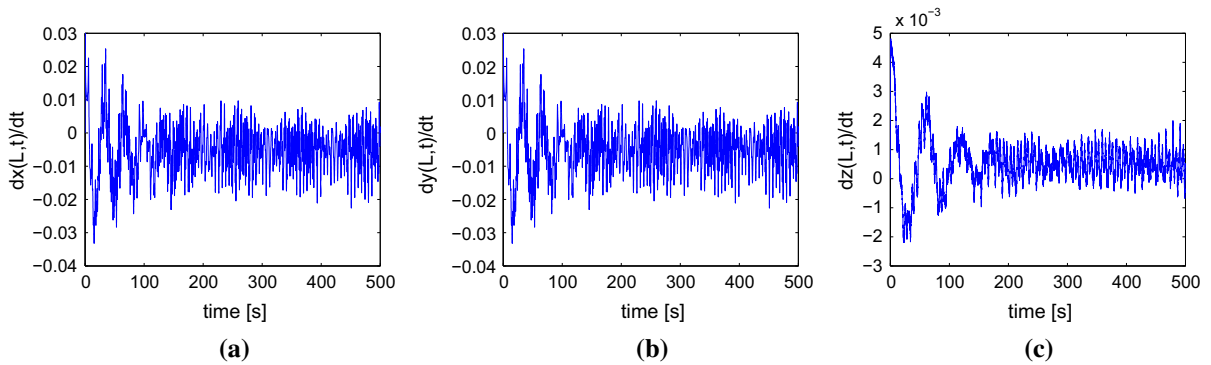


Fig. 16 Boundary slopes of the riser with the proposed control under new disturbances (68) and (69): **a** $x'(L, t)$; **b** $y'(L, t)$; **c** $z'(L, t)$

Table 2 Settling time for different disturbances

Boundary disturbances	Settling time for $x(L, t)$ (s)	Settling time for $y(L, t)$ (s)	Settling time for $z(L, t)$ (s)
(62) and (63)	419.1919	419.1919	363.6364
(64) and (65)	419.1919	419.1919	363.6364
(66) and (67)	429.2929	429.2929	424.2424

the proposed control laws can also ensure the boundary slopes of the riser $|x'_L| \leq C_x$, $|y'_L| \leq C_y$ and $|z'_L| \leq C_z$. However, the control gains become larger.

In addition, we have investigated the settling time for boundary displacements, i.e., the time required for the response of boundary displacements $x(L, t)$, $y(L, t)$ and $z(L, t)$ to settle within 5 % of their final values. The results are listed in Table 2.

From Table 2, we can see that the settling time for disturbances (64) and (65) are exactly the same with the settling time for disturbances (62) and (63), illustrating that changing the frequency of the disturbances will not affect the converging speed. However, increasing the amplitudes of the disturbances, all boundary displacements $x(L, t)$, $y(L, t)$ and $z(L, t)$ converge slower.

5 Conclusion

This paper has presented the boundary control design for a flexible marine riser system subjected to the environmental disturbances. With the proposed control, the vibration of the riser has been suppressed and the constraints of the joint angles have been handled. The bounded stability has been proved based on the Lyapunov's direct method. The numerical simulations have been carried out to illustrate the performance of the con-

trolled system. In this paper, effects of internal flow have been neglected for simplicity. In the future, we will discuss the model analysis and the control design of a flexible riser system with internal flow in three-dimensional space, which includes a more complicated dynamic model. In addition, the input nonlinearities will be under consideration for the riser system.

References

- Dai, S.-L., Wang, C., Luo, F.: Identification and learning control of ocean surface ship using neural networks. *IEEE Trans. Ind. Inf.* **8**(4), 801–810 (2012)
- How, B.V.E., Ge, S.S., Choo, Y.S.: Active control of flexible marine risers. *J. Sound Vib.* **320**, 758–776 (2009)
- Zulli, D., Luongo, A.: Nonlinear energy sink to control vibrations of an internally nonresonant elastic string. *Mechanica* **50**(3), 781–794 (2015)
- Luongo, A., Rega, G., Vestroni, F.: Planar non-linear free vibrations of an elastic cable. *Int. J. Non-Linear Mech.* **19**(1), 39–52 (1984)
- Oueini, S.S., Nayfeh, A.H., Pratt, J.R.: A nonlinear vibration absorber for flexible structures. *Nonlinear Dyn.* **15**(3), 259–282 (1998)
- Nayfeh, S.A., Nayfeh, A.H., Mook, D.T.: Nonlinear response of a taut string to longitudinal and transverse end excitation. *J. Vib. Control* **1**(3), 307–334 (1995)
- Do, K.D., Pan, J.: Boundary control of transverse motion of marine risers with actuator dynamics. *J. Sound Vib.* **318**, 768–791 (2008)
- Do, K.D., Pan, J.: Boundary control of three-dimensional inextensible marine risers. *J. Sound Vib.* **327**(3–5), 299–321 (2009)
- He, W., Sun, C., Ge, S.S.: Top tension control of a flexible marine riser by using integral-barrier Lyapunov function. *IEEE/ASME Trans. Mechatron.* **2**(20), 497–505 (2015)
- Wu, H.-N., Wang, J.-W.: Observer design and output feedback stabilization for nonlinear multivariable systems with diffusion PDE-governed sensor dynamics. *Nonlinear Dyn.* **72**(3), 615–628 (2013)

11. Wang, J.-W., Wu, H.-N., Li, H.-X.: Fuzzy control design for nonlinear ODE-hyperbolic PDE cascaded systems: a fuzzy and entropy-like Lyapunov function approach. *IEEE Trans. Fuzzy Syst.* **22**, 1313–1324 (2014)
12. Wang, J.-W., Wu, H.-N., Li, H.-X.: Stochastically exponential stability and stabilization of uncertain linear hyperbolic pde systems with Markov jumping parameters. *Automatica* **48**, 569–576 (2012)
13. Luo, B., Wu, H.-N., Li, H.-X.: Adaptive optimal control of highly dissipative nonlinear spatially distributed processes with neuro-dynamic programming. *IEEE Trans. Neural Netw. Learn. Syst.* **26**(4), 684–696 (2015)
14. Wang, N., Wu, H.-N., Guo, L.: Coupling-observer-based nonlinear control for flexible air-breathing hypersonic vehicles. *Nonlinear Dyn.* **1**(1), 1–24 (2014)
15. Ge, S.S., Lee, T.H., Zhu, G.: A nonlinear feedback controller for a single-link flexible manipulator based on a finite element model. *J. Robot. Syst.* **14**(3), 165–178 (1997)
16. He, W., Ouyang, Y., Hong, J.: Vibration control of a flexible robotic manipulator in the presence of input deadzone. *IEEE Trans. Ind. Inform.* (2016). doi:[10.1109/TII.2016.2608739](https://doi.org/10.1109/TII.2016.2608739)
17. Armaou, A., Christofides, P.: Wave suppression by nonlinear finite-dimensional control. *Chem. Eng. Sci.* **55**(14), 2627–2640 (2000)
18. Christofides, P., Armaou, A.: Global stabilization of the Kuramoto–Sivashinsky equation via distributed output feedback control. *Syst. Control Lett.* **39**(4), 283–294 (2000)
19. Balas, M.J.: Feedback control of flexible systems. *IEEE Trans. Autom. Control* **23**, 673–679 (1978)
20. Vandegrift, M.W., Lewis, F.L., Zhu, S.Q.: Flexible-link robot arm control by a feedback linearization/singular perturbation approach. *J. Robot. Syst.* **11**(7), 591–603 (1994)
21. Sun, C., He, W., Hong, J.: Neural network control of a flexible robotic manipulator using the lumped spring-mass mode. *IEEE Trans. Syst. Man Cybern. Syst.* (2016). doi:[10.1109/TSMC.2016.2562506](https://doi.org/10.1109/TSMC.2016.2562506)
22. Balas, M.J.: Active control of flexible systems. *J. Optim. Theory Appl.* **23**(3), 415–436 (1978)
23. Nguyen, Q.C., Hong, K.S.: Simultaneous control of longitudinal and transverse vibrations of an axially moving string with velocity tracking. *J. Sound Vib.* **331**(13), 3006–3019 (2012)
24. Nguyen, Q.C., Hong, K.-S.: Transverse vibration control of axially moving membranes by regulation of axial velocity. *IEEE Trans. Control Syst. Technol.* **20**(4), 1124–1131 (2012)
25. Guo, B.-Z., Jin, F.-F.: Output feedback stabilization for one-dimensional wave equation subject to boundary disturbance. *IEEE Trans. Autom. Control* **60**(3), 824–830 (2015)
26. Jin, F.-F., Guo, B.-Z.: Lyapunov approach to output feedback stabilization for the Euler–Bernoulli beam equation with boundary input disturbance. *Automatica* **52**(1), 95–102 (2015)
27. Yang, K.-J., Hong, K.-S., Matsuno, F.: Robust boundary control of an axially moving string by using a PR transfer function. *IEEE Trans. Autom. Control* **50**(12), 2053–2058 (2005)
28. Wu, Y., Xue, X., Shen, T.: Absolute stability of the Kirchhoff string with sector boundary control. *Automatica* **50**(7), 1915–1921 (2014)
29. He, W., Zhang, S., Ge, S.S.: Robust adaptive control of a thruster assisted position mooring system. *Automatica* **50**(7), 1843–1851 (2014)
30. Guo, Q., Yu, T., Jiang, D.: Robust h_∞ positional control of 2-DOF robotic arm driven by electro-hydraulic servo system. *ISA Trans.* **59**, 55–64 (2015)
31. Kang, Y., Zhai, D.-H., Liu, G.-P., Zhao, Y.-B., Zhao, P.: Stability analysis of a class of hybrid stochastic retarded systems under asynchronous switching. *IEEE Trans. Autom. Control* **59**(6), 1511–1523 (2014)
32. Kang, Y., Zhai, D.-H., Liu, G.-P., Zhao, Y.-B.: On input-to-state stability of switched stochastic nonlinear systems under extended asynchronous switching. *IEEE Trans. Cybern.* **46**(5), 1092–1105 (2016)
33. Guo, Q., Yu, T., Jiang, D.: High-gain observer-based output feedback control of single-rod electro-hydraulic actuator. *IET Control Theory Appl.* **9**(16), 2395–2404 (2015)
34. Li, Y., Ge, S.S.: Human-robot collaboration based on motion intention estimation. *IEEE/ASME Trans. Mechatron.* **19**(3), 1007–1014 (2014)
35. Yang, C., Li, Z., Cui, R., Xu, B.: Neural network-based motion control of an underactuated wheeled inverted pendulum model. *IEEE Trans. Neural Netw. Learn. Syst.* **25**(11), 2004–2016 (2014)
36. Gong, D., Lewis, F.L., Wang, L., Xu, K.: Synchronization for an array of neural networks with hybrid coupling by a novel pinning control strategy. *Neural Netw.* **77**, 41–50 (2016)
37. Zhang, S., He, W., Huang, D.: Active vibration control for a flexible string system with input backlash. *IET Control Theory Appl.* **10**(7), 800–805 (2016)
38. Wang, J.-M., Liu, J.-J., Ren, B., Chen, J.: Sliding mode control to stabilization of cascaded heat PDE–ODE systems subject to boundary control matched disturbance. *Automatica* **52**, 23–34 (2015)
39. Paranjape, A.A., Guan, J., Chung, S.-J., Krstic, M.: PDE boundary control for flexible articulated wings on a robotic aircraft. *IEEE Trans. Robotics* **29**(3), 625–640 (2013)
40. Bernard, P., Krstic, M.: Adaptive output-feedback stabilization of non-local hyperbolic pdes. *Automatica* **50**(10), 2692–2699 (2014)
41. Liu, Z., Liu, J.-K., He, W.: Adaptive boundary control of a flexible manipulator with input saturation. *Int. J. Control* **89**(6), 1191–1202 (2016)
42. He, W., Zhang, S.: Control design for nonlinear flexible wings of a robotic aircraft. *IEEE Trans. Control Syst. Technol.* (2016). doi:[10.1109/TCST.2016.2536708](https://doi.org/10.1109/TCST.2016.2536708)
43. Zhao, Z., Liu, Y., He, W., Fei, L.: Adaptive boundary control of an axially moving belt system with high acceleration/deceleration. *Int. J. Syst. Sci.* **10**(11), 1299–1306 (2016)
44. Wu, H.-N., Wang, J.-W.: Static output feedback control via pde boundary and ode measurements in linear cascaded ode-beam systems. *Automatica* **50**(11), 2787–2798 (2014)
45. He, W., Ge, S.S., How, B.V.E., Choo, Y.S., Hong, K.-S.: Robust adaptive boundary control of a flexible marine riser with vessel dynamics. *Automatica* **47**(4), 722–732 (2011)
46. He, W., He, X., Ge, S.S.: Vibration control of flexible marine riser systems with input saturation. *IEEE/ASME Trans. Mechatron.* **21**(1), 254–265 (2016)
47. Gao, Y., Wu, H., Wang, J., Guo, L.: Feedback control design with vibration suppression for flexible air-breathing hypersonic vehicles. *Sci. China Inf. Sci.* **57**(3), 1–14 (2014)

48. He, W., Nie, S., Meng, T., Liu, Y.-J.: Modeling and vibration control for a moving beam with application in a drilling riser. *IEEE Trans. Control Syst. Technol.* (2016). doi:[10.1109/TCST.2016.2577001](https://doi.org/10.1109/TCST.2016.2577001)
49. He, W., Ge, S.S.: Cooperative control of a nonuniform gantry crane with constrained tension. *Automatica* **66**(4), 146–154 (2016)
50. He, W., Ge, S.S., Huang, D.: Modeling and vibration control for a nonlinear moving string with output constraint. *IEEE/ASME Trans. Mechatron.* **20**(4), 1886–1897 (2015)
51. Tee, K.P., Ge, S.S., Tay, E.: Barrier Lyapunov functions for the control of output-constrained nonlinear systems. *Automatica* **45**(4), 918–927 (2009)
52. He, W., Yin, Z., Sun, C.: Adaptive neural network control of a marine vessel with constraints using the asymmetric barrier Lyapunov function. *IEEE Trans. Cybern.* (2016). doi:[10.1109/TCYB.2016.2554621](https://doi.org/10.1109/TCYB.2016.2554621)
53. Tee, K.P., Ren, B., Ge, S.S.: Control of nonlinear systems with time-varying output constraints. *Automatica* **47**(11), 2511–2516 (2011)
54. He, W., Chen, Y., Yin, Z.: Adaptive neural network control of an uncertain robot with full-state constraints. *IEEE Trans. Cybern.* **46**(3), 620–629 (2016)
55. He, W., Ge, S.S.: Vibration control of a flexible beam with output constraint. *IEEE Trans. Ind. Electron.* **62**(8), 5023–5030 (2015)
56. Goldstein, H.: *Classical Mechanics*. Addison-Wesley, Reading, Mass (1951)
57. Queiroz, M.S., Dawson, D.M., Nagarkatti, S.P., Zhang, F.: *Lyapunov Based Control of Mechanical Systems*. Birkhauser, Boston (2000)
58. Faltinsen, O.M.: *Sea Loads on Ships and Offshore Structures*. Cambridge University Press, New York (1990)