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Continuous finite time control for static var compensator with mismatched disturbances

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Abstract In this paper, the composite anti-disturbance control problem is addressed for a single machine bus system with static var compensator. A finite time disturbance observer is designed to estimate the external disturbances. Then based on disturbance estimation value, a continuous finite time anti-disturbance controller is proposed. It is proved that the proposed scheme can guarantee that the system outputs converge to zero in finite time. Finally, a simulation result is presented to demonstrate the effectiveness of the developed method.

Keywords Finite time control · Finite time disturbance observer · Mismatched disturbances · Static var compensator

1 Introduction

In the past decades, static var compensator (SVC) has been used in power system to regulate the system voltage and improve power system stability [1–3]. SVC has many virtues over traditional reactive power system compensators. Many meaningful results have been reported for SVC in the past decades. Based on exact linearization scheme, the SVC controllers have been developed in [4,5]. Although these controllers can guarantee system have good control performance, they may cause unsatisfactory performances when operating states are far from given operating points. Furthermore, on the basis of nonlinear system model, some control approaches have been proposed. In [6], the adaptive fuzzy controller is presented for SVCs based on backstepping control methods. The adaptive fault-tolerant controller is proposed in [7]. When system subjects to external disturbances, the methods in [6,7] may obtain unsatisfactory performance. In [8], an adaptive backstepping sliding mode H_∞ controller is given, where H_∞ control scheme is used to attenuate external disturbances. In [9], a nonlinear robust controller is investigated for the SVC system with external disturbances and parameter uncertainties using modified adaptive backstepping and minmax scheme.

Although these methods have good control performance of the SVC system, the disturbance rejection and robustness performance of these controllers are achieved at a price of sacrificing the nominal control performance. When subjecting to strong disturbances, these approaches may lead to poor performance, for example, the dynamic process of the closed-loop system may become sluggish and even unstable. It is because most of above control schemes reject disturbances merely via the action of feedback regulation in a relatively slow way and do not consider

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active and direct disturbance rejection in the controller design. Disturbance observer based control (DOBC) is an effective method to reject external disturbances and improve robustness against uncertainties [10–25]. So far, DOBC method can be used to cope with both matched disturbances [10–18] and mismatched disturbances [22–25].

In order to guarantee system have a faster convergence rate and a stronger disturbance rejection performance of systems with mismatched disturbances, some finite time composite anti-disturbance control schemes are proposed. Based on finite time disturbance observer and non-singular terminal sliding mode control techniques, a finite time composite controller is developed for rejecting mismatched disturbances in [26]. In [27], a continuous finite time composite anti-disturbance controller is proposed for a class of linear system with mismatched disturbance via finite time disturbance observer and added power integrator methods. Inspired by the above reference, we devote to investigating the finite time anti-disturbance control problem for SVC system with mismatched disturbances. Because the SVC system is a complex nonlinear system with mismatched disturbances, the previous results are difficult to directly apply. This motives us to develop this study.

In this paper, the problem of finite time composite anti-disturbance control for SVC system with mismatched disturbances is addressed. By using finite time disturbance observer, the mismatched disturbances are estimated. Based on disturbance estimation, some novel virtual control laws are constructed to compensate the mismatched disturbances. Then finite time stability is established via Lyapunov function theory. With the proposed composite control method, the system output can converge to zero in finite time in spite of mismatched disturbances and the disturbance rejection ability of system is improved without sacrificing the nominal performance of the original control strategy. Finally, a simulation result is employed to demonstrated the effectiveness of the proposed control scheme.

2 Model and problem formulation

The dynamics of single-machine infinite-bus (SMIB) electrical power system with SVC can be depicted by the following nonlinear equation [4]

$$\begin{aligned}\dot{\delta} &= \omega - \omega_0,\\ \dot{\omega} &= -\frac{D}{H}(\omega-\omega_0) + \frac{\omega_0}{H}\left(P_m - E_q' V_s y_{\mathrm{SVC}} \sin\delta\right) + w_2,\\ \dot{y}_{\mathrm{SVC}} &= \frac{1}{T_{\mathrm{SVC}}}(-y_{\mathrm{SVC}} + y_{\mathrm{SVC}0} + u) + w_3,\end{aligned} \tag{1}$$

where δ and ω represent the angle and speed of the generator rotor, respectively; H, P_m, D, E_q', V_s and T_{SVC} are the inertia constant, the mechanical power on the generator shaft, the damping coefficient, the inner generator voltage and infinite bus voltage, the time constant of SVC regulator, respectively. $y_{\mathrm{SVC}} = \frac{1}{X_1} + X_2 + X_1X_2(B_L + B_C)$ denotes the susceptance of the overall system, and $y_{\mathrm{SVC}0}$ is the initial stable value of y_{SVC}; $X_1 = X_d' + X_T + X_L$, $X_2 = X_L$, X_d' X_T, and X_L mean the direct axis transient reactance of the generator, the reactance of the transformer, and the line reactance, respectively; B_L, B_C, and $B_L + B_C$ show the susceptance of the inductor in SVC, the susceptance of the capacitor in SVC, and the equivalence reactance of SVC, respectively; u denotes the equivalence input of SVC regulator; w_2 and w_3 mean the external disturbances.

Let $(\delta_0, \omega_0, y_{\mathrm{SVC}0})$ denote an operating point of the power system. Define the system state variables as $x_1 = \delta - \delta_0$, $x_2 = \omega - \omega_0$, $x_3 = y_{\mathrm{SVC}} - y_{\mathrm{SVC}0}$. Furthermore, letting $\frac{\omega_0}{H}P_m = a_0$, $-\frac{\omega_0}{H}E_q'V_s = k$, system (1) is rewritten as

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \theta x_2 + a_0 + k(x_3 + y_{\mathrm{SVC}0})\sin(x_1+\delta_0) + w_2,\\ \dot{x}_3 &= -\frac{1}{T_{\mathrm{SVC}}}x_3 + \frac{1}{T_{\mathrm{SVC}}}u + w_3,\\ y &= \begin{bmatrix} q_1x_1 \\ q_2x_2\end{bmatrix},\end{aligned} \tag{2}$$

where y is the regulated output, q_1 and q_2 are nonnegative weighted coefficients, $\theta = -\frac{D}{H}$.

Assumption 1 The disturbances satisfy the following condition $|\dddot{w}_i(t)| \le L_i$, where L_i are known constants, $i = 2, 3$.

According to [28], the following assumption is required for controller design.

Assumption 2 The angle δ satisfies $0° < \delta < 180°$.

Remark 1 If $\sin(x_1 + \delta_0) = 0$, then $\delta = k\pi$, $k = 0, 1, 2, 3, \ldots$, which implies that the power systems do not maintain synchronism. Therefore, the normal region of the power system is $0° < \delta < 180°$.

In order to obtain a finite-time composite controller, some lemmas that will play a key role in the subsequent control development and analysis are revisited as follows.

Lemma 1 [29] *If* $0 < \ell = \frac{\ell_1}{\ell_2} < 1$, *then* $|x^\ell - y^\ell| \leq 2^{1-\ell}|x-y|^\ell$, *where* ℓ_1 *and* ℓ_2 *are positive odd integers.*

Lemma 2 [30] *The inequality* $(|x_1| + \cdots + |x_n|)^p \leq |x_1|^p + \cdots + |x_n|^p$ *holds for* $x_i \in R$, $i = 1, \ldots, n$, *where* p *is a real number satisfying* $0 < p \leq 1$.

Control object In this paper, the problem of finite time output regulation is investigated for system (2) under mismatched disturbances using the finite time disturbance observer and finite time control technique. We aim to design a composite controller such that system outputs converge to zero in finite time with the mismatched disturbances.

3 Composite controller design and stability analysis

3.1 Composite controller design

The problem of finite time output regulation for system (2) with mismatched disturbances is investigated by using a composite finite time controller. The detailed design method is given step by step as follows.

Part I Finite time disturbance observer design.

Borrowed from [31,32], a finite time disturbance observer (FTDO) is presented as

$$\begin{aligned}
\dot{z}_{0j} &= v_{0j} + \hat{x}_{j+1}, \dot{z}_{ij} = v_{ij}, \ldots, \dot{z}_{3j} \\
&= -\lambda_3 L_j \mathrm{sign}(z_{3j} - v_{2j}), \\
v_{0j} &= -\lambda_0 L_j^{\frac{1}{4}} |z_{0j} - x_j|^{\frac{3}{4}} \mathrm{sign}(z_{0j} - x_j) + z_{1j}, \\
v_{ij} &= -\lambda_i L_j^{1/(4-i))} |z_{ij} - v_{(i-1)j}|^{\frac{3-i}{4-i}} \mathrm{sign}(z_{ij} \\
&\quad - v_{(i-1)j}) + z_{(i+1)j},
\end{aligned} \tag{3}$$

where $i = 1, 2$, $j = 1, 2, 3$, $\hat{x}_2 = x_2$, $\hat{x}_3 = \theta x_2 + a_0 + k(x_3 + y_{\mathrm{SVC0}}) \sin(x_1 + \delta_0)$, $\hat{x}_4 = -\frac{1}{T_{\mathrm{SVC}}} x_3 + \frac{1}{T_{\mathrm{SVC}}} u$, $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ are the observer coefficients to be designed, $z_{0j}, z_{1j}, z_{2j}, z_{3j}$ are the estimates of $x_j, w_j, \dot{w}_j, \ddot{w}_j$, respectively.

Define the observer errors $e_{0j} = z_{0j} - x_j$, $e_{1j} = z_{1j} - w_j$, $e_{2j} = z_{2j} - \dot{w}_j$, and $e_{3j} = z_{3j} - \ddot{w}_j$, where $w_1 = \dot{w}_1 = \ddot{w}_1 = 0$. The observer error dynamics are presented as

$$\begin{aligned}
\dot{e}_{0j} &= -\lambda_0 L_j^{\frac{1}{4}} |e_{0j}|^{\frac{3}{4}} \mathrm{sign}(e_{0j}) + e_{1j}, \\
\dot{e}_{ij} &= -\lambda_i L_j^{\frac{1}{4-i}} |e_{ij} - \dot{e}_{(i-1)j}|^{\frac{3-i}{4-i}} \mathrm{sign}(e_{ij} - \dot{e}_{(i-1)j}) \\
&\quad + e_{(i+1)j}, \\
\dot{e}_{3j} &= -\lambda_3 L_j \mathrm{sign}(e_{3j} - \dot{e}_{2j}) - \ddot{w}_j.
\end{aligned} \tag{4}$$

It can be obtained from [31,32] that the observer error system is finite time stable, that is, there exists a finite time instant t_1 such that $e_{0j} \equiv 0$, $e_{1j} \equiv 0$, $e_{2j} \equiv 0$, $e_{3j} \equiv 0$, $j = 1, 2, 3$, for $t > t_1$.

Remark 1 According to [31], the parameters λ_0, λ_1, λ_2, λ_3 are selected. The convergence rate of FTDO is determined by the values of L_j, i.e., the faster convergence rate of the FTDO, the larger the parameters L_j required. However, the parameters L_j can not be selected too large to avoid resulting in an excessive transient peaking.

When $t > t_1$, the system (3) is changed to

$$\dot{z}_{0j} = z_{1j} + \hat{x}_{j+1}, \quad \dot{z}_{1j} = z_{2j}, \quad \dot{z}_{2j} = z_{3j}, \tag{5}$$

where $j = 1, 2, 3$.

Part II Composite controller design.

Combining disturbance estimation values, system (2) is rewritten as

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \theta x_2 + a_0 + k(x_3 + y_{\mathrm{SVC0}}) \sin(x_1 + \delta_0) + z_{12} - e_{12}, \\
\dot{x}_3 &= -\frac{1}{T_{\mathrm{SVC}}} x_3 + \frac{1}{T_{\mathrm{SVC}}} u + z_{13} - e_{13}, \\
y &= \begin{bmatrix} q_1 x_1 \\ q_2 x_2 \end{bmatrix}.
\end{aligned} \tag{6}$$

Since the disturbance estimation errors satisfy $e_{12} = 0$, $e_{13} = 0$ for $t > t_1$, system (6) boils down to

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \theta x_2 + a_0 + k(x_3 + y_{\mathrm{SVC0}}) \sin(x_1 + \delta_0) + z_{12}, \\
\dot{x}_3 &= -\frac{1}{T_{\mathrm{SVC}}} x_3 + \frac{1}{T_{\mathrm{SVC}}} u + z_{13}, \\
y &= \begin{bmatrix} q_1 x_1 \\ q_2 x_2 \end{bmatrix}.
\end{aligned} \tag{7}$$

In the next, the composite finite time controller will be developed for system (7). The stability analysis will be given in the next subsection.

Step 1 Consider the first equation in (7), i.e.,

$$\dot{x}_1 = x_2. \tag{8}$$

Choose a Lyapunov function as

$$V_1 = \int_0^{x_1} (s^{\frac{1}{r_1}})^{2-r_2} \mathrm{d}s, \tag{9}$$

where $r_1 = 1, r_2 = r_1 + \tau$, $-\frac{1}{4} < \tau = -\frac{q}{p} < 0$, q is a positive even integer, p is a positive odd integer. Computing the first derivative of (9) along system (8), yields

$$\dot{V}_1 = x_1^{\frac{2-r_2}{r_1}} x_2 = x_1^{\frac{2-r_2}{r_1}} x_2^* + x_1^{\frac{2-r_2}{r_1}} (x_2 - x_2^*), \tag{10}$$

where x_2^* is a virtual control law. The virtual control law x_2^* is designed as

$$x_2^* = -\beta_1 x_1^{r_2}, \tag{11}$$

where $\beta_1 > \beta_1^* = 3$. Combining (10) and (11), we have

$$\dot{V}_1 \le -3x_1^2 + x_1^{\frac{2-r_2}{r_1}} (x_2 - x_2^*). \tag{12}$$

Step 2 Consider

$$\dot{x}_2 = \theta x_2 + a_0 + k(x_3 + y_{SVC0}) \sin(x_1 + \delta_0) + z_{12}. \tag{13}$$

The Lyapunov function is selected as

$$V_2 = V_1 + \int_{x_2^*}^{x_2} \left(s^{\frac{1}{r_2}} - (x_2^*)^{\frac{1}{r_2}} \right)^{2-r_2-\tau} \mathrm{d}s. \tag{14}$$

Computing the first derivative of (14), we have

$$\begin{aligned}\dot{V}_2 \le& -3x_1^2 + x_1^{\frac{2-r_2}{r_1}} (x_2 - x_2^*) + \xi_2^{2-r_2-\tau} (\theta x_2 \\ &+ a_0 + k(x_3 + y_{SVC0}) \sin(x_1 + \delta_0) + z_{12}) \\ &- (2 - r_2 - \tau) \int_{x_2^*}^{x_2} \left(s^{\frac{1}{r_2}} - (x_2^*)^{\frac{1}{r_2}} \right)^{1-r_2-\tau} \\ &\times \mathrm{d}s \frac{\partial x_2^{*\frac{1}{r_2}}}{\partial x_1} x_2,\end{aligned} \tag{15}$$

where $\xi_2 = x_2^{\frac{1}{r_2}} - (x_2^*)^{\frac{1}{r_2}}$.

Note that $0 < r_2 \le 1$, by Lemma 1 and Young's inequality, we have

$$\begin{aligned}& x_1^{\frac{2-r_2}{r_1}} (x_2 - x_2^*) \le 2^{1-r_2} |x_1|^{\frac{2-r_2}{r_1}} |\xi_2|^{r_2} \le \frac{1}{2} x_1^2 + \hat{c}_{21} \xi_2^2, \\ & - (2 - r_2 - \tau) \int_{x_2^*}^{x_2} \left(s^{\frac{1}{r_2}} - (x_2^*)^{\frac{1}{r_2}} \right)^{1-r_2-\tau} \mathrm{d}s \frac{\partial x_2^{*\frac{1}{r_2}}}{\partial x_1} x_2 \\ & \le (2 - r_2 - \tau) \left| \frac{\partial x_2^{*\frac{1}{r_2}}}{\partial x_1} \right| |\xi_2|^{1-r_2-\tau} (|x_2 - x_2^*|)(|x_2 \\ & - x_2^*| + |x_2^*|) \\ & \le (2 - r_2 - \tau) 2^{1-r_2} \left| \frac{\partial x_2^{*\frac{1}{r_2}}}{\partial x_1} \right| |\xi_2|^{1-\tau} (2^{1-r_2} |\xi_2|^{r_2} \\ & + \beta_1 |x_1|^{r_2}) \\ & \le \frac{1}{2} |x_1|^2 + \hat{c}_{22} |\xi_2|^2,\end{aligned} \tag{16}$$

where $\hat{c}_{21} > 0$ and $\hat{c}_{22} > 0$. Combining (15) and (16), yields

$$\begin{aligned}\dot{V}_2 \le& -2x_1^2 + \xi_2^{2-r_2-\tau} (\theta x_2 + a_0 + k(x_3 + y_{SVC0}) \\ & \sin(x_1 + \delta_0) + z_{12}) + (\hat{c}_{21} + \hat{c}_{22}) \xi_2^2 \\ =& -2x_1^2 + \xi_2^{2-r_2-\tau} (\theta x_2 + a_0 + k(x_3 - x_3^* + x_3^* \\ & + y_{SVC0}) \sin(x_1 + \delta_0) + z_{12}) + (\hat{c}_{21} + \hat{c}_{22}) \xi_2^2,\end{aligned} \tag{17}$$

where x_3^* is a virtual control law. The virtual control law x_3^* is designed as

$$\begin{aligned}x_3^* =& \frac{1}{k \sin(x_1 + \delta_0)} (-\theta x_2 - a_0 - k y_{SVC} \sin(x_1 + \delta_0) \\ & - z_{12} - \beta_2 \xi_2^{r_3}),\end{aligned} \tag{18}$$

where $\beta_2 > \beta_2^* = 2 + \hat{c}_{21} + \hat{c}_{22}$, $r_3 = r_2 + \tau$. Substituting (18) into (17), leads to

$$\dot{V}_2 \le -2x_1^2 - 2\xi_2^2 + \xi_2^{2-r_2-\tau} k(x_3 - x_3^*) \sin(x_1 + \delta_0). \tag{19}$$

Step 3 Consider

$$\dot{x}_3 = -\frac{1}{T_{SVC}} x_3 + \frac{1}{T_{SVC}} u + z_{13}. \tag{20}$$

We choose the following Lyapunov function

$$V_3 = V_2 + \int_{\bar{x}_3^*}^{\bar{x}_3} \left(s^{\frac{1}{r_3}} - (\bar{x}_3^*)^{\frac{1}{r_3}} \right)^{2-r_3-\tau} \mathrm{d}s, \tag{21}$$

where

$$\begin{aligned}\bar{x}_3 =& k \sin(x_1 + \delta_0) x_3 + \theta x_2 + a_0 \\ & + k y_{SVC} \sin(x_1 + \delta_0) + z_{12}, \\ \bar{x}_3^* =& k \sin(x_1 + \delta_0) x_3^* + \theta x_2 + a_0 \\ & + k y_{SVC} \sin(x_1 + \delta_0) + z_{12} = -\beta_2 \xi_2^{r_3}.\end{aligned} \tag{22}$$

Computing the first derivative of (21) gives rise to

$$\begin{aligned}\dot{V}_3 \le& -2x_1^2 - 2\xi_2^2 + \xi_2^{2-r_2-\tau} k(x_3 - x_3^*) \sin(x_1 + \delta_0) \\ & + \xi_3^{2-r_3-\tau} \dot{\bar{x}}_3 - (2 - r_3 - \tau) \int_{\bar{x}_3^*}^{\bar{x}_3} \\ & \times \left(s^{\frac{1}{r_3}} - (\bar{x}_3^*)^{\frac{1}{r_3}} \right)^{1-r_3-\tau} \mathrm{d}s \left(\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_1} \dot{x}_1 + \frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_2} \dot{x}_2 \right) \\ =& -2x_1^2 - 2\xi_2^2 + \xi_2^{2-r_2-\tau} k(x_3 - x_3^*) \sin(x_1 + \delta_0) \\ & + \xi_3^{2-r_3-\tau} \left(\frac{\partial \bar{x}_3}{\partial x_1} x_2 + \frac{\partial \bar{x}_3}{\partial x_2} (k \sin(x_1 + \delta_0) x_3 \right. \\ & + \theta x_2 + a_0 + k y_{SVC} \sin(x_1 + \delta_0) + z_{12}) \\ & \left. + \frac{\partial \bar{x}_3}{\partial x_3} \left(-\frac{1}{T_{SVC}} x_3 + \frac{1}{T_{SVC}} u + z_{13} \right) + \frac{\partial \bar{x}_3}{\partial z_{12}} z_{22} \right)\end{aligned}$$

$$-(2-r_3-\tau)\int_{\bar{x}_3^*}^{\bar{x}_3}\left(s^{\frac{1}{r_3}}-(\bar{x}_3^*)^{\frac{1}{r_3}}\right)^{1-r_3-\tau}\times \mathrm{d}s\left(\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_1}\dot{x}_1+\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_2}\dot{x}_2\right), \quad (23)$$

where $\xi_3=\bar{x}_3^{\frac{1}{r_3}}-(\bar{x}_3^*)^{\frac{1}{r_3}}$, $\frac{\partial \bar{x}_3}{\partial x_1}=k\cos(x_1+\delta_0)(x_3+y_{\mathrm{SVC}})$, $\frac{\partial \bar{x}_3}{\partial x_2}=\theta$, $\frac{\partial \bar{x}_3}{\partial x_3}=k\sin(x_1+\delta_0)$, $\frac{\partial \bar{x}_3}{\partial z_{12}}=1$.

Combining (22) and Lemma 1 plus Young's inequality, we obtain

$$\begin{aligned}&\xi_2^{2-r_2-\tau}k(x_3-x_3^*)\sin(x_1+\delta_0)\\&=\xi_2^{2-r_2-\tau}(\bar{x}_3-\bar{x}_3^*)\\&\le 2^{1-r_2}|\xi_2|^{2-r_2-\tau}|\xi_3|^{r_3}\le\frac{1}{2}|\xi_2|^2+\hat{c}_{31}|\xi_3|^2,\end{aligned} \quad (24)$$

where $\hat{c}_{31}>0$. Now, we estimate the last term on the right-hand side of (23).

First, it follows Lemma 1 and Young's inequality that

$$\begin{aligned}&-(2-r_3-\tau)\int_{\bar{x}_3^*}^{\bar{x}_3}\left(s^{\frac{1}{r_3}}-(\bar{x}_3^*)^{\frac{1}{r_3}}\right)^{1-r_3-\tau}\mathrm{d}s\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_1}\dot{x}_1\\&\le(2-r_3-\tau)\left|\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_1}\right||\xi_3|^{1-r_3-\tau}|\bar{x}_3-\bar{x}_3^*||x_2|\\&\le(2-r_3-\tau)\left|\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_1}\right||\xi_3|^{1-r_3-\tau}|\bar{x}_3-\bar{x}_3^*|\\&\quad\times\left(|x_2-x_2^*|+|x_2^*|\right)\\&\le(2-r_3-\tau)\left|\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_1}\right||\xi_3|^{1-\tau}\\&\quad\times\left(2^{1-r_2}|\xi_2|^{r_2}+\beta_1|x_1|^{r_2}\right)\\&\le\frac{1}{3}x_1^2+\frac{1}{4}\xi_2^2+\hat{c}_{31}\xi_3^2+\hat{c}_{32}\xi_3^2,\end{aligned} \quad (25)$$

where $\hat{c}_{32}>0$. And

$$\begin{aligned}&-(2-r_3-\tau)\int_{\bar{x}_3^*}^{\bar{x}_3}\left(s^{\frac{1}{r_3}}-(\bar{x}_3^*)^{\frac{1}{r_3}}\right)^{1-r_3-\tau}\mathrm{d}s\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_2}\dot{x}_2\\&\le(2-r_3-\tau)\left|\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_2}\right||\xi_3|^{1-r_3-\tau}|\bar{x}_3-\bar{x}_3^*|\beta_2^{r_3}\\&\quad\times|x_2|^{\frac{1}{r_2}-1}|\theta x_2+a_0+k(x_3+y_{\mathrm{SVC0}})\sin(x_1+\delta_0)+z_{12}|\\&=\beta_2^{\frac{1}{r_3}}\beta_2^{r_3}(2-r_3-\tau)|\xi_3|^{1-r_3-\tau}|\bar{x}_3-\bar{x}_3^*||x_2|^{\frac{1}{r_2}-1}|\bar{x}_3|\\&=\beta_2^{\frac{1}{r_3}}\beta_2^{r_3}(2-r_3-\tau)|\xi_3|^{1-r_3-\tau}|\bar{x}_3-\bar{x}_3^*||x_2|^{\frac{1}{r_2}-1}\\&\quad\left(|\bar{x}_3-\bar{x}_3^*|+|\bar{x}_3^*|\right)\\&\le\beta_2^{\frac{1}{r_3}}\beta_2^{r_3}(2-r_3-\tau)2^{1-r_3}|\xi_3|^{1-\tau}|x_2|^{\frac{1}{r_2}-1}\\&\quad\left(2^{1-r_3}|\xi_3|^{r_3}+\beta_2|\xi_2|^{r_3}\right).\end{aligned} \quad (26)$$

Using Lemmas 1 and 2, we have

$$\begin{aligned}|x_2|^{\frac{1}{r_2}-1}&=|x_2-x_2^*+x_2^*|^{\frac{1}{r_2}-1}\\&\le|x_2-x_2^*|^{\frac{1}{r_2}-1}+|x_2^*|^{\frac{1}{r_2}-1}\\&\le 2^{1-r_2}|\xi_2|^{1-r_2}+\beta_1^{\frac{1}{r_2}-1}|x_1|^{1-r_2}.\end{aligned} \quad (27)$$

Substituting (27) into (26) and using Lemma 1 leads to

$$\begin{aligned}&\beta_2^{\frac{1}{r_3}}(2-r_3-\tau)\int_{\bar{x}_3^*}^{\bar{x}_3}\left(s^{\frac{1}{r_3}}-(\bar{x}_3^*)^{\frac{1}{r_3}}\right)^{1-r_3-\tau}\mathrm{d}s\frac{\partial \bar{x}_3^{*\frac{1}{r_3}}}{\partial x_2}\dot{x}_2\\&\le\beta_2^{\frac{1}{r_3}}\beta_2^{r_3}(2-r_3-\tau)2^{1-r_3}|\xi_3|^{1-\tau}\Big(2^{1-r_2}|\xi_2|^{1-r_2}\\&\quad+\beta_1^{\frac{1}{r_2}-1}|x_1|^{1-r_2}\Big)\left(2^{1-r_3}|\xi_3|^{r_3}+\beta_2|\xi_2|^{r_3}\right)\\&\le\beta_2^{\frac{1}{r_3}}\beta_2^{r_3}(2-r_3-\tau)2^{1-r_3}\Big(2^{2-r_2-r_3}|\xi_2|^{1-r_2}|\xi_3|^{1-\tau}\\&\quad+2^{1-r_2}|\xi_2|^{1-r_2+r_3}|\xi_3|^{1-\tau}\\&\quad+2^{1-r_3}\beta_1^{\frac{1}{r_2}-1}|x_1|^{1-r_2}|\xi_3|^{1-\tau+r_3}+\beta_1^{\frac{1}{r_2}-1}\beta_2\\&\quad\times|x_1|^{1-r_2}|\xi_2|^{r_3}|\xi_3|^{1-\tau}\Big)\\&\le\frac{1}{4}\xi_2^2+\hat{c}_{33}\xi_3^2+\frac{1}{4}\xi_2^2+\hat{c}_{34}\xi_3^2+\frac{1}{3}x_1^2+\hat{c}_{35}\xi_3^2\\&\quad+\frac{1}{3}x_1^2+\frac{1}{4}\xi_2^2+\hat{c}_{36}\xi_3^2,\end{aligned} \quad (28)$$

where $\hat{c}_{33}, \hat{c}_{34}, \hat{c}_{35}$, and $\hat{c}_{36}$ are positive constants. Combining (23), (25) and (28), yields

$$\begin{aligned}\dot{V}_3&\le -x_1^2-\xi_2^2-lx_1^{2-\tau}-l\xi_2^{2-\tau}+\xi_3^{2-r_3-\tau}\Big(\frac{\partial \bar{x}_3}{\partial x_1}x_2\\&\quad+\frac{\partial \bar{x}_3}{\partial x_2}\bar{x}_3+\frac{\partial \bar{x}_3}{\partial x_3}\left(-\frac{1}{T_{\mathrm{SVC}}}x_3+\frac{1}{T_{\mathrm{SVC}}}u+z_{13}\right)\\&\quad+\frac{\partial \bar{x}_3}{\partial z_{12}}z_{22}\Big)\\&\quad+(\hat{c}_{31}+\hat{c}_{32}+\hat{c}_{33}+\hat{c}_{34}+\hat{c}_{35}+\hat{c}_{36})\xi_3^2.\end{aligned} \quad (29)$$

The controller u is designed as

$$\begin{aligned}u&=x_3-T_{\mathrm{SVC}}z_{13}-\frac{T_{\mathrm{SVC}}}{\frac{\partial \bar{x}_3}{\partial x_3}}\Big(\frac{\partial \bar{x}_3}{\partial x_1}x_2+\frac{\partial \bar{x}_3}{\partial x_2}\bar{x}_3\\&\quad+\frac{\partial \bar{x}_3}{\partial z_{12}}z_{22}+\beta_3\xi_3^{r_4}\Big),\end{aligned} \quad (30)$$

where $\beta_3\ge\beta_3^*=1+\hat{c}_{31}+\hat{c}_{32}+\hat{c}_{33}+\hat{c}_{34}+\hat{c}_{35}+\hat{c}_{36}$, $r_4=r_3+\tau$.

Remark 2 In order to obtain the finite time controller, the auxiliary state $\bar{x}_3$ and the auxiliary virtual control law $\bar{x}_3^*$ in (22) are defined.

3.2 Stability analysis

Theorem 1 *Consider system* (2)*. If Assumptions* 1 *and* 2 *hold, the composite controller* (30) *can guarantee the closed-loop system* (2)*,* (4) *and composite controller* (30) *is globally finite time stable.*

Proof The stability analysis of the closed-loop system is divided into two parts. First, the finite time stability of system (7) and (30) is established when $t > t_1$. Next, we will prove that system states of (3) and (6) will not escape to the infinity in any time interval $[0, t_1]$.

At first step: substituting (30) into (29) gives rise to

$$\dot{V}_3 \le -x_1^2 - \xi_2^2 - \xi_3^2. \tag{31}$$

It can be verify that

$$V_3 \le \frac{1}{c}\left(x_1^{2-\tau} + \xi_2^{2-\tau} + \xi_n^{2-\tau}\right), \tag{32}$$

where $c > 0$. Let $\lambda_1 = \frac{1}{2}c^{\frac{2}{2-\tau}}$. Using Lemma 2, we can derive from (31) and (32) that

$$\begin{aligned}\dot{V}_3 + \lambda_1 V_3^{\frac{2}{2-\tau}} &\le -(x_1^2 + \xi_2^2 + \xi_3^2) + \frac{\lambda_1}{c^{\frac{2}{2-\tau}}}(x_1^2 + \xi_2^2 + \xi_3^2)\\ &= -\frac{1}{2}(x_1^2 + \xi_2^2 + \xi_3^2).\end{aligned} \tag{33}$$

According to the finite time stability definition [33], we obtain that system (7) and (30) is finite time stable.

Next, we will prove that system states of (3) and (6) will not escape to the infinity in any time interval $[0, t_1]$. By coordinate transform, we obtain the following system

$$\begin{aligned}\dot{x}_1 &= x_2 = x_2 - x_2^* + x_2^*,\\ \dot{\xi}_2 &= x_2^{\frac{1}{r_2}-1}(\bar{x}_3 - e_{12}) + \beta_1^{\frac{1}{r_2}} x_2\\ &= x_2^{\frac{1}{r_2}-1}\left(\bar{x}_3 - \bar{x}_3^* - \beta_2\xi_2^{r_3} - e_{12}\right) + \beta_1^{\frac{1}{r_2}} x_2,\\ \dot{\xi}_3 &= \bar{x}_3^{\frac{1}{r_3}-1}\left(-\rho_3\xi_3^{r_4} - e_{13}\right) + \beta_2^{\frac{1}{r_3}-1}\Bigg(x_2^{\frac{1}{r_2}-1}\\ &\qquad \left(\bar{x}_3 - \bar{x}_3^* - \beta_2\xi_2^{r_3} - e_{12}\right) + \beta_1^{\frac{1}{r_2}} x_2\Bigg).\end{aligned} \tag{34}$$

A finite time bounded function is selected as

$$B(x_1, \xi_2, \xi_3) = \frac{1}{2}(x_1^2 + \xi_2^2 + \xi_3^2). \tag{35}$$

Taking the first derivative of (35), we have

$$\begin{aligned}&\dot{B}(x_1, \xi_2, \xi_3)\\ &= x_1\dot{x}_1 + \xi_2\dot{\xi}_2 + \xi_3\dot{\xi}_3\\ &= x_1(x_2 - x_2^* + x_2^*) + \xi_2\left(x_2^{\frac{1}{r_2}-1}(\bar{x}_3 - e_{12}) + \beta_1^{\frac{1}{r_2}} x_2\right)\\ &\quad + \xi_3\Bigg(\bar{x}_3^{\frac{1}{r_3}-1}\left(-\beta_3\xi_3^{r_4} - e_{13}\right) + \beta_2^{\frac{1}{r_3}-1}\Bigg(x_2^{\frac{1}{r_2}-1}(\bar{x}_3\\ &\quad - \bar{x}_3^* - \beta_2\xi_2^{r_3} - e_{12}) + \beta_1^{\frac{1}{r_2}} x_2\Bigg)\Bigg).\end{aligned} \tag{36}$$

Using Lemma 1 plus Young's inequality, the following inequalities are true

$$\begin{aligned}&x_1(x_2 - x_2^*) \le 2^{1-r_2}|x_1||\xi_2|^{r_2} \le |x_1|^{2+\tau} + h_{11}|\xi_2|^{2+\tau},\\ &x_1x_2^* \le |x_1||\beta x_1^{r_2}| = \beta_1|x_1|^{2+\tau},\\ &\xi_2 x_2^{\frac{1}{r_2}-1}\\ &\quad \le 2^{1-r_2}|\xi_2|(|\xi_2|^{1-r_2} + \beta_1^{\frac{1}{r_2}-1}|x_1|^{1-r_2})\\ &\quad \le 2^{1-r_2}(|\xi_2|^{2-r_2} + \beta_1^{\frac{1}{r_2}-1}|x_1|^{1-r_2}|\xi_2|)\\ &\quad \le 2^{1-r_2}(2|\xi_2|^{2-r_2} + h_{12}|x_1|^{2-r_2}),\\ &\xi_2 x_2^{\frac{1}{r_2}-1}(\bar{x}_3 - e_{12})\\ &\quad \le 2^{1-r_2}(2|\xi_2|^{2-r_2} + h_{12}|x_1|^{2-r_2})(2^{1-r_3}|\xi_3|^{r_3}\\ &\qquad + \beta_3|\xi_2|^{r_3} + e_{12})\\ &\quad \le x_1^{2+\tau} + h_{21}\xi_2^{2+\tau} + h_{22}\xi_3^{2+\tau} + 2^{1-r_2}(2|\xi_2|^{2-r_2}\\ &\qquad + h_{12}|x_1|^{2-r_2})|e_{12}|,\\ &\xi_2\beta_1^{\frac{1}{r_2}} x_2\\ &\quad \le \beta_1^{\frac{1}{r_2}}|\xi_2|(2^{1-r_2}|\xi_2|^{r_2} + \beta_1|x_1|^{r_2})\\ &\quad \le x_1^{2+\tau} + h_{23}|\xi_2|^{2+\tau},\\ &\xi_3\bar{x}_3^{\frac{1}{r_3}-1}\\ &\quad \le 2^{1-r_3}|\xi_3|(|\xi_3|^{1-r_3} + \beta_2^{\frac{1}{r_3}-1}|\xi_2|^{1-r_3})\\ &\quad \le 2^{1-r_3}(|\xi_3|^{2-r_3} + \beta_2^{\frac{1}{r_3}-1}|\xi_2|^{1-r_3}|\xi_3|)\\ &\quad \le 2^{1-r_3}(2|\xi_3|^{2-r_3} + h_{31}|\xi_2|^{2-r_3}),\\ &\xi_3\bar{x}_3^{\frac{1}{r_3}-1}(-\beta_3\xi_3^{r_4} - e_{13})\\ &\quad \le |\xi_2|^{2+\tau} + h_{32}|\xi_3|^{2+\tau} + 2^{1-r_3}(2|\xi_3|^{2-r_3}\\ &\qquad + h_{31}|\xi_2|^{2-r_3})|e_{13}|,\\ &\xi_3\left(\beta_2^{\frac{1}{r_3}-1}(x_2^{\frac{1}{r_2}-1}(\bar{x}_3 - \bar{x}_3^* - \beta_2\xi_2^{r_3} - e_{12}) + \beta_1^{\frac{1}{r_2}} x_2)\right)\end{aligned}$$

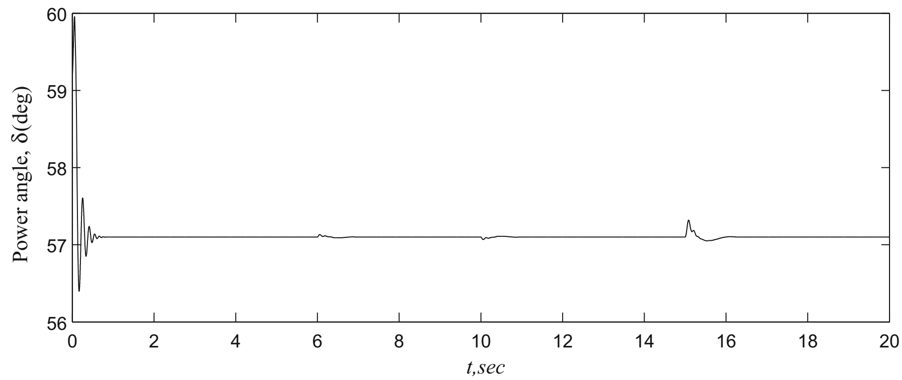


Fig. 1 Response curves of system state δ with external disturbances

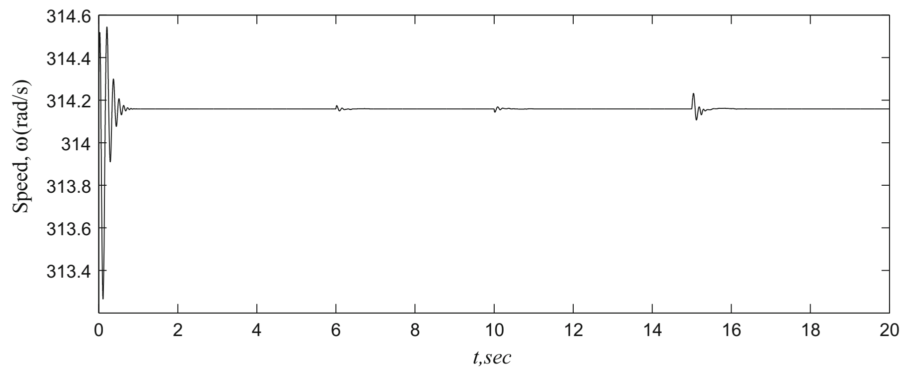


Fig. 2 Response curves of system state ω with external disturbances

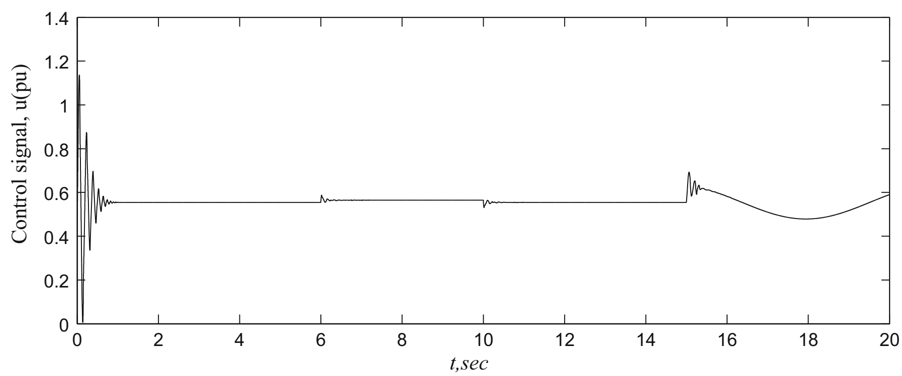


Fig. 3 Curves of system input with external disturbances

$$\le x_1^{2+\tau} + h_{41}\xi_2^{2+\tau} + h_{42}\xi_3^{2+\tau} + (x_1^{2-r_2} + h_{43}\xi_2^{2-r_2} + h_{44}\xi_3^{2-r_2})|e_{12}|, \quad (37)$$

where $h_{11}, h_{12}, h_{21}, h_{22}, h_{31}, h_{32}, h_{41}, h_{42}, h_{43}$, and h_{44} are positive numbers. Since w_2 and w_3 are estimated in finite time, i.e., the estimation errors e_{12} and e_{13} converge to zero in finite time, then e_{1i} is bounded. We denote $e_{1i} \le \bar{e}_i \le \bar{e}$, where $\bar{e} > 0$ is a constant. On the one hand if

$$\eta = \sqrt{x_1^2 + \xi_2^2 + \xi_3^2} \ge \hat{\eta} > 1, \quad (38)$$

then we obtain $|x_1|^{2+\tau} \le \eta^{2+\tau} \le \eta^2$, $|\xi_2|^{2+\tau} \le \eta^{2+\tau} \le \eta^2$, $|\xi_3|^{2+\tau} \le \eta^{2+\tau} \le \eta^2$, $|x_1|^{2-r_i} \le \eta^2$, $|\xi_2|^{2-r_i} \le \eta^2$, $|\xi_3|^{2-r_i} \le \eta^2$. With this in mind, we obtain

$$\dot{B}(x_1, \xi_2, \xi_3) \le K\eta^2 = KB(x_1, \xi_2, \xi_3), \quad (39)$$

where $K = \beta_1 + 4 + h_{11} + h_{21} + h_{23} + h_{41} + h_{22} + h_{32} + h_{42} + 2 * 2^{1-r_2}\bar{e} + 2^{1-r_2}\bar{e}h_{12} + 2 * 2^{1-r_3}\bar{e} + 2^{1-r_3}\bar{e}h_{31} + (1 + h_{43} + h_{44})\bar{e}$. On the other hand, if $\eta < 1$, there exists a constant $\bar{L}$ such that

Fig. 4 Curves of disturbances and disturbance estimation

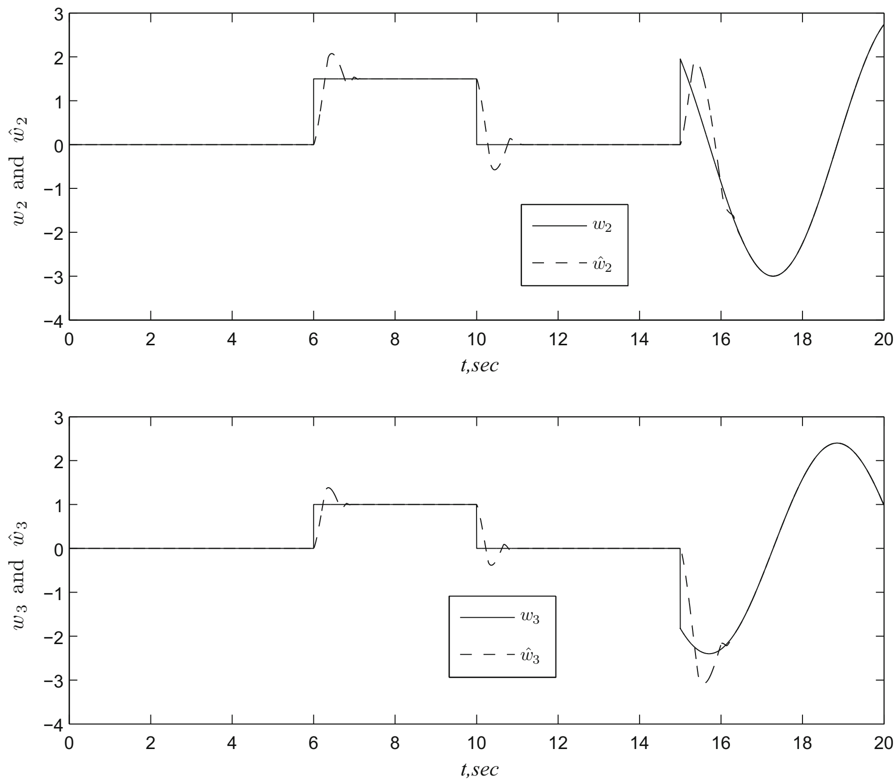


Fig. 5 Response curves of system state δ with model uncertainties

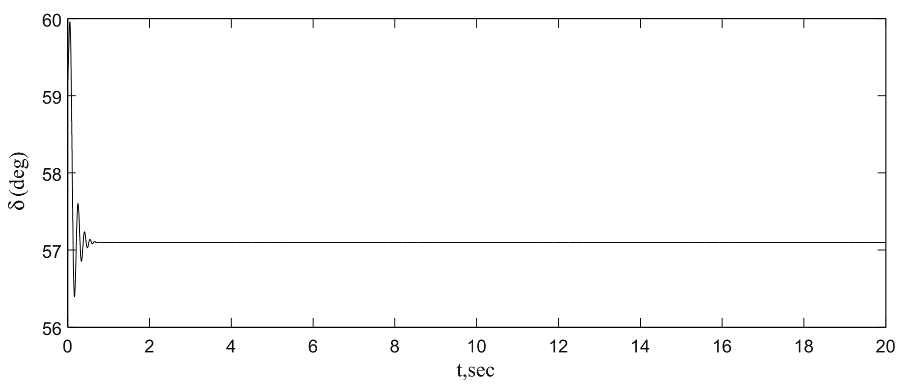
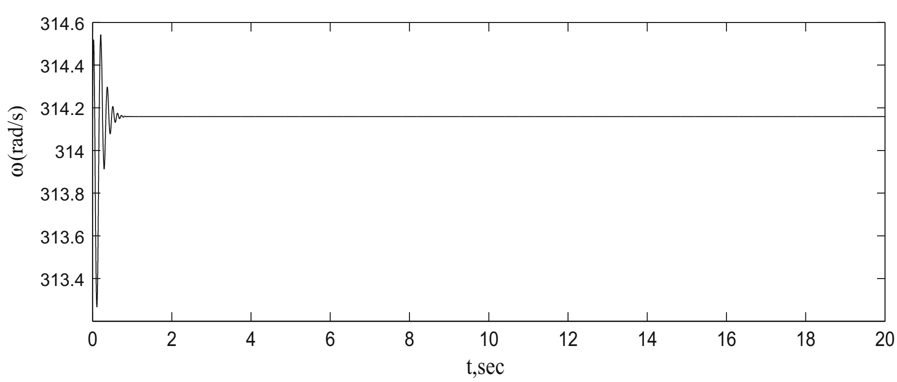


Fig. 6 Response curves of system state ω with model uncertainties



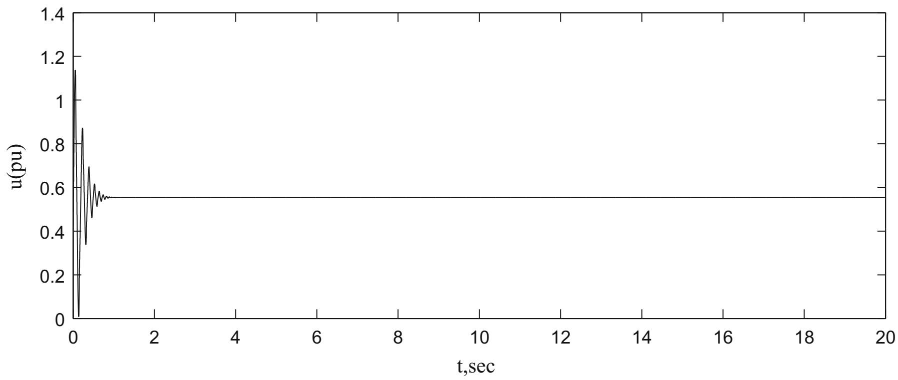


Fig. 7 Curves of system input with model uncertainties

$$\dot{B}(x_1, \xi_2, \xi_3) \le \bar{L}. \tag{40}$$

Thus we obtain

$$\dot{B}(x_1, \xi_2, \xi_3) \le KB(x_1, \xi_2, \xi_3) + \bar{L}. \tag{41}$$

Solving the inequality (41), we have $B(x_1, \xi_2, \xi_3) \le (B(x_1(0), \xi_2(0), \xi_3(0)) + \frac{\bar{L}}{K})e^{Kt} - \frac{\bar{L}}{K}$. When $t \le t_1$, the system states x_1, ξ_2 ξ_3 of (34) are bounded.

According to the above analysis and [34], we obtain that the system consisting of system (6), the estimation error (4) and the control law (30) is finite time stable.

Remark 3 In the absence of disturbances, it is derived from (3) and (4) that

$$\begin{aligned}
\dot{e}_{0j} &= -\lambda_0 L_j^{1/4} |e_{0j}|^{\frac{n}{n+1}} \text{sign}(e_{0j}) + z_{1j}, \\
\dot{z}_{1j} &= -\lambda_1 L_j^{1/3} |z_{1j} - \dot{e}_{0j}|^{\frac{2}{3}} \text{sign}(z_{1j} - \dot{e}_{0j}) + z_{2j}, \\
\dot{z}_{2j} &= -\lambda_2 L_j^{1/2} |z_{2j} - \dot{z}_{1j}|^{\frac{1}{2}} \text{sign}(z_{2j} - \dot{z}_{1j}) + z_{3j}, \\
\dot{z}_{3j} &= -\lambda_3 L_j \text{sign}(z_{3j} - \dot{z}_{2j}),\ j = 1, 2, 3,
\end{aligned}$$

which implies that z_1, z_2, z_3 equal to zero all the time. Then the composite controller (30) degenerates to the traditional finite time controller, which means that the proposed method does not sacrifice the nominal performance. This good property will be verified via simulation results in the next section.

4 Simulation result

The SVC system has the following parameters [8,35]:

$$\begin{aligned}
&H = 5.9\,\text{s}, \quad D = 1.0, \quad V_s = 1.0\,\text{pu}, \\
&T_{\text{SVC}} = 0.02, \quad X_1 = 0.84\,\text{pu}, \\
&X_2 = 0.52\,\text{pu}, \quad B_L + B_C = 0.3\,\text{pu}, \\
&q_1 = 0.4, \quad q_2 = 0.6.
\end{aligned}$$

The controller parameters are chosen as

$$\begin{aligned}
&r_1 = 1, \quad \tau = -\frac{2}{15}, \quad \beta_1 = 5, \quad \beta_2 = 10, \\
&\beta_3 = 25, \quad L_1 = L_2 = L_3 = 10.
\end{aligned}$$

The following operating point is considered

$$\begin{aligned}
&\delta_0 = 57.1\,\text{deg}, \quad \omega_0 = 314.159\,\text{rad/s}, \\
&y_{\text{SVC0}} = 0.4\,\text{pu}.
\end{aligned} \tag{42}$$

Case I External disturbance rejection ability

The external disturbances in the SVC system are taken as

$$w_1(t) = \begin{cases} 0, & t < 6 \text{ and } 10 \le t < 15, \\ 1.5, & 6 \le t < 10, \\ 3\sin(t), & t > 15, \end{cases},$$

$$w_2(t) = \begin{cases} 0, & t < 6 \text{ and } 10 \le t < 15, \\ 1, & 6 \le t < 10, \\ 2.4\cos(t), & t > 15. \end{cases}$$

The simulation results are presented in Figs. 1, 2, 3 and 4. Response curves of system states δ and ω are shown in Figs. 1 and 2. It can be observed that the system outputs can achieve their control object in the presence of mismatched disturbances. In order to illustrate the effectiveness of disturbance observer, the curves of disturbances and disturbance estimation are presented in Fig. 4. The control input is depicted in Fig. 3.

Case II: Robustness against model uncertainties

The robustness against model uncertainties of the proposed scheme is verified in this part. To investigate the performance of robustness, we choose the model uncertainties as follows. θ has variation of +20 %.

Curve of system outputs under the proposed control method is given in Figs. 5 and 6. It can be seen that the closed-loop system has a good robustness performance and the outputs have a satisfactory performance. The control input is presented in Fig. 7.

5 Conclusion

In this paper, the problem of finite time composite anti-disturbance control for SVC system has been investigated. Based on finite time disturbance observer and finite time control techniques, a finite time composite controller has been proposed. Using Lyapunov function theory, the finite time stability of the closed-loop system has been analyzed. Finally, the simulation result has been presented to show the effectiveness of the developed method.

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