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ORIGINAL PAPER

Attitude tracking control for Mars entry vehicle via T-S model with time-varying input delay

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Abstract This paper focuses on the attitude tracking control problem of Mars entry vehicle (MEV) with time-varying input delay. The original attitude dynamics of MEV is divided into slow subsystem and fast subsystem. For slow subsystem, the dynamic inversion method is used to generate the angular velocity command. For fast subsystem, a T-S fuzzy model is used to approximate it, and delays-dependent H_∞ attitude tracking control is applied to reduce the effects of delay on attitude dynamics. Specially, a decomposition coefficient of delay integral inequality is introduced in our proposed results, which may further reduce the design algorithm conservatism. Finally, numerical simulations are used to verify the effectiveness of the proposed method.

Keywords Mars entry vehicle · Time-varying input delay · T-S fuzzy model · H_∞ control

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1 Introduction

Since the human first completed to reach Mars, Mars landing exploration has been more and more frequent. Until now, Mars exploration activities have been taken more than 40 times all around the world [1,2]. Among many tasks of Mars landing missions, the most difficult one is entry, descent and landing (EDL) phase [3,4]. The Mars entry is the longest phase during the EDL process. For a success of the EDL, the entry phase should be accurate firstly [5]. Therefore, many control approaches have been proposed to improve the precision of Mars entry [6–8].

In order to achieve precise entry, an active guided entry trajectory is very necessary, which can accurately steer the Mars entry vehicle through the Martian atmosphere [4]. The guided entry problem includes two parts, one is guidance design and the other is control design. The designed guidance system can minimize the position error by changing the lift vector through bank angle commands [9], which is then fed into the control system as a reference attitude to calculate the desired control torques that the actuators need to produce [10,11]. Thus, the Mars entry control problem can be considered as an attitude tracking problem.

In recent years, the T-S fuzzy model has been applied for nonlinear systems widely [12–16]. Some T-S fuzzy models have been used in solving the stabilization problem of the systems successfully [17–19]. The tracking controller via T-S fuzzy model has been designed by some researchers [20,21]. Unfortunately, in the existing literature for T-S fuzzy model, research on the tracking control of MEV problem is scarce [22], and this is the main motivation for the work presented in this paper. Time delay is the property of a physical system by which the response to an applied force (action) is delayed in its effect [23,24]. Whenever material, information or energy is physically transmitted from one place to another, there is a delay associated with the transmission. The presence of delays makes system analysis and control design much more complex. On the other hand, in order to get high accuracy and stability of MEV, the time delay should be considered in the high- precision attitude tracking control system. The similar idea had been considered in synchronization or state estimation of nonidentical chaotic systems [25] and complex networks [24,26]. But, to the best of our knowledge, little attention has been paid toward attitude tracking control for MEV with time-varying input delay.

Motivated by the above, in this paper, the MEV model with time-varying input delay is considered, the attitude dynamics system of MEV is divided into slow subsystem and fast subsystem, the slow subsystem consists of the attitude dynamics, and the fast subsystem consists of the angular velocity dynamics. A T-S fuzzy model is applied for the fast subsystem. A delays-dependent H_∞ tracking control is designed by means of Lyapunov stability theory. Finally, numerical simulations are used to verify the effectiveness of the proposed method and to show the effects of time delay bound and decomposition coefficient on the fuzzy tracking error system performance. The main contribution of this paper is listed as follows:

1. Different from the existing T-S systems in the literature [12–16], the modeling and tracking control of MEV problem via T-S fuzzy theory is scarce, which is the main motivation for this paper.
2. Compared with the model of MEV in [22], time delay is considered in modeling T-S fuzzy system, which is more real than the original one in [22].
3. A decomposition coefficient of delay integral inequality is introduced in the process of solving functional. It can not only provide flexibility in tracking controller design, but also reduce time delay influences, which can be seen in subsequent simulation part.

Notation Throughout this paper, R^n represents the n-dimensional Euclidean space, $R^{n\times m}$ denotes the $n\times m$ real matrices, for matrix $W\in R^{n\times n}$, W^{-1} denotes the inverse of W, and W^{T} denotes the transpose of W. Zero matrix is 0, unit matrix is I, and the notation $Q>0$ (≥ 0) means that the matrix Q is positive definite(semi-definite). The symmetric term in a matrix is denoted by "*". For a vector $S(t)$, its norm is given by $\|S(t)\|_2^2=S^{\mathrm{T}}(t)S(t)$. diag{...} represents a block diagonal matrix. Matrix, if its dimensions are not explicitly stated, is assumed to have compatible dimensions.

2 Problem formulation and preliminaries

2.1 Model of Mars entry vehicles

The system model of rotational motion for MEV [22] can be described as

$$\dot{\sigma}(t)=\frac{1}{4}G(\sigma(t))\omega(t) \tag{1}$$

$$\begin{aligned}\dot{\omega}(t)=&-J^{-1}S(\omega(t))J\omega(t)\\&+J^{-1}u(t-d(t))+J^{-1}M_{\text{aero}}(t)\end{aligned} \tag{2}$$

where $\sigma(t)=[\sigma_1(t)\ \ \sigma_2(t)\ \ \sigma_3(t)]^{\mathrm{T}}\in R^3$ is the modified Rodrigues parameter (MRP) vector and $G(\sigma(t))\in R^3$ is the nonlinear transformation matrix

$$G(\sigma(t))=\left(\frac{1-\sigma^{\mathrm{T}}(t)\sigma(t)}{2}I_3+\sigma(t)\sigma^{\mathrm{T}}(t)+2S(\sigma(t))\right) \tag{3}$$

$\omega(t)=[\omega_1(t)\ \ \omega_2(t)\ \ \omega_3(t)]^{\mathrm{T}}\in R^3$ is the angular velocity vector, $S(\omega(t))\in R^{3\times 3}$ is a skew symmetric matrix, where the form is defined in [22]. $J\in R^3$ is positive definite symmetric inertia matrix

$$J=\begin{bmatrix}J_1&0&0\\0&J_2&0\\0&0&J_3\end{bmatrix}=\begin{bmatrix}J_{xx}&0&0\\0&J_{yy}&0\\0&0&J_{zz}\end{bmatrix} \tag{4}$$

in which J_{xx}, J_{yy} and J_{zz} are the moments of inertia when rotating about three axis of the body reference frame, respectively. $u(t-d(t))=[u_1(t-d(t)\ \ u_2(t-d(t)\ \ u_3(t-d(t)]\in R^3$ is the vector of control torques exerted on the principal axes. $M_{\text{aero}}(t)=[\mathcal{I}\ \ \mathcal{M}\ \ \mathcal{N}]^{\mathrm{T}}$ is the aerodynamic moment vector, where $\mathcal{I}=\bar{q}S_{\text{ref}}l_{\text{ref}}c_{lb}\beta$, $\mathcal{M}=\bar{q}S_{\text{ref}}l_{\text{ref}}c_{ma}\alpha$,

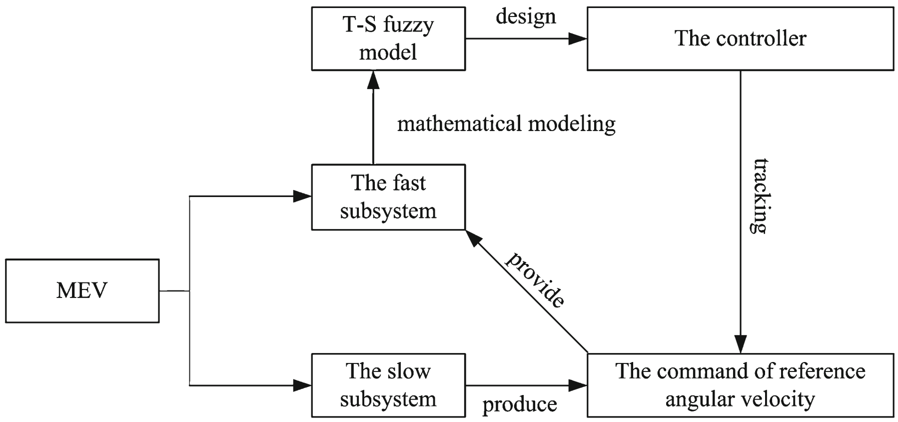


Fig. 1 The structure of MEVs system

$\mathcal{N} = \bar{q} S_{\text{ref}} l_{\text{ref}} c_{nb} \beta$, which is supposed $M_{\text{aero}}(t)$ belongs to $l_2[0, \infty)$ and satisfied $||M_{\text{aero}}(t)|| \leq \delta$. It is assumed that time-varying delay $d(t)$ satisfies $0 \leq d(t) \leq \tau$ and $\dot{d}(t) \leq \bar{d} < 1$, τ is the upper bound for time-varying delay, and $\bar{d}$ is the upper bound of time-varying delay derivative.

Remark 1 Time delay exists commonly in dynamic systems and is frequently a source of instability and poor performance. However, it is noted that the model of MEV in [22] is not considered time delay case. Thus, in this paper, the model of MEV with time-varying delay is firstly presented.

In order to reduce the design complexity, the system (1) and (2) is divided into slow subsystem (1) and fast subsystem (2). The slow subsystem consists of the attitude dynamics, and the fast subsystem consists of the angular velocity dynamics. Figure 1 shows the structure of MEVs system.

2.2 Control design of slow subsystem

The slow subsystem is described as follows:

$$\dot{\sigma(t)} = \frac{1}{4} G(\sigma(t)) \omega_{d(t)} \tag{5}$$

where $\omega_{d(t)}$ is the control-like angular velocity and needs to be designed in the subsequent part. Letting the attitude tracking error as $e_{\sigma(t)} = \sigma(t) - \sigma_{d(t)}$, where $\sigma_{d(t)}$ is the given MRP vector, we may choose the matrices K_a and K_b to satisfy the following condition

$$\dot{e}_\sigma(t) + K_a e_\sigma(t) + K_b \int e_\sigma(t) \mathrm{d}t = 0 \tag{6}$$

Thus, the desired attitude tracking can be obtained. By using the dynamic inversion [22], $\omega_d(t)$ is generated by

$$\omega_d(t) = 4G^{-1}(\sigma(t))[\dot{\sigma}_d(t) - K_a e_\sigma(t) - K_b \int e_\sigma(t) \mathrm{d}t] \tag{7}$$

The above-detailed method can be found in [22]. Our main work is to track the $\omega_d(t)$ in the fast subsystem.

2.3 Control design of fast subsystem

Defining $x_d(t) = \omega_d(t), x_1(t) = \omega_1(t), x_2(t) = \omega_2(t)$, $x_3(t) = \omega_3(t), x(t) = [x_1(t) \;\; x_2(t) \;\; x_3(t)]^{\mathrm{T}}$, it is noted that the fast subsystem has direct connection with the control input $u(t - d(t))$, so the control $u(t - d(t))$ is designed such that $\lim_{t \longrightarrow \infty} (x(t) - x_d(t)) = 0$. For this purpose, we choose the angular velocity vector as the output of MEV

$$\begin{cases} \dot{x}(t) = A(x(t))x(t) + B(x(t))u(t - d(t)) \\ \quad + B(x(t))M_{\text{aero}}(t) \\ y(t) = C(x(t))x(t) \end{cases} \tag{8}$$

where $y(t)$ is the reference output and

$$A(x(t)) \begin{bmatrix} 0 & 0 & c_2 x_2(t) \\ c_3 x_3(t) & 0 & 0 \\ 0 & c_1 x_1(t) & 0 \end{bmatrix},$$
$$B(x(t)) = J^{-1}, C(x(t)) = I^{3\times 3}$$

with $c_1 = J_3^{-1}(J_1 - J_2), c_2 = J_1^{-1}(J_2 - J_3)$, $c_3 = J_2^{-1}(J_3 - J_1)$. A T-S fuzzy model is used to approximate the nonlinear fast subsystem (8). Moreover, the system states $x_i(t)$, $(i = 1, 2, 3)$ are chosen as the premise variables, and $x_i(t)$ is assumed to bound during the whole Martian atmospheric entry phase, this means that

$$x_i(t) \in [r_i,\ R_i] \tag{9}$$

where $r_i < 0$ is the lower bound of $x_i(t)$, $R_i > 0$ is the upper bound of $x_i(t)$, respectively. For each $x_i(t)$, the corresponding membership functions are defined in the following [22]:

$$MB_{i1}(x_i(t)) = \frac{R_i - x_i(t)}{R_i - r_i},\ MB_{i2}(x_i(t)) = \frac{x_i(t) - r_i}{R_i - r_i} \tag{10}$$

Then, the nonlinear fast subsystem (8) can be represented by the following eight fuzzy rules:

Plant rule i IF $x_1(t)$ is $MB_{1n}(x_1(t))$, $x_2(t)$ is $MB_{2k}(x_2(t))$, $x_3(t)$ is $MB_{3l}(x_3(t))$, then

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t - d(t)) + B_i M_{\text{aero}}(t) \\ y(t) = C_i x(t) \end{cases} \tag{11}$$

where

$$A_i = \begin{bmatrix} 0 & 0 & c_2\alpha_{2k} \\ c_3\alpha_{3l} & 0 & 0 \\ 0 & c_1\alpha_{1n} & 0 \end{bmatrix},$$

$$B_i = J^{-1},\ C_i = I^{3\times 3}$$

and $n, k, l = 1, 2,\ i = l + 2(k-1) + 4(n-1)$, $(i = 1, 2, \ldots, 8)$, $\alpha_{m1} = r_m$, $\alpha_{m2} = R_m$, $(m = 1, 2, 3)$. The overall fuzzy systems can obtained as follows

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} h_i(x(t))[A_i x(t) + B_i u(t - d(t)) \\ \quad + B_i M_{\text{aero}}(t)] \\ y(t) = \sum_{i=1}^{8} h_i(x(t)) C_i x(t) \end{cases} \tag{12}$$

where $h(x(t)) = \mu_i(x(t))/\sum_{i=1}^{8}\mu_i(x(t))$, $\mu_i(x(t)) = MB_{1n}(x_1(t)) \cdot MB_{2k}(x_2(t)) \cdot MB_{3l}(x_3(t))$. Hence, $h_i(x(t))$ satisfies $h_i(x(t)) \geq 0$ and $\sum_{i=1}^{8} h_i(x(t)) = 1$ for all t. In order to facilitate the description, the system (12) can be described as follows

$$\begin{cases} \dot{x}(t) = A(x)x(t) + B(x)u(t - d(t)) + B(x)M_{\text{aero}}(t) \\ y(t) = C(x)x(t) \end{cases} \tag{13}$$

where $A(x) = \sum_{i=1}^{8} h_i(x(t))A_i$, $B(x) = \sum_{i=1}^{8} h_i(x(t))B_i$, $C(x) = \sum_{i=1}^{8} h_i(x(t))C_i$. Define the angular velocity tracking error as

$$e_x(t) = x(t) - x_d(t) \tag{14}$$

Combing with (13), the time derivative of $e_x(t)$ is given by

$$\begin{aligned} \dot{e}_x(t) &= A(x)e_x(t) + B(x)u(t - d(t)) + B(x)M_{\text{aero}}(t) \\ &\quad + A(x)x_d(t) - \dot{x}_d(t) \end{aligned} \tag{15}$$

Next, the controller $u(t-d(t))$ can be described as [27]

$$u(t - d(t)) = \bar{u}(t - d(t)) - B^{-1}(x)[A(x)x_d(t) - \dot{x}_d(t)] \tag{16}$$

where $\bar{u}(t - d(t)) \in R^3$ is a new vector to be designed. According to (13), (15) and (16), we have the following fuzzy error system

$$\begin{cases} \dot{e}_x(t) = A(x)e_x(t) + B(x)\bar{u}(t - d(t)) + B(x)M_{\text{aero}}(t) \\ \bar{y}(t) = C(x)e_x(t) \end{cases} \tag{17}$$

2.4 Object of this paper

We consider the following state-feedback controller:

Plant rule i IF $x_1(t)$ is $MB_{1n}(x_1(t))$, $x_2(t)$ is $MB_{2k}(x_2(t))$, $x_3(t)$ is $MB_{3l}(x_3(t))$, then

$$\bar{u}(t - d(t)) = K_i e_x(t - d(t)) \tag{18}$$

where $K_i \in R^{3\times 3}$ are needed to design controller gain, $e_x(t - d(t))$ is the state delay error, $i = l + 2(k-1) + 4(n-1)$, $(i = 1, 2, \ldots, 8)$. Therefore, the overall fuzzy can be described as

$$\bar{u}(t - d(t)) = \sum_{i=1}^{8} h_i(x(t))K_i e_x(t - d(t)) \tag{19}$$

Now, combing with system (17), the objective is to design $K_j (j = 1, 2, \ldots, 8)$, such that the following fuzzy error system is asymptotically stable.

$$\begin{cases} \dot{e}_x(t) = A(x)e_x(t) + B(x)K(x)e_x(t - d(t)) \\ \quad + B(x)M_{\text{aero}}(t) \\ \bar{y}(t) = C(x)e_x(t) \end{cases} \tag{20}$$

where $K(x) = \sum_{j=1}^{8} h_i(x(t))K_j$. The objective of this paper is:

- Designing the controller gains K_j makes the fuzzy error system (20) with $M_{\text{aero}}(t) = 0$ asymptotical,
- In the case when $M_{\text{aero}}(t) \neq 0$, the H_∞ performance of system (20) satisfies $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\text{aero}}(t)\|_2^2$ for any nonzero $M_{\text{aero}}(t) \in l_2\,[0, \infty)$, where $\gamma > 0$ is a prescribed scalar.

To obtain our main results, we need the following lemmas.

Lemma 1 [28] *Let $Z(t) \in R^n$ have continuous derived function $\dot{Z}(t)$ on interval $[-\tau, 0]$. Then for any $n \times n - matrix$ $T_1 > 0$, scalar $\tau > 0$, the following inequality holds:*

$$
\begin{aligned}
&-\int_{t-\tau}^{t} \dot{Z}^{\mathrm{T}}(s) T_1 \dot{Z}(s) ds \\
&\leq -\frac{2}{\tau^3}\left(\int_{t-\tau}^{t} Z(s)ds\right)^{\mathrm{T}} T_1 \left(\int_{t-\tau}^{t} Z(s)ds\right) \\
&\quad -\frac{2}{\tau} Z^{\mathrm{T}}(t-\tau) T_1 Z(t-\tau) \\
&\quad +\frac{4}{\tau^2}\left(\int_{t-\tau}^{t} Z(s)ds\right)^{\mathrm{T}} T_1 Z(t-\tau)
\end{aligned} \tag{21}
$$

Lemma 2 [29] *For any matrix $T_2 > 0$, scalars $\tau_1 > \tau_2$, if there exists a Lebesque vector $w(s)$, then the following inequality holds*

$$
\begin{aligned}
&-\int_{\tau_2}^{\tau_1} w^{\mathrm{T}}(s) T_2 w(s) ds \leq \\
&-\frac{1}{\tau_1 - \tau_2} \int_{\tau_2}^{\tau_1} w^{\mathrm{T}}(s) ds T_2 \int_{\tau_2}^{\tau_1} w(s) ds
\end{aligned} \tag{22}
$$

3 Main result

In this section, we will propose the designed method to the controller gains of the augmented system (20).

Theorem 1 *Given scalars $\gamma > 0$, τ, a and $\bar{d}$, if there exist matrices $R > 0$, $P > 0$, $Q > 0$, M_1, M_2, M_3, M_4, M_5, then the following inequalities hold*

$$\Lambda_{ii} < 0, \quad i = 1, 2, \ldots, 8 \tag{23}$$

$$\Lambda_{ij} + \Lambda_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, 8 \tag{24}$$

where

$$
\Lambda_{ij} = \begin{bmatrix}
\Delta_{11} - aQ & M_1 B_i K_j + A_i^{\mathrm{T}} M_4^{\mathrm{T}} + aQ & M_1 B_i & A_i^{\mathrm{T}} M_3^{\mathrm{T}} & A_i^{\mathrm{T}} M_5^{\mathrm{T}} & P - M_1 + A_i^{\mathrm{T}} M_2^{\mathrm{T}} & C_i^{\mathrm{T}} \\
 & \Delta_{22} - 2aQ & M_4 B_i & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_3^{\mathrm{T}} + aQ & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_5^{\mathrm{T}} & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_2^{\mathrm{T}} - M_4 & 0 \\
 & * & -\gamma^2 I & B_i^{\mathrm{T}} M_3^{\mathrm{T}} & B_i^{\mathrm{T}} M_5^{\mathrm{T}} & B_i^{\mathrm{T}} M_2^{\mathrm{T}} & 0 \\
 & * & * & (-2+a)Q & \frac{2}{\tau}(1-a)Q & -M_3 & 0 \\
 & * & * & * & -\frac{2}{\tau^2}(1-a)Q & -M_5 & 0 \\
 & * & * & * & * & -M_2 - M_2^{\mathrm{T}} + \tau^2 Q & 0 \\
 & * & * & * & * & & -I
\end{bmatrix}
$$

and

$$\Delta_{11} = M_1 A_i + A_i^{\mathrm{T}} M_1^{\mathrm{T}} + R \tag{25}$$

$$\Delta_{22} = -(1-\bar{d})R + M_4 B_i K_j + K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_4^{\mathrm{T}} \tag{26}$$

Then, the system (20) is asymptotically stable and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\text{aero}}(t)\|_2^2$.

Proof For the system (20), choose the following Lyapunov–Krasovskii functional:

$$
\begin{aligned}
V(e_x(t), t) = e_x^{\mathrm{T}}(t) P e_x(t) &+ \tau \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{e}_x^{\mathrm{T}}(s) Q \dot{e}_x(s) \mathrm{d}s \mathrm{d}\beta \\
&+ \int_{t-d(t)}^{t} e_x^{\mathrm{T}}(s) R e_x(s) \mathrm{d}s
\end{aligned} \tag{27}
$$

The time derivative of $V(e_x(t), t)$ is given by

$$
\begin{aligned}
\dot{V}(e_x(t), t) = \dot{e}_x^{\mathrm{T}}(t) P e_x(t) &+ e_x^{\mathrm{T}}(t) P \dot{e}_x(t) + \tau^2 \dot{e}_x^{\mathrm{T}}(t) Q \dot{e}_x(t)
\end{aligned}
$$

$$-\tau\int_{t-\tau}^{t}\dot{e}_x^{\mathrm{T}}(s)Q\dot{e}_x(s)\mathrm{d}s+e_x^{\mathrm{T}}(t)Re_x(t)$$
$$-[1-\dot{d}(t)]e_x^{\mathrm{T}}(t-d(t))Re_x(t-d(t)) \tag{28}$$

The equivalent decomposition of $-\tau\int_{t-\tau}^{t}\dot{e}_x^{\mathrm{T}}(s)Q\dot{e}_x(s)$ ds can be described as

$$-\tau\int_{t-\tau}^{t}\dot{e}_x^{\mathrm{T}}(s)Q\dot{e}_x(s)\mathrm{d}s=Z_1+Z_2 \tag{29}$$

where $Z_1=-a\tau(\int_{t-\tau}^{t-d(t)}\dot{e}_x^{\mathrm{T}}(s)Q\dot{e}_x(s)\mathrm{d}s+\int_{t-d(t)}^{t}\dot{e}_x^{\mathrm{T}}(s)Q\dot{e}_x(s)\mathrm{d}s)$, $Z_2=-(1-a)\tau(\int_{t-\tau}^{t}\dot{e}_x^{\mathrm{T}}(s)Q\dot{e}_x(s)\mathrm{d}s)$, where a is called as the decomposition coefficient of delay integral inequality. It may further reduce the design algorithm conservatism and a satisfies $0\le a\le 1$. From Lemma 1, then we have

$$Z_1\le\Psi_1 \tag{30}$$

where

$$\Psi_1=\begin{bmatrix}e_x(t)\\e_x(t-d(t))\\e_x(t-\tau)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-aQ & aQ & 0\\ & -2aQ & aQ\\ & * & -aQ\end{bmatrix}\times\begin{bmatrix}e_x(t)\\e_x(t-d(t))\\e_x(t-\tau)\end{bmatrix}$$

where

$$\Psi_2=-\frac{2}{\tau^2}(1-a)\left(\int_{t-\tau}^{t}e_x(s)\mathrm{d}s\right)^{\mathrm{T}}Q\left(\int_{t-\tau}^{t}e_x(s)\mathrm{d}s\right)-2(1-a)e_x^{\mathrm{T}}(t-\tau)Qe_x(t-\tau)+\frac{4}{\tau}(1-a)\left(\int_{t-\tau}^{t}e_x(s)\mathrm{d}s\right)^{\mathrm{T}}Qe_x(t-\tau)$$

Therefore,

$$\dot{V}(e_x(t),t)\le\dot{e}_x^{\mathrm{T}}(t)Pe_x(t)+e_x^{\mathrm{T}}(t)P\dot{e}_x(t)+\tau^2\dot{e}_x^{\mathrm{T}}(t)Q\dot{e}_x(t)+\Psi_1+\Psi_2+e_x^{\mathrm{T}}(t)Re_x(t)-[1-\bar{d}]e_x^{\mathrm{T}}\times(t-d(t))Re_x(t-d(t)) \tag{32}$$

First, we consider the stability of system (20) when $M_{\mathrm{aero}}(t)=0$. Note that

$$2[e_x^{\mathrm{T}}(t)M_1+\dot{e}_x^{\mathrm{T}}(t)M_2+e_x^{\mathrm{T}}(t-\tau)M_3+e_x^{\mathrm{T}}(t-d(t))M_4+\int_{t-\tau}^{t}e_x^{\mathrm{T}}(s)\mathrm{d}sM_5]\times[-\dot{e}_x(t)+A(x)e_x(t)+B(x)K(x)e_x(t-d(t))]=0 \tag{33}$$

where M_1, M_2, M_3, M_4 and M_5 are arbitrary matrices with appropriate dimensions. Combing (32) and (33),we arrive at

$$\dot{V}(e_x(t),t)\le\alpha_1^{\mathrm{T}}(t)\Omega(x)\alpha_1(t) \tag{34}$$

where

$$\alpha_1(t)=[e_x^{\mathrm{T}}(t)\;\;e_x^{\mathrm{T}}(t-d(t))\;\;e_x^{\mathrm{T}}(t-\tau)\;\;\int_{t-\tau}^{t}e_x^{\mathrm{T}}(s)\mathrm{d}s\;\;\dot{e}_x^{\mathrm{T}}(t)]^{\mathrm{T}}$$

$$\Omega(x)=\begin{bmatrix}\Delta_{11}(x)-aQ & M_1B(x)K(x)+A^{\mathrm{T}}(x)M_4^{\mathrm{T}}+aQ & A^{\mathrm{T}}(x)M_3^{\mathrm{T}} & A^{\mathrm{T}}(x)M_5^{\mathrm{T}} & P-M_1+A^{\mathrm{T}}(x)M_2^{\mathrm{T}}\\ & \Delta_{22}(x)-2aQ & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_3^{\mathrm{T}}+aQ & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_5^{\mathrm{T}} & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_2^{\mathrm{T}}-M_4\\ & * & (-2+a)Q & \frac{2}{\tau}(1-a)Q & -M_3\\ & * & * & -\frac{2}{\tau^2}(1-a)Q & -M_5\\ & * & * & & -M_2-M_2^{\mathrm{T}}+\tau^2Q\end{bmatrix}$$

From Lemma 2, then we have

$$Z_2\le\Psi_2 \tag{31}$$

and

$$\Delta_{11}(x)=M_1A(x)+A^{\mathrm{T}}(x)M_1^{\mathrm{T}}+R$$
$$\Delta_{22}(x)=-(1-\bar{d})R+M_4B(x)K(x)+K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_4^{\mathrm{T}}$$

Furthermore,

$$\Omega(x)=\sum_{i=1}^{8}\sum_{j=1}^{8}h_i(x(t))h_j(x(t))\Omega_{ij}=\sum_{i=1}^{8}h_i^2(x(t))\Omega_{ii}+\sum_{i<j}^{8}h_i(x(t))h_j(x(t))(\Omega_{ij}+\Omega_{ji}) \quad (35)$$

By using the Schur complement lemma and from (23) and (24), we have known that

$$\Omega_{ii}<0,\quad i=1,2,\ldots,8 \quad (36)$$

$$\Omega_{ij}+\Omega_{ji}<0,\quad i<j,\quad i,j=1,2,\ldots,8 \quad (37)$$

where

$$\Omega_{ij}=\begin{bmatrix}\Delta_{11}-aQ & \Delta_{12}+aQ & A_i^{\mathrm{T}}M_3^{\mathrm{T}} & A_i^{\mathrm{T}}M_5^{\mathrm{T}} & P-M_1+A_i^{\mathrm{T}}M_2^{\mathrm{T}}\\ & \Delta_{22}-2aQ & K_j^{\mathrm{T}}B_i^{\mathrm{T}}M_3^{\mathrm{T}}+aQ & K_j^{\mathrm{T}}B_i^{\mathrm{T}}M_5^{\mathrm{T}} & K_j^{\mathrm{T}}B_i^{\mathrm{T}}M_2^{\mathrm{T}}-M_4\\ & * & (-2+a)Q & \frac{2}{\tau}(1-a)Q & -M_3\\ & * & * & -\frac{2}{\tau^2}(1-a)Q & -M_5\\ & * & * & * & -M_2-M_2^{\mathrm{T}}+\tau^2Q\end{bmatrix}$$

and Δ_{11}, Δ_{22} are defined in (25), (26), $\Delta_{12}=M_1B_iK_j+A_i^{\mathrm{T}}M_4^{\mathrm{T}}$. Thus, $\Omega(x)<0$, which means we have $\dot{V}(e_x(t),t)<0$. So the system (20) is asymptotically stable. Similar to (33), when $M_{\mathrm{aero}}(t)\neq 0$, the following equality is held.

$$2\Psi_3\Psi_4=0 \quad (38)$$

where

$$\Psi_3=e_x^{\mathrm{T}}(t)M_1+\dot{e}_x^{\mathrm{T}}(t)M_2+e_x^{\mathrm{T}}(t-\tau)M_3+e_x^{\mathrm{T}}(t-d(t))M_4+\int_{t-\tau}^{t}e_x^{\mathrm{T}}(s)\mathrm{d}s\,M_5$$

$$\Psi_4=-\dot{e}_x(t)+A(x)e_x(t)+B(x)K(x)e_x(t-d(t))+B(x)M_{\mathrm{aero}}(t)$$

Combing (32) and (38), we arrive at

$$\dot{V}(e_x(t),t)\le\alpha^{\mathrm{T}}(t)\Upsilon(x)\alpha(t) \quad (39)$$

where

$$\alpha(t)=[e_x^{\mathrm{T}}(t)\ \ e_x^{\mathrm{T}}(t-d(t))\ \ M_{\mathrm{aero}}^{\mathrm{T}}(t)\ \ e_x^{\mathrm{T}}(t-\tau)\ \ \int_{t-\tau}^{t}e_x^{\mathrm{T}}(s)\mathrm{d}s\ \ \dot{e}_x^{\mathrm{T}}(t)]^{\mathrm{T}}$$

$$\Upsilon(x)=\begin{bmatrix}\Delta_{11}(x)-aQ & \Delta_{12}(x)+aQ & M_1B(x) & A^{\mathrm{T}}(x)M_3^{\mathrm{T}} & A^{\mathrm{T}}(x)M_5^{\mathrm{T}} & P-M_1+A^{\mathrm{T}}(x)M_2^{\mathrm{T}}\\ & \Delta_{22}(x)-2aQ & M_4B(x) & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_3^{\mathrm{T}}+aQ & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_5^{\mathrm{T}} & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_2^{\mathrm{T}}-M_4\\ & * & 0 & B^{\mathrm{T}}(x)M_3^{\mathrm{T}} & B^{\mathrm{T}}(x)M_5^{\mathrm{T}} & B^{\mathrm{T}}(x)M_2^{\mathrm{T}}\\ & * & * & (-2+a)Q & \frac{2}{\tau}(1-a)Q & -M_3\\ & * & * & * & -\frac{2}{\tau^2}(1-a)Q & -M_5\\ & * & * & * & & -M_2-M_2^{\mathrm{T}}+\tau^2Q\end{bmatrix}$$

and Δ_{11} and Δ_{22} are defined in (34), $\Delta_{12}(x)=M_1B(x)K(x)+A^{\mathrm{T}}(x)M_4^{\mathrm{T}}$. By using the Schur complement lemma, we have known that $\Upsilon(x)\le 0$ from (23) and (24). Next, we will prove the H_∞ performance of the system. The following auxiliary function is considered

$$J(e_x(t)) = \int_0^t (\|\bar{y}(s)\|_2^2 - \gamma^2 \|M_{\text{aero}}(s)\|_2^2)\mathrm{d}s \tag{40}$$

Therefore,

$$J(e_x(t)) \leq \int_0^t (\|\bar{y}(s)\|_2^2 - \gamma^2 \|M_{\text{aero}}(s)\|_2^2 + \dot{V}(e_x(s), s))\mathrm{d}s \tag{41}$$

$$\varUpsilon_1(x) = \begin{bmatrix} C^{\mathrm{T}}(x)C(x) & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & * & -\gamma^2 I & 0 & 0 & 0 \\ & * & * & 0 & 0 & 0 \\ & * & * & * & 0 & 0 \\ & * & * & * & * & 0 \end{bmatrix}$$

Then, we can have that

$$\varUpsilon(x) + \varUpsilon_1(x) = \begin{bmatrix} \varDelta_{11}(x) - aQ + C^{\mathrm{T}}(x)C(x) & \varDelta_{12}(x) + aQ & M_1 B(x) & A^{\mathrm{T}}(x)M_3^{\mathrm{T}} & A^{\mathrm{T}}(x)M_5^{\mathrm{T}} & P - M_1 + A^{\mathrm{T}}(x)M_2^{\mathrm{T}} \\ & \varDelta_{22}(x) - 2aQ & M_4 B(x) & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_3^{\mathrm{T}} + aQ & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_5^{\mathrm{T}} & K^{\mathrm{T}}(x)B^{\mathrm{T}}(x)M_2^{\mathrm{T}} - M_4 \\ & * & -\gamma^2 I & B^{\mathrm{T}}(x)M_3^{\mathrm{T}} & B^{\mathrm{T}}(x)M_5^{\mathrm{T}} & B^{\mathrm{T}}(x)M_2^{\mathrm{T}} \\ & * & * & (-2+a)Q & \frac{2}{\tau}(1-a)Q & -M_3 \\ & * & * & & -\frac{2}{\tau^2}(1-a)Q & -M_5 \\ & * & * & & * & -M_2 - M_2^{\mathrm{T}} + \tau^2 Q \end{bmatrix}$$

It is noted that $\|\bar{y}(s)\|_2^2 - \gamma^2 \|M_{\text{aero}}(s)\|_2^2 + \dot{V}(e_x(s), s) \leq \alpha^{\mathrm{T}}(s)(\varUpsilon(x) + \varUpsilon_1(x))\alpha(s)$, where

According to the Schur complement lemma, we have known $\varUpsilon(x) + \varUpsilon_1(x)$ is equivalent to

$$\Lambda(x) = \begin{bmatrix} \Delta_{11}(x) - aQ & \Delta_{12}(x) + aQ & M_1 B(x) & A^{\mathrm{T}}(x) M_3^{\mathrm{T}} & A^{\mathrm{T}}(x) M_5^{\mathrm{T}} & P - M_1 + A^{\mathrm{T}}(x) M_2^{\mathrm{T}} & C^{\mathrm{T}}(x) \\ & \Delta_{22}(x) - 2aQ & M_4 B(x) & K^{\mathrm{T}}(x) B^{\mathrm{T}}(x) M_3^{\mathrm{T}} + aQ & K^{\mathrm{T}}(x) B^{\mathrm{T}}(x) M_5^{\mathrm{T}} & K^{\mathrm{T}}(x) B^{\mathrm{T}}(x) M_2^{\mathrm{T}} - M_4 & 0 \\ & * & -\gamma^2 I & B^{\mathrm{T}}(x) M_3^{\mathrm{T}} & B^{\mathrm{T}}(x) M_5^{\mathrm{T}} & B^{\mathrm{T}}(x) M_2^{\mathrm{T}} & 0 \\ & * & * & (-2+a)Q & \frac{2}{\tau}(1-a)Q & -M_3 & 0 \\ & * & * & * & -\frac{2}{\tau^2}(1-a)Q & -M_5 & 0 \\ & * & * & * & & -M_2 - M_2^{\mathrm{T}} + \tau^2 Q & 0 \\ & * & * & * & & * & I \end{bmatrix} \tag{42}$$

Furthermore,

$$\begin{aligned} \Lambda(x) &= \sum_{i=1}^{8} \sum_{j=1}^{8} h_i(x(t)) h_j(x(t)) \Lambda_{ij} \\ &= \sum_{i=1}^{8} h_i^2(x(t)) \Lambda_{ii} + \sum_{i<j}^{8} \\ &\quad \times h_i(x(t)) h_j(x(t)) (\Lambda_{ij} + \Lambda_{ji}) \end{aligned} \tag{43}$$

From (23) and (24), we have known that $\Lambda(x) < 0$, which means $\Upsilon(x) + \Upsilon_1(x) < 0$. Therefore, $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\text{aero}}(t)\|_2^2$. So the system is asymptotically stable, the proof is thus completed.

In Theorem 1, a decomposition coefficient of delay integral inequality a is introduced, which can adjust the ratio between Lemmas 1 and 2. When $a = 1$ or $a = 0$, it is obvious that they are two special cases. The following corollaries are given for these two cases.

Corollary 1 *($a = 1$ case) Given scalars $\gamma > 0$, $\bar{d}$ and τ, if there exist matrices $R > 0$, $P > 0$, $Q > 0$, M_1, M_2, M_3, M_4, then the following inequalities hold*

$$\bar{\Phi}_{ii} < 0, \quad i = 1, 2, \ldots, 8 \tag{44}$$

$$\bar{\Phi}_{ij} + \bar{\Phi}_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, 8 \tag{45}$$

where

$$\bar{\Phi}_{ij} = \begin{bmatrix} \Delta_{11} - Q & \Delta_{12} + Q & M_1 B_i & A_i^{\mathrm{T}} M_3^{\mathrm{T}} & P - M_1 + A_i^{\mathrm{T}} M_2^{\mathrm{T}} & C_i^{\mathrm{T}} \\ & \Delta_{22} - 2Q & M_4 B_i & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_3^{\mathrm{T}} + Q & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_2^{\mathrm{T}} - M_4 & 0 \\ & * & -\gamma^2 I & B_i^{\mathrm{T}} M_3^{\mathrm{T}} & B_i^{\mathrm{T}} M_2^{\mathrm{T}} & 0 \\ & * & * & -Q & -M_3 & 0 \\ & * & * & * & -M_2 - M_2^{\mathrm{T}} + \tau^2 Q & 0 \\ & * & * & * & * & -I \end{bmatrix}$$

and Δ_{11}, Δ_{12} are defined in Theorem 1. Then, the system (20) is asymptotically stable, and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\text{aero}}(t)\|_2^2$.

Corollary 2 *($a = 0$ case) Given scalars $\gamma > 0$, $\bar{d}$ and τ, if there exist matrices $R > 0$, $P > 0$, $Q > 0$, M_1, M_2, M_3, M_4 and M_5, then the following inequalities hold*

$$\tilde{\Phi}_{ii} < 0, \quad i = 1, 2, \ldots, 8 \tag{46}$$

$$\tilde{\Phi}_{ij} + \tilde{\Phi}_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, 8 \tag{47}$$

where

$$\tilde{\Phi}_{ij} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & M_1 B_i & A_i^{\mathrm{T}} M_3^{\mathrm{T}} & A_i^{\mathrm{T}} M_5^{\mathrm{T}} & P - M_1 + A_i^{\mathrm{T}} M_2^{\mathrm{T}} & C_i^{\mathrm{T}} \\ & \Delta_{22} & M_4 B_i & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_3^{\mathrm{T}} & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_5^{\mathrm{T}} & K_j^{\mathrm{T}} B_i^{\mathrm{T}} M_2^{\mathrm{T}} - M_4 & 0 \\ & * & -\gamma^2 I & B_i^{\mathrm{T}} M_3^{\mathrm{T}} & B_i^{\mathrm{T}} M_5^{\mathrm{T}} & B_i^{\mathrm{T}} M_2^{\mathrm{T}} & 0 \\ & * & * & -2Q & \frac{2}{\tau} Q & -M_3 & 0 \\ & * & * & * & -\frac{2}{\tau^2} Q & -M_5 & 0 \\ & * & * & * & * & -M_2 - M_2^{\mathrm{T}} + \tau^2 Q & 0 \\ & * & * & * & * & * & -I \end{bmatrix}$$

When the controller $K_j = S_j X^{-T}$, the system (20) is asymptotically stable and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\mathrm{aero}}(t)\|_2^2$.

Then, the system (20) is asymptotically stable and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\mathrm{aero}}(t)\|_2^2$.

Because the Theorem 1 is unable to solve the gain of the controller K_j, on the basis of Theorem 1, we can further obtain the following theorem.

Theorem 2 *Given scalars $\gamma > 0$, $\bar{d}$, τ, ε_1, ε_2, ε_3 and ε_4, if there exist matrices $T > 0$, $G > 0$, $H > 0$ and a nonsingular matrix X, then the following LMIs hold*

$$\bar{\Lambda}_{ii} < 0, \quad i = 1, 2, \ldots, 8 \tag{48}$$

$$\bar{\Lambda}_{ij} + \bar{\Lambda}_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, 8 \tag{49}$$

where

$$\bar{\Lambda}_{ij} = \begin{bmatrix} \bar{\Delta}_{11} - aG & \bar{\Delta}_{12} + aG & B_i & \varepsilon_2 X A_i^{\mathrm{T}} & \varepsilon_4 X A_i^{\mathrm{T}} & H - X^{\mathrm{T}} + \varepsilon_1 X A_i^{\mathrm{T}} & X C_i^{\mathrm{T}} \\ & \bar{\Delta}_{22} - 2aG & \varepsilon_3 B_i & \varepsilon_2 S_j^{\mathrm{T}} B_i^{\mathrm{T}} + aG & \varepsilon_4 S_j^{\mathrm{T}} B_i^{\mathrm{T}} & \varepsilon_1 S_j^{\mathrm{T}} B_i^{\mathrm{T}} - \varepsilon_3 X^{\mathrm{T}} & 0 \\ & * & -\gamma^2 I & \varepsilon_2 B_i^{\mathrm{T}} & \varepsilon_4 B_i^{\mathrm{T}} & \varepsilon_1 B_i^{\mathrm{T}} & 0 \\ & * & * & (-2+a)G & \frac{2}{\tau}(1-a)G & -\varepsilon_2 X^{\mathrm{T}} & 0 \\ & * & * & * & -\frac{2}{\tau^2}(1-a)G & -\varepsilon_4 X^{\mathrm{T}} & 0 \\ & * & * & * & * & -\varepsilon_1 X^{\mathrm{T}} - \varepsilon_1 X + \tau^2 G & 0 \\ & * & * & * & * & * & -I \end{bmatrix}$$

Proof The proof is based on the conditions of Theorem 1. M_1 is nonsingular. Letting $X = M_1^{-1}$, $M_2 = \varepsilon_1 M_1$, $M_3 = \varepsilon_2 M_1$, $M_4 = \varepsilon_3 M_1$, $M_5 = \varepsilon_4 M_1$, then pre- and post-multiplying both sides of (48) and (49) with $\mathrm{diag}\{X^{-1}, X^{-1}, IX^{-1}, X^{-1}, X^{-1}, I\}$ and its transpose, and defining some matrices as follows:

$$T = XRX^{\mathrm{T}}, \; G = XQX^{\mathrm{T}}, \; H = XPX^{\mathrm{T}},$$
$$S_j = K_j X^{\mathrm{T}} (j = 1, 2, \ldots, 8)$$

we can arrive at (23) and (24). According to Theorem 1, we have known that the system (20) is asymptotically stable and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\mathrm{aero}}(t)\|_2^2$. This completes the proof.

and

$$\bar{\Delta}_{11} = A_i X^{\mathrm{T}} + X A_i^{\mathrm{T}} + T \tag{50}$$

$$\bar{\Delta}_{12} = B_i S_j + \varepsilon_3 X A_i^{\mathrm{T}} \tag{51}$$

$$\bar{\Delta}_{22} = -(1 - \bar{d})T + \varepsilon_3 B_i S_j + \varepsilon_3 S_j^{\mathrm{T}} B_i^{\mathrm{T}} \tag{52}$$

Corresponding with Corollaries 1 and 2, we can obtain the following corollaries by using the similar methods of Theorem 2.

Corollary 3 ($a = 1$ case) *Given scalars $\gamma > 0$, $\bar{d}$, τ, ε_1, ε_2 and ε_3, if there exist matrices $T > 0$, $G > 0$,*

$H > 0$ and a nonsingular matrix X, then the following LMIs hold

$$\bar{\Xi}_{ii} < 0, \quad i = 1, 2, \ldots, 8 \tag{53}$$

$$\bar{\Xi}_{ij} + \bar{\Xi}_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, 8 \tag{54}$$

where

$$\bar{\Xi}_{ij} = \begin{bmatrix} \bar{\Delta}_{11} - G & \bar{\Delta}_{12} + G & B_i & \varepsilon_2 X A_i^{\mathrm{T}} & H - X^{\mathrm{T}} + \varepsilon_1 X A_i^{\mathrm{T}} & X C_i^{\mathrm{T}} \\ & \bar{\Delta}_{22} - 2G & \varepsilon_3 B_i & \varepsilon_2 S_j^{\mathrm{T}} B_i^{\mathrm{T}} + G & \varepsilon_1 S_j^{\mathrm{T}} B_i^{\mathrm{T}} - \varepsilon_3 X^{\mathrm{T}} & 0 \\ & * & -\gamma^2 I & \varepsilon_2 B_i^{\mathrm{T}} & \varepsilon_1 B_i^{\mathrm{T}} & 0 \\ & * & * & -G & -\varepsilon_2 X^{\mathrm{T}} & 0 \\ & * & * & * & -\varepsilon_1 X^{\mathrm{T}} - \varepsilon_1 X + \tau^2 G & 0 \\ & * & * & * & * & -I \end{bmatrix}$$

and $\bar{\Delta}_{11}$, $\bar{\Delta}_{12}$ and $\bar{\Delta}_{22}$ are defined in 50, 51 and 52. When the controller $K_j = S_j X^{-T}$, the system (20) is asymptotically stable and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\text{aero}}(t)\|_2^2$.

Corollary 4 *($a = 0$ case) Given scalars $\gamma > 0$, $\bar{d}$, τ, ε_1, ε_2, ε_3 and ε_4, if there exist matrices $T > 0$, $G > 0$, $H > 0$ and a nonsingular matrix X, then the following LMIs hold*

$$\tilde{\Xi}_{ii} < 0, \quad i = 1, 2, \ldots, 8 \tag{55}$$

$$\tilde{\Xi}_{ij} + \tilde{\Xi}_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, 8 \tag{56}$$

where

$$\tilde{\Xi}_{ij} = \begin{bmatrix} \bar{\Delta}_{11} & \bar{\Delta}_{12} & B_i & \varepsilon_2 X A_i^{\mathrm{T}} & \varepsilon_4 X A_i^{\mathrm{T}} & H - X^{\mathrm{T}} + \varepsilon_1 X A_i^{\mathrm{T}} & X C_i^{\mathrm{T}} \\ & \bar{\Delta}_{22} & \varepsilon_3 B_i & \varepsilon_2 S_j^{\mathrm{T}} B_i^{\mathrm{T}} & \varepsilon_4 S_j^{\mathrm{T}} B_i^{\mathrm{T}} & \varepsilon_1 S_j^{\mathrm{T}} B_i^{\mathrm{T}} - \varepsilon_3 X^{\mathrm{T}} & 0 \\ & * & -\gamma^2 I & \varepsilon_2 B_i^{\mathrm{T}} & \varepsilon_4 B_i^{\mathrm{T}} & \varepsilon_1 B_i^{\mathrm{T}} & 0 \\ & * & * & -2G & \frac{2}{\tau} G & -\varepsilon_2 X^{\mathrm{T}} & 0 \\ & * & * & * & -\frac{2}{\tau^2} G & -\varepsilon_4 X^{\mathrm{T}} & 0 \\ & * & * & * & * & -\varepsilon_1 X^{\mathrm{T}} - \varepsilon_1 X + \tau^2 G & 0 \\ & * & * & * & * & * & -I \end{bmatrix}$$

When the controller $K_j = S_j X^{-T}$, the system (20) is asymptotically stable and satisfies H_∞ performance $\|\bar{y}(t)\|_2^2 < \gamma^2 \|M_{\text{aero}}(t)\|_2^2$.

Remark 2 In Theorem 2, a tracking control design algorithm is firstly proposed for MEV with time-varying delay. In order to provide flexibility in tracking controller design and reduce time delay influences, a decomposition coefficient a is introduced. Corollaries 3 and 4 are two special cases of decomposition coefficient a. In the next section, we will further show a decomposition coefficient a is helpful in the reduction of conservatism.

4 Numerical simulation

In this section, we will show the effectiveness of our proposed attitude tracking control method for MEV and will analyze the effects of decomposition coefficient and time delay bound on the fuzzy error system performance. The reference attitude trajectory and Mars atmospheric density model are the same as [30]. The parameters are chosen as $J_{xx} = 2983\,\text{kg m}^2$, $J_{yy} = 4909\,\text{kg m}^2$, $J_{zz} = 5683\,\text{kg m}^2$, $l_{\text{ref}} = 6.323\,\text{m}$, $S_{\text{ref}} = 11.045\,\text{m}^2$, $c_{n\beta} = 0.015$, $c_{m\alpha} = 0$, $c_{l\beta} = -2.414$, $\bar{d} = 0.1$.

The initial attitude conditions are set to $\omega(0) = [0.1000 \;\; 0.2000 \;\; -0.1000]^{\mathrm{T}}$ (rad/s), $\sigma(0) = [0.3670 \;\; 0.1480 \;\; -0.1493]^{\mathrm{T}}$, the design matrices K_a and K_b in (6) for the slow subsystem are $K_a = \text{diag}\{2, 2, 2\}$, $K_b = \text{diag}\{0.0100, 0.0100, 0.0100\}$. The upper bounds for angular velocity on each axis are $R_1 = R_2 = R_3 = 5$ (rad/s), The lower bounds for angular velocity on each axis are $r_1 = r_2 = r_3 = -5$ (rad/s). In order to verify

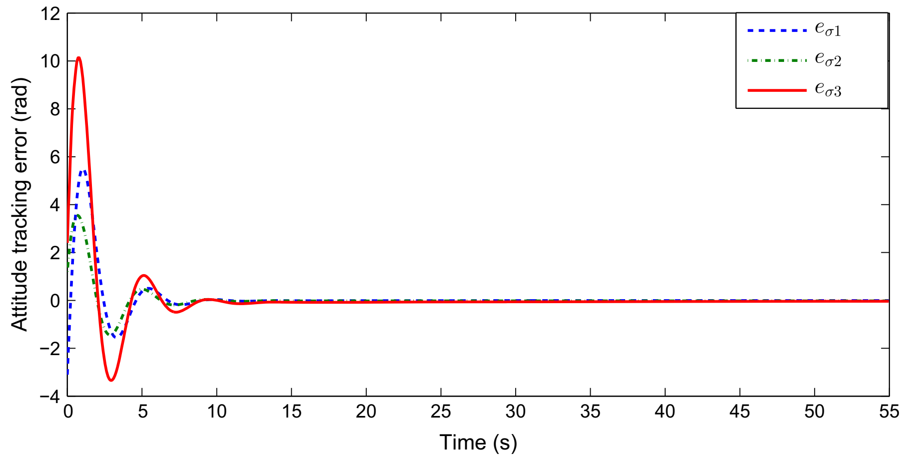


Fig. 2 Attitude tracking errors

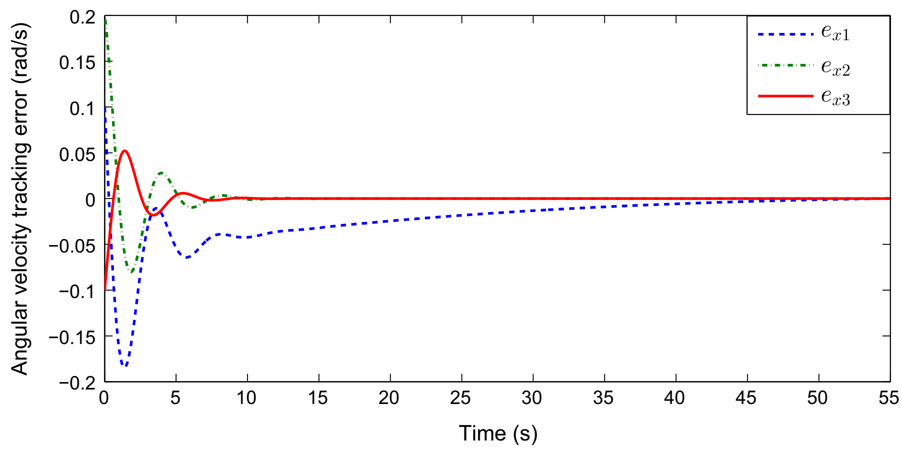


Fig. 3 Angular velocity tracking errors

the effectiveness of the proposed method, by using Theorem 2, simulation research on attitude tracking error control under MATLAB environment.

Firstly, given $\gamma = 18.8531$, $\varepsilon_1 = \varepsilon_3 = 10$, $\varepsilon_2 = \varepsilon_4 = 0.1$, letting the decomposition coefficient $a = 0.1$, $\tau = 100\,\text{ms}$, the controller gains can be obtained as

$$K_1 = \begin{bmatrix} -18.0010 & -0.0273 & -17.8809 \\ 64.3439 & -30.9517 & 0.0437 \\ 0.1638 & -44.5357 & -36.0293 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -18.0611 & 0.0491 & 17.9082 \\ 64.3057 & -31.0172 & -0.0382 \\ -0.0437 & -44.5466 & -36.1058 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -18.0611 & -0.0491 & -17.9082 \\ -64.3057 & -44.5466 & -0.0382 \\ 0.0437 & -44.5466 & -36.1058 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} -18.0010 & 0.0273 & 17.8809 \\ -64.3439 & -30.9517 & 0.0437 \\ -0.1638 & -44.5357 & -36.1058 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} -18.0611 & 0.0491 & -17.9082 \\ 64.3057 & -31.0172 & 0.0382 \\ 0.0437 & 44.5466 & -36.1058 \end{bmatrix},$$

$$K_6 = \begin{bmatrix} -18.0010 & 0.0273 & -17.8809 \\ -64.3439 & -30.9517 & -0.0437 \\ 0.1638 & 44.5357 & -36.0293 \end{bmatrix},$$

$$K_7 = \begin{bmatrix} -18.0010 & -0.0273 & 17.8809 \\ 64.3439 & -30.9517 & -0.0437 \\ -0.1638 & 44.5357 & -36.0293 \end{bmatrix},$$

$$K_8 = \begin{bmatrix} -18.0611 & -0.0491 & 17.9082 \\ -64.3057 & -31.0172 & 0.0382 \\ -0.0437 & 44.5466 & -36.1058 \end{bmatrix}.$$

Figure 2 shows the attitude tracking errors of slow subsystem, and Fig. 3 shows the angular velocity track-

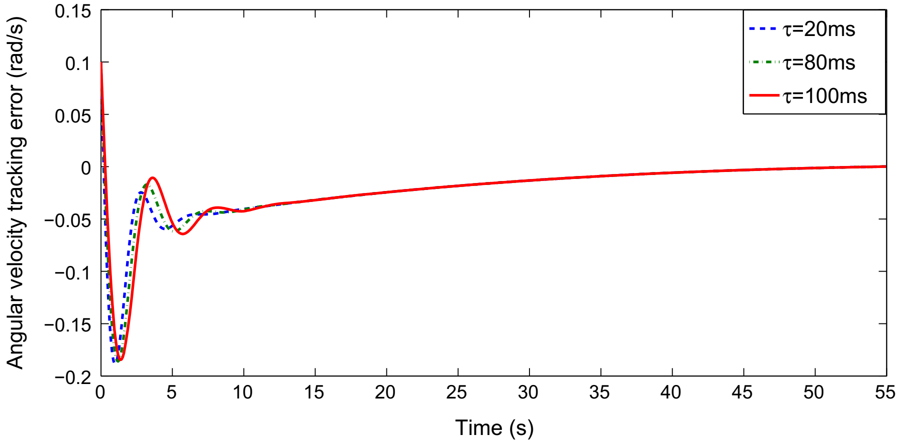


Fig. 4 e_{x1} under different time delay bounds

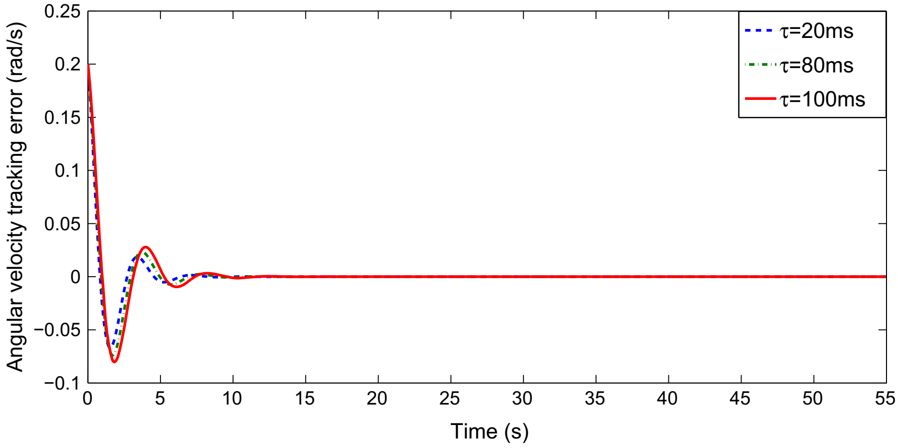


Fig. 5 e_{x2} under different time delay bounds

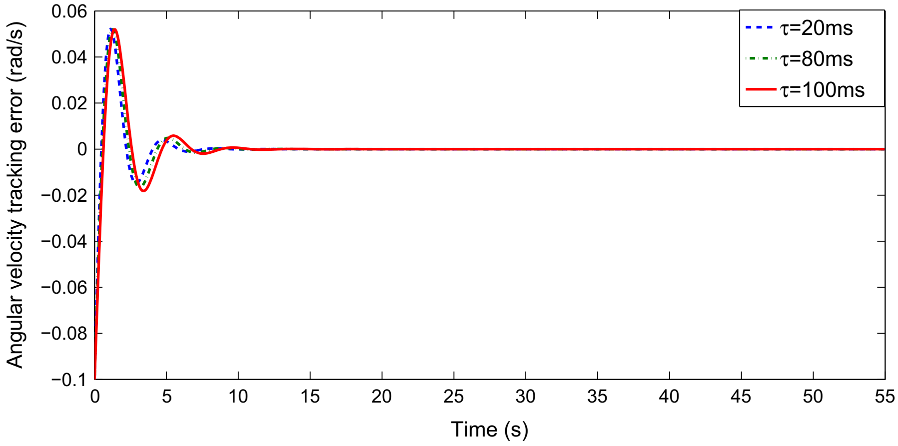


Fig. 6 e_{x3} under different time delay bounds

ing errors of fast subsystem. From Figs. 2 and 3 we can clearly see that the attitude tracking errors and the angular velocity tracking errors are tending to zero gradually, which means they are asymptotically stable.

4.1 The effect of time delay on fast subsystem

When the decomposition coefficient $a = 0.1$, Figs. 4, 5 and 6 show e_{x1}, e_{x2} and e_{x3} under different time delay

Fig. 7 e_{x1} under different decomposition coefficients

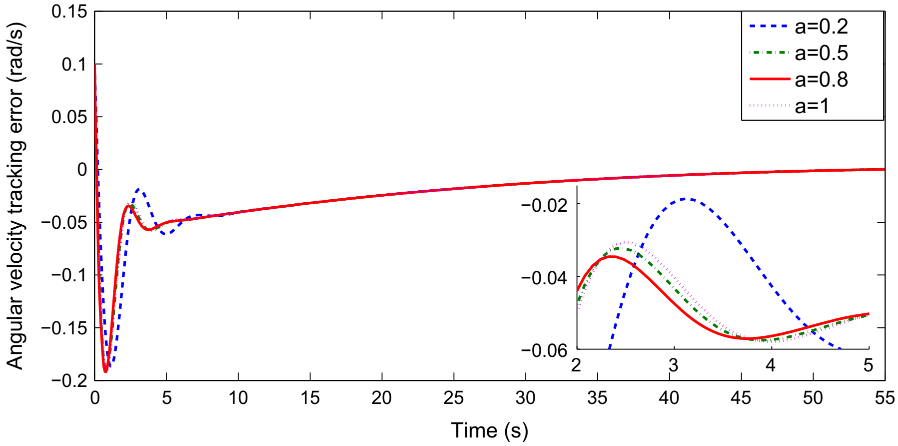


Fig. 8 e_{x2} under different decomposition coefficients

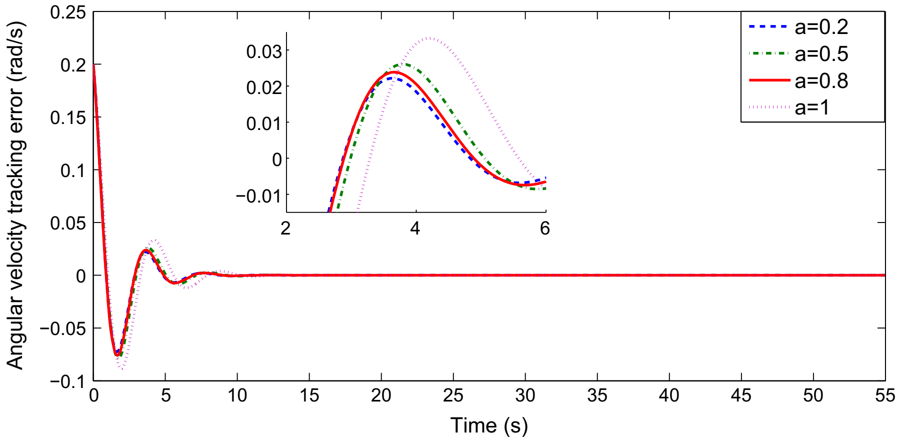
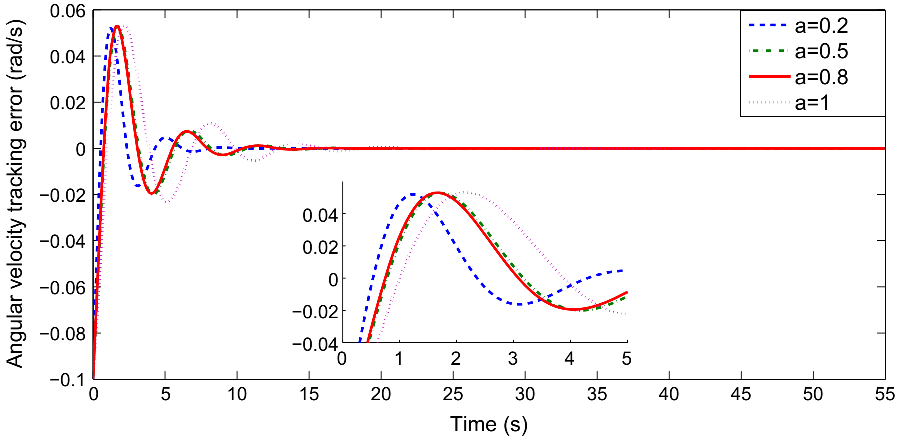


Fig. 9 e_{x3} under different decomposition coefficients



bounds, respectively. From Figs. 4, 5 and 6, we can see that time delay is passive for attitude tracking control.

4.2 The effect of decomposition coefficient on fast subsystem

When the time delay $\tau = 100\,\text{ms}$, Figs. 7, 8 and 9 show e_{x1}, e_{x2} and e_{x3} under different decomposition coefficients, respectively. According to Figs. 7, 8 and 9,it is clear that the decomposition coefficient is influential for the angular velocity. For two special cases ($a = 1$ case, $a = 0$ case), $a = 0$ case is infeasible, $a = 1$ case is not the worst case, which means the decomposition coefficient is an important role of reducing conservatism.

5 Conclusion

This paper considered the attitude tracking control of MEV with time-varying input delay. Firstly, T-S fuzzy method was applied for modeling the fast system of MEV. Secondly, the decomposition coefficient of delay integral inequality was introduced to reduce the delay effect about tracking control. Finally, numerical simulations were used to show the effects of time delay bound and decomposition coefficient on the fuzzy tracking error system performance. It is noted that some disturbances coming from the Martian atmosphere and modeling errors were not deeply considered in this paper. However, they are unavoidable in practical system, which should be considered to improve the tracking control precision of MEV. Thus, robust tracking control of MEV with multi-source disturbances will be studied in future work.

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