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Adaptive synchronization of coupled nonidentical chaotic systems with complex variables and stochastic perturbations

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Abstract This paper considers asymptotic synchronization in an array of complex-variable chaotic systems, where node systems exhibit nonidentical nonlinear dynamics and subject to different stochastic perturbations. The effects of the differences among the node systems are overcome by designing a special adaptive discontinuous controller. By using Lyapunov stability theorem and stochastic properties, sufficient conditions are obtained to guarantee the synchronization. Finally, numerical simulations are given to show the effectiveness of the theoretical results.

Keywords Coupled nonidentical chaotic systems · Stochastic perturbation · Synchronization · Complex variables · Adaptive control

1 Introduction

In the literature, synchronization of coupled chaotic systems (CCSs) with real variables has attracted considerable attention in different areas including biological, information processing and secure communications [1–5]. At the same time, many effective control methods have been proposed to investigate synchronization of CCSs with real variables, such as impulsive control [6], state feedback control [7–9] and adaptive control [10–12], among which adaptive control receives widespread attention since its control gains can be automatically adjusted according to some designed update law.

Recently, increasing attention has been attracted to synchronization and control of CVCSs due to the fact that the CVCSs can evolve in different directions with a constant intersection angle and have wide applications in many fields such as optoelectronics, filtering, imaging, speech synthesis, computer vision and remote sensing [13–15]. For example, Wu et al. [16] investigated complex projective synchronization in coupled dynamical systems with complex variables based on a proper feedback controller. Liu et al. [17] investigated robust adaptive full-state hybrid synchronization of chaotic complex-valued systems with unknown parameters and nonidentical external disturbances. Combination synchronization of three chaotic systems with complex variables was investigated in [18]. Drive-response synchronization for a class of complex-variable chaotic systems with uncertain parameters was studied in [19] via adaptive and impulsive controls.

It should be noted that the node systems in the above-mentioned papers concerning CVCSs are identical. From practical point of view, it is not always reasonable to assume that all the nodes in a network are identical since some real-world complex networks may

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consist of different type nodes and the nodes usually have different physical parameters [20]. Although there are some results on synchronization of coupled real-valued chaotic systems with nonidentical node systems in the literature [9,21], to the best of our knowledge, few papers consider the issue of synchronization in an array of CVCSs with nonidentical nodes. On the other hand, stochastic perturbations to node systems are always unavoidable and may also be nonidentical since the effects of environments on each node system are different. Theoretically, it is difficult to synchronize a network with both nonidentical node systems and nonidentical stochastic perturbations, which can further improve the security of the transmitted signals in a network. However, seldom author considers synchronization of networks coupled with nonidentical complex-valued chaotic systems suffered to nonidentical stochastic perturbations.

Motivated by the above discussions, this paper aims to investigate synchronization in an array of coupled nonidentical CSCVs and nonidentical stochastic perturbations. A simple adaptive discontinuous controller is designed to overcome the effects of the differences among the coupled nodes and synchronize the network onto a nonidentical chaotic system with complex variables. Based on Lyapunov stability theorem and stochastic theory, sufficient conditions are obtained to ensure the synchronization. Numerical simulations are given to show the effectiveness of the theoretical results.

The rest of this paper is organized as follows. In Sect. 2, a network coupled with nonidentical CSCVs with different stochastic perturbations is proposed. Some necessary assumptions and lemmas are also given in this section. In Sect. 3, synchronization of the network is studied. Then, numerical simulations are given in Sect. 4 to demonstrate the effectiveness of theoretical results. Finally, Sect. 5 gives conclusions.

2 Notations

The notations in this paper are quite standard. $\mathbb{C}^n$ denotes a set of n-dimensional complex vectors. For $x \in \mathbb{C}^n$, x^R and x^I denote the real and imaginary parts of x, respectively, $\bar{x}$ denotes the complex conjugate of x, $\|.\|$ is the Euclidean norm, i.e., $\|x\| = \sqrt{x^T \bar{x}}$, and $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The superscript T denotes transposition of a matrix or vector. I_n is the $n \times n$ identity matrix. $A = (a_{ij})_{N\times N}$ denotes matrix of N-dimension, $\|A\| = \sqrt{\lambda_{\max}(A^T \bar{A})}$, $\lambda_{\max}(A)$ means the largest eigenvalue of A, $A^s = \frac{1}{2}(\bar{A} + A^T)$. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ that satisfy the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L^P_{\mathcal{F}_0}([-\kappa, 0]; \mathbb{C}^n)$ the family of all $\mathcal{F}_0$-measurable $C([-\kappa, 0]; \mathbb{C}^n)$-valued random variables $\zeta = \{\zeta(s) : -\kappa \leq s \leq 0\}$ such that $\sup_{-\kappa\leq s\leq 0} \mathbb{E}\|\zeta(s)\|^p < \infty$, where $\mathbb{E}\{.\}$ stands for the mathematical expectation with respect to the given probability measure P.

3 Preliminaries

Consider a network coupled with CVCSs which is described as follows:

$$\mathrm{d}x_i(t) = \left[f_i(x_i(t)) + \varrho \sum_{j=1}^{N} a_{ij} \Lambda x_j(t) + U_i(t) \right] \mathrm{d}t + \delta_i(t) \mathrm{d}\omega_i(t), \; i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{C}^n$ is an n-dimensional complex vector with $x_{il}(t) = x^R_{il}(t) + j x^I_{il}(t)$, $l = 1, 2, \ldots, n$, $x^R_{il}(t)$ and $x^I_{il}(t)$ are real and imaginary parts of x_{il}, respectively, $j = \sqrt{-1}$; $f_i : \mathbb{C}^n \to \mathbb{C}^n$ is nonlinear complex-valued vector function, where f_i can be different from f_j if $i \neq j$. $\delta_i(t)$ is the noise intensity matrix function; $\omega_i(t) = (\omega_{i1}, \omega_{i2}, \ldots, \omega_{in}) \in R^n$ is a vector-form Wiener process defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_{t\geq 0}, P)$; $U_i(t)$ is the controller to be designed; the constant matrix $\Lambda = (\Lambda_{ij})_{n\times n}$ describes the inner-coupling matrix of the network; ϱ represents the coupling strength; matrix $A = (a_{ij})_{N\times N}$ stands for the coupling of the whole network. If there is a connection from node i to node j $(i \neq j)$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$ $(i \neq j)$ and the diagonal elements of matrix A are defined as $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$.

Our goal is to synchronize the states of the network (1) onto the complex-variable manifold:

$$\mathrm{d}y(t) = g(y(t))\mathrm{d}t, \tag{2}$$

where $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in \mathbb{C}^n$, $g(y(t)) = (g_1(y(t)), g_2(y(t)), \ldots, g_n(y(t))) \in \mathbb{C}^n$.

Definition 1 [22]. The coupled network (1) is said to be globally asymptotically synchronized onto (2) if

$$\lim_{t\to+\infty} \mathbb{E}\{\|x_i(t) - y(t)\|\} = 0, \; i = 1, 2, \ldots, N,$$

hold for any given initial condition.

Let $e_i(t) = x_i(t) - y(t)$, $i = 1, 2, \ldots, N$. It can obtain the following error system by subtracting (2) from (1).

$$\begin{aligned} \mathrm{d}e_i(t) &= \mathrm{d}x_i(t) - \mathrm{d}y(t) \\ &= \Big[\overline{f}_i(e_i(t)) + \varrho \sum_{j=1}^{N} a_{ij}\Lambda e_j(t) \\ &\quad + U_i(t) + \Sigma_i(t)\Big]\mathrm{d}t \\ &\quad + \delta_i(t)\mathrm{d}\omega_i(t), i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$

where $\overline{f}_i(e_i(t)) = f_i(x_i(t)) - f_i(y(t))$, $\Sigma_i(t) = f_i(y(t)) - f(y(t))$.

For convenience of study, the error system (3) is separated into real and imaginary parts. Let $e_i(t) = e_i^R(t) + je_i^I(t)$, $\overline{f}_i(e_i(t)) = \overline{f}_i^R(e_i(t)) + j\overline{f}_i^I(e_i(t))$, and $U_i(t) = U_i^R(t) + jU_i^I(t)$, $\Sigma_i(t) = \Sigma_i^R(t) + j\Sigma_i^I(t)$, $\delta_i(t) = \delta_i^R(t) + j\delta_i^I(t)$. Then the following two real-valued systems can be obtained from (3):

$$\begin{aligned} \mathrm{d}e_i^R(t) &= \Big[\overline{f}_i^R(e_i(t)) + \varrho \sum_{j=1}^{N} a_{ij}\Lambda e_j^R(t) + U_i^R(t) \\ &\quad + \Sigma_i^R(t)\Big]\mathrm{d}t + \delta_i^R(t)\mathrm{d}\omega_i(t), \quad i = 1, 2, \ldots, N, \end{aligned} \tag{4}$$

$$\begin{aligned} \mathrm{d}e_i^I(t) &= \Big[\overline{f}_i^I(e_i(t)) + \varrho \sum_{j=1}^{N} a_{ij}\Lambda e_j^R(t) + U_i^I(t) \\ &\quad + \Sigma_i^I(t)\Big]\mathrm{d}t + \delta_i^I(t)\mathrm{d}\omega_i(t), \quad i = 1, 2, \ldots, N. \end{aligned} \tag{5}$$

Denote $z(t) = (z_1(t), \ldots, z_N(t), z_{N+1}(t), \ldots, z_{2N}(t))^T = ((e^R(t))^T, (e^I(t))^T)^T$. It follows from (4) and (5) that

$$\begin{aligned} \mathrm{d}z_k(t) &= \Big[\hat{f}_k(z_k(t)) + \varrho \sum_{j=1}^{2N} \bar{a}_{kj}\Lambda z_j(t) \\ &\quad + \bar{U}_k(t) + \overline{\Sigma}_k(t)\Big]\mathrm{d}t \\ &\quad + \bar{\delta}_k(t)\mathrm{d}\bar{\omega}_k(t), k = 1, 2, \ldots, 2N, \end{aligned} \tag{6}$$

where $(\hat{f}_1(z_1(t)), \ldots, \hat{f}_1(z_N(t)), \hat{f}_{N+1}(z_{N+1}(t)), \ldots, \hat{f}_{2N}(z_{2N}(t)))^T = (\overline{f}_1^R(e_1^R(t)), \ldots, \overline{f}_N^R(e_N^R(t)), \overline{f}_1^I(e_1^I(t)), \ldots, \overline{f}_N^I(e_N^I(t))$, $\overline{A} = (\overline{a}_{kj})_{2N\times 2N} = \mathrm{diag}(A, A)$, $(\bar{U}_1(t), \ldots, \bar{U}_N(t), \bar{U}_{N+1}(t), \ldots, \bar{U}_{2N}(t)) = (U_1^R(t), \ldots, U_N^R(t), U_1^I(t), \ldots, U_N^I(t)$, $(\overline{\Sigma}_1(t), \ldots, \overline{\Sigma}_N(t), \overline{\Sigma}_{N+1}(t), \ldots, \overline{\Sigma}_{2N}(t)) = (\Sigma_1^R(t), \ldots, \Sigma_N^R(t), \Sigma_1^I(t), \ldots, \Sigma_N^I(t))$.

As far as the authors' knowledge, most of the existing results on synchronization of coupled CVCSs only focus on identical node systems. Obviously, synchronization control of coupled nonidentical CVCSs is more difficult than that of coupled CVCSs with identical nodes. In order to obtain synchronization criterion for the network (1), the complex-variable error system (3) has been transformed into real-variable error system (6). To proceed our study, the following assumptions are needed.

(H_1) There exist nonnegative constants h_k such that $\|\hat{f}_k(u)\| \le h_k\|u\|$, where $u \in \mathbb{R}^n$, $k = 1, 2, \ldots, 2N$.

(H_2) There exist nonnegative constants μ_{kj} such that, for $k = 1, 2, \ldots, 2N$,

$$\mathrm{trace}(\overline{\sigma}_k^T(t)\overline{\sigma}_k(t)) \le \sum_{j=1}^{2N} \mu_{kj} z_j^T(t) z_j(t).$$

(H_3) Systems (1) and (2) are chaotic, and there exist positive constants M_{kj}, $\overline{M}_j$ such that $|f_{kj}(y(t))| \le M_{kj}$, $|f_j(y(t))| \le \overline{M}_j$, $k = 1, 2, \ldots, 2N$, $j = 1, 2, \ldots, n$.

As for Wiener process, the following properties are useful [2,23].

(1) $\mathbb{E}\{\bar{\sigma}_k(t)\mathrm{d}\bar{\omega}_k\} = 0$, $(\bar{\sigma}_k(t)\mathrm{d}\bar{\omega}_k)^T(\bar{\sigma}_k(t)\mathrm{d}\bar{\omega}_k) = \mathrm{trace}(\bar{\sigma}_k(t)^T\bar{\sigma}(t))\mathrm{d}t$;

(2) Suppose that $V = V(x(t))$ is a scalar function, which $x = (x_1, x_2, \ldots, x_n)^T$. The differential form of V is obtained as

$$\mathrm{d}V = \frac{\partial V}{\partial t}dt + \sum_{i=1}^{n}\frac{\partial V}{\partial x_i}dx_i + \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 V}{\partial x_i \partial x_j}dx_i dx_j.$$

In expansion of the above equation, the following algebraic operation is used: $\mathrm{d}t\mathrm{d}t = 0$, $\mathrm{d}t\mathrm{d}\bar{\omega}_{kj} = 0$, $\mathrm{d}\bar{\omega}_{kj}\mathrm{d}\bar{\omega}_{kj} = dt$, $\mathrm{d}\bar{\omega}_{kj}\mathrm{d}\bar{\omega}_{kl} = 0$ $(j \ne l)$.

4 Main results

In this section, based on the Lyapunov stability theorem, general criterion for synchronization of coupled nonidentical CVCSs will be obtained. Rigorous mathematical proofs are also given.

Theorem 1 *Suppose that conditions* $(H_1) - (H_3)$ *are satisfied. Then, with the controller*

$$U_k(t) = -l_k(t)z_k(t) - \alpha\beta_k(t)\mathrm{sign}(z_k(t)),\quad k = 1, 2, \ldots, 2N, \tag{7}$$

and adaptive law

$$\begin{cases} \dot{l}_k(t) = \varepsilon_k z_k^T(t)z_k(t), & k = 1, 2, \ldots, 2N, \\ \dot{\beta}_k(t) = \eta_k \sum_{j=1}^{n} |z_{kj}(t)|, & k = 1, 2, \ldots, 2N, \end{cases} \tag{8}$$

the network (1) *can be globally asymptotically synchronized onto* (2), *where* $\alpha > 1$, $\varepsilon_k > 0$, $\eta_k > 0$ *are small constants, and* $\mathrm{sign}(\cdot)$ *is the sign function.*

Proof Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{k=1}^{2N} z_k^T(t)z_k(t) + \sum_{k=1}^{2N}\frac{1}{2\varepsilon_k}(l_k(t) - p_k)^2 + \sum_{k=1}^{2N}\frac{1}{2\eta_k}(q_k - \beta_k(t))^2.$$

Differentiating $V(t)$ along the solution of (6) and taking the expectations on both sides, one obtains that

$$\begin{aligned}
\mathbb{E}\left\{\frac{\mathrm{d}V(t)}{dt}\right\} &= \mathbb{E}\Bigg\{\sum_{k=1}^{2N} z_k^T(t)\mathrm{d}z_k(t) \\
&\quad + \sum_{k=1}^{2N}(l_k(t) - p_k)z_k^T(t)z_k(t) \\
&\quad - \sum_{k=1}^{2N}\sum_{j=1}^{n}(q_k - \beta_k(t))|z_{kj}(t)|\Bigg\} \\
&= \mathbb{E}\Bigg\{\sum_{k=1}^{2N} z_k^T(t)\Bigg[\hat{f}_k(z_k(t)) \\
&\quad + \varrho\sum_{j=1}^{2N}\bar{a}_{kj}\Lambda z_j(t) - l_k(t)z_k(t) \\
&\quad - \alpha\beta_k(t)\mathrm{sign}(z_k(t)) + \overline{\Sigma}_k \\
&\quad + \frac{1}{2}\mathrm{trace}\bar{\delta}_k^T(t)\bar{\delta}_k(t)\Bigg] \\
&\quad + \sum_{k=1}^{2N}(l_k(t) - p_k)z_k^T(t)z_k(t) \\
&\quad - \sum_{k=1}^{2N}\sum_{j=1}^{n}(q_k - \beta_k(t))|z_{kj}(t)|\Bigg\}.
\end{aligned} \tag{9}$$

It can be obtained from (H_1) and (H_2) that

$$\begin{aligned}
\mathbb{E}\left\{\frac{\mathrm{d}V(t)}{dt}\right\} &\le \mathbb{E}\Bigg\{\sum_{k=1}^{2N} z_k^T(t)h_k z_k(t) \\
&\quad + \varrho\sum_{k=1}^{2N}\sum_{j=1}^{2N}\overline{a}_{kj}z_k^T(t)\Lambda z_k(t) \\
&\quad + \sum_{k=1}^{2N} z_k^T(t)\overline{\Sigma}_k + \sum_{k=1}^{2N} z_k^T(t)l_k(t)z_k(t) \\
&\quad + \alpha\sum_{k=1}^{2N} z_k^T(t)\beta_k(t)\mathrm{sign}(z_k(t)) \\
&\quad + \frac{1}{2}\sum_{k=1}^{2N}\sum_{j=1}^{2N}\mu_{kj}z_j^T(t)z_j(t) \\
&\quad + \sum_{k=1}^{2N}(l_k(t) - p_k)z_k^T(t)z_k(t) \\
&\quad + \sum_{k=1}^{2N}\sum_{j=1}^{n}(q_k - \beta_k(t))|z_{kj}|\Bigg\} \\
&\le \mathbb{E}\Bigg\{\sum_{k=1}^{2N} h_k\|z_k(t)\|^2 \\
&\quad + \varrho\sum_{k,j=1,k\neq j}^{2N}\overline{a}_{kj}\|\Lambda\|\|z_k(t)\|\|z_j(t)\| \\
&\quad + \varrho\sum_{k}^{2N}\rho_{min}\overline{a}_{kk}z_k^T(t)z_k(t) \\
&\quad + \sum_{k=1}^{2N} z_k^T(t)\overline{\Sigma}_k + \frac{1}{2}\sum_{k=1}^{2N}\sum_{j=1}^{2N}\mu_{kj}\|z_j(t)\|^2 \\
&\quad - \sum_{k=1}^{2N} p_k\|z_k(t)\|^2 \\
&\quad + \alpha\sum_{k=1}^{2N} z_k^T(t)\beta_k(t)\mathrm{sign}(z_k(t)) \\
&\quad + \sum_{k=1}^{2N}\sum_{j=1}^{n}(q_k - \beta_k(t))|z_{kj}|\Bigg\},
\end{aligned} \tag{10}$$

where $\rho_{\min}$ is the minimum eigenvalue of Λ^s.

By (10) and (H_3), one has

$$\sum_{k=1}^{2N} z_k^T(t)\overline{\Sigma}_k + \alpha\sum_{k=1}^{2N} z_k^T(t)\beta_k(t)\mathrm{sign}(z_k(t)) - \sum_{k=1}^{2N}\sum_{j=1}^{n}(q_k - \beta_k(t))|z_{kj}(t)|$$

$$\le \sum_{k=1}^{2N}\sum_{j=1}^{n}|z_{kj}^T(t)|(M_{kj}+\overline{M}_j(t))$$
$$-\alpha\sum_{k=1}^{2N}\beta_k(t)|z_k(t)|$$
$$-\sum_{k=1}^{2N}\sum_{j=1}^{n}(q_k-\beta_k(t))|z_{kj}(t)|$$
$$\le \sum_{k=1}^{2N}\sum_{j=1}^{n}|z_{kj}^T(t)|(M_{kj}+\overline{M}_j(t)-q_k)-(\alpha-1)$$
$$\times\sum_{k=1}^{2N}\beta_k(t)|z_k(t)|\le 0, \tag{11}$$

where $q_k \ge \max(M_{kj}+\overline{M}_j)$, $k=1,2,\ldots,2N$, $j=1,2,\ldots,n$.

The inequalities (10) and (11) imply that

$$\mathbb{E}\left\{\frac{\mathrm{d}V(t)}{dt}\right\} \le \mathbb{E}\left\{\sum_{k=1}^{2N}h_k\|z_k(t)\|^2\right.$$
$$+\varrho\sum_{k,j=1,k\neq j}^{2N}\overline{a}_{kj}\|\Lambda\|\|z_k(t)\|\|z_j(t)\|$$
$$+\varrho\sum_{k}^{2N}\rho_{\min}\overline{a}_{kk}z_k^T(t)z_k(t)$$
$$+\frac{1}{2}\sum_{k=1}^{2N}\sum_{j=1}^{2N}\mu_{ij}z_j^T(t)z_j(t)$$
$$\left.-\sum_{k=1}^{2N}p_k\|z_k(t)\|^2\right\}$$
$$\le \mathbb{E}\left\{\overline{z}^T(t)\big(H+\|\Lambda\|\widetilde{A}+\Phi-\bar{P}\big)\overline{z}(t)\right\}, \tag{12}$$

where $\overline{z}(t)=(\|z_1(t)\|,\|z_2(t)\|,\ldots,\|z_{2N}(t)\|)^T$, $\widetilde{A}=(\tilde{a}_{kj})_{2N\times 2N}$, $\tilde{a}_{kj}=\bar{a}_{kj}, k\neq j$, $\tilde{a}_{kk}=\frac{\rho_{\min}}{\|\Lambda\|}\bar{a}_{kk}$, $H=\mathrm{diag}(h_1,h_2,\ldots,h_{2N})$, $\bar{P}=\mathrm{diag}(p_1,p_2,\ldots,p_{2N})$, $\Phi=(\mu_{kj})_{2N\times 2N}$.

Let $p_k = h_k+\lambda_{\max}(\|\Lambda\|\widetilde{A}+\Phi)+1$, $k=1,2,\ldots,2N$. One has

$$\mathbb{E}\left\{\frac{\mathrm{d}V(t)}{dt}\right\} \le -\mathbb{E}\{\overline{z}^T(t)\overline{z}(t)\}\le 0. \tag{13}$$

Hence,

$$\lim_{t\to\infty}\mathbb{E}\{\|z_k(t)\|\}=0, k=1,2,\ldots,2N. \tag{14}$$

This completes the proof. □

When the node systems in the network (1) and the isolated system (2) are identical, the following corollary 1 can be directly obtained from Theorem 1.

Corollary 1 *Assume that the conditions* $(H_1)-(H_3)$ *are satisfied and* $\hat{f}_1(z_1(t))=\hat{f}_2(z_2(t))=\cdots=\hat{f}_{2N}(z_{2N}(t))$. *Then the coupled system* (1) *can be globally asymptotically synchronized onto* (2) *with the controller* (7) *and adaptive law* (8)

Remark 1 From the proof of Theorem 1, one can see that the discontinuous term $\alpha\beta_k(z_k(t))$ in controller (7) plays an important role in realizing the synchronization. The inequality (11) shows that the discontinuous term can overcome the effect of nonidentical dynamics. Since node systems in real-world networks are always nonidentical, Theorem 1 is more general than those in [16–19].

5 Numerical examples

In this section, we provide one example to show that our theoretical results obtained above are effective.

Consider Chen system complex variables which are described as [24]:

$$\dot{y}(t)=g(t,y(t))=Cy(t)+\overline{g}(y(t)), \tag{15}$$

where $y(t)=(y_1(t),y_2(t),y_3(t))^T$, $y_1(t)$ and $y_2(t)$ are complex variables, $y_3(t)$ is real, $\overline{g}(y(t))=[0,-y_1(t)y_3(t),\frac{\overline{y}_1(t)y_2(t)+y_1(t)\overline{y}_2(t)}{2}]^T$,

$$C=\begin{pmatrix}-27 & 27 & 0\\ -3 & 23 & 0\\ 0 & 0 & -1\end{pmatrix}.$$

Figure 1 shows the chaotic trajectory of (15) with initial value $y(0)=(12+j,10+4*j,15)^T$.

Complex-valued Lorenz system is presented as [25]:

$$\dot{x}(t)=f(t,x(t))=\tilde{C}x(t)+\tilde{f}(x(t)), \tag{16}$$

where $x(t)=(x_1(t),x_2(t),x_3(t))^T$, $x_1(t)$ and $x_2(t)$ are complex variables, $x_3(t)$ is real, $\tilde{f}(x(t))=[0,-x_1(t)x_3(t),\frac{\overline{x}_1(t)x_2(t)+x_1(t)\overline{x}_2(t)}{2}]^T$,

$$\tilde{C}=\begin{pmatrix}-10 & 10 & 0\\ 28 & -1 & 0\\ 0 & 0 & -8/3\end{pmatrix}.$$

Figure 2 presents the chaotic trajectory of (16) with initial value $x(0)=(4+j,3+5*j,8)^T$.

Complex-valued L$\ddot{u}$ system is described by [26]

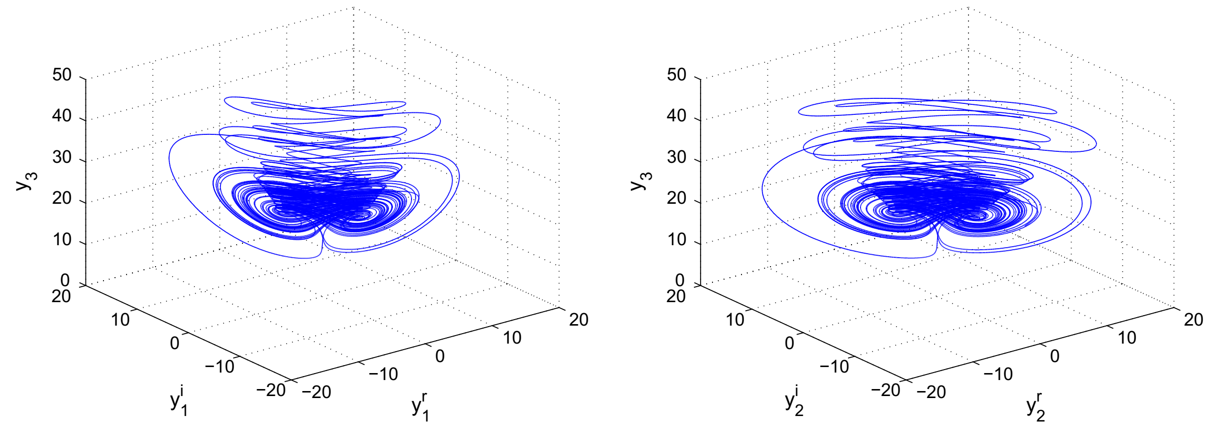

Fig. 1 Trajectory of (15) with initial conditions: $y_1(0) = 12 + j$, $y_2(0) = 10 + 4 * j$, $y_3(0) = 15$

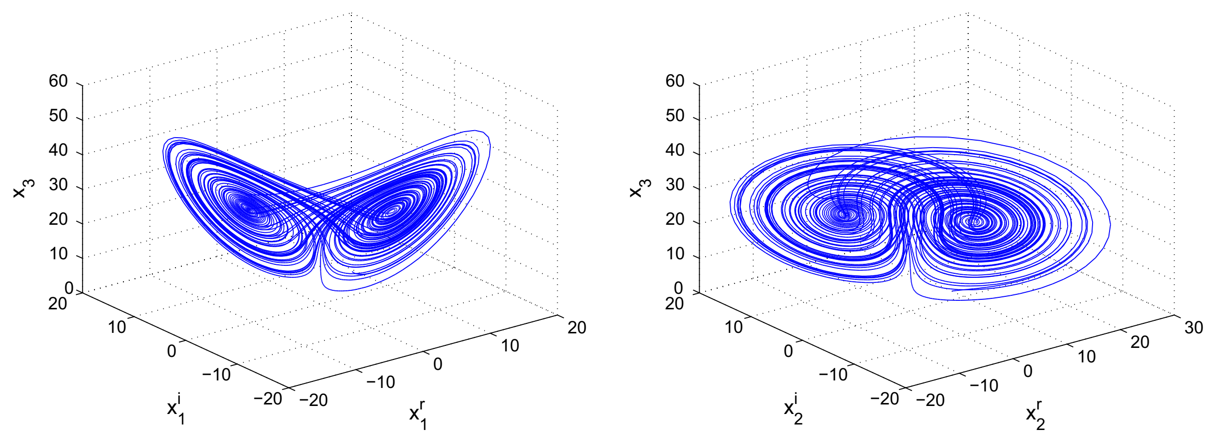

Fig. 2 Trajectory of (16) with initial conditions: $x_1(0) = 4 + j$, $x_2(0) = 3 + 5 * j$, $x_3(0) = 8$

$$\dot{x}(t) = \overline{f}(t, x(t)) = \check{C}x(t) + \check{f}(x(t)), \qquad (17)$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$, $x_1(t)$ and $x_2(t)$ are complex variables, $x_3(t)$ is real, $\check{f}(x(t)) = [0, -x_1(t)x_3(t), \frac{\overline{x}_1(t)x_2(t)+x_1(t)\overline{x}_2(t)}{2}]^T$,

$$\check{C} = \begin{pmatrix} -35 & 35 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & -3 \end{pmatrix},$$

Figure 3 describes the chaotic trajectory of (17) with initial value $x(0) = (8 + j, 5 + 4j, 7)^T$.

Consider a controlled network consisting of the above two types of nonidentical chaotic nodes (16) and (17) as follows:

$$\mathrm{d}x_i(t) = \left[f_i(x_i(t)) + \varrho \sum_{j=1}^{10} a_{ij} \Lambda x_j(t) + U_i(t) \right] \mathrm{d}t + \delta_i(t) \mathrm{d}\omega_i(t), \quad i = 1, 2, \ldots, 10, \qquad (18)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, $\Lambda = \mathrm{diag}(1, 1, 1)$, $\varrho = 1$, the noise intensity function matrix is $\delta_i(t) = \mathrm{diag}(x_{i1}(t) - x_{i+1,1}(t), x_{i2}(t) - x_{i+1,2}(t), x_{i3}(t) - x_{i+1,3}(t))$, and the outer coupling matrix A is

$$A = \begin{pmatrix} -2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & -6 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & -4 & 0 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & -4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & -3 \end{pmatrix},$$

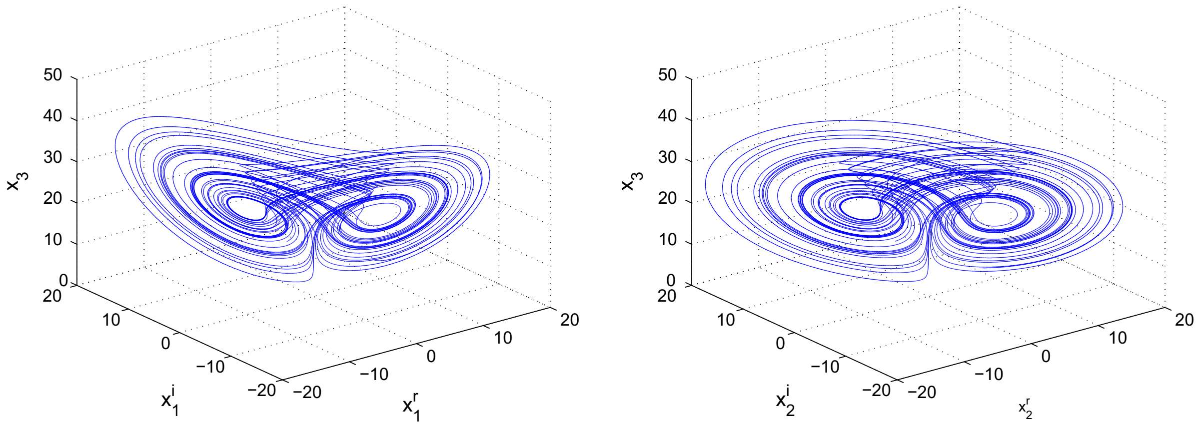

Fig. 3 Trajectory of (17) with initial conditions: $x_1(0) = 8 + j, x_2(0) = 5 + 4 * j, x_3(0) = 7$

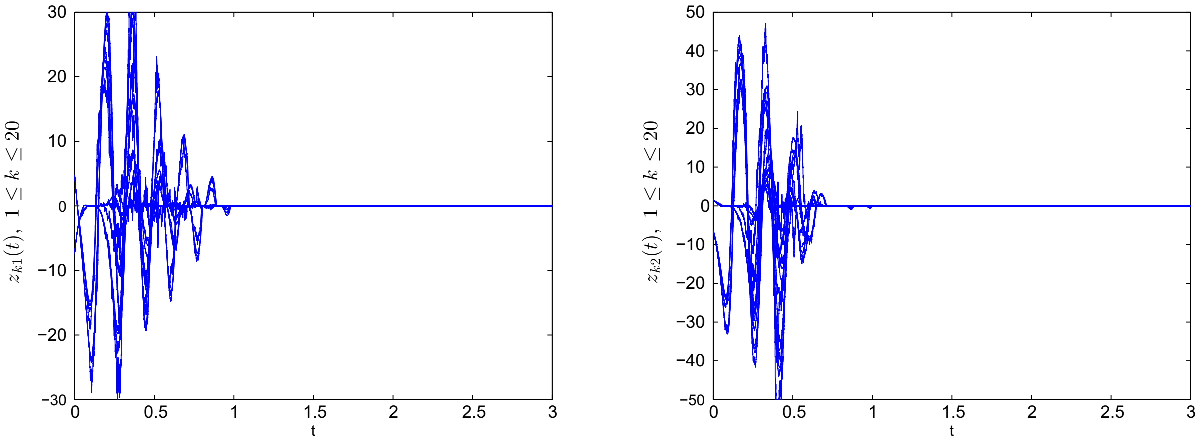

Fig. 4 The time responses of the synchronization errors $z_{k1}(t)$ (*left*) and $z_{k2}(t)$ (*right*), $k = 1, 2, \ldots, 20$ in (19)

and

$$f_i(x_i(t)) = \tilde{C}x(t) + \tilde{f}(x(t)), i = 1, 2, \ldots, 5,$$
$$f_i(x_i(t)) = \check{C}x(t) + \check{f}(x(t)), i = 6, 7, \ldots, 10.$$

It follows that

$$dz_k(t) = \left[\hat{f}_k z_k(t) + \varrho \sum_{j=1}^{20} \bar{a}_{kj} \Lambda a_j(t) + \bar{U}_k(t) + \bar{\Sigma}\right]dt + \bar{\delta}_k(t) d\bar{\omega}_k(t), k = 1, 2, \ldots, 20. \quad (19)$$

It is easy to see that $\hat{f}_i(z_i(t))$ satisfies the condition (H_1). Figures 1, 2 and 3 show that the states of the considered systems are finite, which implies that the assumption (H_3) is satisfied. Now we verify the assumption (H_2). From the noise intensity function matrix in (18), one has that

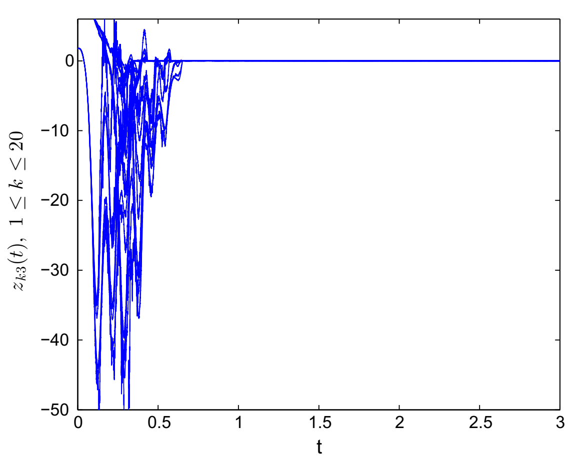

Fig. 5 The time responses of the synchronization errors $z_{k3}(t)$, $k = 1, 2, \ldots, 20$ in (19)

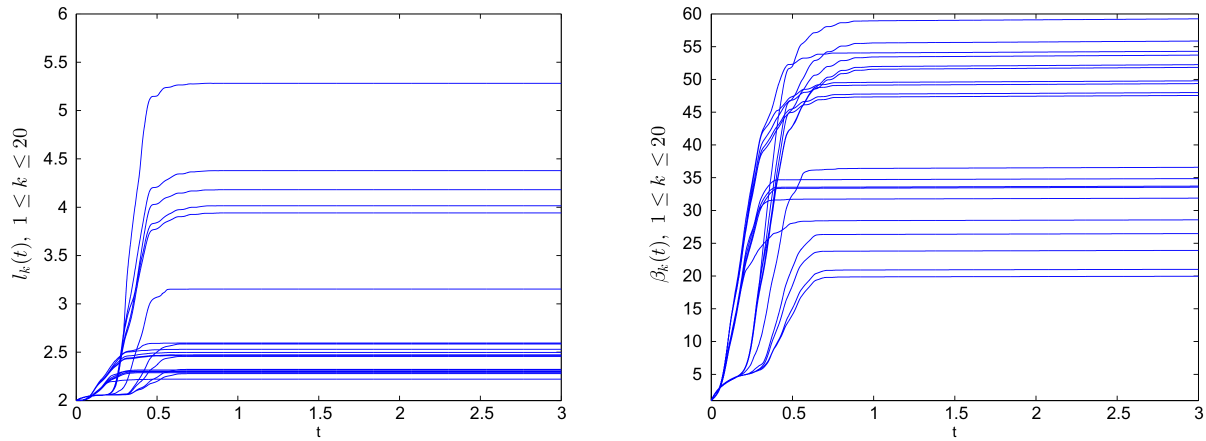

Fig. 6 The trajectories of control gains $l_k(t)$ (*left*) and $\beta_k(t)$ (*right*), $k = 1, 2, \ldots, 20$. of (7)

$$\begin{aligned}\mathrm{trace}(\delta_i^T(t)\delta_i(t)) &= (x_{i1}(t) - x_{i+1,1}(t))^2 + (x_{i2}(t)\\ &\quad - x_{i+1,2}(t))^2 + (x_{i3}(t) - x_{i+1,3}(t))^2\\ &\le 2(x_i^2(t) + x_{i+1}^2(t)).\end{aligned}$$

Hence (H_2) is satisfied.

According to Theorem 1, (18) can be synchronized onto (15) under the controller (7) with update law (8).

Taken parameters in the numerical simulations are: Step length is 0.0001, $\Lambda = \mathrm{diag}(1, 1, 1)$, $l_k = 2$, $\beta_k = 1$, $k = 1, 2, \ldots, 20$, $\varepsilon_k = 0.005$, $\eta_k = 2.5$, $k = 1, 2, \ldots, 10$, $\varepsilon_k = 0.001$, $\eta_k = 2.2$, $k = 11, 12, \ldots, 20$ $\alpha = 6$. Choosing the initial values of chaotic system randomly in the interval $[-3, 3]$, we obtain the simulation results shown in Figs. 4 and 5, which demonstrate that the synchronization is realized. Figure 6 presents the time evolution of the control gains $l_k(t)$, $\beta_k(t)$, $k = 1, 2, \ldots, 20$, from which one can see that all the control gains approach to some constants when the synchronization has been realized.

6 Conclusions

Synchronization of coupled nonidentical complex-valued chaotic systems suffered to different stochastic perturbations has been investigated in this paper. The designed adaptive controller can restrict the effects of the nonidentical dynamics and nonidentical stochastic perturbations. Based on Lyapunov stability theorem and the properties of stochastic differential equations, several synchronization criteria have been derived. Some existing results on synchronization of coupled identical chaotic systems with complex variables are extended. Numerical simulations verify the effectiveness of the theoretical results.

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