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Neural adaptive control for uncertain MIMO systems with constrained input via intercepted adaptation and single learning parameter approach

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Abstract This paper focuses on the problem of semi-globally stable neural adaptive control for a class of uncertain multi-input/multi-output nonlinear systems in the presence of strong interconnection, input saturation, and external disturbance. Radial basis function neural networks are utilized in the online learning of uncertain dynamics. The features of the scheme developed can be briefly summarized as follows: (1) The problem of "explosion of complexity" caused by the repeated differentiations of virtual controllers in traditional backstepping design is circumvented via the pioneering dynamic surface control technique; (2) the subsystem in the whole system can be any order, and only one scalar is needed to be online updated when dealing with uncertain dynamics and external disturbance, which is computationally inexpensive from the perspective of practical application; and (3) the bounds of transient and ultimate tracking errors are adjusted by the design parameters in an explicit form with input saturation in effect by virtue of the novel intercepted adaptation approach. It is proved via Lyapunov stability theory that all the closed-loop signals are guaranteed semi-globally uniformly ultimately bounded, and simulation results are presented to verify the effectiveness of the proposed method.

Keywords Neural adaptive control · Nonlinear MIMO system · Dynamic surface control · Intercepted adaptation

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1 Introduction

Approximation-based adaptive control has gone through booming developments and advances recent years, where NNs and fuzzy logic systems are widely used as online approximators by virtue of their parallel processing property and function approximation capacity [1–4]. Although fruitful results on approximation-based adaptive control have been reported in existing literature, "dimension curse" is a barrier that restricts the real application of this methodology, that is to say, in order to achieve good function approximation performance, as large as possible number of neural nodes or fuzzy rules are used in theoretical study and analysis, which results in too large number of parameters that must be online tuned in practical application. As a result, the approximation times tend to be unacceptably large and the time-consuming process is unavoidable when approximation-based control are implemented. To remove the requirement of matching conditions, backstepping design, a Lyapunov-based integration and analysis methodology, has been widely advanced to control high-order nonlinear systems, see [5–7] and the references therein as examples. Although numerous theoretical works have been done, the problem of "explosion of complexity" limits the practical application of backstepping methodology, which is caused by the repeated differentiation of so-called virtual controllers in the backstepping recursive design procedure [8], which leads to an extremely complicated control scheme, especially for high-order systems. Fortunately, Yang et al. and Yip et al. [9,10] and [8,11] solved the problem of "dimension curse" and "explosion of complexity," respectively, in their pioneering works. In [9], small gain-based adaptive fuzzy robust tracking controller was designed, which achieved that only one function is needed to be approximated by fuzzy systems with any numbers of states and rules in fuzzy systems and thus reduced the computation resource since only two parameters needed to be adapted online. Such a technique was formally named minimal learning parameter (MLP) algorithm in subsequent literature [12,13]. In [10], MLP-based robust adaptive tracking control was developed for a class of strict-feedback uncertain nonlinear systems. In [14], direct adaptive fuzzy tracking control was developed for a class of perturbed strict-feedback nonlinear systems by virtue of merits of MLP. To further simplify the MLP algorithm, Chen et al. [15] proposed direct adaptive fuzzy control for nonlinear strict-feedback systems, where the number of adaptation law is reduced to one, and further, in [16], adaptive neural control for MIMO nonlinear time-delay systems is obtained using Lyapunov–Krasovskii function and one parameter adaptation. Fuzzy-approximation-based adaptive control (SISO [17] and MIMO [18]) is developed for nonlinear systems with time delays, where the control is independent of the choice of the fuzzy membership functions and requires one adaptive law for nth-order system. In [8], in order to simplify the adaptive backstepping design, adaptive dynamic surface control (DSC) was proposed, where n first-order low-pass filters were added which prevent the differentiation of model nonlinearities from existing. This result was further extended in [11] for non-Lipschitz systems. Inspired by this success, NNs were incorporated into DSC technique for strict-feedback nonlinear systems in [19]. Considerations of input dead zone and state delay were later made in [20] and [21], respectively.

Since it is impossible for physical actuator equipped to provided unlimited control input, input saturation should be explicitly considered in control system design, especially for adaptive control system. Without proper consideration of effect of input saturation, adaptation laws would act aggressively to seek the satisfactory performance [22–24]. On dealing with input saturation control problem, several interesting methods were reported in existing literature. In [22], a concept of augmented error signal (AES)-based adaptive control was developed for systems with hard saturation. According to the applications in flight control [25] and flight vehicle control [26], it can be concluded that the AES-based method is effective in dealing with input saturation. Partially inspired by this success, an online approximation control of uncertain nonlinear systems under control input saturation was designed in [23], which was designed such that input saturation does not destroy the adaptation capabilities in feedback adaptive control systems, and it was pointed out that the method in [23] can be trivially extended to high-order nonlinear systems using the recursive design of backstepping method. Therefore, backstepping-based AES design can be found in [25,27]. In [28], Takagi–Sugeno fuzzy models and linear matrix inequality optimization are used to the robust control of nonlinear systems in the presence of actuator saturation. In [29], model reference adaptive control for SISO time-invariant continuous-time plants with control saturation was proposed. In [30], backstepping-based variable structure control using Lyapunov synthesis was proposed for MIMO nonlinear systems with control input nonlinearities, first-order filters were utilized to the virtual control laws so that the extra computations of time derivatives of virtual controllers were circumvented. In [31], robust adaptive backstepping control is designed for uncertain nonlinear systems with control saturation and external disturbance, where Nussbaum function is utilized to compensate the nonlinear dynamics caused by control saturation. In [32], adaptive fuzzy output feedback control is developed for nonlinear systems in the presence of input dead-zone. In [33], output feedback adaptive fuzzy control is designed for output constrained nonlinear systems together with input saturation. In spite of the reported methods reported in the literature, there is extra space to improve the above-mentioned methods, that is, the problem of "dimension curse" and "explosion of complexity" exists in the literature. As can be seen from the methods in [30,34], an alternative method is to use fuzzy logic system or NNs to approximate the combination of unknown dynamics and differentiations of virtual controllers, which can solve the "explosion of complexity" but still suffer the "dimension curse," that is to say, the methods in [30,34] further augment the computational volume due to the information of reference signal and its derivatives must be incorporated into the input of fuzzy logic system or neural networks. In [35,36], adaptive fuzzy/neural tracking control is developed for stochastic nonlinear systems with input constraints, where smooth nonlinear function is utilized to approximate the saturation function and one adaptive parameter is independent of the number of fuzzy rule bases, and the plants are SISO stochastic nonlinear systems.

This paper is motivated by the neural adaptive control of uncertain MIMO nonlinear systems with strong interconnection, input saturation, and external disturbance to overcome the problem of "dimension curse" and "explosion of complexity." Novel intercepted adaptation approach is designed to attenuate the effect caused by input saturation, and the signals used to intercept the adaptation signal are generated by properly built auxiliary systems. The intercepted adaptation approach allows the online approximation goes on and can prevent the presence of input saturation from destroying the adaptation capacity and memory of approximators. MLP and DSC techniques are utilized to tackle the problem of "dimension curse" and "explosion of complexity," respectively. However, such a combination of these two techniques is non-trivial since input saturation and external disturbance are considered simultaneously in the plant, and extra efforts must be done to guarantee the closed-loop stability, which can be seen from the following design procedures and stability analysis sections. Furthermore, the MLP proposed in the pioneering works [9,10] is further simplified partially inspired by [15], where only one parameter is to be adapted online in the method proposed in this paper. At the same time, the bound values of transient and ultimate tracking errors can be adjusted to arbitrarily small by choosing proper design parameters in an explicit way even with input saturation in effect. Comparing with existing methods where smooth function is used to approximate saturation function, see [31,36] and references therein, the new features of intercepted adaptation approach lie in that the effect caused by input saturation is handled directly; in the meantime, the amplitude of effect caused by constrained input can be attenuated to arbitrarily small in an explicit form.

The rest of the paper is organized as follows. In Sect. 2, formulated problem and some necessary preliminaries are presented. The main results of this paper are given in Sect. 3, and the stability analysis of the closed-loop generated by the proposed control method is shown in Sect. 4. Section 5 presents the simulation examples to demonstrate the effectiveness of the developed control scheme. Section 6 ends this paper with concluding remarks.

2 Problem formulation and preliminaries

Consider the following MIMO nonlinear systems composed of N subsystems with constrained input and external disturbance:

$$\begin{cases} \dot{x}_{1,i_1} = x_{1,i_1+1} + f_{1,i_1}(\bar{x}_{1,i_1}, \ldots, \bar{x}_{j,i_j}, \ldots, \bar{x}_{N,i_N}) \\ \qquad + \Delta_{1,i_1}(X,t) \\ \qquad \cdots \\ \dot{x}_{1,n_1} = v_1 + f_{1,n_1}(X) + \Delta_{1,n_1}(X,t) \\ \quad v_1 = \mathrm{sat}(u_1) \\ \quad y_1 = x_{1,1} \\ \qquad \vdots \\ \dot{x}_{j,i_j} = x_{j,i_j+1} + f_{j,i_j}(\bar{x}_{1,i_1}, \ldots, \bar{x}_{j,i_j}, \ldots, \bar{x}_{N,i_N}) \\ \qquad + \Delta_{j,i_j}(X,t) \\ \qquad \cdots \\ \dot{x}_{j,n_j} = v_j + f_{j,n_j}(X, \bar{u}_{j-1}) + \Delta_{j,i_j}(X,t) \\ \quad v_j = \mathrm{sat}(u_j) \\ \quad y_j = x_{j,1} \end{cases} \tag{1}$$

where x_{j,i_j} is the i_jth state variable in subsystem j, $j = 1, \ldots, N$, N is the number of subsystems, $i_j = 1, \ldots, n_j$, n_j is the order of the subsystem j, $\bar{x}_{j,i_j} = [x_{j,1}, \ldots, x_{j,n_j}]^{\mathrm{T}}$, $X = [x_{1,n_1}^{\mathrm{T}}, \ldots, x_{N,n_N}^{\mathrm{T}}]^{\mathrm{T}}$ is the state variable vector of whole system, $f_{j,i_j}(\cdot)$ is unknown smooth function in its arguments, $\Delta_{j,i_j}(\cdot)$ is coupled external disturbance in whole state vector and time, v_j is the input to the subsystem j, u_j is the designed control law for subsystem j, and y_j is the output of subsystem j, respectively. The relationship of v_j and u_j is as follows:

$$v_j = \mathrm{sat}(u_j) = \begin{cases} \mathrm{sign}(u_j)u_j^+, & \text{if } |u_j| > u_j^+, \quad j = 1, \ldots, N \\ u_j, & \text{else} \end{cases} \tag{2}$$

where u_j^+ is a positive constant that represents the bound value of the maximum output of actuator in subsystem j.

The target is to design neural adaptive control law u_j, $j = 1, \ldots, N$ for system given by Eq. (1) such that (1) all the signals in the closed-loop system remain semi-globally uniformly ultimately bounded and (2) the output y_j follows the reference signal $y_{j,r}$ with small tracking errors which can be adjusted to arbitrarily small by choosing proper design parameters.

To this end, the following assumptions and lemma are used throughout this paper.

Assumption 1 [31] The jth subsystem, $j = 1, \ldots, N$, in Eq. (1) is input-to-state stable.

Assumption 2 The external disturbances $\Delta_{j,i_j}(\cdot)$ are bounded by unknown constant.

Assumption 3 [13] The desired trajectories $y_{j,r}$, $j = 1, \ldots, N$ are smooth enough such that there exists a positive constant $B_{j,0}$ satisfying $\Pi_{j,0} := \left\{(y_{j,r}, \dot{y}_{j,r}, \ddot{y}_{j,r}) \middle| y_{j,r}^2 + \dot{y}_{j,r}^2 + \ddot{y}_{j,r}^2 \leq B_{j,0}^2\right\}$.

Remark 1 Since it is quite a challenge to establish the stability property of an unstable plant under input saturation in a general form [37], therefore Assumption 1 is an assumption to guarantee the stability of a saturated nonlinear system as in [31], which is used in this paper to establish the closed-loop stability. Assumption 2 and Assumption 3 are quite standard ones in existing literature.

Lemma 1 *[12] For any given continuous function $f(x)$, $x \in R^n$ with $f(0) = 0$, by applying the continuous function separation in [38] and the RBF NNs approximation techniques, $f(x)$ can be reconstructed*

$$f(x) = S(x)Wx + \epsilon \tag{3}$$

where x is system arguments, $S(x) = [s_1(x), s_2(x), \ldots, s_l(x)]$ is a Gaussian basis function vector, and W is a weight matrix,

$$s_i(x) = \frac{1}{\sqrt{2\pi}a_i}\exp\left(-\frac{(x-\mu_i)^{\mathrm{T}}(x-\mu_i)}{2a_i^2}\right),$$

$$i = 1, \ldots, l, \quad W = \begin{bmatrix} w_{11} & w_{12} \cdots w_{1n} \\ w_{21} & w_{22} \cdots w_{2n} \\ \vdots & \vdots \quad \cdots \quad \vdots \\ w_{l1} & w_{l1} \cdots w_{ln} \end{bmatrix},$$

where μ_i and a_i denote the center of the receptive field and the width of the Gaussian function, respectively.

3 Main results

This section presents the major design procedures of jth subsystem, $j = 1, \ldots, N$, ϑ_j is the estimated value of $\max\left\{\beta_{j,i_j}^2, \eta_{j,i_j}^2\right\}$, with β_{j,i_j} and η_{j,i_j} specified later.

Construct the following system to generate signal $\chi_j = [\chi_{j,1}, \ldots, \chi_{j,n_j}]^{\mathrm{T}}$, which is used to intercept the adaption laws:

$$\begin{cases} \dot{\chi}_{j,i_j} = \chi_{j,i_j+1} - \alpha_{j,i_j}\chi_{j,i_j} \\ \qquad i_j = 1, \ldots, n_j - 1 \\ \dot{\chi}_{j,n_j} = \Delta u_j - \alpha_{j,n_j}\chi_{j,n_j} \\ \qquad j = 1, 2, \ldots, N \end{cases} \tag{4}$$

where α_{j,i_j} is positive design parameter, Δu_j is defined as $\Delta u_j := v_j - u_j$. In order to facilitate the design procedure and stability analysis, we make the following coordinates changes:

$$\begin{cases} z_{j,1} = y_{j,1} - y_{j,r} - \chi_{j,1} \\ z_{j,i_j} = x_{j,i_j} - r_{j,i_j-1} - \chi_{j,i_j} \\ \qquad i_j = 2, \ldots, n_j \end{cases} \tag{5}$$

In the following, neural adaptive control scheme is developed via intercepted adaptation and single learning parameter approach. To better present the main design idea, the first step $(j, 1)$, the intermediate step (j, i_j), and the final step (j, n_j) of jth subsystem are elaborated with detailed explanations.

Step $(j, 1)$: Differentiating both sides of $z_{j,1} = y_{j,1} - y_{j,r} - \chi_{j,1}$ gives

$$\begin{aligned} \dot{z}_{j,1} &= x_{j,2} + f_{j,1}(\bar{x}_{1,i_1}, \ldots, \bar{x}_{j,i_j}, \ldots, \bar{x}_{N,i_N}) \\ &\quad + \Delta_{j,1}(X, t) - \dot{y}_{j,r} - \chi_{j,2} + \alpha_{j,1}\chi_{j,1} \\ &= z_{j,2} + r_{j,1} + f_{j,1}(\bar{x}_{1,i_1}, \ldots, \bar{x}_{j,i_j}, \ldots, \bar{x}_{N,i_N}) \\ &\quad + \Delta_{j,1}(X, t) - \dot{y}_{j,r} + \alpha_{j,1}\chi_{j,1} \end{aligned} \tag{6}$$

Using the RBF NNs approximation technique in Lemma 1, the unknown function $f_{j,1}(\cdot)$ is remodeled as follows:

$$\begin{aligned} f_{j,1}(X_{j,1}) &= S_{j,1}(X_{j,1})W_{j,1}X_{j,1} + \epsilon_{j,1} \\ &= S_{j,1}(X_{j,1})W_{j,1}x_{j,1} \\ &\quad + S_{j,1}(X_{j,1})W_{j,1}X_{j,1}^* \\ &\quad + \epsilon_{j,1} \\ &= S_{j,1}(X_{j,1})W_{j,1}z_{j,1} \\ &\quad + S_{j,1}(X_{j,1})W_{j,1}(y_{j,r} \\ &\quad + \chi_{j,1}) + S_{j,1}(X_{j,1})W_{j,1}X_{j,1}^* + \epsilon_{j,1} \end{aligned} \tag{7}$$

where

$$X_{j,1}=\left[\bar{x}_{1,i_1}^{\mathrm{T}},\ldots,\bar{x}_{j,i_j}^{\mathrm{T}},\ldots,\bar{x}_{N,i_N}^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$X_{j,1}^{*}=\left[\bar{x}_{1,i_1}^{\mathrm{T}},\ldots,\bar{x}_{j,i_j}^{*\mathrm{T}},\ldots,\bar{x}_{N,i_N}^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\bar{x}_{j,i_j}^{*\mathrm{T}}=\left[0,x_{j,2},\ldots,x_{j,i_j}\right]^{\mathrm{T}}$$

Inspired by the technique used in [13], we define the following notations: $\beta_{j,1}:=\|W_{j,1}\|$, $m(W_{j,1}):=W_{j,1}/\beta_{j,1}$, $\phi_{j,1}:=m(W_{j,1})z_{j,1}$ with $m(W_{j,1})$ being a normalized term, it follows

$$f_{j,1}(X_{j,1})=\beta_{j,1}S_{j,1}(X_{j,1})\phi_{j,1}+S_{j,1}(X_{j,1})W_{j,1}(y_{j,r}+\chi_{j,1})+S_{j,1}(X_{j,1})W_{j,1}X_{j,1}^{*}+\epsilon_{j,1}\quad(8)$$

Integrating Eq. (6) and Eq. (8) gives

$$\dot{z}_{j,1}=z_{j,2}+r_{j,1}+\beta_{j,1}S_{j,1}(X_{j,1})\phi_{j,1}+\zeta_{j,1}-\dot{y}_{j,r}+\alpha_{j,1}\chi_{j,1}\quad(9)$$

where $\zeta_{j,1}:=S_{j,1}(X_{j,1})W_{j,1}(y_{j,r}+\chi_{j,1})+S_{j,1}(X_{j,1})W_{j,1}X_{j,1}^{*}+\epsilon_{j,1}+\Delta_{j,1}(X,t)$ is bounded, i.e.,

$$\|\zeta_{j,1}\|\leq\eta_{j,1}\rho_{j,1}(X_{j,1})\quad(10)$$

with $\eta_{j,1}=\max\left\{\|W_{j,1}(y_{j,r}+\chi_{j,1})\|,\|W_{j,1}X_{j,1}^{*}\|,\epsilon_{j,1}^{*},\Delta_{j,1}^{*}\right\}$, $\rho_{j,1}(X_{j,1})=1+\|S_{j,1}(X_{j,1})\|$.

Select the following virtual control law:

$$r_{j,1}=-\alpha_{j,1}(x_{j,1}-y_{j,r})+\dot{y}_{j,r}-\hat{\vartheta}_j\Theta_{j,1}(X_{j,1})z_{j,1}\quad(11)$$

where the definition of $\Theta_{j,1}(X_{j,1})$ and adaptation law of $\hat{\vartheta}_j$ will be specified at step (j,n_j).

To implement the DSC technique in [11], let $r_{j,1}$ pass through the following inertial filter with time constant $\tau_{j,1}$ to obtain $\varphi_{j,1}$; thus, the differentiation of $r_{j,1}$ is circumvented:

$$\tau_{j,1}\dot{\varphi}_{j,1}+\varphi_{j,1}=r_{j,1},\ \varphi_{j,1}(0)=r_{j,1}(0)\quad(12)$$

Step (j,i_j): The differentiation of z_{j,i_j} is calculated as

$$\begin{aligned}\dot{z}_{j,i_j}&=x_{j,i_j}+f_{j,i_j}(\bar{x}_{1,i_1},\ldots,\bar{x}_{j,i_j},\ldots,\bar{x}_{N,i_N})\\&\quad+\Delta_{j,i_j}(X,t)-\dot{r}_{j,i_j-1}-\chi_{j,i_j+1}+\alpha_{j,i_j}\chi_{j,i_j}\\&=z_{j,i_j+1}+r_{j,i_j}+f_{j,i_j}(\bar{x}_{1,i_1},\ldots,\bar{x}_{j,i_j},\\&\quad\ldots,\bar{x}_{N,i_N})+\Delta_{j,i_j}\\&\quad(X,t)-\dot{r}_{j,i_j-1}+\alpha_{j,i_j}\chi_{j,i_j}\end{aligned}\quad(13)$$

Using the RBF NNs approximation technique in Lemma 1 and defining the same notations with emendatory subscript in step $(j,1)$, $f_{j,i_j}(X_{j,i_j})$ is rewritten as

$$\begin{aligned}f_{j,i_j}(X_{j,i_j})&=S_{j,i_j}(X_{j,i_j})W_{j,i_j}X_{j,i_j}+\epsilon_{j,i_j}\\&=S_{j,i_j}(X_{j,i_j})W_{j,i_j}x_{j,i_j}\\&\quad+S_{j,i_j}(X_{j,i_j})W_{j,i_j}X_{j,i_j}^{*}+\epsilon_{j,i_j}\\&=S_{j,i_j}(X_{j,i_j})W_{j,i_j}z_{j,i_j}\\&\quad+S_{j,i_j}(X_{j,i_j})W_{j,i_j}(r_{j,i_j-1}+\chi_{j,i_j})\\&\quad+S_{j,i_j}(X_{j,1})W_{j,i_j}X_{j,i_j}^{*}+\epsilon_{j,i_j}\\&=\beta_{j,i_j}S_{j,i_j}(X_{j,i_j})\phi_{j,i_j}\\&\quad+S_{j,i_j}(X_{j,i_j})W_{j,i_j}(r_{j,i_j-1}+\chi_{j,i_j})\\&\quad+S_{j,i_j}(X_{j,i_j})W_{j,i_j}X_{j,i_j}^{*}+\epsilon_{j,i_j}\end{aligned}\quad(14)$$

It follows

$$\dot{z}_{j,i_j}=z_{j,i_j+1}+r_{j,i_j}+\beta_{j,i_j}S_{j,i_j}(X_{j,i_j})\phi_{j,i_j}+\zeta_{j,i_j}-\dot{r}_{j,i_j-1}+\alpha_{j,i_j}\chi_{j,i_j}\quad(15)$$

where $\zeta_{j,i_j}:=S_{j,i_j}(X_{j,i_j})W_{j,i_j}(r_{j,i_j-1}+\chi_{j,i_j})+S_{j,i_j}(X_{j,i_j})W_{j,i_j}X_{j,i_j}^{*}+\epsilon_{j,i_j}+\Delta_{j,i_j}(X,t)$ is bounded, i.e.,

$$\|\zeta_{j,i_j}\|\leq\eta_{j,i_j}\rho_{j,i_j}(X_{j,i_j})\quad(16)$$

with $\eta_{j,i_j}=\max\left\{\|W_{j,i_j}(r_{j,i_j-1}+\chi_{j,i_j})\|,\|W_{j,i_j}X_{j,i_j}^{*}\|,\epsilon_{j,i_j}^{*},\Delta_{j,i_j}^{*}\right\}$, $\rho_{j,i_j}(X_{j,i_j})=1+\|S_{j,i_j}(X_{j,i_j})\|$

Select the following virtual control law:

$$r_{j,i_j}=-\alpha_{j,i_j}(x_{j,i_j}-r_{j,i_j-1})-\hat{\vartheta}_j\Theta_{j,i_j}(X_{j,i_j})z_{j,i_j}+\dot{\varphi}_{j,i_j-1}\quad(17)$$

where the definition of $\Theta_{j,i_j}(X_{j,i_j})$ and adaptation law of $\hat{\vartheta}_j$ will be specified at step (j,n_j).

Let r_{j,i_j} pass through the following inertial filter with time constant τ_{j,i_j} to obtain φ_{j,i_j}:

$$\tau_{j,i_j}\dot{\varphi}_{j,i_j}+\varphi_{j,i_j}=r_{j,i_j},\ \varphi_{j,i_j}(0)=r_{j,i_j}(0)\quad(18)$$

Step (j,n_j): By defining the same notations with emendatory subscript in above step, invoking the RBF NNs approximation technique, the differentiation of z_{j,n_j} is obtained as

$$\begin{aligned}\dot{z}_{j,n_j}&=v_j+f_{j,n_j}(X,\bar{u}_{j-1})+\Delta_{j,n_j}(X,t)\\&\quad-\dot{r}_{j,n_j-1}-\Delta u_j+\alpha_{j,n_j}\chi_{j,n_j}\\&=u_j+f_{j,n_j}(X,\bar{u}_{j-1})+\Delta_{j,n_j}(X,t)\\&\quad-\dot{r}_{j,n_j-1}+\alpha_{j,n_j}\chi_{j,n_j}\\&=u_j+\beta_{j,n_j}S_{j,n_j}(X,\bar{u}_{j-1})\\&\quad\phi_{j,n_j}+\zeta_{j,n_j}-\dot{r}_{j,n_j-1}+\alpha_{j,n_j}\chi_{j,n_j}\end{aligned}\quad(19)$$

where $\beta_{j,n_j}:=\|W_{j,n_j}\|$, $\phi_{j,n_j}:=m(W_{j,i_j})z_{j,n_j}$, $m(W_{j,n_j}):=W_{j,n_j}/\beta_{j,n_j}$, $\zeta_{j,n_j}:=S_{j,n_j}(X_{j,n_j},\bar{u}_{j-1})W_{j,n_j}(\chi_{j,n_j}+r_{j,n_j-1})+S_{j,n_j}(X_{j,n_j},\bar{u}_{j-1})W_{j,n_j}X_{j,n_j}^{*}$

$+ S_{j,n_j}(X_{j,n_j}, \bar{u}_{j-1}) W_{j,n_j} \bar{u}_{j-1} + \epsilon_{j,n_j} + \Delta_{j,n_j}(X,t)$, $\|\zeta_{j,n_j}\| \leq \eta_{j,n_j} \rho_{j,n_j}(X_{j,n_j})$, $\rho_{j,n_j}(X_{j,n_j}) = 1 + \|S_{j,n_j}(X_{j,n_j})\|$, and $\eta_{j,n_j} := \max\{\|W_{j,n_j}(r_{j,n_j-1} + \chi_{j,n_j})\|, \|W_{j,n_j} X^*_{j,n_j}\|, \|W_{j,n_j}\bar{u}_{j-1}\|, \epsilon^*_{j,n_j}, \Delta^*_{j,n_j}\}$.

Select the following controller u_j, adaptation law $\dot{\hat{\vartheta}}_j$, and known function $\Theta_{j,i_j}(X_{j,i_j})$:

$$u_j = -\alpha_{j,n_j}(x_{j,n_j} - r_{j,n_j-1}) - \hat{\vartheta}_j \Theta_{j,n_j} (X_{j,n_j}) z_{j,n_j} + \dot{\varphi}_{j,n_j-1} \tag{20a}$$

$$\dot{\hat{\vartheta}}_j = \Gamma_j \sum_{i_j=1}^{n_j} \left[\Theta_{j,i_j}(X_{j,i_j}) z^2_{j,i_j} \right] - c_j \hat{\vartheta}_j \tag{20b}$$

$$\Theta_{j,i_j}(X_{j,i_j}) = \frac{1}{4\kappa^2_{j,i_j}} \rho_{j,i_j}(X_{j,i_j})^2 \tag{20c}$$

where Γ_j, c_j, and κ_{j,i_j} are positive design parameters.

The aforementioned design procedures are summarized in the following theorem.

Theorem 1 *For uncertain MIMO systems Eq. (1) with satisfied Assumption 1–Assumption 3, if the following initial conditions are satisfied in addition:*

$$\sum_{j=1}^{N} \sum_{i_j=1}^{n_j} \left(z^2_{j,i_j}(0) \right) + \sum_{j=1}^{N} \sum_{i_j=1}^{n_j-1} \left(e^2_{j,i_j}(0) \right) + \sum_{j=1}^{N} \left(\tilde{\vartheta}_j(0) \Gamma_j^{-1} \tilde{\vartheta}_j(0) \right) \leq 2p$$

where p is any positive number, and the control scheme given by Eq. (20) guarantees the following statements:

1. *The signals in the closed-loop system remain semi-globally uniformly ultimately bounded;*
2. *The adjustable ultimate tracking error $z_{j,1}$ is given by:*

$$\lim_{t\to\infty} |y_{j,1} - y_{j,r} - \chi_{j,1}| \leq \sqrt{\frac{2\gamma^*}{b}}$$

3. *The adjustable transient tracking error $y_{j,1} - y_{j,r}$ is given by:*

$$|y_j - y_{j,r}| \leq \sqrt{\frac{2\gamma^*}{b} + \sum_{j=1}^{N} \left(\tilde{\vartheta}_j(0) \Gamma_j^{-1} \tilde{\vartheta}_j(0) \right)} + \frac{\sum_{j=1}^{N} |\Delta u_j|}{2\sqrt{k_0}}$$

and the definitions of γ^*, b, and k_0 will be specified later.

Remark 2 The pioneering DSC [11] and MLP [9] techniques have been resoundingly synthesized into the traditional backstepping design method. It can be observed easily that the controller in Eq. (20) is very simple, and the new features of the control scheme can be briefly summarized as follows: (1) Input saturation, MIMO structure, and external disturbance are further considered compared with the pioneering works that proposed the DSC technique [8,11,19] and (2) we further simply the MLP technique in pioneering works [9,10], i.e., two parameters need to be online adjusted in [9] and [10], while only one parameter needs to be online adjusted in the control scheme proposed in this paper, which further simplify the controller structure.

Remark 3 Set the initial value $\chi_j(0)$ zero and if no input saturation happens, the variable χ_j remains zero state, that is to say, no supererogatory computation happens without input saturation. When input saturation happens, χ_j responses with changing Δu_j, and the original signal $y_{j,1} - y_{j,r}$ and $x_{j,i_j} - r_{j,i_j-1}$ used in the parameter learning is therefore intercepted by χ_{j,i_j} to prevent aggressive action, performance degradation, even instability in the presence of input saturation, and such a method is thus named after "intercepted adaptation" approach.

Remark 4 In view of the fact that the RBF NNs approximation technique in Lemma 1 is established in some compact set, the stability property obtained in this work is thus semi-global. It is also noticeable that other kinds of linearly parameterized approximation techniques, such as spline functions, fuzzy systems, and high-order NNs, can replace the RBF NNs with remaining design procedures and stability analysis being trivially obtained.

Remark 5 In order to better utilize the design methodology of dynamic surface control in pioneering works [11,19], the subsystems in Eq. (1) is of the same form of the SISO system in [19], but we consider extra external disturbance and input saturation simultaneously in this work. By virtue of the mean value theorem [39] and other necessary assumptions, the method developed in this paper is applicable to the general MIMO non-affine nonlinear systems; one refers to [39] and [40] for more details on this technique. In the following simulation studies, an example that the control law in Theorem 1 is used to control the MIMO non-affine nonlinear system in [40] with extra consideration of input saturation demonstrates this statements.

4 Stability analysis

In this section, the closed-loop stability by choosing proper design parameters is rigorously proved. The closed-loop dynamics resulting from the control law in Theorem 1 can be obtained as follows:

$$\begin{cases} \dot{z}_{j,1} = z_{j,2} - \alpha_{j,1} z_{j,1} + \beta_{j,1} S_{j,1}(X_{j,1})\phi_{j,1} \\ \quad + \zeta_{j,1} - \hat{\vartheta}_j \Theta_{j,1}(X_{j,1}) z_{j,1} \\ \dot{z}_{j,i_j} = z_{j,i_j+1} - \alpha_{j,i_j} z_{j,i_j} + \beta_{j,i_j} S_{j,i_j}(X_{j,i_j})\phi_{j,i_j} \\ \quad + \zeta_{j,i_j} - \hat{\vartheta}_j \Theta_{j,i_j}(X_{j,i_j}) z_{j,i_j} \\ \quad + \dot{\varphi}_{j,i_j-1} - \dot{r}_{j,i_j-1} \\ \quad j = 2, \ldots, N, \; i_j = 2, \ldots, n_j - 1 \\ \dot{z}_{j,n_j} = -\alpha_{j,n_j} z_{j,n_j} + \beta_{j,n_j} S_{j,n_j}(X_{j,n_j})\phi_{j,n_j} \\ \quad + \zeta_{j,n_j} - \hat{\vartheta}_j \Theta_{j,n_j}(X_{j,n_j}) z_{j,n_j} \\ \quad + \dot{\varphi}_{j,n_j-1} - \dot{r}_{j,n_j-1} \\ r_{j,i_j} = \tau_{j,i_j} \dot{\varphi}_{j,i_j} + \varphi_{j,i_j}, \; \varphi_{j,i_j}(0) = r_{j,i_j}(0) \\ \dot{\hat{\vartheta}}_j = \Gamma_j \sum_{i_j=1}^{n_j} \left[\Theta_{j,i_j}(X_{j,i_j}) z_{j,i_j}^2 - c_j \hat{\vartheta}_j \right] \end{cases} \tag{21}$$

Defining $e_{j,i_i} := \varphi_{j,i_j} - r_{j,i_j}$, $j = 1, \ldots, N$, $i_j = 1, \ldots, n_j$, and differentiating both sides gives

$$\dot{e}_{j,i_j} = -\frac{e_{j,i_j}}{\tau_{j,i_j}} + B_{j,i_j}(\cdot)$$

with

$$\begin{cases} B_{j,1}(\cdot) := \alpha_{j,1}(x_{j,2} - \dot{y}_{j,r}) - \ddot{y}_{j,r} + \frac{\partial r_{j,1}}{\partial z_{j,1}} \dot{z}_{j,1} \\ \quad + \frac{\partial r_{j,1}}{\partial X_{j,1}} \dot{X}_{j,1} + \frac{\partial r_{j,1}}{\partial \hat{\vartheta}_j} \dot{\hat{\vartheta}}_j \\ B_{j,i_j}(\cdot) := \alpha_{j,i_j}(x_{j,i_j+1} - \dot{r}_{j,i_j-1}) + \frac{\partial r_{j,1}}{\partial z_{j,1}} \dot{z}_{j,1} \\ \quad + \frac{\partial r_{j,1}}{\partial X_{j,1}} \dot{X}_{j,1} + \frac{\partial r_{j,1}}{\partial \hat{\vartheta}_j} \dot{\hat{\vartheta}}_j - \ddot{\varphi}_{j,i_j-1} \end{cases} \tag{22}$$

It follows

$$\begin{cases} \dot{z}_{j,1} = z_{j,2} - \alpha_{j,1} z_{j,1} + \beta_{j,1} S_{j,1}(X_{j,1})\phi_{j,1} \\ \quad + \zeta_{j,1} - \hat{\vartheta}_j \Theta_{j,1}(X_{j,1}) z_{j,1} \\ \dot{z}_{j,i_j} = z_{j,i_j+1} - \alpha_{j,i_j} z_{j,i_j} + \beta_{j,i_j} S_{j,i_j}(X_{j,i_j}) \\ \quad \phi_{j,i_j} + \zeta_{j,i_j} - \hat{\vartheta}_j \Theta_{j,i_j}(X_{j,i_j}) z_{j,i_j} \\ \quad - \frac{e_{j,i_j-1}}{\tau_{j,i_j-1}} + B_{j,i_j-1}(\cdot) \\ \quad j = 2, \ldots, N, \; i_j = 2, \ldots, n_j - 1 \\ \dot{z}_{j,n_j} = -\alpha_{j,n_j} z_{j,n_j} + \beta_{j,n_j} S_{j,n_j}(X_{j,n_j})\phi_{j,n_j} \\ \quad + \zeta_{j,n_j} - \hat{\vartheta}_j \Theta_{j,n_j}(X_{j,n_j}) z_{j,n_j} \\ \quad - \frac{e_{j,n_j-1}}{\tau_{j,n_j-1}} + B_{j,n_j-1}(\cdot) \\ r_{j,i_j} = \tau_{j,i_j} \dot{\varphi}_{j,i_j} + \varphi_{j,i_j}, \; \varphi_{j,i_j}(0) = r_{j,i_j}(0) \\ \dot{\hat{\vartheta}}_j = \Gamma_j \sum_{i_j=1}^{n_j} \left[\Theta_{j,i_j}(X_{j,i_j}) z_{j,i_j}^2 \right] - c_j \hat{\vartheta}_j \end{cases} \tag{23}$$

From Assumption 3, it is known that $\Pi_{j,0}$ is compact in space R^3. The following set is compact in space $R^{(\sum_{i_k=1}^{i_k}(2i_k))}$ for any $p > 0$:

$$\Pi_{j,i_k} = \left\{ \sum_{i_k=1}^{i_j} \left(z_{j,i_k}^2 \right) + \sum_{i_k=1}^{i_j-1} e_{j,i_k}^2 + \tilde{\vartheta}_j \Gamma_j^{-1} \tilde{\vartheta}_j \right\},$$
$$i_k = 1, \ldots, n_j$$

Therefore, the set $\Pi_{j,0} \times \Pi_{j,i_k}$ is compact in $R^{(\sum_{i_k=1}^{i_k}(2i_k+3))}$. As a result, B_{j,i_j}, $j = 1, \ldots, N$, $i_j = 1, \ldots, n_j - 1$ are bounded on $\Pi_{j,0} \times \Pi_{j,i_k}$, that is to say, there exists B_{j,i_j}^+ such that $|B_{j,i_j}(\cdot)| \leq B_{j,i_j}^+$. One can refer to [13] and [19] for more details on the existence of B_{j,i_j}^+.

Choose the following Lyapunov candidate:

$$V = \frac{1}{2} \sum_{j=1}^{N} \sum_{i_j=1}^{n_j} \left(z_{j,i_j}^2 \right) + \frac{1}{2} \sum_{j=1}^{N} \sum_{i_j=1}^{n_j-1} \left(e_{j,i_j}^2 \right) + \frac{1}{2} \sum_{j=1}^{N} \tilde{\vartheta}_j \Gamma_j^{-1} \tilde{\vartheta}_j \tag{24}$$

Its derivative along Eq. (23) is obtained as:

$$\begin{aligned} \dot{V} &= \sum_{j=1}^{N} \sum_{i_j=1}^{n_j} \left(z_{j,i_j} \dot{z}_{j,i_j} \right) + \sum_{j=1}^{N} \sum_{i_j=1}^{n_j-1} \left(e_{j,i_j} \dot{e}_{j,i_j} \right) \\ &\quad + \sum_{j=1}^{N} \tilde{\vartheta}_j \Gamma_j^{-1} \dot{\hat{\vartheta}}_j \\ &= \sum_{j=1}^{N} \sum_{i_j=1}^{n_j} \\ &\quad \left(-\alpha_{j,i_j} z_{j,i_j}^2 + \beta_{j,i_j} S_{j,i_j}(X_{j,i_j}) \phi_{j,i_j} z_{j,i_j} \right. \\ &\quad \left. + \zeta_{j,i_j} z_{j,i_j} - \hat{\vartheta}_j \Theta_{j,i_j}(X_{j,i_j}) z_{j,i_j}^2 \right) \\ &\quad + \sum_{j=1}^{N} \sum_{i_j=1}^{n_j-1} \left(z_{j,i_j} z_{j,i_j+1} \right) \\ &\quad + \sum_{j=1}^{N} \sum_{i_j=2}^{n_j} \left(-z_{j,i_j} \frac{e_{j,i_j-1}}{\tau_{j,i_j-1}} + z_{j,i_j} B_{j,i_j-1}(\cdot) \right) \\ &\quad + \sum_{j=1}^{N} \sum_{i_j=1}^{n_j-1} \left(-\frac{e_{j,i_j}^2}{\tau_{j,i_j}} + e_{j,i_j} B_{j,i_j}(\cdot) \right) \\ &\quad + \sum_{j=1}^{N} \left(\tilde{\vartheta}_j \sum_{i_j=1}^{n_j} \left[\Theta_{j,i_j}(X_{j,i_j}) z_{j,i_j}^2 \right] - c_j \tilde{\vartheta}_j \hat{\vartheta}_j \right) \\ &\leq \sum_{j=1}^{N} \left(-\alpha_{j,1} z_{j,1}^2 + \frac{1}{2} z_{j,1}^2 \right) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}\Bigg(-\alpha_{j,i_j}z_{j,i_j}^2+z_{j,i_j}^2\\
&+\frac{z_{j,i_j}^2}{2\tau_{j,i_j-1}}+|z_{j,i_j}B_{j,i_j-1}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\Bigg(-\alpha_{j,n_j}z_{j,n_j}^2+\frac{1}{2}z_{j,n_j}^2\\
&+\frac{z_{j,n_j}^2}{2\tau_{j,n_j-1}}+|z_{j,n_j}B_{j,n_j-1}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\beta_{j,i_j}S_{j,i_j}(X_{j,i_j})\phi_{j,i_j}z_{j,i_j}+\zeta_{j,i_j}z_{j,i_j}\right)\\
&+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\Bigg(-\frac{e_{j,i_j}^2}{2\tau_{j,i_j}}+|e_{j,i_j}B_{j,i_j}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\Bigg(-\vartheta_j\sum_{i_j=1}^{n_j}\Big[\Theta_{j,i_j}(X_{j,i_j})z_{j,i_j}^2\Big]-c_j\tilde{\vartheta}_j\hat{\vartheta}_j\Bigg)
\end{aligned}
\tag{25}
$$

Using the following facts

$$
\begin{aligned}
&\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\beta_{j,i_j}S_{j,i_j}(X_{j,i_j})\phi_{j,i_j}z_{j,i_j}\right)\\
&\quad\le\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\Bigg(\frac{\beta_{j,i_j}^2}{4\kappa_{j,i_j}^2}\|S_{j,i_j}(X_{j,i_j})\|^2z_{j,i_j}^2\Bigg)\\
&\quad\quad+\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2
\end{aligned}
\tag{26a}
$$

$$
\begin{aligned}
&\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\zeta_{j,i_j}z_{j,i_j}\right)\\
&\quad\le\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\Bigg(\frac{\eta_{j,i_j}^2}{4\kappa_{j,i_j}^2}\rho_{j,i_j}^2(X_{j,i_j})z_{j,i_j}^2+\kappa_{j,i_j}^2\Bigg)
\end{aligned}
\tag{26b}
$$

$$
\begin{aligned}
&\sum_{j=1}^{N}\left(-c_j\tilde{\vartheta}_j\hat{\vartheta}_j\right)\\
&\quad\le\sum_{j=1}^{N}\left(-\frac{c_j}{2}\tilde{\vartheta}_j^2+\frac{c_j}{2}\vartheta_j^2\right)\\
&\quad\le\sum_{j=1}^{N}\Bigg(-\frac{c_j}{2\max\{\Gamma_j^{-1}\}}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j+\frac{c_j}{2}\vartheta_j^2\Bigg)
\end{aligned}
\tag{26c}
$$

gives

$$
\begin{aligned}
\dot{V}\le&\sum_{j=1}^{N}\Bigg(-\alpha_{j,1}z_{j,1}^2+\frac{1}{2}z_{j,1}^2\Bigg)\\
&+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}\Bigg(-\alpha_{j,i_j}z_{j,i_j}^2+z_{j,i_j}^2\\
&+\frac{z_{j,i_j}^2}{2\tau_{j,i_j-1}}+|z_{j,i_j}B_{j,i_j-1}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\Bigg(-\alpha_{j,n_j}z_{j,n_j}^2+\frac{1}{2}z_{j,n_j}^2\\
&+\frac{z_{j,n_j}^2}{2\tau_{j,n_j-1}}+|z_{j,n_j}B_{j,n_j-1}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\Bigg(\max\left\{\beta_{j,i_j}^2,\eta_{j,i_j}^2\right\}\frac{1}{4\kappa_{j,i_j}^2}\\
&\times\rho_{j,i_j}^2(X_{j,i_j})z_{j,i_j}^2+\kappa_{j,i_j}^2+\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\Bigg)\\
&+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\Bigg(-\frac{e_{j,i_j}^2}{2\tau_{j,i_j}}+|e_{j,i_j}B_{j,i_j}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\Bigg(-\vartheta_j\sum_{i_j=1}^{n_j}\Big[\Theta_{j,i_j}(X_{j,i_j})z_{j,i_j}^2\Big]\\
&-\frac{c_j}{2\max\{\Gamma_j^{-1}\}}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j+\frac{c_j}{2}\vartheta_j^2\Bigg)
\end{aligned}
\tag{27}
$$

Choose the design parameters $\alpha_{j,1}$ and k_{j0} in the following way:

$$\alpha_{j,1}=\frac{1}{2}+k_{j0} \tag{28a}$$

$$k_{j0}=\min\left\{\frac{c_j}{2\max\{\Gamma_j^{-1}\}}\right\}>0 \tag{28b}$$

It follows

$$
\begin{aligned}
\dot{V}\le&\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)\\
&+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}\Bigg(-\alpha_{j,i_j}z_{j,i_j}^2+z_{j,i_j}^2\\
&+\frac{z_{j,i_j}^2}{2\tau_{j,i_j-1}}+|z_{j,i_j}B_{j,i_j-1}(\cdot)|\Bigg)\\
&+\sum_{j=1}^{N}\Big(-\alpha_{j,n_j}z_{j,n_j}^2
\end{aligned}
$$

$$+\frac{1}{2}z_{j,n_j}^2+\frac{z_{j,n_j}^2}{2\tau_{j,n_j-1}}+|z_{j,n_j}B_{j,n_j-1}(\cdot)|\Bigg)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2+\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(-\frac{e_{j,i_j}^2}{2\tau_{j,i_j}}+|e_{j,i_j}B_{j,i_j}(\cdot)|\right)$$
$$+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j+\frac{c_j}{2}\vartheta_j^2\right) \tag{29}$$

Define

$$\gamma:=\sum_{j=1}^{N}\left(\frac{c_j}{2}\vartheta_j^2+\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\right)\right) \tag{30}$$

and note that the following inequality is true for any $\varepsilon>0$:

$$\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}|e_{j,i_j}B_{j,i_j}(\cdot)| \le \sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(\frac{e_{j,i_j}^2B_{j,i_j}^2(\cdot)}{2\varepsilon}+\frac{\varepsilon}{2}\right) \tag{31}$$

Choose design parameter τ_{j,i_j} in the way such that $\frac{1}{2\tau_{j,i_j}}=\frac{B_{j,i_j}^{+2}}{2\varepsilon}+k_{j0}$, it gives

$$\dot{V}\le\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}\Bigg(-\alpha_{j,i_j}z_{j,i_j}^2+z_{j,i_j}^2$$
$$+\frac{z_{j,i_j}^2}{2\tau_{j,i_j-1}}+|z_{j,i_j}B_{j,i_j-1}(\cdot)|\Bigg)$$
$$+\sum_{j=1}^{N}\Bigg(-\alpha_{j,n_j}z_{j,n_j}^2+\frac{1}{2}z_{j,n_j}^2$$
$$+\frac{z_{j,n_j}^2}{2\tau_{j,n_j-1}}+|z_{j,n_j}B_{j,n_j-1}(\cdot)|\Bigg)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\Bigg(-\left(\frac{B_{j,i_j}^{+2}}{2\varepsilon}+k_{j0}\right)e_{j,i_j}^2$$
$$+\frac{e_{j,i_j}^2B_{j,i_j}^2(\cdot)B_{j,i_j}^{+2}}{2B_{j,i_j}^{+2}\varepsilon}+\frac{\varepsilon}{2}\Bigg)$$
$$+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)+\gamma$$
$$=\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}$$
$$\left(-\alpha_{j,i_j}z_{j,i_j}^2+z_{j,i_j}^2+\frac{z_{j,i_j}^2}{2\tau_{j,i_j-1}}+|z_{j,i_j}B_{j,i_j-1}(\cdot)|\right)$$
$$+\sum_{j=1}^{N}\Bigg(-\alpha_{j,n_j}z_{j,n_j}^2+\frac{1}{2}z_{j,n_j}^2$$
$$+\frac{z_{j,n_j}^2}{2\tau_{j,n_j-1}}+|z_{j,n_j}B_{j,n_j-1}(\cdot)|\Bigg)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}$$
$$\left(-k_{j0}e_{j,i_j}^2-\left(1-\frac{B_{j,ij}^2}{B_{j,i_j}^{+2}}\right)\frac{e_{j,i_j}^2B_{j,i_j}^{+2}}{2\varepsilon}+\frac{\varepsilon}{2}\right)$$
$$+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)+\gamma \tag{32}$$

Since B_{j,i_j}^{+} is the bound value of B_{j,i_j}, i.e., $|B_{j,i_j}|\le B_{j,i_j}^{+}$, $1-\frac{B_{j,ij}^2}{B_{j,i_j}^{+2}}$ is therefore positive, and further $-\left(1-\frac{B_{j,ij}^2}{B_{j,i_j}^{+2}}\right)\frac{e_{j,i_j}^2B_{j,i_j}^{+2}}{2\varepsilon}$ is negative,

$$\dot{V}\le\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}$$
$$\left(-\alpha_{j,i_j}z_{j,i_j}^2+z_{j,i_j}^2+\frac{z_{j,i_j}^2}{2\tau_{j,i_j-1}}+|z_{j,i_j}B_{j,i_j-1}(\cdot)|\right)$$
$$+\sum_{j=1}^{N}\Bigg(-\alpha_{j,n_j}z_{j,n_j}^2+\frac{1}{2}z_{j,n_j}^2$$
$$+\frac{z_{j,n_j}^2}{2\tau_{j,n_j-1}}+|z_{j,n_j}B_{j,n_j-1}(\cdot)|\Bigg)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)$$

$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(-k_{j0}e_{j,i_j}^2\right)$$
$$+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)+\gamma+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(\frac{\varepsilon}{2}\right) \tag{33}$$

Choose the design parameters α_{j,i_j} in the following way:

$$\alpha_{j,i_j}=1+\frac{1}{2\tau_{j,i_j-1}}+\frac{B_{j,i_j-1}^{+2}}{2\varepsilon}+k_{j0},\quad j=1,\ldots,N,\ i_j=2,\ldots,n_j \tag{34}$$

it yields

$$\dot{V}\le\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j-1}\left(-k_{j,0}z_{j,i_j}^2-\frac{B_{j,i_j-1}^{+2}}{2\varepsilon}z_{j,i_j}^2+\frac{z_{j,i_j}^2B_{j,i_j-1}^2(\cdot)B_{j,i_j-1}^{+2}}{2\varepsilon B_{j,i_j-1}^{+2}}+\frac{\varepsilon}{2}\right)$$
$$+\sum_{j=1}^{N}\left(-k_{j0}z_{j,n_j}^2-\frac{B_{j,n_j-1}^{+2}}{2\varepsilon}z_{j,n_j}^2+\frac{z_{j,n_j}^2B_{j,n_j-1}^2(\cdot)B_{j,n_j-1}^{+2}}{2\varepsilon B_{j,n_j-1}^{+2}}+\frac{\varepsilon}{2}\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(-k_{j0}e_{j,i_j}^2\right)+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)$$
$$+\gamma+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(\frac{\varepsilon}{2}\right)$$
$$=\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j}\left(-k_{j,0}z_{j,i_j}^2-\left(1-\frac{B_{j,i_j-1}^2(\cdot)}{B_{j,i_j-1}^{+2}}\right)\frac{B_{j,i_j-1}^{+2}z_{j,i_j}^2}{2\varepsilon}\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(-k_{j0}e_{j,i_j}^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)$$
$$+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)+\gamma+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}(\varepsilon)$$
$$\le\sum_{j=1}^{N}\left(-k_{j0}z_{j,1}^2\right)+\sum_{j=1}^{N}\sum_{i_j=2}^{n_j}\left(-k_{j,0}z_{j,i_j}^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(-k_{j0}e_{j,i_j}^2\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)+\gamma^*$$
$$=\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(-k_{j0}\left(z_{j,i_j}^2\right)\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(-k_{j0}e_{j,i_j}^2\right)+\sum_{j=1}^{N}\left(-k_{j0}\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)+\gamma^*$$
$$\le-k_0\left(\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(z_{j,i_j}^2\right)+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(e_{j,i_j}^2\right)+\sum_{j=1}^{N}\left(\tilde{\vartheta}_j\Gamma_j^{-1}\tilde{\vartheta}_j\right)\right)$$
$$+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)+\gamma^*$$
$$=-2k_0V+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\kappa_{j,i_j}^2\|\phi_{j,i_j}\|^2\right)+\gamma^* \tag{35}$$

where k_0 is chosen such that $k_0=\min\{k_{10},\ldots,k_{N0}\}$, γ^* is defined as $\gamma^*:=\gamma+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}(\varepsilon)$. From the definition of ϕ_{j,i_j}, it is known that

$$\|\phi_{j,i_j}\|\le\|m(W_{j,i_j})\||z_{j,i_j}|\le|z_{j,i_j}| \tag{36}$$

Choose design parameter κ_{j,i_j} such that $\max\left\{\kappa_{j,i_j}\right\}\le\frac{1}{\sqrt{2}}$, it follows

$$\dot{V}\le-2k_0V+\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\frac{1}{2}z_{j,i_j}^2\right)+\gamma^*$$
$$\le-bV+\gamma^* \tag{37}$$

with b defined as $b := 2k_0 - 1$. Choose the design parameter k_0 such that $k_0 > \gamma^*/(2p) + 1/2$, it gives $\dot{V} \leq 0$ on $V = p$, i.e., $V \leq p$ is an invariant set; in this sense, for any initial value $V(0)$ satisfying $V(0) \leq p$, $V(t) \leq p$ is true for all $t \geq 0$.

Equation (37) yields

$$0 \leq V(t) \leq \frac{\gamma^*}{b} + \left(V(0) - \frac{\gamma^*}{b}\right)\exp^{-bt} \tag{38}$$

This implies that there exists a time moment T such that z_{j,i_i}, e_{j,i_j}, and ϑ_j are bounded in the following compact sets for any $t > T$:

$$\Omega_z = \left\{ z_{j,i_j} \middle| |z_{j,i_j}| \leq \sqrt{\frac{2\gamma^*}{b}} \right\} \tag{39a}$$

$$\Omega_e = \left\{ e_{j,i_j} \middle| |e_{j,i_j}| \leq \sqrt{\frac{2\gamma^*}{b}} \right\} \tag{39b}$$

$$\Omega_\vartheta = \left\{ \vartheta_j \middle| |\vartheta_j| \leq \sqrt{\frac{2\gamma^*}{|\Gamma_j^{-1}|b}} \right\} \tag{39c}$$

From Eq. (38), it is known that the transient bound value of $V(t)$ is $\frac{\gamma^*}{b} + V(0)$ with

$$V(0) := \frac{1}{2}\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(z_{j,i_j}^2(0)\right) + \frac{1}{2}\sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(e_{j,i_j}^2(0)\right) + \frac{1}{2}\sum_{j=1}^{N}\tilde{\vartheta}_j(0)\Gamma_j^{-1}\tilde{\vartheta}_j(0) \tag{40}$$

From Eq. (12) and Eq. (18), it is known that $e_{j,i_j}(0) = 0$, $j = 1, \ldots, N, i_j = 1, \ldots, n_j - 1$. Setting the initial values $z_{j,i_j}(0)$ to be zero gives

$$V(0) = \frac{1}{2}\sum_{j=1}^{N}\left(\tilde{\vartheta}_j(0)\Gamma_j^{-1}\tilde{\vartheta}_j(0)\right) \tag{41}$$

It can be observed that $V(0)$ is a decreasing function of Γ_j. The bound value of transient $z_{j,1}$ is therefore obtained as

$$|z_{j,1}| = |y_j - y_{j,r} - \chi_{j,1}| \leq \sqrt{\frac{2\gamma^*}{b} + \sum_{j=1}^{N}\left(\tilde{\vartheta}_j(0)\Gamma_j^{-1}\tilde{\vartheta}_j(0)\right)} \tag{42}$$

Now, we will find out the bound value of $\chi_{j,1}$ to seek the bound value of tracking error $y_{j,1} - y_{j,r}$. To that end, we choose the following Lyapunov function

$$V_\chi = \frac{1}{2}\sum_{j=1}^{N}\sum_{i_j}^{n_j}\chi_{j,i_j}^2 \tag{43}$$

and its derivative is obtained as

$$\begin{aligned}\dot{V}_\chi &= \sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(-\alpha_{j,i_i}\chi_{j,i_j}^2\right) \\ &\quad + \sum_{j=1}^{N}\sum_{i_j=1}^{n_j-1}\left(\chi_{j,i_j}\chi_{j,i_j+1}\right) + \sum_{j=1}^{N}\left(\chi_{j,n_j}\Delta u_j\right) \\ &\leq \sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(-\alpha_{j,i_i}\chi_{j,i_j}^2\right) \\ &\quad + \frac{1}{2}\sum_{j=1}^{N}\chi_{j,1}^2 + \sum_{j=1}^{N}\sum_{i_j=2}^{n_j}\chi_{j,i_j}^2 + \frac{1}{2}\sum_{j=1}^{N}\Delta u_j^2\end{aligned} \tag{44}$$

From Eq. (34), it follows

$$\begin{aligned}\dot{V}_\chi &\leq -k_0\sum_{j=1}^{N}\sum_{i_j=1}^{n_j}\left(\chi_{j,i_j}^2\right) + \frac{1}{2}\sum_{j=1}^{N}\Delta u_j^2 \\ &= -2k_0V_\chi + \frac{1}{2}\sum_{j=1}^{N}\Delta u_j^2\end{aligned} \tag{45}$$

then

$$\begin{aligned}0 &\leq V_\chi \\ &\leq \frac{\sum_{j=1}^{N}\Delta u_j^2}{4k_0} + \left(V_\chi(0) - \frac{\sum_{j=1}^{N}\Delta u_j^2}{4k_0}\right)\exp^{-2k_0t}\end{aligned} \tag{46}$$

Setting the initial value $V_\chi(0)$ to be zero gives the following inequality in finite time:

$$\begin{aligned}0 \leq V_\chi &\leq \frac{\sum_{j=1}^{N}\Delta u_j^2}{4k_0} - \left(\frac{\sum_{j=1}^{N}\Delta u_j^2}{4k_0}\right)\exp^{-2k_0t} \\ &\leq \frac{\sum_{j=1}^{N}\Delta u_j^2}{4k_0}\end{aligned} \tag{47}$$

In view of the definition of V_χ, we have

$$\frac{1}{2}\chi_{j,1} \leq \frac{\sum_{j=1}^{N}\Delta u_j^2}{4k_0} \Rightarrow |\chi_{j,1}| \leq \frac{\sum_{j=1}^{N}|\Delta u_j|}{2\sqrt{k_0}} \tag{48}$$

and therefore, the bound of $y_j - y_{j,r}$ is obtained as follows:

$$\begin{aligned}|y_j - y_{j,r}| &\leq \sqrt{\frac{2\gamma^*}{b} + \sum_{j=1}^{N}\left(\tilde{\vartheta}_j(0)\Gamma_j^{-1}\tilde{\vartheta}_j(0)\right)} \\ &\quad + \frac{\sum_{j=1}^{N}|\Delta u_j|}{2\sqrt{k_0}}\end{aligned} \tag{49}$$

From Eq. (49), it can be observed that the tracking error $y_j - y_{j,r}$ in jth subsystem can be adjusted to arbitrarily small by choosing large enough b and k_0 and small enough γ^*; at the same time, the effects of initial estimation errors $\tilde{\vartheta}_j(0)$ can be attenuated by choosing large enough Γ_j.

Remark 6 The parameter-choosing techniques are partially inspired by the pioneering works [11] and [19]. Since the MIMO structure, effect of input saturations, neural approximation errors, external disturbances are explicitly contained simultaneously in our closed-loop system, we have done extra efforts to obtain Eq. (37), which facilitates the derivations of the ultimate and transient convergence sets of tracking error.

Remark 7 From the above analysis, it is known that the tracking error $y_j - y_{j,r}$ can be tuned to arbitrarily small by choosing proper design parameters; at the same time, the effect of initial estimation errors can be attenuated by choosing large enough Γ_j, an independent parameter of b and k_0. Although such a merit, extra attention must be paid if put the method into practice, since too large sets of b and k_0 may lead to a high-gain control, which will cause chattering phenomenon in practical applications. What is more, the proposed method involves several design parameters, and it is a challenge and an open problem to choose an optimal set of these parameters, and in the following simulation section, a trial-and-error method is used.

Remark 8 It is noted that the initial values of state variables in V must be confined in a ball with a radius of $\sqrt{2p}$. By choosing large enough p and it is in fact that the state variables in practice are impossible to be infinite, the initial conditions, in this sense, are quite easy to satisfy and are not restrictive indeed. The arguments in [11] are applicable in our work on how to set these state variables in the desired ball, which is not discussed in details here.

5 Simulation results

5.1 Example 1

Consider the following uncertain MIMO nonlinear systems:

$$\begin{cases} \dot{x}_{1,1} = 0.5(x_{1,1}+x_{2,1})+0.1x_{1,1}^2x_{2,1}^2+x_{1,2}+\Delta_{1,1} \\ \dot{x}_{1,2} = (x_{1,1}x_{1,2}) + \cos(x_{1,1}x_{2,1}) + v_1 + \Delta_{1,2} \\ v_1 = \mathrm{sat}(u_1),\ u_1^+ = 2.8 \\ y_1 = x_{1,1} \\ \dot{x}_{2,1} = x_{1,1}x_{2,1} + \sin(x_{1,1}x_{2,1}) + x_{2,2} + \Delta_{2,1} \\ \dot{x}_{2,2} = (x_{1,1}x_{1,2} + x_{2,1}x_{2,2}) \\ \qquad + e^{x_{1,1}} + e^{-x_{2,1}} + v_2 + \Delta_{2,2} \\ v_2 = \mathrm{sat}(u_2),\ u_2^+ = 2.8 \\ y_2 = x_{2,1} \end{cases} \tag{50}$$

where $\Delta_{1,1} = 0.2x_{1,1}x_{1,2}x_{2,1}x_{2,2}\sin(t)$, $\Delta_{1,2} = 0.5\sin(x_{1,1}^2 + x_{1,2}^2)\cos(x_{2,1}^2x_{2,2}^2)$, $\Delta_{2,1} = 0.3\sin(x_{1,1}x_{1,2}x_{2,1}x_{2,2})$, $\Delta_{2,2} = 0.4(x_{1,1} + x_{1,2})\cos(x_{2,1}x_{2,2})\sin^2(t)$. Input saturations $u_1^+ = 2.8$ and $u_2^+ = 2.8$ are imposed on the 1st and 2nd subsystems, respectively.

The references y_j, $j = 1, 2$ are generated by the van der Pol oscillator, which is described by:

$$\begin{cases} \dot{y}_{1,r} = \dot{y}_{2,r} \\ \dot{y}_{2,r} = -y_{1,r} + \beta_v(1 - y_{1,r}^2)y_{2,r} \end{cases}$$

if β_v is chosen as positive constant, the outputs of the van der Pol oscillator get close to a limit cycle. In this example, β_v is chosen as 0.001.

The initial values of the plant are as follows. $x_{1,1}(0) = 0.3$, $x_{1,2}(0) = 0.1$, $x_{2,1}(0) = 0.1$, $x_{2,2}(0) = 0.2$, $\varphi_{11}(0) = 0$, $\varphi_{21}(0) = 0$, $y_{1r}(0) = 0.2$, $y_{2r}(0) = -0.1$, $\hat{\vartheta}_1(0) = \hat{\vartheta}_2(0) = 0$. The design parameters are chosen as follows. $\alpha_{1,1} = 16$, $\alpha_{1,2} = 26$, $\alpha_{2,1} = 16$, $\alpha_{2,2} = 26$, $\kappa_{j,i_j} = 0.1$, $j = 1, 2$, $i_j = 1, 2$, $\tau_{1,1} = \tau_{2,1} = 0.005$, $c_1 = c_2 = 0.1$, $\Gamma_1 = \Gamma_2 = 5$.

The RBF NNs (1, 1) contain 20 nodes with widths $a_{1,1} = 1.5$ and centers $\mu_{1,1}$ evenly spaced in $[-2, 2] \times [-1.5, 1.5]$; RBF NNs (1, 2) contain 30 nodes with widths $a_{1,2} = 1.5$ and centers $\mu_{1,2}$ evenly spaced in $[-2.5, 2.5] \times [-1.5, 1.5] \times [-2, 2] \times [-1.5, 1.5]$; RBF NNs (2, 1) contain 20 nodes with widths $a_{2,1} = 1.5$ and centers $\mu_{2,1}$ evenly spaced in $[-2, 2] \times [-1.5, 1.5]$; RBF NNs (2, 2) contain 30 nodes with widths $a_{2,2} = 1.5$ and centers $\mu_{2,2}$ evenly spaced $[-2.5, 2.5] \times [-1.5, 1.5] \times [-2, 2] \times [-1.5, 1.5]$. The initial values of all RBF NNs are set to be zero.

The simulation results of this example are shown in Fig. 1. Figure 1a, b presents the tracking performance of subsystems, and Fig. 1c shows the tracking errors. It is clear that the results are satisfactory. From Fig. 1d, boundedness of x_{12} and x_{22} is observed. From Fig. 1e, f, it follows that the constrained input signals become periodic after about 1s. Figure 1g, h illustrates that the adaptive parameters ($\hat{\vartheta}_1$ and $\hat{\vartheta}_2$) and auxiliary signals

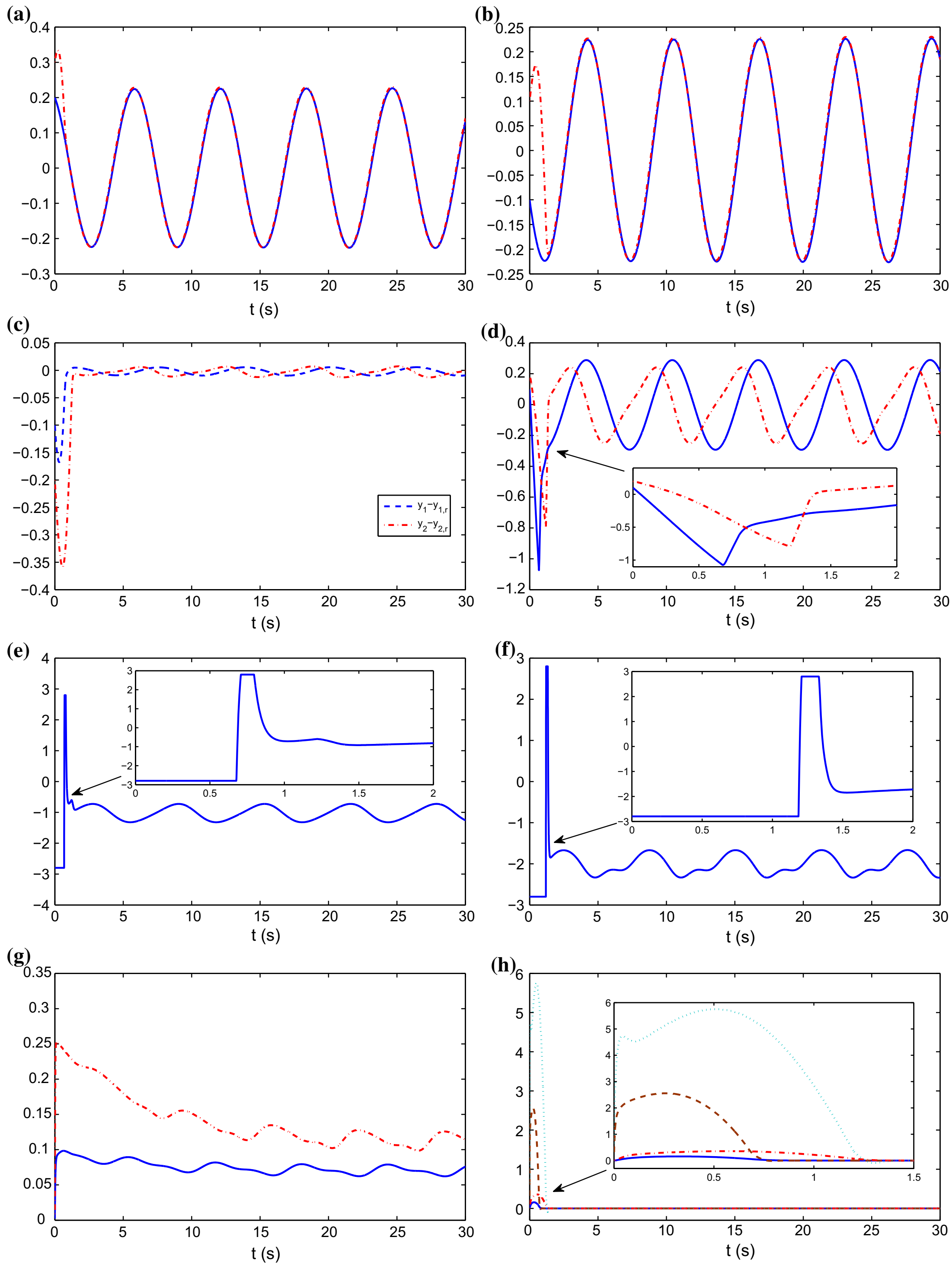

Fig. 1 Simulation results of example 1. **a** Output y_1 (*dot-dash line*) follows $y_{1,r}$ (*solid line*). **b** Output y_2 (*dot-dash line*) follows $y_{2,r}$ (*solid line*). **c** Trajectories of tracking errors. **d** Trajectories of x_{12} (*solid line*) and x_{22} (*dot-dash line*). **e** Control input v_1. **f** Control input v_2. **g** Trajectories of $\hat{\vartheta}_1$ (*solid line*) and $\hat{\vartheta}_2$ (*dot-dash line*). **h** Trajectories of $\chi_{1,1}$ (*solid line*), $\chi_{1,2}$ (*dash line*), $\chi_{2,1}$ (*dot-dash line*) and $\chi_{2,2}$ (*dot line*)

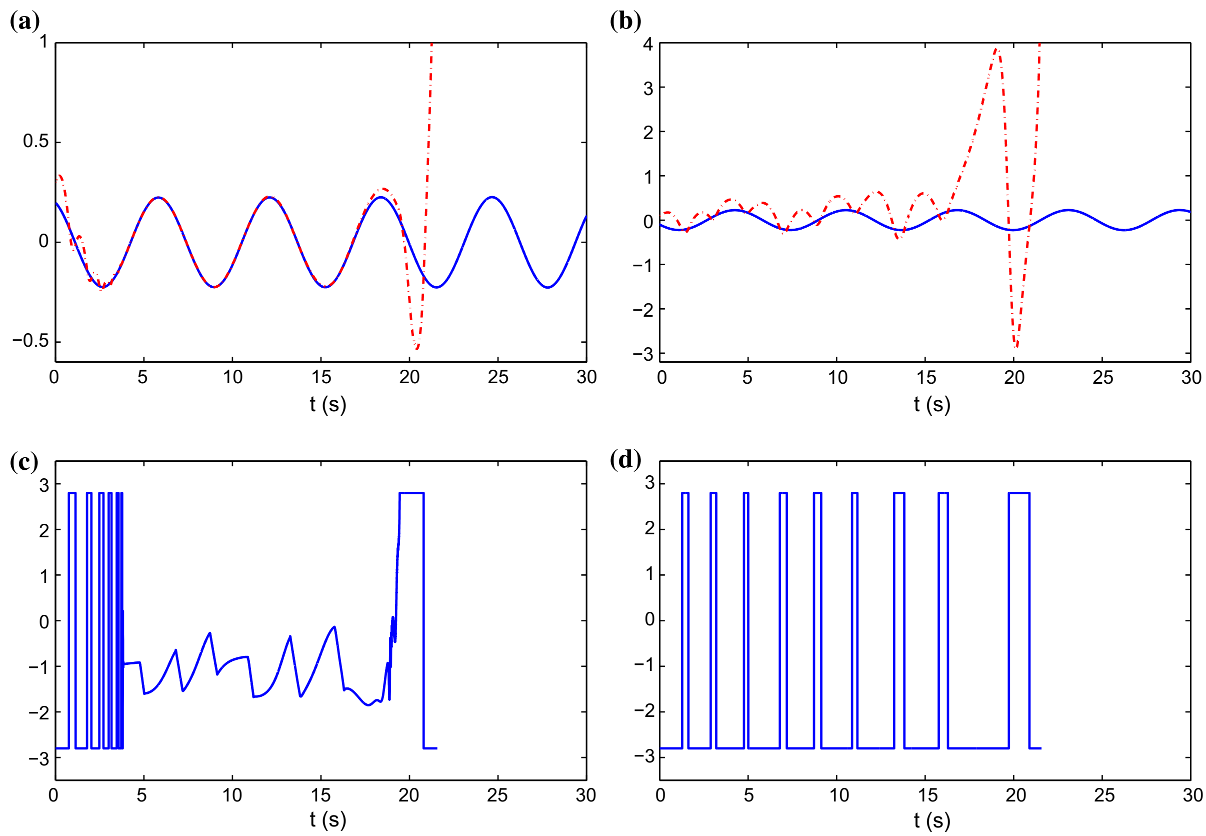

Fig. 2 Simulation results of example 1: $\chi = 0$. **a** Output y_1 (*dot-dash line*) follows $y_{1,r}$ (*solid line*). **b** Output y_2 (*dot-dash line*) follows $y_{2,r}$ (*solid line*). **c** Trajectories of v_1. **d** Trajectories of v_2

($\chi_{1,1}$, $\chi_{1,2}$, $\chi_{2,1}$ and $\chi_{2,2}$) are bounded. It is concluded that the closed-loop signals are all bounded.

For unprejudiced comparison, we will set $\chi_j = 0$ to check the system response since χ_j is the key point to attenuate the effect caused by input saturation and guarantee systematic performance according to above theoretical analysis. Actually, when $\chi_j = 0$, Eq. (5) becomes $z_{j,1} = y_{j,1} - y_{j,r}$ and $z_{j,i_j} = x_{j,i_j} - r_{j,i_j-1}$, which are widely used in existing literature [40,41]. The results when $\chi_j = 0$ are given in Figure 2, and the closed-loop stability is ruined.

5.2 Example 2

Consider the following non-affine MIMO nonlinear system in [40]. To verify the effectiveness of the proposed method, input saturations characterized by $u_1^+ = 1.1$ and $u_2^+ = 0.5$ are imposed on the first and second subsystems, respectively.

$$\begin{cases} \dot{x}_{1,1} = x_{1,1} + x_{2,1} + \frac{x_{1,2}^2}{5} \\ \dot{x}_{1,2} = x_{1,1}x_{1,2} + x_{2,1} + v_1 + \frac{v_1^3}{7} + \Delta_{1,2} \\ \quad v_1 = \mathrm{sat}(u_1),\ u_1^+ = 1.1 \\ \quad y_1 = x_{1,1} \\ \dot{x}_{2,1} = x_{1,1}x_{1,2} + x_{2,1} + v_1 + v_2 + \frac{v_2^2}{7} + \Delta_{2,1} \\ \quad v_2 = \mathrm{sat}(u_2),\ u_2^+ = 0.5 \\ \quad y_2 = x_{2,1} \end{cases} \tag{51}$$

The references are the output of van del Pol oscillator with $\beta_v = 0.002$. The initial values of the plant are as follows. $x_{1,1}(0) = 0.3$, $x_{1,2} = 0.2$, $x_2(0) = 0$, $\varphi_{11}(0) = 0$, $y_{1r}(0) = 0.2$, $y_{2r}(0) = 0.5$, $\hat{\vartheta}_1(0) = \hat{\vartheta}_2(0) = 0$. The design parameters are chosen as follows. $\alpha_{1,1} = 6$, $\alpha_{1,2} = 12$, $\alpha_2 = 25$, $\kappa_{1,1} = \kappa_{1,2} = \kappa_2 = 0.1$, $\tau_{1,1} = 0.005$, $c_1 = c_2 = 0.1$, $\Gamma_1 = \Gamma_2 = 5$.

The RBF NNs (1, 1) contain 20 nodes with widths $a_{1,1} = 1.5$ and centers $\mu_{1,1}$ evenly spaced in $[-2, 2] \times [-1.5, 1.5]$; RBF NNs (1, 2) contain 30 nodes with widths $a_{1,2} = 1.5$ and centers $\mu_{1,2}$ evenly spaced in

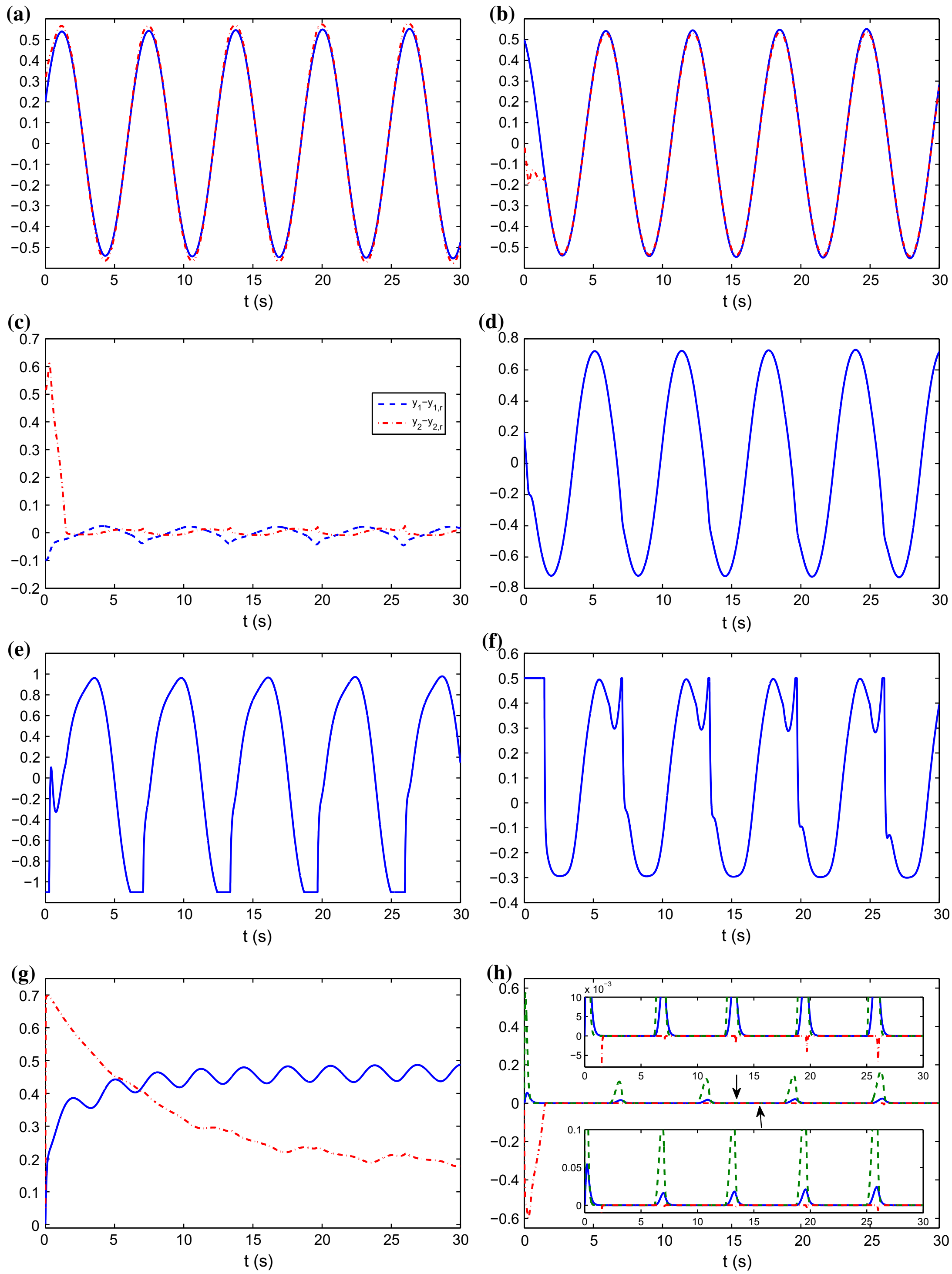

Fig. 3 Simulation results of example 2. **a** Output y_1 (*dot-dash line*) follows $y_{1,r}$ (*solid line*). **b** Output y_2 (*dot-dash line*) follows $y_{2,r}$ (*solid line*). **c** Trajectories of tracking errors. **d** Trajectories of x_{12}. **e** Control input v_1. **f** Control input v_2. **g** Trajectories of $\hat{\vartheta}_1$ (*solid line*) and $\hat{\vartheta}_2$ (*dot-dash line*). (h) Trajectories of $\chi_{1,1}$ (*solid line*), $\chi_{1,2}$ (*dash line*), and χ_2 (*dot-dash line*)

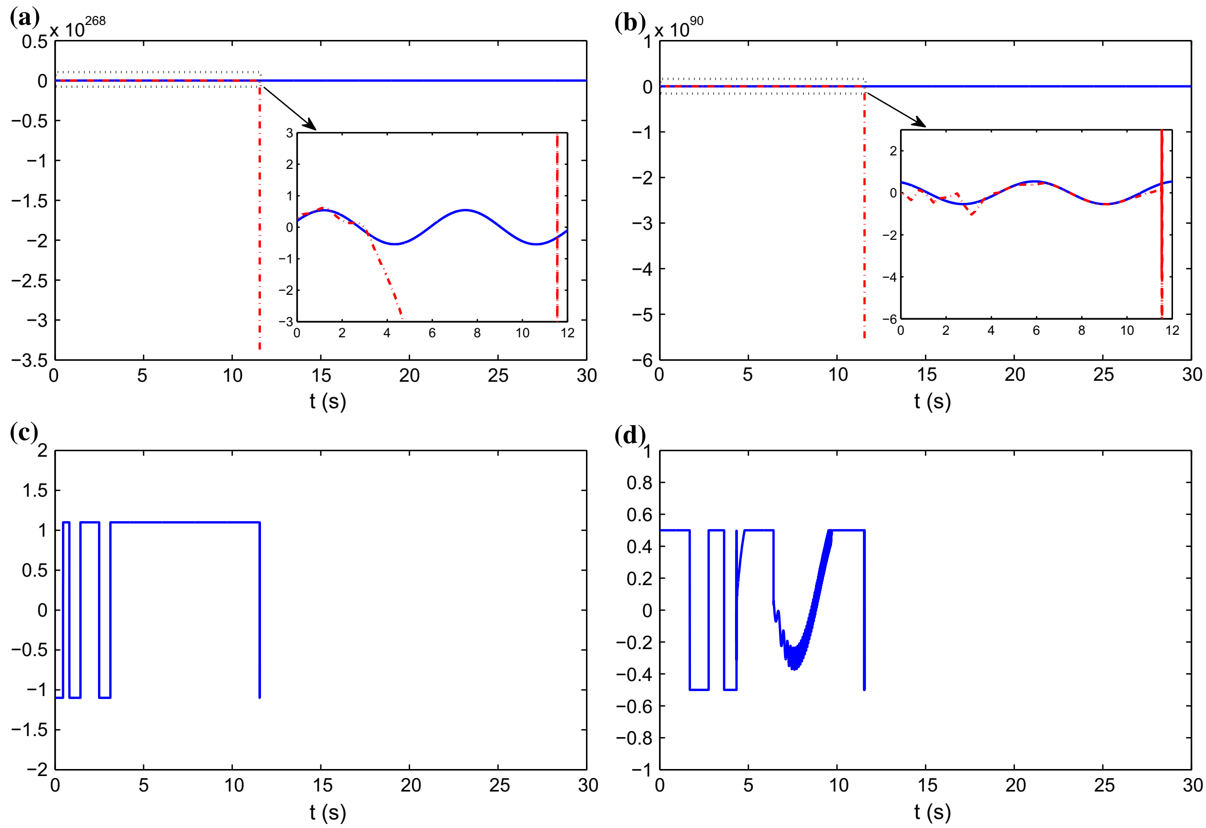

Fig. 4 Simulation results of example 2: $\chi = 0$. **a** Output y_1 (*dot-dash line*) follows $y_{1,r}$ (*solid line*). **b** Output y_2 (*dot-dash line*) follows $y_{2,r}$ (*solid line*). **c** Trajectories of v_1. **d** Trajectories of v_2

$[-2.5, 2.5] \times [-1.5, 1.5] \times [-2, 2] \times [-1.5, 1.5]$; RBF NNs (2) contain 30 nodes with widths $a_2 = 1.5$ and centers μ_2 evenly spaced $[-2.5, 2.5] \times [-1.5, 1.5] \times [-2, 2] \times [-1.5, 1.5] \times [-2, 2]$. The initial values of all RBF NNs are set to be zero.

The simulation results of this example are shown in Fig. 3. Figure 3a, b presents the tracking performance of subsystems, and Fig. 3c shows the tracking errors, and the results are satisfactory. From Fig. 3d, boundedness of x_{12} is observed. From Fig. 3e, f, it follows that the constrained input signals become periodic after about 2s. Figure 3g, h illustrates that the adaptive parameters ($\hat{\vartheta}_1$ and $\hat{\vartheta}_2$) and auxiliary signals ($\chi_{1,1}$, $\chi_{1,2}$ and χ_2) are bounded. It is concluded that the closed-loop signals are all bounded. The stability is ruined if χ_j is set to be zero, see Fig. 4.

6 Concluding remarks

In this paper, neural adaptive control is proposed for a class of uncertain MIMO systems in the presence of constrained input. Both the problems of "explosion of complexity" and "dimension curse" are circumvented simultaneously in the developed method via DSC and MLP algorithms, respectively. Novel intercepted adaptation approach is developed to attenuate the effects caused by input saturation. Comparing with the pioneering MLP algorithm, only one parameter needs to be online learning. Simulation results are used to demonstrate the effectiveness of the proposed approach.

References

1. Chen, B., Liu, K., Liu, X., Shi, P., Lin, C., Zhang, H.: Approximation-based adaptive neural control design for a class of nonlinear systems. IEEE Trans. Cybern. **44**(5), 610–619 (2014)
2. Nekoukar, V., Erfanian, A.: Adaptive fuzzy terminal sliding mode control for a class of MIMO uncertain nonlinear systems. Fuzzy Sets Syst. **179**(1), 34–49 (2011)
3. Tong, S., Li, H.X.: Fuzzy adaptive sliding-mode control for MIMO nonlinear systems. IEEE Trans. Fuzzy Syst. **11**(3), 354–360 (2003)

4. Li, H.X., Tong, S.: A hybrid adaptive fuzzy control for a class of nonlinear MIMO systems. IEEE Trans. Fuzzy Syst. **11**(1), 24–34 (2003)
5. Tong, S.C., He, X.L., Zhang, H.G.: A combined backstepping and small-gain approach to robust adaptive fuzzy output feedback control. IEEE Trans. Fuzzy Syst. **17**(5), 1059–1069 (2009)
6. Jagannathan, S., Lewis, F.L.: Robust backstepping control of a class of nonlinear systems using fuzzy logic. Inf. Sci. **123**(3), 223–240 (2000)
7. Shaocheng, T., Changying, L., Yongming, L.: Fuzzy adaptive observer backstepping control for MIMO nonlinear systems. Fuzzy Sets Syst. **160**(19), 2755–2775 (2009)
8. Yip, P.P., Hedrick, J.K.: Adaptive dynamic surface control: a simplified algorithm for adaptive backstepping control of nonlinear systems. Int. J. Control **71**(5), 959–979 (1998)
9. Yang, Y., Ren, J.: Adaptive fuzzy robust tracking controller design via small gain approach and its application. IEEE Trans. Fuzzy Syst. **11**(6), 783–795 (2003)
10. Yang, Y., Feng, G., Ren, J.: A combined backstepping and small-gain approach to robust adaptive fuzzy control for strict-feedback nonlinear systems. IEEE Trans. Syst. Man Cybern. A Syst. Hum. **34**(3), 406–420 (2004)
11. Swaroop, D., Hedrick, J.K., Yip, P.P., Gerdes, J.C.: Dynamic surface control for a class of nonlinear systems. IEEE Trans. Automa. Control **45**(10), 1893–1899 (2000)
12. Yang, Y.-S., Wang, X.-F.: Adaptive h_∞ tracking control for a class of uncertain nonlinear systems using radial-basis-function neural networks. Neurocomputing **70**(4), 932–941 (2007)
13. Li, T.-S., Wang, D., Feng, G., Tong, S.-C.: A DSC approach to robust adaptive NN tracking control for strict-feedback nonlinear systems. IEEE Trans. Syst. Man Cybern. B Cybern. **40**(3), 915–927 (2010)
14. Wang, M., Chen, B., Dai, S.-L.: Direct adaptive fuzzy tracking control for a class of perturbed strict-feedback nonlinear systems. Fuzzy Sets Syst. **158**(24), 2655–2670 (2007)
15. Chen, B., Liu, X., Liu, K., Lin, C.: Direct adaptive fuzzy control of nonlinear strict-feedback systems. Automatica **45**(6), 1530–1535 (2009)
16. Chen, B., Liu, X., Liu, K., Lin, C.: Novel adaptive neural control design for nonlinear MIMO time-delay systems. Automatica **45**(6), 1554–1560 (2009)
17. Chen, B., Liu, X., Liu, K., Lin, C.: Fuzzy-approximation-based adaptive control of strict-feedback nonlinear systems with time delays. IEEE Trans. Fuzzy Syst. **18**(5), 883–892 (2010)
18. Chen, B., Liu, X., Liu, K., Lin, C.: Adaptive control for nonlinear MIMO time-delay systems based on fuzzy approximation. Inf. Sci. **222**, 576–592 (2013)
19. Wang, D., Huang, J.: Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form. IEEE Trans. Neural Netw. **16**(1), 195–202 (2005)
20. Zhang, T.-P., Ge, S.: Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form. Automatica **44**(7), 1895–1903 (2008)
21. Yoo, S.J., Park, J.B., Choi, Y.H.: Adaptive dynamic surface control for stabilization of parametric strict-feedback nonlinear systems with unknown time delays. IEEE Trans. Autom. Control **52**(12), 2360–2365 (2007)
22. Monopoli, R.: Adaptive control for systems with hard saturation, in: Decision and Control including the 14th Symposium on Adaptive Processes, 1975 IEEE Conference on, Vol. 14, pp. 841–843. IEEE (1975)
23. Polycarpou, M., Farrell, J., Sharma, M.: On-line approximation control of uncertain nonlinear systems: issues with control input saturation. In: American Control Conference 2003. Proceedings of the 2003, Vol. 1, pp. 543–548. IEEE (2003)
24. Polycarpou, M., Farrell, J., Sharma, M.: Robust on-line approximation control of uncertain: nonlinear systems subject to constraints. In: IEEE International Conference on Engineering of Complex Computer Systems, pp. 66–74 (2004)
25. Farrell, J., Sharma, M., Polycarpou, M.: Backstepping-based flight control with adaptive function approximation. J. Guid. Control. Dyn. **28**(6), 1089–1102 (2005)
26. Johnson, E.N., Calise, A.J.: Limited authority adaptive flight control for reusable launch vehicles. J. Guid. Control. Dyn. **26**(6), 906–913 (2003)
27. Lin, D., Wang, X., Yao, Y.: Fuzzy neural adaptive tracking control of unknown chaotic systems with input saturation. Nonlinear Dyn. **67**(4), 2889–2897 (2012)
28. Cao, Y.-Y., Lin, Z.: Robust stability analysis and fuzzy-scheduling control for nonlinear systems subject to actuator saturation. IEEE Trans. Fuzzy Syst. **11**(1), 57–67 (2003)
29. Zhong, Y.-S.: Globally stable adaptive system design for minimum phase SISO plants with input saturation. Automatica **41**(9), 1539–1547 (2005)
30. Chen, M., Ge, S.S., How, B.Voon Ee: Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities. IEEE Trans. Neural Netw. **21**(5), 796–812 (2010)
31. Wen, C., Zhou, J., Liu, Z., Su, H.: Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance. IEEE Trans. Autom. Control **56**(7), 1672–1678 (2011)
32. Tong, S., Li, Y.: Adaptive fuzzy output feedback control of MIMO nonlinear systems with unknown dead-zone input. IEEE Trans. Fuzzy Syst. **21**(1), 134–146 (2013)
33. Li, Y., Tong, S., Li, T.: Adaptive fuzzy output-feedback control for output constrained nonlinear systems in the presence of input saturation. Fuzzy Sets Syst. **248**, 138–155 (2014)
34. Tong, S., Sui, S.S., Li, Y.: Fuzzy adaptive output feedback control of MIMO nonlinear systems with partial tracking errors constrained. IEEE Trans. Fuzzy Syst. (2015). doi:10.1109/TFUZZ.2014.2327987
35. Wang, H., Chen, B., Liu, X., Liu, K., Lin, C.: Robust adaptive fuzzy tracking control for pure-feedback stochastic nonlinear systems with input constraints. IEEE Trans. Cybern. **43**(6), 2093–2104 (2013)
36. Wang, H., Chen, B., Liu, X., Liu, K., Lin, C.: Adaptive neural tracking control for stochastic nonlinear strict-feedback systems with unknown input saturation. Inf. Sci. **269**, 300–315 (2014)
37. Wang, H. S., et al.: Modified model reference adaptive control with saturated inputs, decision and control, 1992, Proceedings of the 31st IEEE Conference on (1992)
38. Lin, W., Qian, C.: Adaptive control of nonlinearly parameterized systems: the smooth feedback case. IEEE Trans. Autom. Control **47**(8), 1249–1266 (2002)

39. Ge, S.S., Wang, C.: Adaptive neural control of uncertain MIMO nonlinear systems. IEEE Trans. Neural Netw. **15**(3), 674–692 (2004)
40. Chen, Z., Ge, S.S., Zhang, Y., Li, Y.: Adaptive neural control of MIMO nonlinear systems with a block-triangular pure-feedback control structure. IEEE Trans. Neural Netw. Learn. Syst. **25**(11), 2017–2029 (2014)
41. Tong, S., Li, Y., Shi, P.: Observer-based adaptive fuzzy backstepping output feedback control of uncertain MIMO pure-feedback nonlinear systems. IEEE Trans. Fuzzy Syst. **20**(4), 771–785 (2012)