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An adaptive chattering-free PID sliding mode control based on dynamic sliding manifolds for a class of uncertain nonlinear systems

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Abstract This paper proposes a new dynamic PID sliding mode control technique for a class of uncertain nonlinear systems. The offered controller is formulated based on the Lyapunov stability theory and guarantees the existence of the sliding mode around the sliding surface in a finite time. Furthermore, this approach can eliminate the chattering phenomenon caused by the switching control action and can realize high-precision performance. Moreover, an adaptive parameter tuning method is proposed to estimate the unknown upper bounds of the disturbances. Simulation results for an inverted pendulum system demonstrate the efficiency and feasibility of the suggested technique.

Keywords Adaptive sliding mode controller · Chattering-free performance · PID sliding surface · Dynamic sliding mode · Nonlinear system

1 Introduction

1.1 Background and motivations

Stabilization and tracking control of uncertain nonlinear systems have significant applications in electronics, mechanics, and robotic systems [1–3]. In particular, various dynamical systems such as flexible-link robots, oscillators, chaotic systems, and synchronous machines may be modeled by an uncertain nonlinear structure. In previous years, investigation of nonlinear control problems has attracted much attention due to the nonlinear structure of most physical and dynamical systems [4]. Nonlinear systems with time-varying and uncertain terms can model many important phenomena and display various behaviors such as different equilibrium points and multiple periodic orbits for non-autonomous systems [5].

As an effective and famous robust control scheme, sliding mode control (SMC) has been extensively applied for the improvement of stability, performance, and robustness of linear and nonlinear systems in the presence of uncertainties [6–8]. The main features of SMC are the fast response, robustness against uncertainties, insensitivity to the bounded disturbances, good transient performance, and computational simplicity with respect to other control techniques [9–11]. The procedure of SMC can be separated into two phases which are the sliding phase and the reaching phase [12,13]. Because of the influence of sliding surface on the stability and transient performance of control systems, the major step is to introduce a proper switching surface so that tracking errors reduce to a satisfactory value [6,14]. In SMC method, the robust tracking performance can be satisfied after the system states reach the sliding surface, and consequently robustness is not guaranteed during the reaching phase. On the other

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hand, SMC suffers from the undesired high-frequency oscillations known as chattering phenomena which is created by the discontinuous control law and is very harmful for actuators used in practical systems [15–17]. To suppress the chattering, a second-order sliding mode (SOSM) control approach has been presented [18–20]. The main idea of SOSM is that the discontinuous sign function is forced to act on the time derivative of the control law, and thus the actual control input resulted after integration is continuous which removes the chattering [18]. The other difficult issue in designing SMC is the requirement of the norm of uncertainty properly which is used for the switching control gain. Perturbation estimation techniques have been investigated in [21–28] to overcome the problem of the knowledge of perturbation upper bounds.

1.2 Literature review

In [22], an observer-based SMC is presented for the excitation control of synchronous generators which employs output feedback and perturbation estimation. In [23], a sliding perturbation observer is offered and integrated into the proportional integral (PI)-controlled system to compensate for the perturbations caused by parameter variations and unknown disturbances. In [24], a variable structure control technique combined with the adaptive perturbation estimation method is proposed for trajectory tracking of the piezoelectric actuators. In [25], a high-order SMC observer is suggested to guarantee the perturbation estimation of the linear time-invariant systems affected by unmatched perturbations. In [26], an adaptive second-order SMC with a PID sliding surface is designed to overcome the faults in the heat recovery steam generator boilers and achieve good performance for the boilers. The difference of our offered method in comparison with what is proposed in [26] is the nonlinear structure of the controlled system; however, besides good tracking performance stated in [26], the controlled heat recovery steam generator boilers tolerate the faults in input and system matrices. In [27], a SMC system with a PID sliding surface is adopted to control the speed of an electromechanical system, and the desired trajectory is tracked in the presence of uncertainties and disturbances. The novelties of the suggested method of this paper in comparison with [27] are the application of adaptive parameter tuning scheme, using a dynamic sliding surface, and finite-time tracking controller design for uncertain nonlinear systems; however, using high-speed computers and some application tools in [27], the experimental results are presented which confirm the efficiency and simplicity of the method. In [28], an adaptive fuzzy SMC methodology with a PID sliding surface is proposed for the control of robot manipulators. Similar to our proposed method, an adaptive SMC approach combined with a PID sliding surface is presented in [28]; however, fuzzy logic control scheme of [28] is used to generate the hitting control signal, and the output gain of the fuzzy SMC is tuned online by a supervisory fuzzy structure, where the chattering problem is avoided. Also, compared with the proposed method in this paper, an adaptive robust PID–SMC controller is presented in [29] which is optimized by a multi-objective genetic algorithm to control a mechanical system in the presence of external disturbances and perturbations. Moreover, the application of PID-SMC to a MIMO system which is proposed in [30] is a more challenging action due to the coupling effects between the control variables. All the methods introduced in the above-mentioned references motivate researchers to actively develop the proposed method of this paper.

1.3 Contributions

To the best of the author's knowledge, very little attempts have been made to design a robust adaptive SMC using a PID sliding surface for the tracking control problem of nonlinear system with time-varying uncertainties. In the recent years, very little attention has been paid to this problem, which is still open in the literature. In this paper, a PID sliding surface-based SMC control approach is proposed for the tracking problem of nonlinear uncertain systems with time-varying uncertainties and nonlinearities. Using the suggested control technique, the tracking errors converge to zero in a finite time and the existence of the sliding mode around the sliding surface in a finite time is guaranteed. This approach eliminates the reaching phase of SMC to improve the global robustness of the system. In addition, this method overcomes the chattering problem caused by the discontinuous sign function and eliminates the need for pre-design information about the upper bound of external disturbances. The stability and robustness of the proposed control method are proved using Lyapunov stability theory. To justify the feasibility and efficiency of the introduced method, the tracking problem of an inverted pendulum system is simulated.

1.4 Paper organization

This paper is organized as follows: the problem description is given in Sect. 2. In Sect. 3, the main results are proposed and the convergence and stability of the closed-loop systems are analyzed via Lyapunov stability theory. Simulation results on an inverted pendulum system are provided in Sect. 4, and the conclusions are presented in Sect. 5.

2 Problem description

The nonlinear second-order system with time-varying uncertainties is described as:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= f(x,t) + \Delta f(x,t) \\ &\quad + (b(x,t) + \Delta b(x,t))\, u(t) + d_0(x,t), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t)]^T$ is the state vector, $u(t)$ is the control input, $b(x,t)$ and $f(x,t)$ are the known bounded nonlinear functions, and $\Delta b(x,t)$, $\Delta f(x,t)$ and $d_0(x,t)$ are the nonlinear functions which introduce the system uncertainties and external disturbances. Defining $d(x,t) = \Delta f(x,t) + \Delta b(x,t)u(t) + d_0(x,t)$, the nonlinear second-order system can be considered as:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= f(x,t) + b(x,t)u(t) + d(x,t). \end{aligned} \tag{2}$$

The uncertain nonlinear system (2) is supposed to track the desired trajectory $x_d(t) = [x_{1d}(t), x_{2d}(t)]^T$, where $x_{2d}(t) = \dot{x}_{1d}(t)$, and $x_{2d}(t)$ is a differentiable function of time. The tracking error is defined as:

$$E(t) = x(t) - x_d(t) = [e(t), \dot{e}(t)]^T, \tag{3}$$

where $e(t) = x_1(t) - x_{1d}(t)$ and $\dot{e}(t) = x_2(t) - x_{2d}(t)$.

The dynamic sliding mode equation for system (2) can be defined as:

$$s(t) = G\,(E(t) - L(t)), \tag{4}$$

where $G = [g_1, g_2]$ are the gain coefficients and $L(t) = [l(t), \dot{l}(t)]^T$ are the terminal functions.

Assumption 1 [31]: Consider the function $l(t) : R^+ \to R$, $l(t) \in C^n[0,\infty)$, $\dot{l}(t) \in L^\infty$, $l(t)$ is finite in interval $[0,T]$, $E(0) = L(0)$, $\dot{E}(0) = \dot{L}(0)$, that is, $l(0) = e(0)$, $\dot{l}(0) = \dot{e}(0)$, and $\ddot{l}(0) = \ddot{e}(0)$. Moreover, for every $t \geq T$, $l(t) = 0$, $\dot{l}(t) = 0$, and $\ddot{l}(t) = 0$. $C^n[0,\infty)$ is the set of n rank differentiable continuous functions defined in $[0,\infty)$.

Remark 1 The sliding surface (4) makes that the state trajectories arrive at the sliding surface right from the beginning. Thus, the reaching interval is eliminated, and the global robustness of the whole system is guaranteed. From (4) and Assumption 1, it follows that:

$$s(0) = G\,(E(0) - L(0)) = 0, \tag{5}$$

$$\dot{s}(0) = G\left(\dot{E}(0) - \dot{L}(0)\right) = 0. \tag{6}$$

The terminal function $l(t)$ is defined as [31]:

$$l(t) = \begin{cases} \left(\frac{a_{20}}{T^5}e_0 + \frac{a_{21}}{T^4}\dot{e}_0 + \frac{a_{22}}{T^3}\ddot{e}_0\right)t^5 \\ \quad + \left(\frac{a_{10}}{T^4}e_0 + \frac{a_{11}}{T^3}\dot{e}_0 + \frac{a_{12}}{T^2}\ddot{e}_0\right)t^4 & \text{if} \quad t \leq T \\ \quad + \left(\frac{a_{00}}{T^3}e_0 + \frac{a_{01}}{T^2}\dot{e}_0 + \frac{a_{02}}{T}\ddot{e}_0\right)t^3 \\ \quad + \frac{1}{2}\ddot{e}_0 t^2 + \dot{e}_0 t + e_0, \\ 0, & \text{if} \quad t \geq T \end{cases} \tag{7}$$

In light of Assumption 1 and the following equations, functions $l(t)$, $\dot{l}(t)$, and $\ddot{l}(t)$ can all be equal to zero at time $t = T$ by designing $a_{ij}(i,j = 0,\ldots,2)$ as:

$$\begin{cases} a_{00} + a_{10} + a_{20} = -1, \\ 3a_{00} + 4a_{10} + 5a_{20} = 0, \\ 6a_{00} + 12a_{10} + 20a_{20} = 0, \end{cases} \tag{8}$$

$$\begin{cases} a_{01} + a_{11} + a_{21} = -1, \\ 3a_{01} + 4a_{11} + 5a_{21} = -1, \\ 6a_{01} + 12a_{11} + 20a_{21} = 0, \end{cases} \tag{9}$$

$$\begin{cases} a_{02} + a_{12} + a_{22} = -\frac{1}{2}, \\ 3a_{02} + 4a_{12} + 5a_{22} = -1, \\ 6a_{02} + 12a_{12} + 20a_{22} = -1. \end{cases} \tag{10}$$

From (8)–(10), the values of the parameters a_{ij} $(i,j = 0,\ldots,2)$ can be obtained as follows:

$$a_{00} = -10,\ a_{01} = -6,\ a_{02} = -1.5,\ a_{10} = 15,$$
$$a_{11} = 8,\ a_{12} = 1.5,\ a_{20} = -6,\ a_{21} = -3,$$
$$a_{22} = -0.5.$$

3 Main results

The proposed PID sliding manifold can be defined as:

$$\sigma(t) = k_p s(t) + k_i \int_0^t s(\tau)d\tau + k_d \dot{s}(t), \tag{11}$$

where k_p, k_i, and k_d are the positive constants representing the proportional, integral, and derivative coefficients, respectively. The time derivative of the PID sliding manifold (11) can be obtained as:

$$\dot{\sigma}(t) = k_p \dot{s}(t) + k_i s(t) + k_d \ddot{s}(t). \tag{12}$$

If the condition $\dot{\sigma}(t) = 0$ is satisfied and the constant gains k_p, k_i, and k_d are designed properly, then the dynamic sliding surface $s(t)$ will converge to zero exponentially, and hence the right-hand side expression of (12) is strictly Hurwitz.

In the following theorem, the tracking action of the desired trajectory $x_d(t)$ is guaranteed and the tracking error $E(t)$ converges to zero in finite time T.

Theorem 1 *Consider the uncertain nonlinear second-order system (2). Suppose that the control law is defined as:*

$$\begin{aligned}\dot{u}(t) = &-(g_2 b(x,t))^{-1}\Big\{k_p\left(g_1\left(\dot{e}(t) - \dot{l}(t)\right)\right.\\ &+ g_2\left(f(x,t) + b(x,t)u(t) - \dot{x}_{2d} - \ddot{l}(t)\right)\Big)\\ &+ k_d\left(g_1\left(f(x,t) + b(x,t)u(t) - \dot{x}_{2d} - \ddot{l}(t)\right)\right.\\ &+ g_2\left(\dot{f}(x,t) + \dot{b}(x,t)u(t) - \ddot{x}_{2d} - \dddot{l}(t)\right)\Big),\\ &+ k_i G\left(E(t) - L(t)\right) + \kappa\, sgn(\sigma)\left|\sigma\right|^{\eta}\\ &+ \gamma\sigma + \chi\, sgn(\sigma)\Big\},\end{aligned} \tag{13}$$

where γ and κ are arbitrary positive constants and χ is a scalar value which satisfies:

$$\chi \geq \max(\Lambda), \tag{14}$$

where $\Lambda = \left(k_p g_2 + k_d g_1\right) d(x,t) + k_d g_2 \dot{d}(x,t)$. Then, the sliding manifold $\sigma(t)$ converges to zero in finite time, the dynamic sliding surface $s(t)$ converges to zero exponentially, and the tracking action of the desired trajectory $x_d(t)$ is guaranteed.

Proof Consider the following candidate Lyapunov function:

$$V_1(t) = \frac{1}{2}\sigma(t)^T\sigma(t). \tag{15}$$

The first and second time derivatives of the dynamic sliding surface can be obtained from (2)–(4) as:

$$\begin{aligned}\dot{s}(t) &= G\left(\dot{E}(t) - \dot{L}(t)\right)\\ &= g_1\left(\dot{e}(t) - \dot{l}(t)\right) + g_2\left(f(x,t) + b(x,t)u(t)\right.\\ &\quad\left. + d(x,t) - \dot{x}_{2d} - \ddot{l}(t)\right),\end{aligned} \tag{16}$$

$$\begin{aligned}\ddot{s}(t) &= G\left(\ddot{E}(t) - \ddot{L}(t)\right)\\ &= g_1\left(f(x,t) + b(x,t)u(t) + d(x,t) - \dot{x}_{2d} - \ddot{l}(t)\right)\\ &\quad + g_2\left(\dot{f}(x,t) + \dot{b}(x,t)u(t) + b(x,t)\dot{u}(t)\right.\\ &\quad\left. + \dot{d}(x,t) - \ddot{x}_{2d} - \dddot{l}(t)\right).\end{aligned} \tag{17}$$

If (4), (16), and (17) are substituted into (12), one gives:

$$\begin{aligned}\dot{\sigma}(t) = &\, k_p\left(g_1\left(\dot{e}(t) - \dot{l}(t)\right)\right.\\ &+ g_2\left(f(x,t) + b(x,t)u(t) + d(x,t) - \dot{x}_{2d} - \ddot{l}(t)\right)\\ &+ k_i G\left(E(t) - L(t)\right) + k_d\left(g_1\left(f(x,t)\right.\right.\\ &+ \left. b(x,t)u(t) + d(x,t) - \dot{x}_{2d} - \ddot{l}(t)\right)\\ &+ g_2\left(\dot{f}(x,t) + \dot{b}(x,t)u(t) + b(x,t)\dot{u}(t)\right.\\ &+ \left.\left.\dot{d}(x,t) - \ddot{x}_{2d} - \dddot{l}(t)\right)\right).\end{aligned} \tag{18}$$

Differentiating $V_1(t)$ and using (18) yields:

$$\begin{aligned}\dot{V}_1(t) &= \sigma(t)^T\dot{\sigma}(t)\\ &= \sigma(e)^T\Big\{k_p\left(g_1\left(\dot{e}(t) - \dot{l}(t)\right) + g_2\left(f(x,t)\right.\right.\\ &\quad\left.\left. + b(x,t)u(t) + d(x,t) - \dot{x}_{2d} - \ddot{l}(t)\right)\right)\\ &\quad + k_i G\left(E(t) - L(t)\right) + k_d\left(g_1\left(f(x,t)\right.\right.\\ &\quad\left. + b(x,t)u(t) + d(x,t) - \dot{x}_{2d} - \ddot{l}(t)\right)\\ &\quad + g_2\left(\dot{f}(x,t) + \dot{b}(x,t)u(t) + b(x,t)\dot{u}(t)\right.\\ &\quad\left.\left. + \dot{d}(x,t) - \ddot{x}_{2d} - \dddot{l}(t)\right)\right)\Big\},\end{aligned} \tag{19}$$

where substituting (13) in (19), one can obtain:

$$\begin{aligned}\dot{V}_1(t) = &-\sigma^T\kappa\,\mathrm{sgn}(\sigma)\left|\sigma\right|^{\eta} - \sigma^T\gamma\sigma - \sigma^T\chi\,\mathrm{sgn}(\sigma)\\ &+ \sigma^T\left(\left(k_p g_2 + k_d g_1\right)d(x,t) + k_d g_2\dot{d}(x,t)\right),\end{aligned} \tag{20}$$

where based on the condition (14) follows that:

$$\begin{aligned}\dot{V}_1(t) &\leq -\gamma\left|\sigma\right|^2 - \kappa\left|\sigma\right|^{\eta+1}\\ &= -\alpha V(\sigma) - \beta V^{\bar{\eta}}(\sigma),\end{aligned} \tag{21}$$

where $\bar{\eta} = (\eta + 1)/2 < 1$, $\alpha = 2\gamma > 0$, and $\beta = 2^{\bar{\eta}}\kappa > 0$. This means that the Lyapunov function (15) decreases gradually and the sliding manifold $\sigma(t)$ converges to zero in finite time. This completes the proof. □

Remark 2 From (5), (6), and (11), one can obtain $\sigma(0) = 0$. Based on the Lyapunov stability analysis, the sliding manifold $\sigma(t) = 0$ can be achieved all the time. This demonstrates that the reaching phase in the sliding mode control can be removed and the global robustness can be satisfied.

Remark 3 Using the discontinuous control law $\dot{u}(t)$ in (13), the sliding manifold $\sigma(t)$ is forced to converge to zero in finite time. Thus, by integrating (13), the actual control law $u(t)$ becomes continuous and eliminates destructive high-frequency oscillations called chattering.

In practice, the upper bound of the system disturbances is often unknown and therefore χ is difficult to determine. In the following theorem, an adaptive parameter tuning technique is proposed to estimate the unknown upper bound of the system disturbances.

Theorem 2 *Let the PID sliding manifold be in the form of (11) and assume that the external disturbances $d(x,t)$ and $\dot{d}(x,t)$ are unknown but bounded, where χ in (14) is an unknown positive constant. Also, suppose that $\hat{\chi}$ is the estimation value of χ which is estimated by using the following adaptation law:*

$$\dot{\hat{\chi}} = \psi\,|\sigma(t)|\,, \tag{22}$$

where ψ is a positive constant. Using the adaptive parameter tuning control law given by:

$$\begin{aligned}\dot{u}(t) = &-(g_2 b(x,t))^{-1}\Big\{k_p\left(g_1\left(\dot{e}(t)-\dot{l}(t)\right)\right.\\ &+\left.g_2\left(f(x,t)+b(x,t)u(t)-\dot{x}_{2d}-\ddot{l}(t)\right)\right)\\ &+k_d\left(g_1\left(f(x,t)+b(x,t)u(t)-\dot{x}_{2d}-\ddot{l}(t)\right)\right.\\ &+\left.g_2\left(\dot{f}(x,t)+\dot{b}(x,t)u(t)-\ddot{x}_{2d}-\dddot{l}(t)\right)\right)\\ &+k_i G\left(E(t)-L(t)\right)+\kappa\ sgn(\sigma)\,|\sigma|^{\eta}\\ &+\gamma\sigma+\hat{\chi}\ sgn(\sigma)\Big\}\,,\end{aligned} \tag{23}$$

then the finite-time convergence of the sliding manifold to zero is guaranteed and also the zero tracking error objective is satisfied.

Proof The positive definite Lyapunov function is determined as:

$$V_2(t) = \frac{1}{2}\mu\tilde{\chi}^T\tilde{\chi} + \frac{1}{2}\sigma^T(t)\sigma(t), \tag{24}$$

where $\tilde{L} = \hat{L} - L$ and μ is a positive coefficient with the condition $\mu < \frac{1}{\psi}$. Taking the time derivative of (24) and using (18) and (22) yield:

$$\begin{aligned}\dot{V}_2(t) &= \mu\tilde{\chi}\dot{\hat{\chi}} + \sigma^T(t)\dot{\sigma}(t)\\ &= \mu\left(\hat{\chi}-\chi\right)\dot{\hat{\chi}} + \sigma^T(t)\left\{k_p\dot{s}(t)+k_i s(t)+k_d\ddot{s}(t)\right\}\\ &= \mu\psi\left(\hat{\chi}-\chi\right)|\sigma(t)| + \sigma^T(t)\Big\{k_p\left(g_1\left(\dot{e}(t)-\dot{l}(t)\right)\right.\\ &\quad+g_2\left(f(x,t)+b(x,t)u(t)\right.\\ &\quad\left.\left.+d(x,t)-\dot{x}_{2d}-\ddot{l}(t)\right)\right)\\ &\quad+k_i G\left(E(t)-L(t)\right)+k_d\left(g_1\left(f(x,t)\right.\right.\\ &\quad\left.+b(x,t)u(t)+d(x,t)-\dot{x}_{2d}-\ddot{l}(t)\right)\\ &\quad+g_2\left(\dot{f}(x,t)+\dot{b}(x,t)u(t)+b(x,t)\dot{u}(t)\right.\\ &\quad\left.\left.+\dot{d}(x,t)-\ddot{x}_{2d}-\dddot{l}(t)\right)\right)\Big\}\,,\end{aligned} \tag{25}$$

where substituting (23) into (25), one can obtain:

$$\begin{aligned}\dot{V}_2(t) &= \mu\psi\left(\hat{\chi}-\chi\right)|\sigma(t)| + \sigma^T(t)\left(-\kappa\ \mathrm{sgn}(\sigma(t))\,|\sigma(t)|^{\eta}\right.\\ &\quad\left.-\gamma\sigma(t)-\hat{\chi}\ \mathrm{sgn}(\sigma(t))+\Lambda\right)\\ &\le \mu\psi\left(\hat{\chi}-\chi\right)|\sigma(t)| + |\Lambda|\,|\sigma(t)| - \kappa\,|\sigma(t)|^{\eta+1}\\ &\quad-\gamma\,|\sigma(t)|^2-\hat{\chi}\,|\sigma(t)|\\ &\le \mu\psi\left(\hat{\chi}-\chi\right)|\sigma(t)| + |\Lambda|\,|\sigma(t)| - \hat{\chi}\,|\sigma(t)|\\ &\quad+\chi\,|\sigma(t)|-\chi\,|\sigma(t)|\\ &= -\left(\chi-|\Lambda|\right)|\sigma(t)| - (1-\mu\psi)\left(\hat{\chi}-\chi\right)|\sigma(t)|\,.\end{aligned} \tag{26}$$

Since $\chi > |\Lambda|$ and $\mu\psi < 1$, therefore (26) can be expressed by:

$$\begin{aligned}\dot{V}_2(t) &\le -\sqrt{2}\left(\chi-|\Lambda|\right)\frac{|\sigma(t)|}{\sqrt{2}}\\ &\quad-\sqrt{\frac{2}{\mu}}\left(1-\mu\psi\right)\frac{\tilde{\chi}}{\sqrt{\frac{2}{\mu}}}\,|\sigma(t)|\\ &\le -\min\left\{\sqrt{2}\left(\chi-|\Lambda|\right),\sqrt{\frac{2}{\mu}}\left(1-\mu\psi\right)|\sigma(t)|\right\}\\ &\quad\times\left(\frac{|\sigma(t)|}{\sqrt{2}}+\frac{\tilde{\chi}}{\sqrt{\frac{2}{\mu}}}\right)\\ &= -\Omega V_2(t)^{\frac{1}{2}},\end{aligned} \tag{27}$$

where $\Omega = \min\left\{\sqrt{2}\left(\chi-|\Lambda|\right),\sqrt{\frac{2}{\mu}}\left(1-\mu\psi\right)|\sigma(t)|\right\} > 0$. In conclusion, using the adaptive tuning control law (23), the finite-time convergence to the sliding manifold $\sigma(t) = 0$ is guaranteed. □

Since the discontinuous switching function sgn(.) presented in (23) can cause the chattering problem, undesired responses can occur for the uncertain nonlinear system. To avoid this problem, the function sgn(.) can be replaced by the following continuous saturation function:

$$\mathrm{sat}(\sigma) = \begin{cases}\mathrm{sgn}(\sigma), & |\sigma| > \Phi\\ \frac{\sigma}{\Phi}, & |\sigma| \le \Phi\end{cases} \tag{28}$$

where Φ is the boundary layer thickness. Furthermore, although the existence of the proposed NFTSMC can be guaranteed outside the Φ, it cannot be satisfied inside

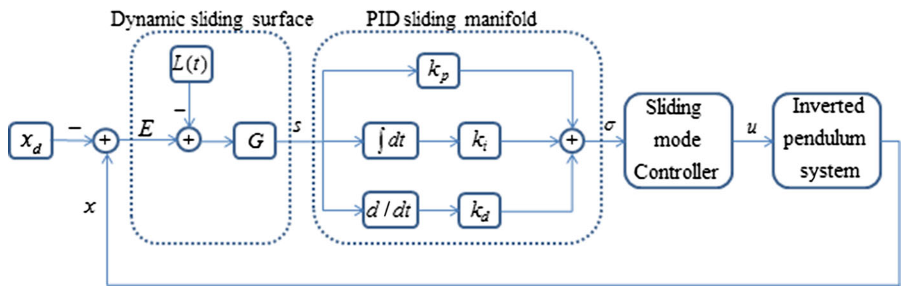

Fig. 1 Schematic diagram of the proposed controller

the Φ. In the worst situation, the state trajectories of the system would just reach Φ. This will obtain a considerable influence on the steady-state characteristics of the system.

4 Simulation results

The inverted pendulum system is a famous test platform for evaluating control techniques [32]. The control goal is to balance the pendulum in the inverted position. This system has particular and significant real-life applications such as position control, aerospace vehicle control, and robotics [33]. In this section, the proposed control technique is applied to an inverted pendulum system. Dynamical equations of the system are denoted as:

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = \frac{g(m_c + m_p)\sin(x_1) - m_p l_p x_2^2 \sin(x_1)\cos(x_1)}{l_p\left(\frac{4}{3}(m_c + m_p) - m_p\cos^2(x_1)\right)} + \frac{\cos(x_1)}{l_p\left(\frac{4}{3}(m_c + m_p) - m_p\cos^2(x_1)\right)}u(t) + d(x,t), \tag{29}$$

where $x = [x_1, x_2]^T$, x_1 and x_2 are the angular position and angular velocity of the pendulum, l_p is the length of the pendulum, m_c is the cart mass, m_p is the pendulum mass, and g is the acceleration due to gravity. The values of the system parameters are set as follows:

$$l_p = 0.5\ \text{m}, \quad m_p = 0.1\ \text{kg}, \quad m_c = 1\ \text{kg}, \quad g = 9.8\ \text{m/s}^2.$$

The external disturbance is assumed as $d(x,t) = 0.3\cos(x_1) + 0.2\cos(\pi t) + 0.1\sin(1.5t)u$. From (2) and (29), the bounded nonlinear functions $f(x,t)$ and $b(x,t)$ are determined as:

$$f(x,t) = \frac{g(m_c + m_p)\sin(x_1) - m_p l_p x_2^2 \sin(x_1)\cos(x_1)}{l_p\left(\frac{4}{3}(m_c + m_p) - m_p\cos^2(x_1)\right)},$$

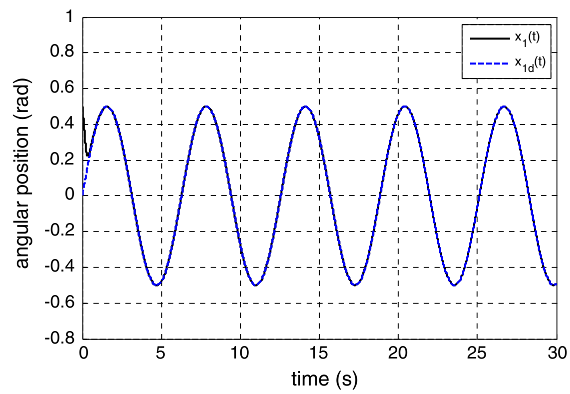

Fig. 2 Angular position tracking trajectory

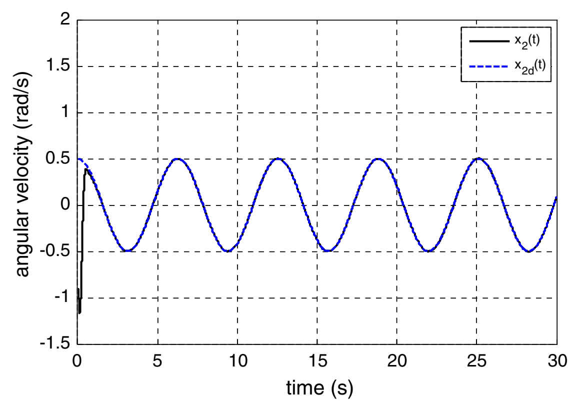

Fig. 3 Angular velocity tracking trajectory

$$b(x,t) = \frac{\cos(x_1)}{l_p\left(\frac{4}{3}(m_c + m_p) - m_p\cos^2(x_1)\right)}.$$

The constant parameters are selected as: $k_p = 4$, $k_i = 2$, $k_d = 1$, $g_1 = 5$, $g_2 = 0.6$, $\psi = 4.5$, $\Phi = 0.05$, $\mu = 0.2$, $\gamma = 3$, $\kappa = 15$, and $\eta = \frac{3}{5}$. The initial conditions of the system states are assumed as:

$$x(0) = [0.5 \quad -1]^T.$$

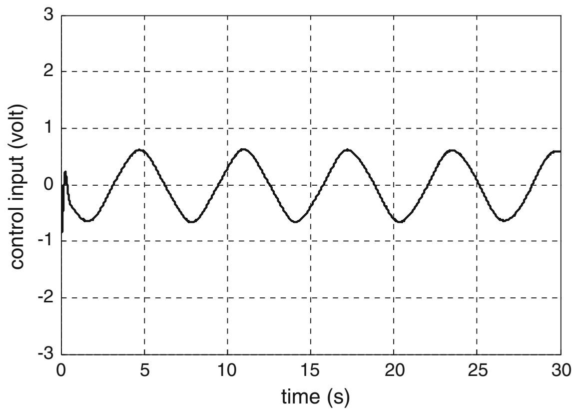

Fig. 4 Control input

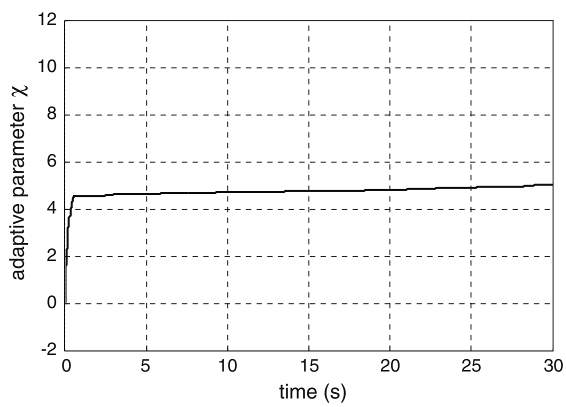

Fig. 5 Adaptive parameter χ

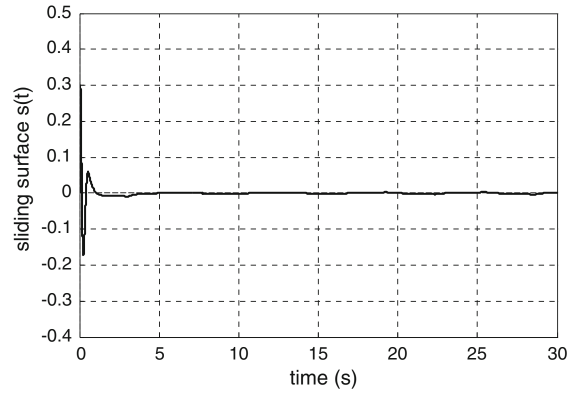

Fig. 6 Dynamic sliding surface $s(t)$

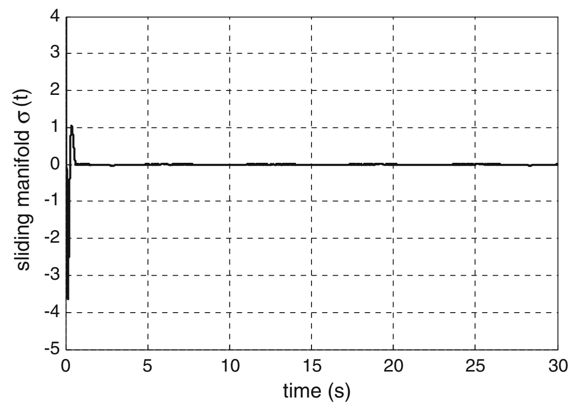

Fig. 7 PID sliding manifold $\sigma(t)$

The desired trajectory is chosen as:

$$x_d(t) = 0.5\sin(t). \tag{30}$$

The schematic diagram of the control configuration is shown in Fig. 1. The angular position and angular velocity tracking trajectories are shown in Figs. 2 and 3. It can be observed from these figures that the position and velocity states appropriately track the desired reference signals. The time responses of the control input and adaptive parameter χ are displayed in Figs. 4 and 5. It is demonstrated that the proposed control law yields the excellent vibration control and robustness features and overcomes the external disturbances and nonlinearities. The time responses of the dynamic sliding surface $s(t)$ and the PID sliding manifold $\sigma(t)$ are shown in Figs. 6 and 7. Clearly, it can be seen that the designed sliding surfaces converge to the origin quickly. Noticeably, the simulations confirm the efficiency and feasibility of the proposed method.

5 Conclusions

In this paper, a novel dynamic PID sliding mode controller is successfully designed for a class of uncertain nonlinear systems. Using the proposed technique, the chattering problem caused by the switching term of the SMC method is eliminated. Also, the proposed control method can guarantee that the system states reach the sliding manifolds at all times, and then the global robustness of the closed-loop system is guaranteed. In addition, an adaptive parameter tuning scheme is presented to estimate the unknown upper bound of the disturbances. Finally, usefulness and effectiveness of the proposed technique for an inverted pendulum system have been verified by simulations.

References

1. Skruch, P.: Feedback stabilization of a class of nonlinear second-order systems. Nonlinear Dyn. **59**, 681–693 (2010)
2. Aghababa, M.P.: Fractional modeling and control of a complex nonlinear energy supply-demand system. Complexity. (2014). doi:10.1002/cplx.21533
3. Yue, M., Wei, X.: Dynamic balance and motion control for wheeled inverted pendulum vehicle via hierarchical sliding mode approach. Proc. Inst. Mech. Eng. Part I J. Syst. Control Eng. **228**(6), 351–358 (2014)
4. Mobayen, S., Majd, V.J.: Robust tracking control method based on composite nonlinear feedback technique for linear systems with time-varying uncertain parameters and disturbances. Nonlinear Dyn. **70**, 171–180 (2012)
5. Rosas, D., Alvarez, J., Alvarez, E.: Robust synchronization of arrays of uncertain nonlinear second-order dynamical systems. Nonlinear Dyn. **67**, 2735–2746 (2012)
6. Mobayen, S.: Design of LMI-based global sliding mode controller for uncertain nonlinear systems with application to Genesio's chaotic system. Complexity. (2014). doi:10.1002/cplx.21545
7. Mobayen, S.: Design of a robust tracker and disturbance attenuator for uncertain systems with time delays. Complexity. (2014). doi:10.1002/cplx.21569
8. Mobayen, S.: Finite-time robust-tracking and model-following controller for uncertain dynamical systems. J. Vib. Control (2014). doi:10.1177/1077546314538991
9. Wang, L., Sheng, Y., Liu, X.: A novel adaptive high-order sliding mode control based on integral sliding mode. Int. J. Control Autom. Syst. **12**, 459–472 (2014)
10. Mobayen, S., Majd, V.J., Sojoodi, M.: An LMI-based composite nonlinear feedback terminal sliding-mode controller design for disturbed MIMO systems. Math. Comput. Simul. **85**, 1–10 (2012)
11. Mobayen, S.: An LMI-based robust controller design using global nonlinear sliding surfaces and application to chaotic systems. Nonlinear Dyn. **79**(2), 1075–1084 (2014)
12. Mobayen, S.: Design of CNF-based nonlinear integral sliding surface for matched uncertain linear systems with multiple state-delays. Nonlinear Dyn. **77**, 1047–1054 (2014)
13. Mobayen, S.: Robust tracking controller for multivariable delayed systems with input saturation via composite nonlinear feedback. Nonlinear Dyn. **76**, 827–838 (2014)
14. Pisano, A., Usai, E.: Sliding mode control: a survey with application in math. Math. Comput. Simul. **81**, 954–79 (2011)
15. Liu, L., Han, Z., Li, W.: Global sliding mode control and application in chaotic systems. Nonlinear Dyn. **56**, 193–198 (2009)
16. Efimov, D., Fridman, L.: Global sliding-mode observer with adjusted gains for locally Lipschitz systems. Automatica **47**, 565–570 (2011)
17. Jing, Y., Mao, Y., Dimirovski, G.M., Zhang, S.: Adaptive global sliding mode control strategy for the vehicle antilock braking systems. In: American Control Conference (ACC '09), St. Louis, MO (2009)
18. Joe, H., Kim, M., Yu, S.C.: Second-order sliding-mode controller for autonomous underwater vehicle in the presence of unknown disturbances. Nonlinear Dyn. **78**(1), 183–196 (2014)
19. Li, X., Yu, X., Han, Q.L.: Stability analysis of second-order sliding mode control systems with input-delay using Poincaré map. IEEE Trans. Autom. Control **58**, 2410–2415 (2013)
20. Chang, J.L.: Dynamic compensator-based second-order sliding mode controller design for mechanical systems. IET Control Theory Appl. **7**, 1675–1682 (2013)
21. Zeinali, M., Notash, L.: Adaptive sliding mode control with uncertainty estimator for robot manipulators. Mech. Mach. Theory **45**(1), 80–90 (2010)
22. Jiang, L., Wu, Q.H., Zhang, C., Zhou, X.X.: Observer-based nonlinear control of synchronous generators with perturbation estimation. Int. J. Elect. Pow. Energy Syst. **23**(5), 359–367 (2001)
23. Lu, Y.S., Cheng, C.M., Cheng, C.H.: Non-overshooting PI control of variable-speed motor drives with sliding perturbation observers. Mechatronics **15**(9), 1143–1158 (2005)
24. Ghafarirad, H., Rezaei, S.M., Abdullah, A., Zareinejad, M., Saadat, M.: Observer-based sliding mode control with adaptive perturbation estimation for micropositioning actuators. Precis. Eng. **35**(2), 271–281 (2011)
25. Loza, A.F., Punta, E., Fridman, L., Bartolini, G., Delprat, S.: Nested backward compensation of unmatched perturbations via HOSM observation. J. Frankl. Inst. **351**(5), 2397–2410 (2014)
26. Aliakbari, S., Ayati, M., Osman, J.H.S., Sam, Y.M.: Second-order sliding mode fault-tolerant control of heat recovery steam generator boiler in combined cycle power plants. Appl. Therm. Eng. **50**(1), 1326–1338 (2013)
27. Eker, I.: Sliding mode control with PID sliding surface and experimental application to an electromechanical plant. ISA Trans. **45**(1), 109–118 (2006)
28. Amer, A.F., Sallam, E.A., Elawady, W.M.: Adaptive fuzzy sliding mode control using supervisory fuzzy control for 3 DOF planar robot manipulators. Appl. Soft. Comput. **11**(8), 4943–4953 (2011)
29. Taherkhorsandi, M., Mahmoodabadi, M.J., Talebipour, M., Castillo-Villar, K.K.: Pareto design of an adaptive robust hybrid of PID and sliding control for a biped robot via genetic algorithm optimization. Nonlinear Dyn. **79**(1), 251–263 (2014)
30. Cao, Y., Chen, X.B.: An output-tracking-based discrete PID-sliding mode control for MIMO systems. IEEE/ASME Trans. Mech. **19**(4), 1183–1194 (2014)
31. Liu, J., Sun, F.: A novel dynamic terminal sliding mode control of uncertain nonlinear systems. J. Control Theory Appl. **5**, 189–193 (2007)
32. Hassanzadeh, I., Mobayen, S.: Controller design for rotary inverted pendulum system using evolutionary algorithms. Math. Probl. Eng. **2011**, 17 pp (2011)
33. Hassanzadeh, I., Mobayen, S.: PSO-based controller design for rotary inverted pendulum system. J. Appl. Sci. **8**(16), 2907–2912 (2008)