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ORIGINAL PAPER

Artificial neural network-based modeling of brain response to flicker light

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Abstract Not only does the modeling of dynamical systems, for instance the biological systems, play an important role in the accurate perception and analysis of these systems, but it also makes the prediction and control of their behavior straightforward. The results of multiple researches in the field of the modeling of biological systems have indicated that the chaotic behavior is a prevalent feature of most complex interactive biological systems. Our results demonstrate that the artificial neural network provides us an effective means to model the underlying dynamics of these systems. In this paper, at first, we represent the results of the use of a multilayer feed-forward neural network to model some famous chaotic systems. The specified neural network is trained with the return maps extracted from the time series. We proceed with the paper by evaluating the accuracy and robustness of our model. The ability of the select neural network to model the dynamics of chosen chaotic systems is verified, even in the presence of noise. Afterwards, we model the brain response to the flicker light. It is known that the brain response to some stimuli such as the flicker light recorded as electroretinogram is an exemplar of chaotic behavior. The need remains, however, for realistic modeling of this behavior of the brain. In this paper, we represent the results of the modeling of this chaotic response by utilizing the proposed neural network. The capability of the neural network to model this specific brain response is confirmed.

Keywords Artificial neural network · Bifurcation diagram · Brain response · Chaotic behavior · Electroretinogram · Modeling

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1 Introduction

When we face with the problem of the description of the behavior of real-world phenomena or systems such as the biological systems, the employment of models is inevitable. The models grant us responses to the questions about the behavior and characteristics of various systems scrutinized under different conditions. The fast advancements in the field of the modeling of dynamical systems, for example the biological systems, which are the result of numerous relevant investigations, are the evidence of its fundamental importance. Biomedical modeling refers to the utilization of various facilities, for instance the computer simulations, discrete mathematics, and numerical methods to mimic the dynamical behavior of biological systems. An extremely potent platform is provided through the biomedical modeling to improve the efficiency of the tasks and projects performed in varied medical fields, for example the experimentations, data acquisitions, knowledge transfers, and technology development, as well as the examination of the effectiveness and safety of medical devices and drugs. It also leads to the reduction in the dependency of medical experimentations on human and animal studies and acceleration of the translation of basic science into clinical medicine and treatment.

The term dynamics refers to the study of change, and dynamical system is a brief term for stating how a system of variables interacts and evolves with time [1]; for instance, we may intend to comprehend the way cancer cells interact, grow, and divide in time, so we could respond to the inquiries into the existence or expansion of a cancer disease, such as "how resistant is the cancer to some treatments" or "what happens if the dosages of chemotherapeutic drugs increase or decrease about 10 %." The majority of researches in various fields such as the biological science, the economy, the stock market, the climate models, or even the reactive or radioactive chemicals in groundwater are conducted to reply to analogous investigations. Although it seems that the named systems, even all the existing systems, are distinct, some unique potent methodologies are generally utilized for their scrutiny. The theory of nonlinear dynamical systems plays a vital role in the modeling of complex phenomena.

Dynamical systems, for example the behavioral patterns in biological systems, population dynamics in biology, chemical kinetics in chemistry, and mechanics in physics, are represented by a variety of mathematical models. These models possess the ability to describe different phenomena whose instantaneous states alter over time. The mathematical theory of dynamical systems has the fundamental aim of determining or characterizing the long-term behavior of systems by using the differential equation and iterative mapping analysis methods. It appears that simple deterministic nonlinear dynamical systems and even piecewise linear systems could surprisingly represent a quite unpredictable behavior, almost similar to the random one. This particular behavior of systems is known as deterministic chaos [2]. The theory of chaos was developed approximately five decades ago and initially applied in the meteorology. The chaotic systems are characterized by nonlinear representations whose long-term behavior is sensitive to initial conditions. Although their behavior may appear to be random, these systems are in fact deterministic and repeatable if all conditions remain the same. In the last decade, chaos theory has turned into a popular analysis method of nonlinear data, for which intractable solutions are produced using the majority of mathematical models [3–6].

The complex patterns such as the chaotic ones are widely observed when the behavior of biological systems, for example brain and heart, and their responses to different internal and external stimuli are precisely assessed [4,7–11]. This characteristic is the consequence of complicated interactions within various components of every biological system as well as with its environment. The human brain response indicates the remarkable inclusive competence of the brain in making proper decisions through analyzing internal and external stimuli in the form of transferred signals to the mind/brain phase-space. During the past several decades, scientists have discovered this phenomenon and proposed some models based on computational, biological, and neuropsychological methods. Despite some advances in researches related to this area of brain study, less effort has been devoted to realistically modeling the complex nonlinear responses of the brain to certain internal and external stimuli [12].

The visual system is one of the most sensitive and intricate senses after the brain. There are about 125 million receptors, named rods and cones, in the retina of each eye. The receptors are nerve cells, which emit electrical signals when they are stimulated by the light radiated from the scene observing. The information about the observed scene, which includes the senses of all its intricacies, for example the form, depth, movement, color, and texture, is extracted from these electrical signals through the analyses performed by the rest of the retina and also the brain proper. The interaction between the visual system and brain causes the system to become more complicated [13].

The human eye could be considered a highly specialized luminance multidetector. As a result of specific anatomy and physiology of the eye, the human's visual perception might be disturbed by the alteration in the visible light. The human's visual perception of the flickering light is affected by:

- Eye viewing field,

- Some parts of eye anatomy such as the photoreceptors (rods and cones) and their dispersal at the retina,
- Some parts of eye physiology such as the eye adaptation procedures, for instance the pupil, photochemical and neural adaptation to the luminous variations, including the modification of spectral luminous efficiency of the photo-receptors.

The visual and flicker perception could be characterized on the basis of various features of visual stimuli, specifically:

(1) Object and background luminance,
(2) Object and background luminance nonuniformity,
(3) Object and background contrast,
(4) Object and background light spectrum,
(5) Object size,
(6) Object location compared to the main axis of view.

The selection of these parameters is based on the fact that the stimulation contrast and radiant density are both of great importance. The flicker perception could be affected by other characteristics of visual stimuli such as [14]:

(1) The alterations in the luminance and color of the stimuli with time,
(2) The movement or spatial changes of the stimuli in the viewing field.

Although understanding the mechanism and characteristics of the brain response to the flicker light could improve the cognition of the visual system [15–17], this brief explanation demonstrates its extreme complexity. It appears that the brain response to some stimuli such as the flicker light is an example of chaotic reaction [17–20]. This characteristic makes the modeling of this response a particularly challenging task, specifically owing to the lack of thorough knowledge about this system. In this paper, we attempt to model the chaotic response of the brain to the flicker light by applying some electroretinogram (ERG) data.

The ERG represents the activity of retinal cells. After stimulating the eye by the flicker light, the electrical potential is produced in the retina. The flicker ERG is an essential means not only for assessing the function of the cone system in ocular diseases, but it also plays a key role in the diagnosis of varying abnormalities such as the schizophrenia, mood disorder, migraine, and epilepsy [17,19–22].

The complexity and nonlinearity of the behavior of bio-systems make its modeling a challenging task [23]. Artificial neural networks (ANNs) are computational models inspired by the structure of central nervous system, particularly the brain, and partially mimic its emergent behavior. In comparison with the brain, the function of ANNs is extremely simplified; however, their capacity to solve various nonlinear problems is almost known to the researchers working on the modeling of systems with complex nonlinear behavior. Multilayer feed-forward neural networks (FNNs) are a kind of ANN that possess the potentiality to solve complex nonlinear functions and therefore are one of the popular modeling techniques [24–33].

The results of numerous contemporary investigations in the biological science have indicated that some famed chaotic systems, for example Logistic Map, Lorenz and Rössler systems, characterize varying chaotic dynamics of bio-systems while considering particular conditions for them, i.e., specific quantities for their parameters [21,23,34–42]. In this paper, at first, we represent the results of the modeling of certain chaotic systems, which are being observed as the behavioral patterns of several bio-systems, by utilizing a multilayer FNN. One of our main objectives in selecting the model was to propose a simple model, without compromising the complex characteristics of chaotic behavior. Our results verify that the specified FNN has the capability to model the chosen chaotic systems, even in the presence of noise. We proceed with the paper by evaluating the ability of the select multilayer FNN to model the chaotic response of the brain to the flicker light. As it turned out, the proposed FNN has the ability to model this complex behavior of the brain. We describe the data and model in more details in the next sections.

2 Materials and method

2.1 Brief introduction to under-study chaotic systems

On the contrary to what is expected, the outcomes of recent scrutiny indicate that not only is the chaotic behavior observed as a dynamic of complex systems, but it could also be a possible dynamic of uncomplicated systems [43]; i.e., the chaotic dynamical systems are widespread. The exemplars of such systems could be spotted in almost every scientific field, including the biological science. In this paper, at first, we model some popular chaotic systems observed as the behavioral patterns of bio-systems. We classified under-study systems into two main categories:

(1) Discrete chaotic systems (chaotic maps), including Logistic map, Henon map, and Rulkov map. These maps have been employed as models in researches related to the electrocardiogram (ECG), neurons, and tumor growth [23,34–36].
(2) Continuous chaotic systems (chaotic flows), including Lorenz system and Rössler system. These flows have been applied as models in investigations associated with the ECG, electroencephalogram (EEG), kidney, neurons, tumor growth, and chronotherapy [21,37–42].

We describe these systems in brief in this section.

1. Logistic Map: The logistic difference equation described as

$$x_{n+1} = ax_n(1 - x_n) \tag{1}$$

is perhaps the most famous chaotic system [44–46]. Despite having just one control parameter, this map could exhibit a variety of behavior, including the chaotic one. Its chaotic template has been spotted in the behavior of many biological systems, e.g., the heart and brain, together with their responses to various internal and external stimuli [35,47].

2. Henon Map: The Henon map written as

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + y_n \\ y_{n+1} = bx_n \end{cases} \tag{2}$$

is a discrete chaotic system introduced by Henon in 1976 [46]. This system has two control parameters, a and b. In spite of its simplicity, the system depicts many features of dynamical behavior of biological systems such as the heart [35].

3. Rulkov Map: The Rulkov map represented by the following difference equations

$$\begin{cases} x_{n+1} = \frac{\alpha}{1+x_n^2} + y_n \\ y_{n+1} = y_n - \mu(x_n - \sigma) \end{cases} \tag{3}$$

is a two-dimensional (2D) discrete chaotic system with three control parameters, α, μ, and σ. This system, which was proposed by Rulkov in 2001, has been widely applied to the computational neuroscience since it could mimic the rich nonlinear dynamical behavior of neurons [23,36].

4. Lorenz system: Lorenz chaotic system which consists of three differential equations as follows

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = x(r - z) - y \\ \dot{z} = -bz + xy \end{cases} \tag{4}$$

originated from the work of Edward Lorenz in 1963. This system has three control parameters, a, r, and b, and two nonlinear terms, xz and xy. The dynamic of the system, i.e. period-doubling cascades conducing to the chaos, is observed in the brain, tumor growth, and metastasis [37,40].

5. Rössler system: Rössler thought about chaotic equations simpler than Lorenz system. He finally presented a 3D chaotic system with just one nonlinear term in 1979 [48], as follows

$$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = b + z(x - c) \end{cases} \tag{5}$$

This system demonstrates numerous characteristics of dynamical behavior of biological systems such as the brain and kidney [39,41,42]. The foresaid systems are introduced in Table 1 in brief.

2.2 Artificial neural network model

The outcomes of researches performed by the use of ANNs indicate that ANNs have the capability to solve complex nonlinear problems. Among them, multilayer FNNs are being utilized further to model nonlinear systems [28,32]. With the aim of modeling the dynamics of described chaotic systems, we manipulate a multilayer feed-forward neural network retaining four hidden layers with 7, 4, 8, and 5 neurons in the layers, as shown in Fig. 1. In common with other ANN-based researches [49,50], the hidden layers and the number of their neurons are selected via the trial-and-error method. Hyperbolic tangent sigmoid (tansig) function is chosen as the activation function of the hidden layers to help the network learn the nonlinear dynamics of the chaotic systems, i.e., the relationships between the inputs and outputs. The activation function of the output layer is assigned linear (purelin function). The

Table 1 Under-study chaotic systems

System (dimension)	Discrete/continuous system	Equation	Parameters
Logistic Map (1D)	Discrete	$x_{n+1} = ax_n(1 - x_n)$	a
Henon Map (2D)	Discrete	$\begin{cases} x_{n+1} = 1 - ax_n^2 + y_n \\ y_{n+1} = bx_n \end{cases}$	a and b
Rulkov Map (2D)	Discrete	$\begin{cases} x_{n+1} = \frac{\alpha}{1+x_n^2} + y_n \\ y_{n+1} = y_n - \mu(x_n - \sigma) \end{cases}$	α and μ and σ
Lorenz System (3D)	Continuous	$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = x(r - z) - y \\ \dot{z} = -bz + xy \end{cases}$	a and r and b
Rössler System (3D)	Continuous	$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = b + z(x - c) \end{cases}$	a and b and c

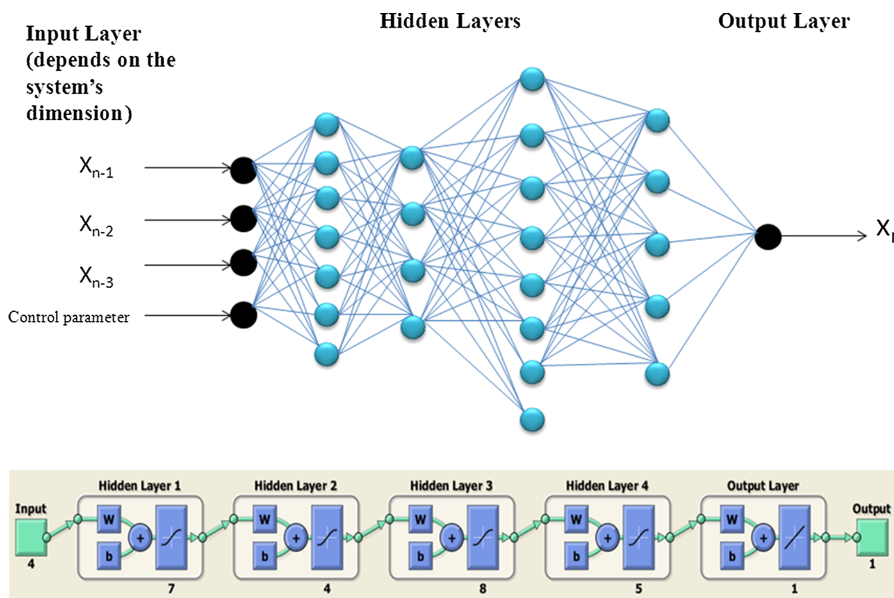

Fig. 1 Schematic of the multilayer feed-forward neural network proposed to model the chaotic systems

time-delay samples are chosen to be the neural network's inputs, as illustrated in Fig. 1. The number of FNN's inputs equates to the dimension of the select system plus one since we also consider one of the system's control parameters as the input. The current state of the system, x_n, is selected to be the neural network's output. Data points are generated from the equations of chaotic systems. We train the neural network by the use of the standard back-propagation (BP) learning algorithm. All the simulations, whose results are presented in the paper, are performed using the MATLAB software (version 7.12).

In view of training the specified neural network, we utilize the MATLAB command "newff," which is recommended for training multilayer FNNs in particular [51,52]. For every continuous system, we extract the discrete time series (return map) by recording the successive local maxima of one of the time variables, e.g., x(t). Afterwards, we plot the bifurcation diagram of the continuous system using the extracted points. We train the neural network with the bifurcation diagram of every chaotic system. We represent the achieved results in the next section.

2.3 Simulation results

We evaluate the proposed model by the use of several discrete chaotic systems (Logistic, Henon, and Rulkov

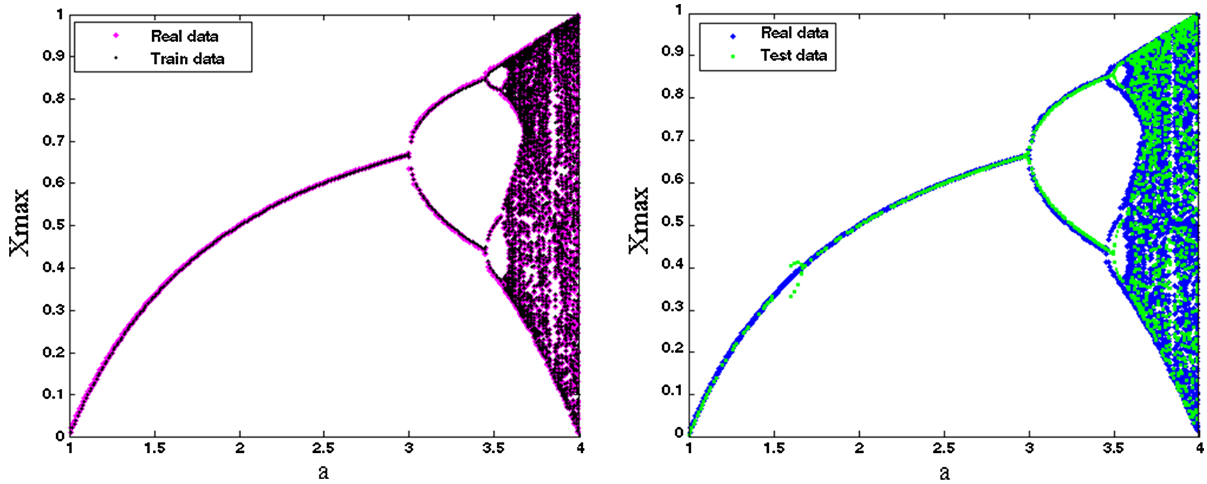

Fig. 2 Bifurcation diagrams of FNN model training and testing with logistic map (control parameter: a varying from 1 to 4)

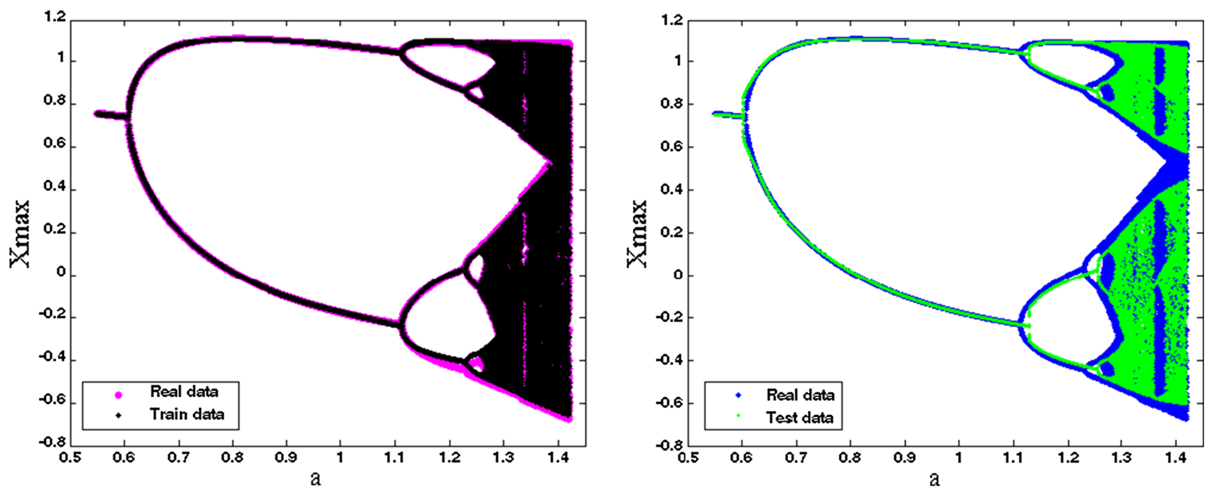

Fig. 3 Bifurcation diagrams of FNN model training and testing with Henon map (control parameter: a varying from 0.5 to 1.4, $b = 0.3$)

maps) and continuous chaotic systems (Lorenz and Rössler systems) previously described in essence. We apply the designated FNN for all the simulations performed.

Since the logistic map is a 1D map with one parameter, a, we consider one time-delay sample (x_{n-1}) and the parameter, a, as the inputs and x_n as the output of the FNN. Figure 2 demonstrates the results of FNN training and testing with the data extracted from the logistic map.

We then assess the ability of our model by using 2D Henon map owning two control parameters, a and b. We consider two time-delay samples (x_{n-1} and x_{n-2}) and one of the control parameters as the inputs and x_n as the output of the network. Figures 3 and 4 illustrate the outcomes of FNN training and testing with the data extracted from the Henon map.

Rulkov map is a 2D map with three control parameters, α, μ, and σ. Thus, we regard two time-delay samples (x_{n-1} and x_{n-2}) and one of the control parameters as the inputs and x_n as the output of the FNN. Figure 5 demonstrates the results of FNN training and testing with the data extracted from this map when the control parameter μ is just considered as one of the inputs.

Although the achieved results verify that the proposed FNN-based model could predict the dynamics of

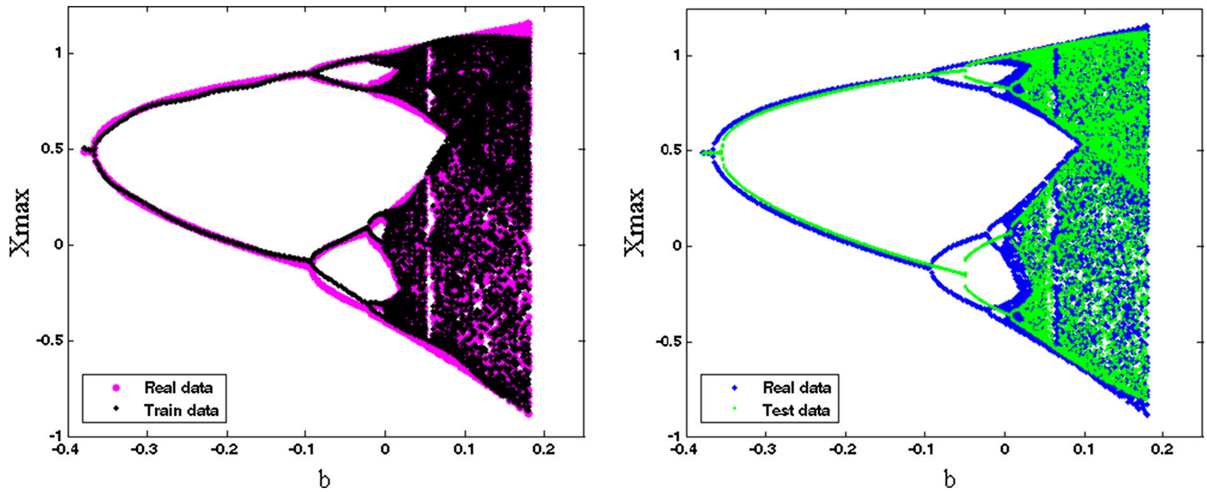

Fig. 4 Bifurcation diagrams of FNN model training and testing with Henon map (control parameter: b varying from -0.4 to 0.2, $a = 1.4$)

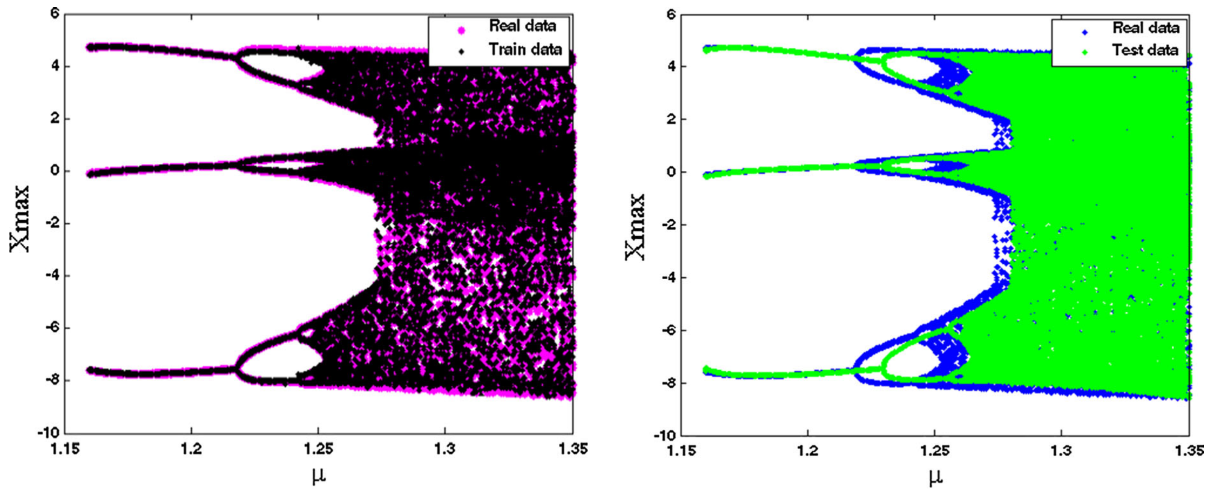

Fig. 5 Bifurcation diagrams of FNN model training and testing with Rulkov map (control parameter: μ varying from 1.15 to 1.35, $\alpha = 6$, $\sigma = -1$)

some nonlinear dynamical systems, we take the analysis further and assess our model by utilizing certain continuous chaotic dynamical systems.

The Lorenz system is a 3D system with three control parameters, a, r, and b, and two nonlinear terms. We therefore regard three time-delay samples (x_{n-1}, x_{n-2}, and x_{n-3}) and one of the control parameters as the inputs and x_n as the output of the FNN. Figure 6 depicts the results of FNN training and testing with the data extracted from this system when the control parameter a is just considered as one of the inputs.

The Rössler system is a 3D system with three control parameters, a, b, and c. We therefore regard three time-delay samples (x_{n-1}, x_{n-2}, and x_{n-3}) and one of the control parameters as the inputs and x_n as the output of the FNN. Figures 7 and 8 demonstrate the results of FNN training and testing with the data extracted from this system when the control parameters b and c are considered as the inputs.

The achieved results exhibited in this section validate the capability of the proposed FNN-based model to mimic the dynamics of various chaotic dynamical

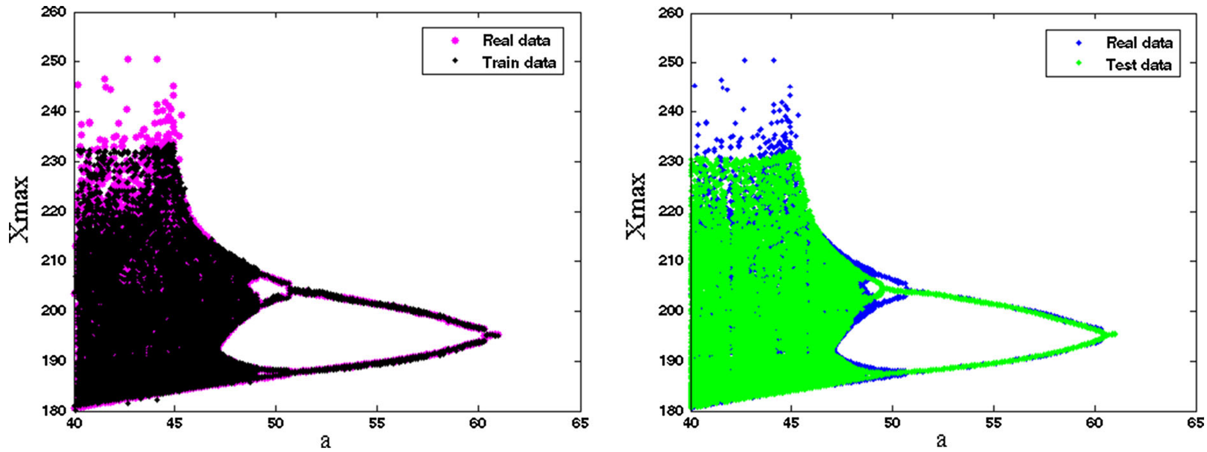

Fig. 6 Bifurcation diagrams of FNN model training and testing with Lorenz system (control parameter: a varying from 40 to 60, $b = 8/3$, $c = 28$)

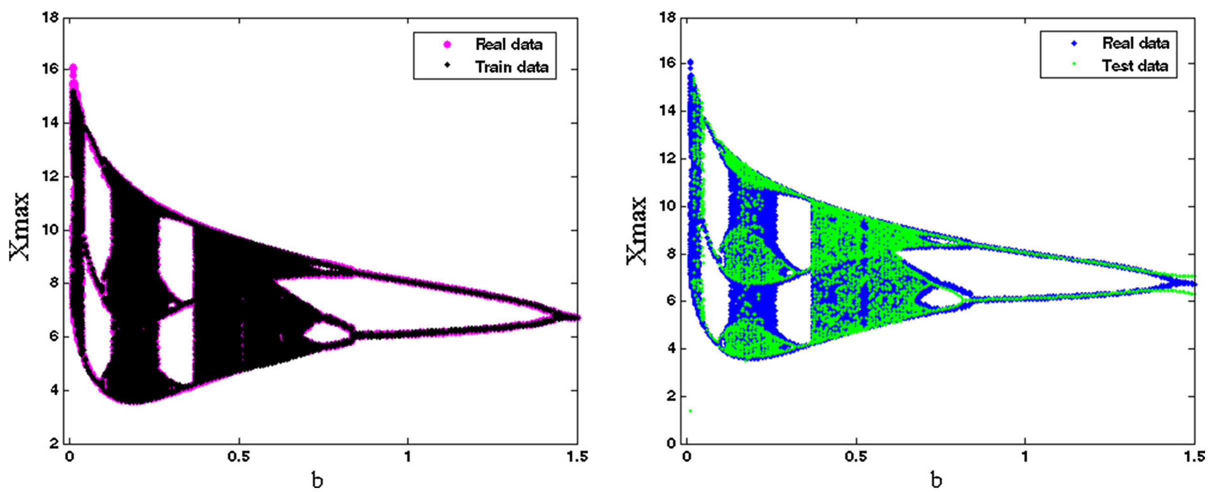

Fig. 7 Bifurcation diagrams of FNN model training and testing with Rössler system (control parameter: b varying from 0 to 1.5, $a = 0.2$, $c = 5.7$)

systems. The outcomes of the assessment of the model by comparing the variations of several samples of time series of the specified systems and the ones generated by the model validate our model as well.

2.4 Assessment of model robustness

In fact, the noise alters the majority of the data recorded from varying dynamical systems such as the biological systems. At the end of this section, we assess the robustness of our model against noise. For this purpose, we add white Gaussian noises with different signal-to-noise ratios (SNRs) to the bifurcation diagrams of the specified systems. Figures 9, 10, 11, 12, and 13 represent the obtained results for Henon map. As an example, it is demonstrated in Fig. 13 that even in the presence of 25 dB Gaussian noise, the proposed FNN model could follow the attractor of Henon map.

3 Experiments

3.1 Electroretinogram data

Since the ERG data are not as conventional as the electroencephalogram (EEG) and electrocardiogram (ECG) data, in the first instance, we briefly explain

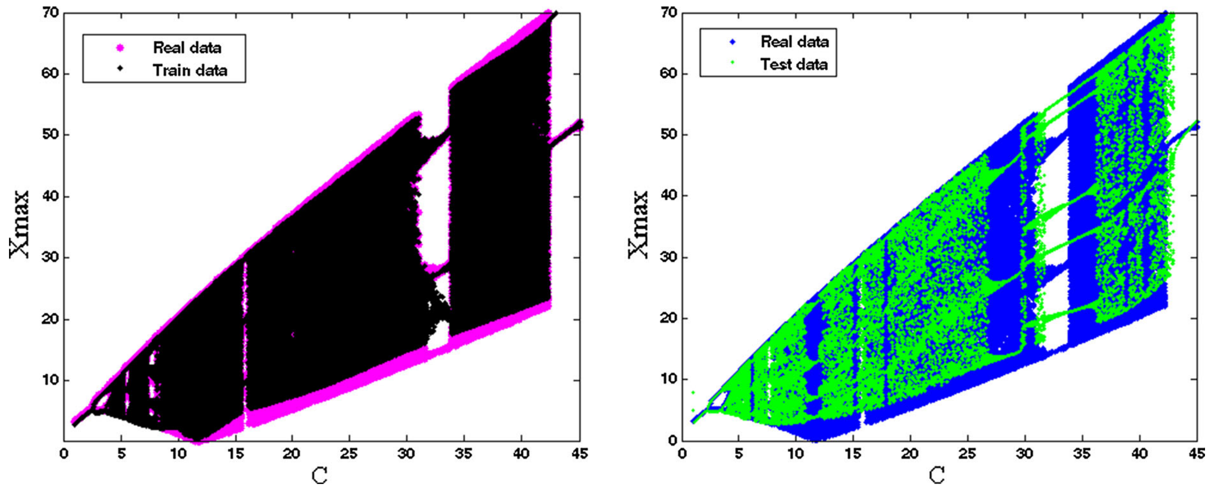

Fig. 8 Bifurcation diagrams of FNN model training and testing with Rössler system (control parameter: c varying from 0 to 45, $a = 0.2$, $b = 0.2$)

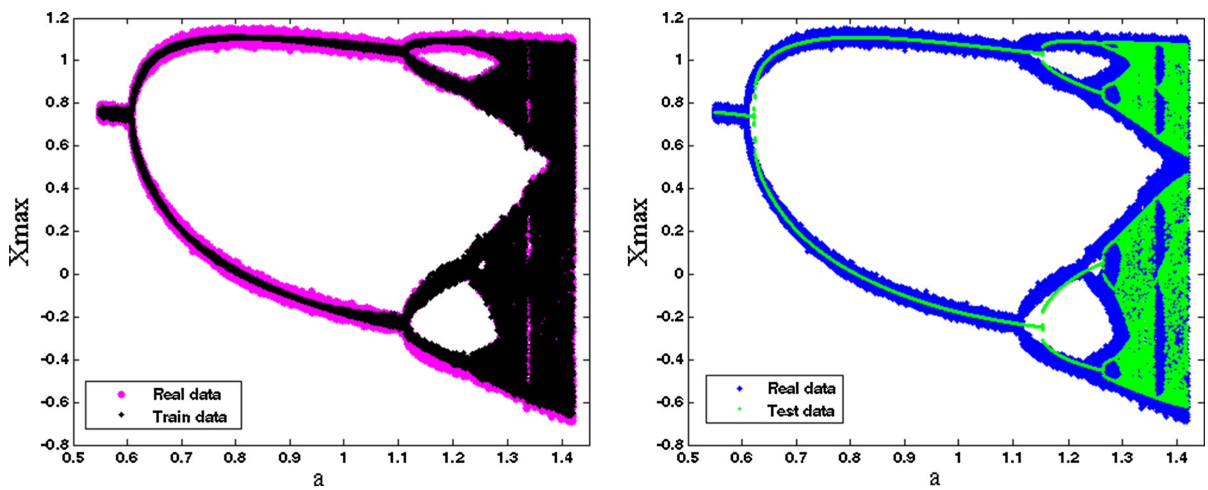

Fig. 9 Effect of white Gaussian noise (40 dB) on the bifurcation diagrams of FNN model training and testing with Henon map (control parameter a, $b = 0.3$)

how it generates in the retina of a vertebrate. The complex modulation of the visual information is accomplished through the successive procedures performed in the retina such as the amplification, feedback, and network adaptation processes carried out in the inner retina, together with the anatomical convergence and divergence of the neurons. The neurons, which sequentially receive the visual information, are classified into three categories:

(1) The photoreceptors, also known as the first-order neurons: the amplification of the electrical signals produced after the activation of photoreceptors by light makes their propagation to the following neurons possible.
(2) The ON and OFF bipolar cells and laterally interacting horizontal cells, also known as the second-order neurons: After the modulation of visual information by these neurons, it is propagated to the succeeding neurons.
(3) The ganglion cells and signal-modulating amacrine cells, also called the third-order neurons: The spike-coded visual information is transmitted to the lateral geniculate and thereafter to the visual cortex via the optic nerve, i.e., the axons of the ganglion cells [53,54].

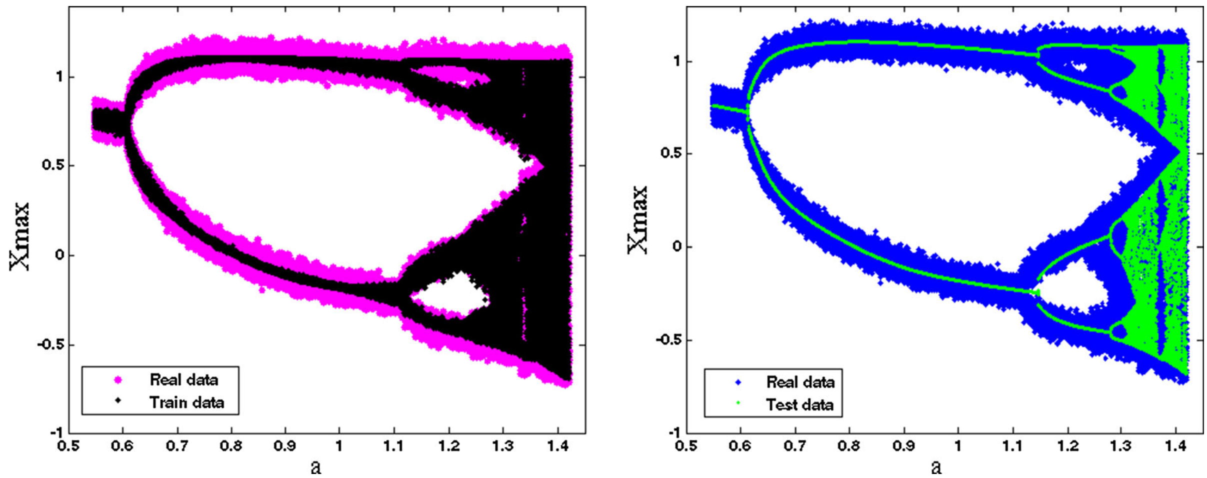

Fig. 10 Effect of white Gaussian noise (30 dB) on the bifurcation diagrams of FNN model training and testing with Henon map (control parameter a, $b = 0.3$)

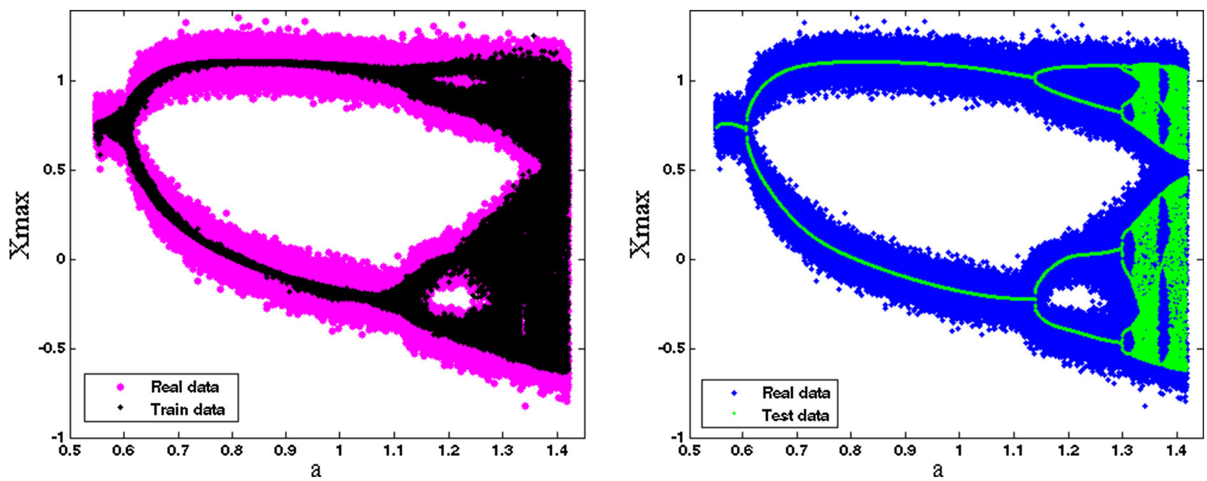

Fig. 11 Effect of white Gaussian noise (25 dB) on the bifurcation diagrams of FNN model training and testing with Henon map (control parameter a, $b = 0.3$)

It is known that significant alterations are occurring in the ionic constitution of the extracellular fluid while single neurons depolarize or hyperpolarize. As a consequence of the specified variations within and along the retinal neurons and along the Müller cells, some electroretinographic potentials are produced [55]. The specific electroretinographic potential, the ERG, is yielded when certain stimulators such as the light flashes temporarily transfigure the electrical current flow, and as a result, the fast and slow components are observed in the ERG. The resultant field potential could be recorded from the cornea since it is transferred to the cornea and even to the eyelids (in the attenuated form) through the passive volume conductors, e.g., the vitreous, lens, and anterior chamber. Although the field potentials generated in this particular condition have small magnitudes, less than 700 μV, as observed in Fig. 14, they have the special characteristic of being recreated with the same time and magnitude attributes in the similar circumstances. The registered ERG could demonstrate some features of different constituents of the eye such as the reaction to various flash intensities, the retinal ability to adapt, the selective rod and cone contributions, and On and Off methodologies [53,54].

Similar to the EEG and ECG, the diagnostic applications of the ERG, when it is recorded in its standard

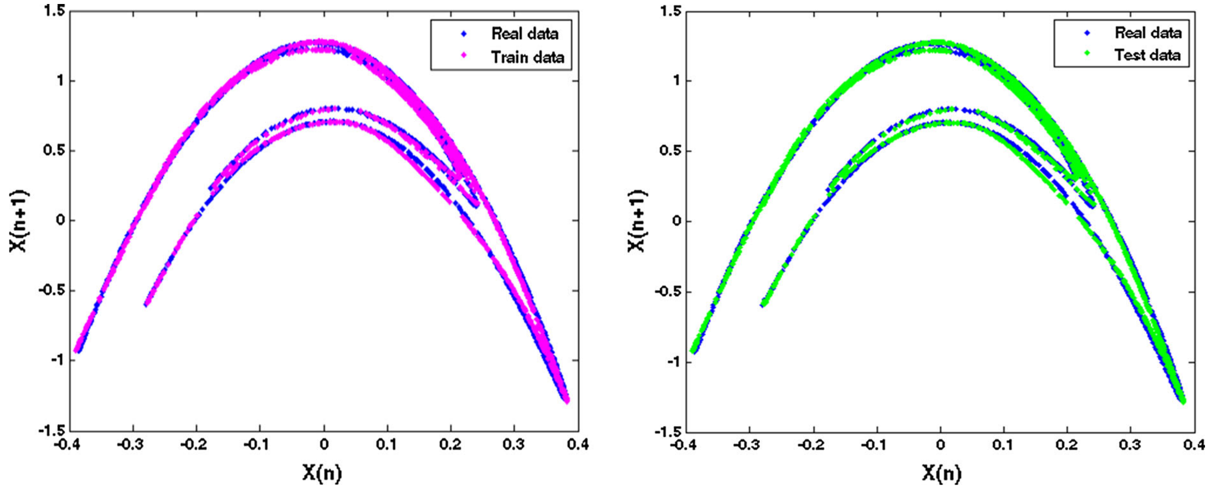

Fig. 12 Attractors of Henon map created while training and testing FNN model in the absence of white Gaussian noise

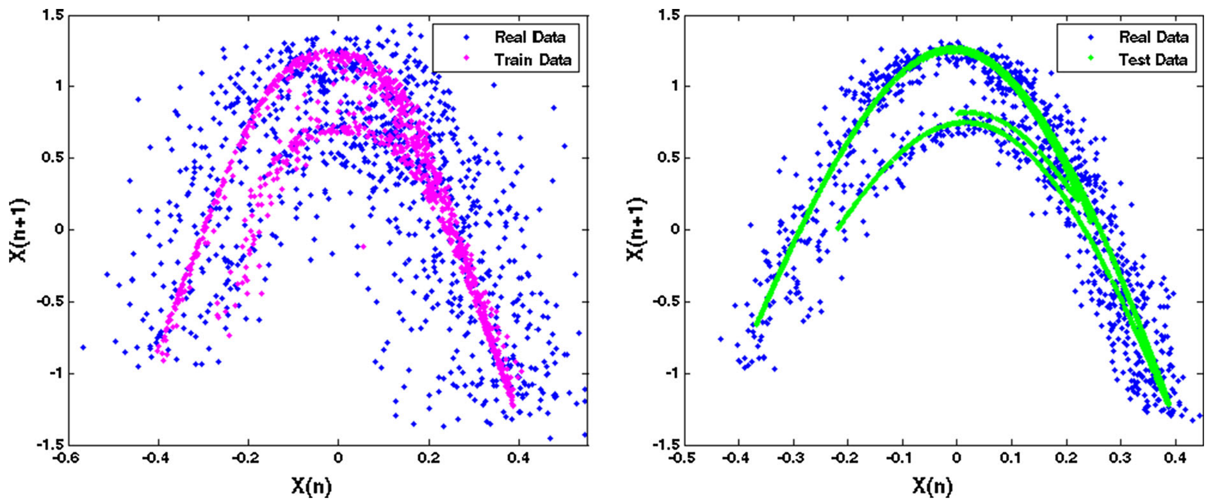

Fig. 13 Attractors of Henon map created while training and testing FNN model in the presence of 25 dB white Gaussian noise

protocol, persuade the clinicians to register and analyze this field potential in spite of its intrinsic complexity. Some congenital or acquired retinal diseases alter the ERG configuration as it is represented in Fig. 15. A "Normal" dark-adapted ERG recorded when the eye is being stimulated by an intense flash (as a result, a mixed rod/cone signal is elicited) is illustrated in this figure [53].

The brain is an extremely complex nonlinear system, and therefore, its response to the flicker light, recorded as the flicker ERG, has rich dynamics. One of the significant characteristics of the flicker ERG that has received little attention in the relevant researches is its period-doubling cascades to chaos [17,19–21]. The period-doubling bifurcation, which occurs in some nonlinear systems such as the biological systems, refers to one of the conditions in which the dynamic of the system switches to a new state [19]. In this bifurcation, the time required for the motion of the system to repeat itself doubles again and again while a parameter, which is describing the system, is changing until the state of the system transforms from the regular periodic motion into the chaos. This pattern has been observed in the human ERG as well as in the ERG of animals, e.g., the salamander, rat, and rabbit [19,20,56].

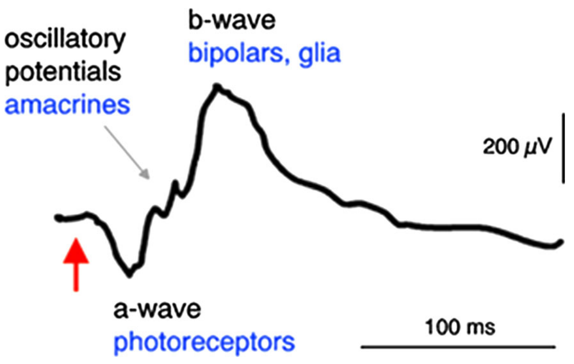

Fig. 14 Exemplar of a light-adapted ERG waveform and its principal components [53,54] (by permission of the author G. Niemeyer)

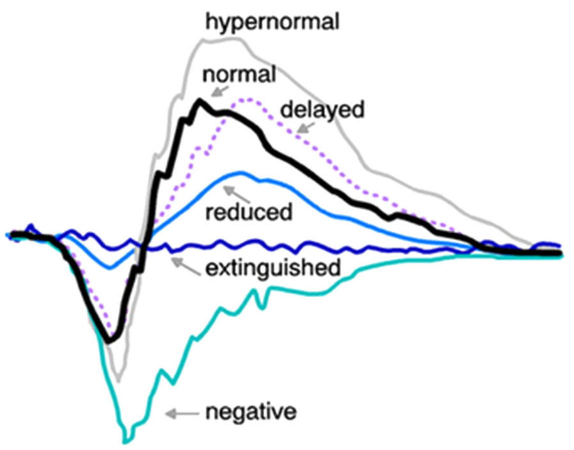

Fig. 15 Sketch of the changes in the ERG waveform as a result of some congenital or acquired retinal diseases [53,54] (by permission of the author G. Niemeyer)

For recording the ERGs employed in our research (provided by Professor Markus Meister), the retina of a salamander was stimulated by the periodic square-wave flashes. The ERGs were recorded under two different circumstances. In one of the recordings, the flash frequencies, f, were changed in the range of 0–30 Hz, while its contrast, c, was constant. In another recording, the flash frequency, f, was kept constant at 16 Hz, while its contrast, c, was altered. The recorded ERG signals were filtered at 1–1000 Hz [17]. As shown in Fig. 16, during stimulating the eye by the rapid flicker (with some specific contrast and frequency attributes), the brain dynamic is varying significantly from a stable periodic state to a chaotic one. The variations of both parameters lead to the change of the brain dynamic.

To model this highly nonlinear reaction, we apply the recommended multilayer FNN since it appears that it has the capability to model the dynamics of complex nonlinear systems. We describe the employed FNN-based model in detail in the next section.

3.2 Neural network structure

Various factors, for example the nature, type and size of the data, the number of variables, and validation method, have an influence on the optimum numbers of hidden layers and their neurons. In order to model the brain response to the flicker light, we choose the numbers of hidden layers and their neurons via the trial-and-error method, which is the most common strategy while ANNs are being utilized. Based on the results of trial-and-error method, we use the same multilayer FNN (with four hidden layers and 7, 4, 8, and 5 neurons in its layers) shown in Fig. 1. The same transfer functions, hyperbolic tangent sigmoid (tansig) and linear (purelin) functions, are applied to the hidden layers (to help the network learn the complex relationships between the input and output) and output, respectively.

The number of inputs of the neural network is selected on the basis of the estimated dimensions of the data. By the use of the false nearest neighbor method, we estimate the dimensions of the ERG data 3 (since the amounts of false nearest neighbor decrease to below 0.1 for the dimension of 3 and the dimension values greater than 3). We therefore choose three time-delay samples (x_{n-1}, x_{n-2}, and x_{n-3}) and one of the parameters, contrast or flash frequency, as the inputs of the network and x_n (the current state) as its output. Considering the two control parameters, contrast and flash frequency, the dimensions of the described ERGs are calculated 3, as it is clearly demonstrated in Figs. 17 and 18.

3.3 Simulation results

Based on our explanation in the previous section, with the aim of modeling the brain response to the flicker light, the identical multilayer FNN is selected to be our model. For training this neural network, first, the maximum points are extracted from the time series (every recorded ERG). Afterwards, the bifurcation diagram of the extracted points (from the ERG) is plotted. The optimal method for appraising a chaotic dynamical system is through its bifurcation diagram, as recommended

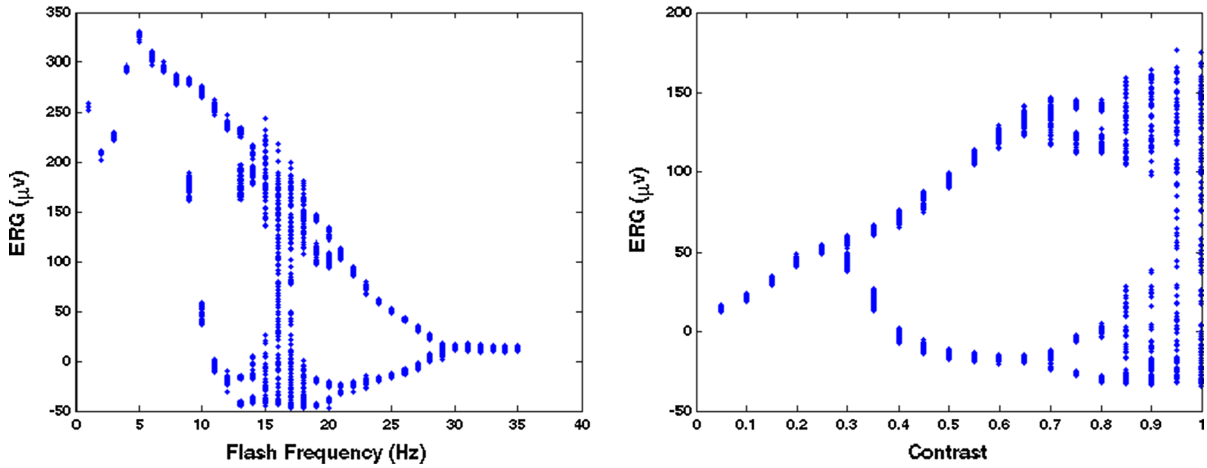

Fig. 16 Bifurcation diagrams of the salamander ERG, when the flash frequency is changing and the contrast is constant ($c = 1.0$) (*left*) and while the contrast is varying and the flash frequency is constant ($f = 16\,\text{Hz}$) (*right*). For depicting the bifurcation diagrams, the maxima of time series are extracted

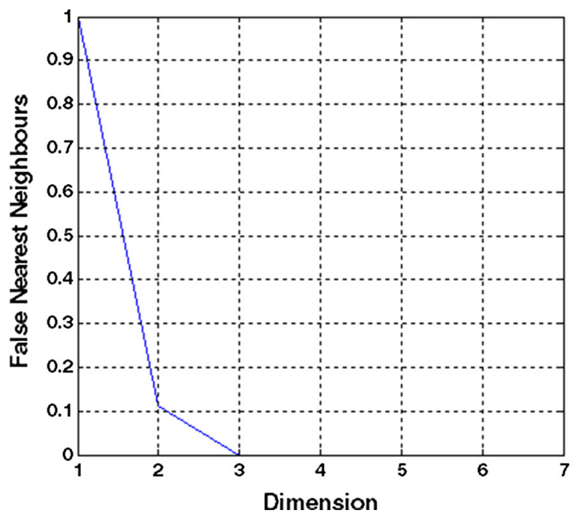

Fig. 17 By the use of the false nearest neighbor method, the dimension of the ERG data is estimated 3 when the flash frequency is the control parameter

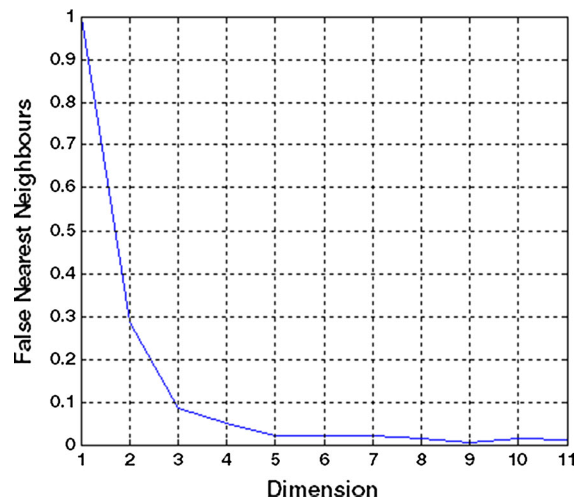

Fig. 18 By the use of the false nearest neighbor method, the dimension of the ERG data is estimated 3 when the contrast is the control parameter

by experts [57–59]. In order to train the neural network with the achieved bifurcation diagram, we apply "newff" command of the MATLAB software (version 7.12). We train the neural network by the use of the back-propagation algorithm as well. For training the neural network, we do not regard the parts of the ERG data correlated with the transient response of the brain.

After training the neural network, we evaluate the validity and accuracy of the suggested model. With the aim of testing the model, various initial points extracted from every bifurcation diagram (with desired resolutions) are defined (as the inputs) for the neural network. The accuracy of the estimated outputs confirms the ability of the model to predict the dynamics of under-study brain response. Figures 19 and 20 demonstrate the results of FNN model training and testing while the control parameters, contrast and flash frequency, are changing. Figures 21 and 22 demonstrate the samples of time series of the real ERG data and the ones generated by the model. The results of the comparison

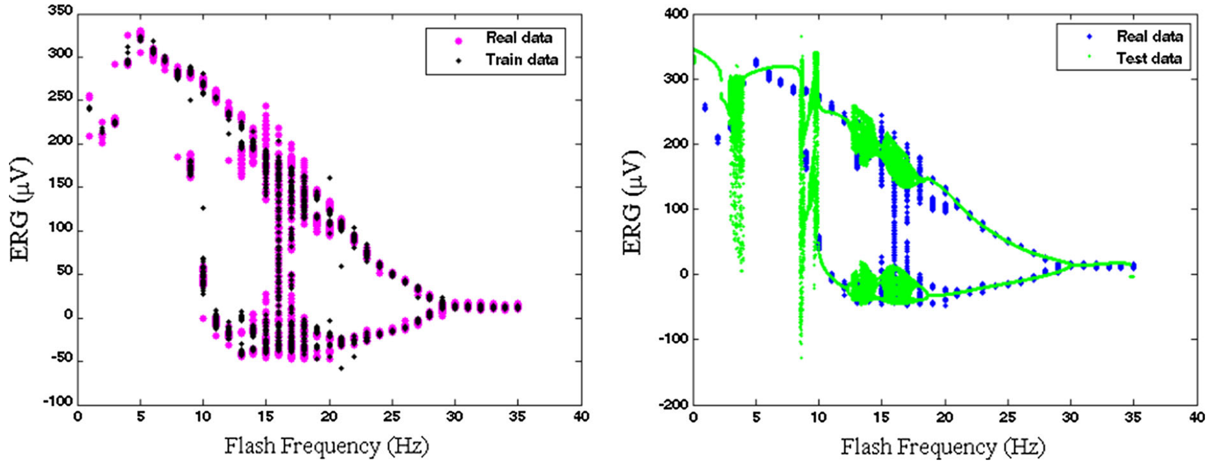

Fig. 19 Bifurcation diagrams of FNN model training and testing, when the flash frequency is considered to be the control parameter

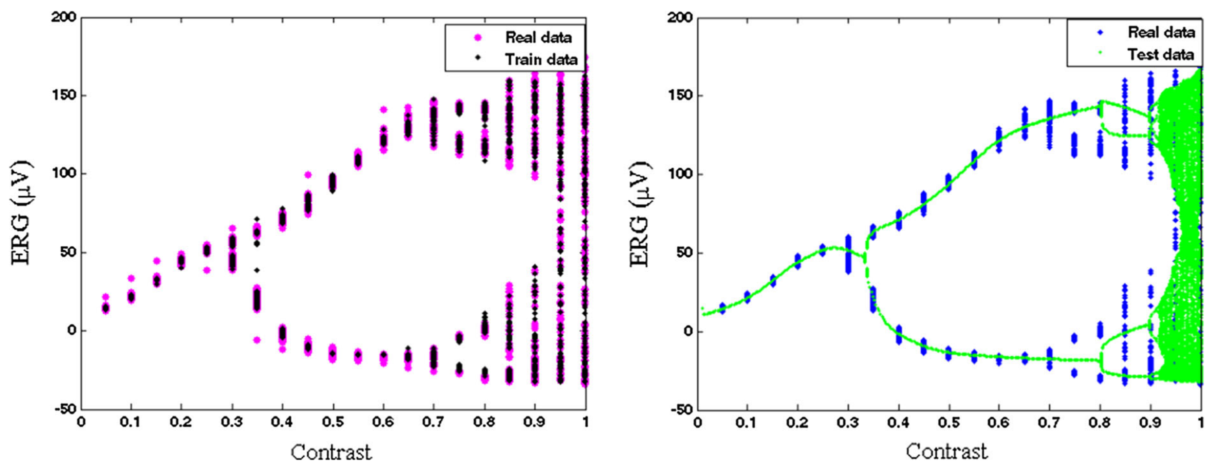

Fig. 20 Bifurcation diagrams of FNN model training and testing, when the contrast is considered to be the control parameter

between the bifurcation diagrams generated by the proposed model and the bifurcation diagrams of the ERG data validate that the select neural network could follow the dynamics of the brain response to the flicker light, even while it is chaotic.

4 Concluding remarks

While the scientists around the world have been advancing on the accurate cognition, analysis, modeling, prediction, and control of different systems, it becomes more perceptible that the nonlinear dynamics, complex systems, and chaos theory have been closely associated together; hence, they all have to be considered cooperatively in the contemporary researches. This union results in the expansion of their applications in the engineering, economic, and sciences, specifically the biological science. Recent fast significant developments in the biological and medical sciences have been the result of substantial efforts devoted to precisely modeling the behavior of biological systems and their responses to various stimuli. The principal complications with delving into the biological systems are their extreme nonlinearity and complexity. The results of recent numerous investigations in the field of biological science have indicated that complex patterns such as the chaotic ones are repeatedly perceived in the behavior of bio-systems and their responses to different internal and external stimuli when carefully assessed;

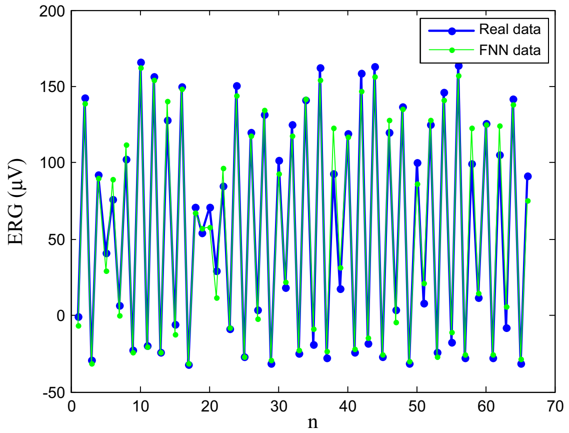

Fig. 21 Comparison of the samples of time series of the real ERG data and the one generated by the proposed FNN model ($c = 0.94$)

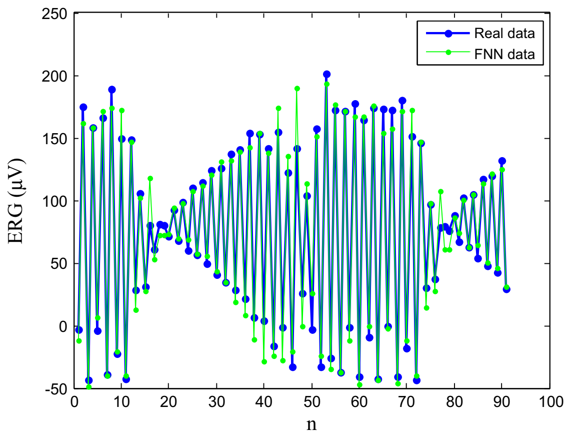

Fig. 22 Comparison of the samples of time series of the real ERG data and the one generated by the proposed FNN model ($f = 16\,\text{Hz}$)

i.e., in consequence of complicated interactions within various components of these systems as well as with their environments, these systems behave in complex nonlinear ways. These features make the modeling of biological systems convoluted.

The results of our investigation, represented in this paper, confirm that the specified FNN possesses the potentiality to model the chaotic response of the brain to the flicker light. The chaotic pattern could be observed when the responses of the brain to some stimuli such as the flicker light are recorded. With the aim of modeling this response of the brain, we employ some ERG data recorded from the salamander's retina while being stimulated by periodic square-wave flashes. The ERGs were recorded in two diverse circumstances: constant contrast, c, and variable flash frequencies, f, and vice versa. In view of evaluating the accuracy and validity of our model, at first, we have modeled some famous chaotic systems and then assessed the robustness of our model against noise. One of our points in selecting the model was to keep our model simple, without compromising the realistic behavior. The distinctive characteristic of our modeling method, which makes it dominant within the modeling techniques, is training the select neural network with the return map extracted from the under-study time series.

Based on the achieved outcomes of our scrutiny, in summary, we could conclude that:

(1) The neural network models have the potential to reflect some dynamics of complicated nonlinear systems, for instance the biological systems, while their bifurcation diagrams are utilized.
(2) The FNN-based model is robust against noise, as we have assessed this feature of our model in this paper.
(3) For the assessment of the modeling results, specifically their proximity to the behavior of the select system, the bifurcation and attractor diagrams could be employed.

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