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Fault-tolerant sampled-data control of flexible spacecraft with probabilistic time delays

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Abstract In this paper, fault-tolerant sampled-data control for flexible spacecraft in the presence of external disturbances, partial actuator failures and probabilistic time delays is investigated. In particular, unlike the common assumptions on continuous-time information on control input, a more realistic sampled-data communication strategy is proposed with probabilistic occurrence of time-varying delays which is modeled by introducing Bernoulli distributed sequences. The main purpose of this paper is to derive fault-tolerant sampled-data control law which makes the closed-loop system robustly asymptotically stable with a prescribed upper bound of the cost function about its equilibrium point for all possible actuator failures. More precisely, by constructing an appropriate Lyapunov–Krasovskii functional involving the lower and upper bound of the probabilistic time delay, a new set of sufficient conditions are derived in terms of linear matrix inequalities for achieving the required result. Numerical simulations are presented by taking the real parameters to the considered aircraft model, which is not only highlighting the ensured closed-loop performance by the proposed control law, but also illustrates its superior fault tolerance, fast convergence and robustness in the presence of external disturbances and actuator faults when compared with the conventional controller. The simulation result reveals the effectiveness and potential of the proposed new design techniques.

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1 Introduction

Flexible spacecraft plays an important role in communications, navigation, remote sensing and so on [1]. Moreover, the design of control systems for flexible spacecraft is challenging issue since its dynamics include a large number of weakly damped elastic modes coupled with the rigid modes, and hence, the model parameters and disturbance inputs act on the spacecraft are not precisely known [6,9]. In particular, when some actuator failures occur in spacecraft systems, the performance of the control system can be degraded and even the closed-loop system becomes unstable. Therefore, to enhance system reliability, it is significant to design fault-tolerant control so that the performance of the system becomes well even in the presence of some actuator failures. That is, a fault-tolerant control possesses the ability to accommodate for system failures automatically and to maintain overall system performance in the presence of component failures [19,21]. Meanwhile, the spacecraft commonly operates in the presence of various disturbances which include aerodynamic torque, radiation torque, gravitational torque and other environmental torques. Further, the design of control scheme in H_∞ setting has good advantages, and it is well known that the H_∞ performance is closely related to the capability of disturbance rejection in dynamical systems [14,16]. Recently, with the rapid development of LMI technique and Lyapunov approach, delay-dependent sufficient conditions for dynamical systems via H_∞ control law have been extensively studied in the literature [5].

In practice, the model parameters of the spacecraft cannot be exactly determined, since the spacecraft system is always subjected to uncertainties. Further, the dynamics of a spacecraft is usually time-varying and highly nonlinear in the presence of various uncertainties due to environmental changes. Moreover, the attitude dynamics of spacecraft are coupled and highly nonlinear which makes that the design of controller will be complicated for achieving the good performance. During the past decades, considerable efforts have been made to design robust control for flexible spacecraft systems for simultaneous attitude control and vibration suppression [10,15]. In recent decades, fault-tolerant control is becoming an increasingly important area of research activities, and many investigations on fault-tolerant control have been carried out for linear or nonlinear system [23]. Moreover, in order to enhance the system reliability and to discuss the robustness issue in a spacecraft attitude system against inertia uncertainties and bounded disturbances, an accurate fault-tolerant control is needed.

However, it should be mentioned that almost all of the existing results mainly rely on the assumption of continuous-time information exchanges on control input, which means that all information is transmitted continuously. More precisely, discrete-time communication is used with sampled-data information instead of continuous-time communication, which is totally different from existing research works and is more useful in realistic situations [2,4,8,11,12]. By adapting the key idea from sampled-data control systems [3,17,20], in this work, only the samples of the control input signals at discrete time instants will be employed for investing the stability of flexible spacecraft system. Furthermore, since the effects of time delays are inevitable, it is important and necessary to consider time delay into account when considering the dynamics of systems [24–26]. Moreover, in practical systems, the study of control scheme with time-varying delay is more important than that with constant delays. Actually, the time delay in many realistic control systems exists in a stochastic fashion [7,13,22]. In particular, a stochastic model is introduced in control input to describe the probabilistic effects of the time-varying delays, where the probability distribution for occurrences of the delays is known a priori [18]. However, to the best of author's knowledge, fault-tolerant sampled-data control scheme for flexible spacecraft with randomly distributed time-varying delay and actuator faults has not been reported in the present literature, which motivates our study.

The main objective of this paper is to design a state feedback fault-tolerant sampled-data controller such that for all possible actuator failures the closed-loop system is asymptotically stable with a prescribed upper bound of the cost function. By constructing a proper Lyapunov–Krasovskii functional and employing LMI technique, a new set of sufficient conditions are obtained to guarantee the asymptotic stability of the considered system and also to design the fault-tolerant sampled-data controller. The main contributions of this paper are organized as follows:

- In the proposed model, a more realistic sampled-data communication strategy is proposed with probabilistic occurrence of time-varying delays.
- A unified framework is established for the fault-tolerant controller design in which the disturbance-rejection attenuation, probabilistic time delays and sampled-data information are enforced.
- The sampling time-varying delay in the control input is considered with random instants, that is, the measurements are sampled at a fixed time but arrived to the processing unit with a random time-varying delays.
- It should be noted that the proposed results are more general because it can guarantee the required result for fixed faults and as well as fault value occurring in a range of interval, and also normal actuator operating case.

Finally, numerical example with simulation result is provided to illustrate the effectiveness and the advantage of our theoretical results.

Notations The notation is fairly standard throughout this paper. $\mathbb{R}^{n\times n}$ denotes the $n\times n$-dimensional Euclidean space; the superscripts "T "and "(-1) "stand for matrix transposition and matrix inverse, respectively; $P>0$ represents that P is real, symmetric and positive definite; I and 0 denote the identity and zero matrices with compatible dimensions, respectively; we use an asterisk $(*)$ to represent a term that is induced by symmetry matrix; and * represents a term that is induced by symmetry. For a vector $w(t)$, its norm is given by $\|w(t)\| = \int_0^\infty w^T(t)w(t)\mathrm{d}t$. Finally, if not explicitly stated, all matrices are assumed to be compatible for algebraic operations.

2 Problem formulation and preliminaries

In this work, we consider the single-axis model obtained from the nonlinear attitude dynamics of the flexible spacecraft [15]. In this paper, we consider the situation in which the actuator experiences failure during the entire attitude maneuvers, and then, the faulty dynamics of the spacecraft with one rigid body and one flexible appendage can be described by [23]:

$$\begin{aligned}&\mathcal{J}\ddot{\theta}(t) + F\ddot{\eta}(t) = u^F(t),\\&\ddot{\eta}(t) + C_m\dot{\eta}(t) + \Lambda\eta(t) + F^T\ddot{\theta}(t) = 0,\end{aligned}\tag{1}$$

where $\theta(t)$ denotes the attitude angle; $\mathcal{J}$ is the moment of inertia of the spacecraft; $\eta(t)$ represents the flexible modal coordinate; F is the rigid-elastic coupling matrix; and $u^F(t)$ is the control torque generated by the reaction wheels that are installed in the flexible spacecraft. Further, C_m and Λ denote damping and stiffness matrices and are defined as $C_m = \mathrm{diag}\{2\xi_1\varpi_1,\ldots,2\xi_n\varpi_n\}$ and $\Lambda = \mathrm{diag}\{\varpi_1{}^2,\ldots,\varpi_n{}^2\}$, where $\xi_i, (i=1,\ldots,n)$ is the damping ratio, $\varpi_i, (i=1,\ldots,n)$ is the modal frequency, and n is their dimensions. Moreover, the vibration energy is concentrated in low-frequency modes in a flexible structure, and its reduced order model can be obtained by modal truncation. So, by taking the first two bending modes into account, the system can be written as

$$\left(\mathcal{J}-FF^T\right)\ddot{\theta}(t)=F(C_m\dot{\eta}(t)) + \Lambda\eta(t) + u^F(t).\tag{2}$$

In order to formulate the fault-tolerant control problem, the fault model must be considered. Now, we consider the reliable control signals in the following form [23]

$$u^F(t) = Gu(t),$$

where G denotes the control effectiveness factor, ie., the actuator fault matrix and satisfies the following condition

$$\begin{aligned}&G = \mathrm{diag}\{g_1, g_2,\ldots,g_n\},\quad g_i\in[\underline{g}_i,\ \overline{g}_i],\\&\quad i=1,2,\ldots,n,\ \ 0\le\underline{g}_i\le g_i\le\overline{g}_i\le 1,\end{aligned}$$

where g_i is an unknown constant; $\underline{g}_i$ and $\overline{g}_i$ represent the known lower and upper bounds of g_i, respectively. Also, for simplicity, we define $\hat{G} = \mathrm{diag}\left\{\hat{g}_1,\hat{g}_2,\ldots\hat{g}_n\right\}$, $\check{G} = \mathrm{diag}\left\{\check{g}_1,\check{g}_2,\ldots\check{g}_n\right\}$, $L = \mathrm{diag}\{l_1,\ldots,l_n\}$, where

$$\begin{aligned}&\hat{g}_i = \frac{\overline{g}_i+\underline{g}_i}{2},\quad \check{g}_i = \frac{\overline{g}_i-\underline{g}_i}{\overline{g}_i+\underline{g}_i},\\&l_i = \frac{g_i-\hat{g}_i}{\hat{g}_i},\quad i=1,2,\ldots,n.\end{aligned}\tag{3}$$

Then, we have

$$\begin{aligned}&G = \hat{G}(I+L),\ |L|\le\check{G}\le I,\ |L|\\&\quad = \mathrm{diag}\{|l_1|,\ldots,|l_n|\}.\end{aligned}\tag{4}$$

It should be mentioned that when $g_i = 0$, then the ith actuator completely fails and that when $g_i = 1$, then the ith actuator is normal. Also, if $0 < g_i < 1$, then the ith actuator has partial failure, ie., the considered fault is a partial loss of control effectiveness. By denoting $x(t) = [\theta^T(t)\,\dot{\theta}^T(t)]^T$, then the system (2) with actuator faults (4) can be written in the following state-space form

$$\begin{aligned}&\dot{x}(t) = Ax(t) + BGu(t) + Bw(t),\\&y(t) = Cx(t),\end{aligned}\tag{5}$$

where $A = \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}$, $B = \begin{bmatrix}0\\ (\mathcal{J}-FF^T)^{-1}\end{bmatrix}$, $C = I$ and $w(t) = F(C_m\dot{\eta}(t) + \Lambda(t))$ is the disturbance from the flexible appendages and belongs to $l_2[0,\ \infty]$ and satisfies $||w(t)||\le\delta$.

It should be noted that time delays are often exist in flexible spacecraft due to the physical structure and energy consumption of the actuators. Further, it should be mentioned that the time delays in control input may be variable due to the complex disturbance or other conditions. Motivated by this fact, in this paper, we consider the sampled-data control input described by variable time delays in the following form

$$\begin{aligned}&u(t) = u_\mathrm{d}(t_k) = u_\mathrm{d}(t-(t-t_k)) = u_\mathrm{d}(t-\tau(t)),\\&\quad t_k\le t\le t_{k+1},\ \tau(t) = t-t_k,\end{aligned}\tag{6}$$

where u_d is a discrete-time control signal, the time-varying delay $0\le\tau(t) = t-t_k$ is piecewise linear with the derivative $\dot{\tau}(t) = 1$, and for $t \neq t_k$, t_k is the sampling instant.

In this work, the control input $u(t)$ is sampled before feedback into the system, which provides way to the sampled-data control problem. Also, we consider the sampled-data control input in the form

$$u(t) = Kx(t_k),$$

where K is the state feedback gain to be determined. The interval between any two sampling instants is assumed to be bounded by τ_M which means that for any $k \geq 0$, $t_{k+1} - t_k = \tau_k \leq \tau_M$ always holds, where τ_M is the maximum upper bound of the sampling interval τ_k. By defining $\tau(t) = t - t_k$, the sampling intervals can be written as $t_k = t - (t - t_k) = t - \tau(t)$ and the control input vector can be written as

$$u(t) = Kx(t_k) = Kx(t - \tau(t)). \tag{7}$$

Under the control input (7), the system (5) can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BGKx(t - \tau(t)) + Bw(t), \\ y(t) &= Cx(t). \end{aligned} \tag{8}$$

It should be mentioned that, the sampling time may contain some uncertainty due to environmental disturbances and other working conditions. These variations in sampling period may result some randomness to sampling period or sampling time delay or may be in both. In order to deal such situation, it is useful to define time delay in a stochastic form by using its probabilistic characteristics. One of the useful way is the introduction of binary stochastic variable in control systems, which was first introduced in [13] and then successfully applied in several papers (see [7,22] and references therein). Also, in a real process, the sampling period itself might be a stochastic variable due to unpredictable environmental changes. To consider such a reality, in this paper, we assume that the sampling periods of each control signal randomly switch between some values, ie., the sampling periods may vary in probabilistic way.

In order to obtain the required result, we assume that the following condition holds:

Assumption 1 The time-varying delay $\tau(t)$ satisfies the condition $0 \leq \tau(t) \leq \tau_M$, where τ_M is a positive scalar denoting the maximum delay. In practice, there exists a constant τ_0 satisfying $0 \leq \tau_0 \leq \tau_M$ such that $\tau(t)$ takes values in $[0, \tau_0]$ or $(\tau_0, \tau_M]$ with certain probability. Therefore, $\tau(t)$ becomes a random variable which takes value in the intervals $[0, \tau_0]$ or $(\tau_0, \tau_M]$, and the probability distribution of $\tau(t)$ is assumed to be Prob $\{\tau(t) \in [0, \tau_0]\} = \alpha_0$ and Prob $\{\tau(t) \in (\tau_0, \tau_M]\} = 1 - \alpha_0$, where $0 \leq \alpha_0 \leq 1$ is a constant.

In order to describe the probability distribution of the time-varying delay, by following the idea proposed in [22], we define the set of functions

$$\begin{aligned} \mathbb{C}_1 &= \{t : \tau(t) \in [0, \tau_0]\} \quad \text{and} \\ \mathbb{C}_2 &= \{t : \tau(t) \in (\tau_0, \tau_M]\}. \end{aligned} \tag{9}$$

Further, we define the two mapping functions as follows:

$$\tau_1(t) = \begin{cases} \tau(t) & \text{for } t \in \mathbb{C}_1 \\ 0 & \text{for else} \end{cases} \quad \tau_2(t) = \begin{cases} \tau(t) & \text{for } t \in \mathbb{C}_2 \\ \tau_0 & \text{for else,} \end{cases}$$

where $\dot{\tau}_1(t) = 1$ and $\dot{\tau}_2(t) = 1$. It follows from (9) that $\mathbb{C}_1 \cup \mathbb{C}_2 = \mathcal{Z}_{\leq 0}$ and $\mathbb{C}_1 \cap \mathbb{C}_2 = \Phi$, where Φ is the empty set. It can be seen that $t \in \mathbb{C}_1$ implies the event $\tau(t) \in [0, \tau_0]$ and $t \in \mathbb{C}_2$ implies the event $\tau(t) \in (\tau_0, \tau_M]$ occurs. Next, define a stochastic variable $\alpha(t)$ as

$$\alpha(t) = \begin{cases} 1 & \text{for } t \in \mathbb{C}_1, \\ 0 & \text{for } t \in \mathbb{C}_2. \end{cases}$$

Moreover, $\alpha(t)$ is a Bernoulli distributed white sequences with Prob $\{\alpha(t) = 1\} = \mathbb{E}[\alpha(t)] = \alpha_0$ and Prob $\{\alpha(t) = 0\} = 1 - \mathbb{E}[\alpha(t)] = 1 - \alpha_0$. Furthermore, we can show that $\mathbb{E}[\alpha(t) - \alpha_0] = 0$ and $\mathbb{E}[(\alpha(t) - \alpha_0)^2] = \alpha_0(1 - \alpha_0)$. By incorporating random delay into the closed-loop system in (8), it can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \alpha(t)BGKx(t - \tau_1(t)) \\ &\quad + (1 - \alpha(t))BGKx(t - \tau_2(t)) + Bw(t), \\ y(t) &= Cx(t) \end{aligned}$$

which is equivalent to

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \alpha_0 BGKx(t - \tau_1(t)) \\ &\quad + (1-\alpha_0)BGKx(t-\tau_2(t)) + (\alpha(t)-\alpha_0)BGK \\ &\quad \left\{x(t - \tau_1(t)) - x(t - \tau_2(t))\right\} + Bw(t), \\ y(t) &= Cx(t). \end{aligned} \tag{10}$$

Our main purpose of this paper is to design the fault-tolerant sampled-data controller gain matrix K such that the system in (10) when $w(t) = 0$ is asymptotically stable with probabilistic delay. Also, the controller minimizes the upper bound of the following cost function

$$J_c = \int_0^\infty x^T(t)Zx(t) + u^T(t)G^T RGu(t)\mathrm{d}t \leq J^*, \tag{11}$$

where $J^* > 0$ is a specified constant, Z and R are given positive definite matrices, and also the closed-loop system (10) satisfies $\|y(t)\|_2 < \gamma \|w(t)\|_2$, for any nonzero $w(t)$ that belongs to $l_2[0, \infty)$ under the initial condition, where $\gamma > 0$ is a prescribed scalar.

Now, we recall the following lemmas which will be used in our main results.

Lemma 2.1 *[14] Given constant matrices Ξ_1, Ξ_2 and Ξ_3 with appropriate dimensions, where $\Xi_1 = \Xi_1^T > 0$; $\Xi_2 = \Xi_2^T > 0$; and $\Xi_1 + \Xi_3^T \Xi_2^{-1} \Xi_3 < 0$ if and only if* $\begin{bmatrix} \Xi_1 & \Xi_3^T \\ \Xi_3 & -\Xi_2 \end{bmatrix} < 0$.

Lemma 2.2 *[14] Let S, E and L be the real matrices of appropriate dimensions with $\|L\| < I$. Then, for any scalar $\epsilon > 0$, we have $SLE + E^T L^T S^T \leq \epsilon^{-1} S S^T + \epsilon E^T E$.*

Lemma 2.3 *[22] Π, Π_{1d} and Π_{2d} $(d = 1, 2)$ are constant matrices of appropriate dimensions; $\tau_d(t)(d = 1, 2)$ satisfies $0 \leq \tau_1(t) \leq \tau_0 \leq \tau_2(t) \leq \tau_M$ and $\Pi + [\tau_1(t)\Pi_{11} + (\tau_0 - \tau_1(t))\Pi_{21}] + [(\tau_2(t) - \tau_0)\Pi_{12} + (\tau_M - \tau_2(t)\Pi_{22}] < 0$ if and only if*

$$\Pi + \tau_0 \Pi_{11} + (\tau_M - \tau_0)\Pi_{12} < 0,$$
$$\Pi + \tau_0 \Pi_{11} + (\tau_M - \tau_0)\Pi_{22} < 0,$$
$$\Pi + \tau_0 \Pi_{21} + (\tau_M - \tau_0)\Pi_{12} < 0,$$
$$\Pi + \tau_0 \Pi_{21} + (\tau_M - \tau_0)\Pi_{22} < 0.$$

3 Fault-tolerant sampled-data control design

In this section, first we will discuss the problem of H_∞ performance analysis and further the result is extended to obtain the desired fault-tolerant sampled-data controller. In particular, based on the Lyapunov technique, a set of delay-dependent sufficient conditions are derived in terms of linear matrix inequalities to check the asymptotic stability of closed-loop system. More precisely, by assuming that the control gain K is known, we will develop the condition in the following theorem in which the closed-loop system (10) is asymptotically stable.

Theorem 3.1 *Assume that the condition (I) is hold. For the given control gain matrix K, actuator failures rate G, positive scalars γ, α_0 and matrices $Z > 0$, $R > 0$, the closed-loop system (10) is asymptotically stable and satisfies $\|y(t)\|_2 < \gamma \|w(t)\|_2$, if there exist symmetric positive definite matrices P, Q_m, S_m, $m = 1, 2$, any matrices N_{1n}, N_{2n}, M_{1n}, M_{2n}, $n = 1, 2, 3, 4$ and invertible matrices H_r, $r = 1, 2, 3, 4$ such that the following LMIs hold:*

$$\begin{bmatrix} \Pi & \Pi_1 & \bar{C}^T & \Omega_k & \Lambda_l \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -S_1 & 0 \\ * & * & * & * & -S_2 \end{bmatrix} < 0, \quad k, l = 1, 2, \tag{12}$$

where

$$\Pi = \left[\Pi_{m,n}\right]_{6\times 6},$$
$$\Pi_{1,1} = H_1 A + A^T H_1^T + Q_1 + N_{21} + N_{21}^T + Z + K^T G^T R G K,$$
$$\Pi_{1,2} = H_1 \alpha_0 B G K + A^T H_2^T + N_{11} - N_{21} + N_{22}^T,$$
$$\Pi_{1,3} = -N_{11} + M_{21},$$
$$\Pi_{1,4} = (1 - \alpha_0) H_1 B G K + M_{11} - M_{21} + N_{23}^T + A^T H_3^T,$$
$$\Pi_{1,5} = -M_{11} + N_{24}^T,$$
$$\Pi_{1,6} = P - H_1 + A^T H_4^T,$$
$$\Pi_{2,2} = N_{12} + N_{12}^T - N_{22} - N_{22}^T + \alpha_0 \left(H_2 B G K + K^T G^T B^T H_2^T\right),$$
$$\Pi_{2,3} = -N_{12} + M_{22},$$
$$\Pi_{2,4} = M_{12} - M_{22} + N_{13}^T - N_{23}^T + H_2 (1 - \alpha_0) B G K + \alpha_0 K^T G^T B^T H_3^T,$$
$$\Pi_{2,5} = -M_{12} + N_{14}^T - N_{24}^T,$$
$$\Pi_{2,6} = -H_2 + \alpha_0 K^T G^T B^T H_4^T,$$
$$\Pi_{3,3} = Q_2 - Q_1,$$
$$\Pi_{3,4} = -N_{13}^T + M_{23}^T,$$
$$\Pi_{3,5} = -N_{14}^T + M_{24}^T,$$
$$\Pi_{3,6} = 0,$$
$$\Pi_{4,4} = M_{13} + M_{13}^T - M_{23} - M_{23}^T + (1 - \alpha_0)\left(H_3 B G K + K^T G^T B^T H_3^T\right),$$
$$\Pi_{4,5} = -M_{13} + M_{14}^T - M_{24}^T,$$
$$\Pi_{4,6} = -H_3 + (1 - \alpha_0) K^T G^T B^T H_4^T,$$
$$\Pi_{5,5} = -Q_2 - M_{14} - M_{14}^T,$$
$$\Pi_{5,6} = 0, \ \Pi_{6,6} = \tau_0 S_1 + (\tau_M - \tau_0) S_2 - H_4 - H_4^T,$$
$$\Pi_1 = \left[(H_1 B)^T \ (H_2 B)^T \ 0 \ (H_3 B)^T \ 0 \ (H_4 B)^T\right]^T,$$
$$\bar{C}^T = \left[C \ 0_{5n}\right]^T, \ \Omega_1 = \sqrt{\tau_0}\left[N_{11}^T \ N_{12}^T \ 0 \ N_{13}^T \ N_{14}^T \ 0\right]^T,$$
$$\Omega_2 = \sqrt{\tau_0}\left[N_{21}^T \ N_{22}^T \ 0 \ N_{23}^T \ \hat{N}_{24}^T \ 0\right]^T,$$

$$\Lambda_1 = \sqrt{(\tau_M-\tau_0)}\left[M_{11}^T\ M_{12}^T\ 0\ M_{13}^T\ M_{14}^T\ 0\right]^T,$$
$$\Lambda_2 = \sqrt{(\tau_M-\tau_0)}\left[M_{21}^T\ M_{22}^T\ 0\ M_{23}^T\ M_{24}^T\ 0\right]^T.$$

Moreover, an upper bound of the performance index (11) *is given by* $J_c \le \lambda_1 \|x(0)\|^2 = J^*$, *where*

$$\lambda_1 = \lambda_{\max}(P) + \tau_0\lambda_{\max}(Q_1) + (\tau_M-\tau_0)\lambda_{\max}(Q_2) + \frac{\tau_0^2}{2}\lambda_{\max}(R_1) + \frac{(\tau_M-\tau_0)^2}{2}\lambda_{\max}(R_2)$$

Proof Consider the Lyapunov–Krasovskii functional candidate for the model (10) in the following form

$$V(t,x(t)) = x^T(t)Px(t) + \int_{t-\tau_0}^{t} x^T(s)Q_1x(s)\mathrm{d}s + \int_{t-\tau_M}^{t-\tau_0} x^T(s)Q_2x(s)\mathrm{d}s + \int_{-\tau_0}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_M}^{-\tau_0}\int_{t+\theta}^{t} \dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta. \quad (13)$$

By calculating the time derivatives $\dot{V}(t,x(t))$ along the trajectories of the system (10), we obtain

$$\dot{V}(t,x(t)) = 2x^T(t)P\dot{x}(t) + x^T(t)Q_1x(t) + x^T(t-\tau_0)\big(-Q_1+Q_2\big)x(t-\tau_0) - x^T(t-\tau_M)Q_2x(t-\tau_M) + \dot{x}^T(t)[\tau_0S_1 + (\tau_M-\tau_0)S_2]\dot{x}(t) - \int_{t-\tau_0}^{t}\dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s - \int_{t-\tau_M}^{t-\tau_0}\dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s. \quad (14)$$

Further, each integral terms in (14) can be written as

$$-\int_{t-\tau_0}^{t}\dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s = -\int_{t-\tau_0}^{t-\tau_1(t)}\dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s - \int_{t-\tau_1(t)}^{t}\dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s,$$
$$-\int_{t-\tau_M}^{t-\tau_0}\dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s = -\int_{t-\tau_M}^{t-\tau_2(t)}\dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s - \int_{t-\tau_2(t)}^{t-\tau_0}\dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s.$$

On the otherhand, by the Newton–Leibniz formula, for any arbitrary matrices $N_1,\ N_2,\ M_1$ and M_2 with compatible dimensions, the following equalities hold:

$$2\zeta^T(t)N_1\left[x(t-\tau_1(t)) - x(t-\tau_0) - \int_{t-\tau_0}^{t-\tau_1(t)}\dot{x}(s)\mathrm{d}s\right] = 0, \quad (15)$$
$$2\zeta^T(t)N_2\left[x(t) - x(t-\tau_1(t)) - \int_{t-\tau_1(t)}^{t}\dot{x}(s)\mathrm{d}s\right] = 0, \quad (16)$$
$$2\zeta^T(t)M_1\left[x(t-\tau_2(t)) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau_2(t)}\dot{x}(s)\mathrm{d}s\right] = 0, \quad (17)$$
$$2\zeta^T(t)M_2\left[x(t-\tau_0) - x(t-\tau_2(t)) - \int_{t-\tau_2(t)}^{t-\tau_0}\dot{x}(s)\mathrm{d}s\right] = 0, \quad (18)$$
$$(\tau_0-\tau_1(t))\zeta^T(t)N_1S_1^{-1}N_1^T\zeta(t) - \int_{t-\tau_0}^{t-\tau_1(t)}\zeta^T(t)N_1S_1^{-1}N_1^T\zeta(t)\mathrm{d}s = 0, \quad (19)$$
$$\tau_1(t)\zeta^T(t)N_2S_1^{-1}N_2^T\zeta(t) - \int_{t-\tau_1(t)}^{t}\zeta^T(t)N_2S_1^{-1}N_2^T\zeta(t)\mathrm{d}s = 0, \quad (20)$$
$$(\tau_M-\tau_2(t))\zeta^T(t)M_1S_2^{-1}M_1^T\zeta(t) - \int_{t-\tau_M}^{t-\tau_2(t)}\zeta^T(t)M_1S_2^{-1}M_1^T\zeta(t)\mathrm{d}s = 0, \quad (21)$$
$$(\tau_2(t)-\tau_0)\zeta^T(t)M_2S_2^{-1}M_2^T\zeta(t) - \int_{t-\tau_2(t)}^{t-\tau_0}\zeta^T(t)M_2S_2^{-1}M_2^T\zeta(t)\mathrm{d}s = 0, \quad (22)$$

where

$$\zeta^T(t) = \left[x^T(t)\ x^T(t-\tau_1(t))\ x^T(t-\tau_0)\ x^T(t-\tau_2(t))\ x^T(t-\tau_M)\ \dot{x}(t)\right]$$
$$N_1 = \left[N_{11}^T\ N_{12}^T\ 0\ N_{13}^T\ N_{14}^T\ 0\right]^T,$$
$$N_2 = \left[N_{21}^T\ N_{22}^T\ 0\ N_{23}^T\ N_{24}^T\ 0\right]^T,$$
$$M_1 = \left[M_{11}^T\ M_{12}^T\ 0\ M_{13}^T\ M_{14}^T\ 0\right]^T$$

and $M_2 = \begin{bmatrix} M_{21}^T & M_{22}^T & 0 & M_{23}^T & M_{24}^T & 0 \end{bmatrix}^T$. □

Also, for any matrix H of appropriate dimensions, the following inequality holds:

$$2\zeta^T(t)H\Big[Ax(t) + \alpha_0 BGKx(t - \tau_1(t)) + (1 - \alpha_0)BGKx(t - \tau_2(t)) + (\alpha(t) - \alpha_0)BGK\{x(t - \tau_1(t)) - x(t - \tau_2(t))\} - \dot{x}(t)\Big] = 0, \tag{23}$$

where $H = \begin{bmatrix} H_1^T & H_2^T & 0 & H_3^T & 0 & H_4^T \end{bmatrix}^T$.

By taking mathematical expectation on $\dot{V}(t, x(t))$ and using (14)–(23), we can obtain

$$\begin{aligned}\mathbb{E}[\dot{V}(t,x(t))] \le \mathbb{E}\Big[\zeta^T(t)\Big[\tilde{\Pi} &+ (\tau_0 - \tau_1(t))N_1S_1^{-1}N_1^T + \tau_1(t)N_2S_1^{-1}N_2^T + (\tau_M - \tau_2(t))M_1S_2^{-1}M_1^T \\ &+ (\tau_2(t) - \tau_0)M_2S_2^{-1}M_2^T\Big]\zeta(t) \\ &- \int_{t-\tau_0}^{t-\tau_1(t)} \Big[\Big(\zeta^T(t)N_1 + \dot{x}(s)S_1\Big) S_1^{-1} \Big(N_1^T\zeta(t) + S_1\dot{x}^T(s)\Big)\Big] \mathrm{d}s \\ &- \int_{t-\tau_1(t)}^{t} \Big[\Big(\zeta^T(t)N_2 + \dot{x}(s)S_1\Big) S_1^{-1} \Big(N_2^T\zeta(t) + S_1\dot{x}^T(s)\Big)\Big] \mathrm{d}s \\ &- \int_{t-\tau_M}^{t-\tau_2(t)} \Big[(\zeta^T(t)M_1 + \dot{x}(s)S_2)S_2^{-1} \Big(M_1^T\zeta(t) + S_2\dot{x}^T(s)\Big)\Big] \mathrm{d}s \\ &- \int_{t-\tau_2(t)}^{t-\tau_0} \Big[\Big(\zeta^T(t)M_2 + \dot{x}(s)S_2\Big) S_2^{-1} \Big(M_2^T\zeta(t) + S_2\dot{x}^T(s)\Big)\Big] \mathrm{d}s\Big] \\ \le \zeta^T(t)\Big[\tilde{\Pi} &+ (\tau_0 - \tau_1(t))N_1S_1^{-1}N_1^T + \tau_1(t)N_2S_1^{-1}N_2^T + (\tau_M - \tau_2(t)) M_1S_2^{-1}M_1^T + (\tau_2(t) - \tau_0)M_2S_2^{-1}M_2^T\Big]\zeta(t) \\ = \zeta^T(t)\breve{\Theta}\zeta(t),\end{aligned} \tag{24}$$

where

$$\begin{aligned}\breve{\Theta} = \tilde{\Pi} &+ (\tau_0 - \tau_1(t))N_1S_1^{-1}N_1^T + \tau_1(t)N_2S_1^{-1}N_2^T + (\tau_M - \tau_2(t))M_1S_2^{-1}M_1^T \\ &+ (\tau_2(t) - \tau_0)M_2S_2^{-1}M_2^T,\end{aligned}$$

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_{1,1} & \Pi_{1,2} & \Pi_{1,3} & \Pi_{1,4} & \Pi_{1,5} & \Pi_{1,6} \\ * & \Pi_{2,2} & \Pi_{2,3} & \Pi_{2,4} & \Pi_{2,5} & \Pi_{2,6} \\ * & * & \Pi_{3,3} & \Pi_{3,4} & \Pi_{3,5} & \Pi_{3,6} \\ * & * & * & \Pi_{4,4} & \Pi_{4,5} & \Pi_{4,6} \\ * & * & * & * & \Pi_{5,5} & \Pi_{5,6} \\ * & * & * & * & * & \Pi_{6,6} \end{bmatrix},$$

with $\tilde{\Pi}_{1,1} = H_1A + A^TH_1^T + Q_1 + N_{21} + N_{21}^T$ and the remaining parameters are defined in (12).

From (24), it is obvious that $\tilde{\Pi} + (\tau_0 - \tau_1(t))N_1S_1^{-1}N_1^T + \tau_1(t)N_2S_1^{-1}N_2^T + (\tau_M - \tau_2(t))M_1S_2^{-1}M_1^T + (\tau_2(t) - \tau_0)M_2S_2^{-1}M_2^T < 0$ is a convex combination of the terms $(\tau_0 - \tau_1(t))$, $\tau_1(t)$, $(\tau_M - \tau_2(t))$ and $(\tau_2(t) - \tau_0)$.

It follows from Lemma 2.3 that the inequality (24) is true if and only if

$$\tilde{\Pi} + \tau_0N_1S_1^{-1}N_1^T + (\tau_M - \tau_0)M_1S_2^{-1}M_1^T < 0, \tag{25}$$

$$\tilde{\Pi} + \tau_0N_1S_1^{-1}N_1^T + (\tau_M - \tau_0)M_2S_2^{-1}M_2 < 0, \tag{26}$$

$$\tilde{\Pi} + \tau_0N_2S_1^{-1}N_2^T + (\tau_M - \tau_0)M_1S_2^{-1}M_1^T < 0, \tag{27}$$

$$\tilde{\Pi} + \tau_0N_2S_1^{-1}N_2^T + (\tau_M - \tau_0)M_2S_2^{-1}M_2^T < 0. \tag{28}$$

By using Schur complement, it is easy to see that inequalities (25)–(28) are equivalent to the following LMIs

$$\begin{bmatrix} \tilde{\Pi} & \Omega_k & \Lambda_l \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \quad k, l = 1, 2,$$

which implies that $\dot{V}(t, x(t)) < 0$. Hence, the closed-loop system (10) with $w(t) = 0$ is asymptotically stable. Next, we consider the upper bound of the cost function (11). Now, we have

$$\dot{V}(t, x(t)) + x^T(t)Zx(t) + u^T(t)G^TRGu(t) \le \zeta^T(t) \begin{bmatrix} \Pi & \Omega_k & \Lambda_l \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} \zeta(t).$$

If (12) holds, then we have

$$\dot{V}(t, x(t)) + x^T(t)Zx(t) + u^T(t)G^TRGu(t) < 0. \tag{29}$$

Integrating both sides of (29) from $t = 0$ to $t = \infty$, we obtain

$$\begin{aligned} J_c &= \int_0^\infty \left(x^T(t)Zx(t) + u^T(t)G^TRGu(t)\right)\mathrm{d}t \\ &\le -\int_0^\infty \dot{V}(t, x(t))\mathrm{d}t \le x^T(0)Px(0) \\ &\quad + \int_{-\tau_0}^0 x^T(s)Q_1x(s)\mathrm{d}s \\ &\quad + \int_{-\tau_M}^{-\tau_0} x^T(s)Q_2x(s)\mathrm{d}s \\ &\quad + \int_{-\tau_0}^0\int_\theta^0 \dot{x}^T(s)S_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \int_{-\tau_M}^{-\tau_0}\int_\theta^0 \dot{x}^T(s)S_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\le \lambda_1\|x(0)\|^2 = J^*, \end{aligned}$$

where $\lambda_1 = \lambda_{\max}(P) + \tau_0\lambda_{\max}(Q_1) + (\tau_M - \tau_0)\lambda_{\max}(Q_2) + \frac{\tau_0^2}{2}\lambda_{\max}(R_1) + \frac{(\tau_M-\tau_0)^2}{2}\lambda_{\max}(R_2)$.

Finally, in order to establish the H_∞ performance of the system, we consider the following relation

$$J_h = \int_0^\infty \left[y^T(t)y(t) - \gamma^2w^T(t)w(t)\right]\mathrm{d}t. \tag{30}$$

We combine the performance J_c and J_h as

$$\begin{aligned} J(t) = J_c + J_h = \int_0^\infty &\Big\{\left[y^T(t)y(t) - \gamma^2w^T(t)w(t)\right] \\ &+ \left[x^T(t)Zx(t) + u^T(t)G^TRGu(t)\right]\Big\}\mathrm{d}t. \end{aligned}$$

By applying the similar procedure used above in the proof of stability, it can be calculated that

$$\begin{aligned} &\mathbb{E}\Big[\left[y^T(t)y(t) - \gamma^2w^T(t)w(t)\right] \\ &\quad + \left[\dot{V}(t, x(t)) + x^T(t)Zx(t) + u^T(t)G^TRGu(t)\right]\Big] \\ &\le \tilde{\zeta}^T(t)\Theta\tilde{\zeta}(t), \end{aligned} \tag{31}$$

where

$$\begin{aligned} &\tilde{\zeta}^T(t) = \left[\zeta^T(t)\ w^T(t)\right],\ \Theta = \Pi + \Pi_1 + \overline{C}^T\overline{C} \\ &\quad + (\tau_0 - \tau_1(t))N_1S_1^{-1}N_1^T + \tau_1(t)N_2S_1^{-1}N_2^T \\ &\quad + (\tau_M - \tau_2(t))M_1S_2^{-1}M_1^T + (\tau_2(t) - \tau_0)M_2S_2^{-1}M_2^T. \end{aligned}$$

By applying the idea of convex combination to (31), we have

$$\begin{aligned} &\Pi + \Pi_1 + \overline{C}^T\overline{C} \\ &\quad + (\tau_0 - \tau_1(t))N_1S_1^{-1}N_1^T + \tau_1(t)N_2S_1^{-1}N_2^T \\ &\quad + (\tau_M - \tau_2(t))M_1S_2^{-1}M_1^T \\ &\quad + (\tau_2(t) - \tau_0)M_2S_2^{-1}M_2^T < 0 \end{aligned}$$

and it can be written equivalently in the form

$$\begin{aligned} &\Pi + \Pi_1 + \overline{C}^T\overline{C} + \tau_0N_1S_1^{-1}N_1^T \\ &\quad + (\tau_M - \tau_0)M_1S_2^{-1}M_1^T < 0, \end{aligned} \tag{32}$$

$$\begin{aligned} &\Pi + \Pi_1 + \overline{C}^T\overline{C} + \tau_0N_1S_1^{-1}N_1^T \\ &\quad + (\tau_M - \tau_0)M_2S_2^{-1}M_2 < 0, \end{aligned} \tag{33}$$

$$\begin{aligned} &\Pi + \Pi_1 + \overline{C}^T\overline{C} + \tau_0N_2S_1^{-1}N_2^T \\ &\quad + (\tau_M - \tau_0)M_1S_2^{-1}M_1^T < 0, \end{aligned} \tag{34}$$

$$\begin{aligned} &\Pi + \Pi_1 + \overline{C}^T\overline{C} + \tau_0N_2S_1^{-1}N_2^T \\ &\quad + (\tau_M - \tau_0)M_2S_2^{-1}M_2^T < 0. \end{aligned} \tag{35}$$

By using Schur complement, it is easy to see that inequalities (32)–(35) are equivalent to the LMIs in (12). According to LMIs in (12), we have $J(t) < 0$ which implies that $||y(t)||_2 < \gamma||w(t)||_2$ holds for any nonzero $w(t) \in l_2[0,\ \infty]$. The proof is completed.

Further, we will present the design method to compute the fault-tolerant sampled-data controller gain based on the results obtained in the previous section. In particular, in the following theorem, when the actuator failure matrix G is unknown but satisfying the constraints (4), we determine the gain matrix of the fault-tolerant sampled-data state feedback controller such that the closed-loop system (10) is robustly asymptotically stable.

Theorem 3.2 *Under the Assumption (I), for given positive scalars γ, α_0, $\beta_q, q = 1, 2, 3$, unknown actuator failures rate G and matrices $Z > 0$, $R > 0$, the system (5) is robustly asymptotically stabilized via the sampled-data control law (7) and satisfies $\|y(t)\|_2 < \gamma\|w(t)\|_2$ for any nonzero $w(t) \in l_2[0,\ \infty)$ with zero initial condition, if there exist symmetric positive definite matrices $\widehat{P}$, $\widehat{Q}_m$, $\widehat{S}_m$, $m = 1, 2$, any matrices Y, $\hat{N}_{1n}$, $\hat{N}_{2n}$, $\hat{M}_{1n}$, $\hat{M}_{2n}, n = 1, 2, 3, 4$, invertible matrix X and there exist positive scalars ϵ_p, $p = 1, 2, 3, 4$ such that the following LMIs hold:*

$$\hat{\Xi}_{kl} = \begin{bmatrix} \breve{\Xi}_{kl} & \epsilon_1\tilde{\Im}_1\hat{G} & \tilde{Y}_1 & \epsilon_2\tilde{\Im}_2\hat{G} & \tilde{Y}_2 & \epsilon_3\tilde{\Im}_3\hat{G} & \tilde{Y}_3 & \epsilon_4\tilde{\Im}_4\hat{G} & \tilde{Y}_4 \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\epsilon_3 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon_3 I & -\epsilon_4\hat{G} & 0 \\ * & * & * & * & * & * & * & -\epsilon_4 I & 0 \\ * & * & * & * & * & * & * & * & -\epsilon_4 I \end{bmatrix} < 0, \tag{36}$$

where

$$\breve{\Xi}_{kl} = \begin{bmatrix} \hat{\Pi} & \hat{\Pi}_1 & \hat{C}^T & \hat{\Omega}_k & \hat{\Lambda}_l & X & Y^T\hat{G} \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -\hat{S}_1 & 0 & 0 & 0 \\ * & * & * & * & -\hat{S}_2 & 0 & 0 \\ * & * & * & * & * & -Z^{-1} & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \ k,l = 1,2$$

with

$$\hat{\Pi} = \left[\hat{\Pi}_{m,n}\right]_{6\times 6},\ \hat{\Pi}_{1,1} = (AX^T + XA^T) + \hat{Q}_1 + \hat{N}_{21} + \hat{N}_{21}^T$$

$$\hat{\Pi}_{1,2} = \alpha_0 B\hat{G}Y + \beta_1 XA^T + \hat{N}_{11} + \hat{N}_{21} + \hat{N}_{22}^T,$$

$$\hat{\Pi}_{1,3} = -\hat{N}_{11} + \hat{M}_{21},$$

$$\hat{\Pi}_{1,4} = (1-\alpha_0)B\hat{G}Y + \hat{M}_{11} - \hat{M}_{21} + \hat{N}_{23}^T + \beta_2 XA^T,$$

$$\hat{\Pi}_{1,5} = -\hat{M}_{11} + \hat{N}_{24}^T,$$

$$\hat{\Pi}_{1,6} = \hat{P} - X^T + \beta_3 XA^T,$$

$$\hat{\Pi}_{2,2} = \hat{N}_{12} + \hat{N}_{12}^T - \hat{N}_{22} - \hat{N}_{22}^T + \beta_1\alpha_0(B\hat{G}Y + Y^T\hat{G}^T B^T),$$

$$\hat{\Pi}_{2,3} = -\hat{N}_{12} + \hat{M}_{22},$$

$$\hat{\Pi}_{2,4} = \hat{M}_{12} - \hat{M}_{22} + \hat{N}_{13}^T - \hat{N}_{23}^T + \beta_1(1-\alpha_0)B\hat{G}Y + \beta_2\alpha_0 Y^T\hat{G}^T B^T,$$

$$\hat{\Pi}_{2,5} = -\hat{M}_{12} + \hat{N}_{14}^T - \hat{N}_{24}^T,$$

$$\hat{\Pi}_{2,6} = -\beta_1 X^T + \beta_3\alpha_0 Y^T\hat{G}^T B^T,$$

$$\hat{\Pi}_{3,3} = \hat{Q}_2 - \hat{Q}_1,$$

$$\hat{\Pi}_{3,4} = -\hat{N}_{13}^T + \hat{M}_{23}^T,$$

$$\hat{\Pi}_{3,5} = -\hat{N}_{14}^T + \hat{M}_{24}^T, \quad \hat{\Pi}_{3,6} = 0,$$

$$\hat{\Pi}_{4,4} = \hat{M}_{13} + \hat{M}_{13}^T - \hat{M}_{23} - \hat{M}_{23}^T + \beta_2(1-\alpha_0)(B\hat{G}Y + Y^T\hat{G}^T B^T),$$

$$\hat{\Pi}_{4,5} = -\hat{M}_{13} + \hat{M}_{14}^T - \hat{M}_{24}^T,$$

$$\hat{\Pi}_{4,6} = -\beta_2 X^T + \beta_3(1-\alpha_0)Y^T\hat{G}^T B^T,$$

$$\hat{\Pi}_{5,5} = -\hat{Q}_2 - \hat{M}_{14} - \hat{M}_{14}^T, \quad \hat{\Pi}_{5,6} = 0,$$

$$\hat{\Pi}_{6,6} = \tau_0\hat{S}_1 + (\tau_M - \tau_0)\hat{S}_2 - \beta_3(X + X^T),$$

$$\hat{\Pi}_1 = [B^T\ \beta_1 B^T\ 0\ \beta_2 B^T\ 0\ \beta_3 B^T]^T,$$

$$\hat{C} = [(XC^T)^T\ \ 0_{5n}]^T,$$

$$\hat{\Omega}_1 = \sqrt{\tau_0}\left[\hat{N}_{11}^T\ \hat{N}_{12}^T\ 0\ \hat{N}_{13}^T\ \hat{N}_{14}^T\ 0\right]^T,$$

$$\hat{\Omega}_2 = \sqrt{\tau_0}\left[\hat{N}_{21}^T\ \hat{N}_{22}^T\ 0\ \hat{N}_{23}^T\ \hat{N}_{24}^T\ 0\right]^T,$$

$$\hat{\Lambda}_1 = \sqrt{(\tau_M - \tau_0)}\left[\hat{M}_{11}^T\ \hat{M}_{12}^T\ 0\ \hat{M}_{13}^T\ \hat{M}_{14}^T\ 0\right]^T,$$

$$\hat{\Lambda}_2 = \sqrt{(\tau_M - \tau_0)}\left[\hat{M}_{21}^T\ \hat{M}_{22}^T\ 0\ \hat{M}_{23}^T\ \hat{M}_{24}^T\ 0\right]^T,$$

$$\tilde{\Im}_1 = \left[\alpha_0 B^T\ \alpha_0\beta_1 B^T\ 0_{10n}\right]^T,$$

$$\tilde{Y}_1 = \left[0\ Y^T\ 0_{10n}\right]^T,$$

$$\tilde{\Im}_2 = \left[(1-\alpha_0)B^T\ (1-\alpha_0)\beta_1 B^T + \alpha_0\beta_1 B^T\ 0_{10n}\right]^T,$$

$$\tilde{Y}_2 = \left[0_{3n}\ Y^T\ 0_{8n}\right]^T,$$

$$\tilde{\Im}_3 = \left[0\ \alpha_0\beta_3 B^T\ 0\ (1-\alpha_0)\beta_2 B^T + (1-\alpha_0)\beta_3 B^T\ 0_{8n}\right]^T,$$

$$\tilde{Y}_3 = \left[0_{5n}\ Y^T\ 0_{6n}\right]^T,$$

$$\tilde{\Im}_4 = \left[0_{12n}\right]^T,$$

$$\tilde{Y}_4 = \left[Y^T\ 0_{11n}\right]^T.$$

Furthermore, the fault-tolerant sampled-data feedback controller gain matrix is designed as $K = YX^{-1^T}$, *and an upper bound of performance index* (11) *is given by* $J_c \le \lambda_2 \|x(0)\|^2 = J^*$, *where*

$$\lambda_2 = \lambda_{\max}\left(X^{-1}\hat{P}X^{-T}\right) + \tau_0\lambda_{\max}\left(X^{-1}\hat{Q}_1X^{-T}\right) + (\tau_M - \tau_0)\lambda_{\max}\left(X^{-1}\hat{Q}_2X^{-T}\right) + \frac{\tau_0^2}{2}\lambda_{\max}\left(X^{-1}\hat{R}_1X^{-T}\right) + \frac{(\tau_M - \tau_0)^2}{2}\lambda_{\max}\left(X^{-1}\hat{R}_2X^{-T}\right)$$

Proof In order to obtain the fault-tolerant sampled-data state feedback control gain matrix, take $X = H_1^{-1}$, $H_2 = \beta_1 H_1$, $H_3 = \beta_2 H_1$ and $H_4 = \beta_3 H_1$, where β_1, β_2 and β_2 are the designing parameters. Pre- and post-multiplying (12) by diag $\{T_1, I, I, X, X\}$ and its transpose, respectively, where $T_1 \in \text{diag}\{X, X, \ldots, X\} \in \mathbb{R}^{6\times 6}$, $H_1 = X^{-1}$ and letting $X^{-1}\hat{P}X^{-1^T} = P$, $X^{-1}\hat{Q}_iX^{-1^T} = Q$, $X^{-1}\hat{S}_iX^{-1^T} = S_i$, $i = 1,2,3$, $X^{-1}\hat{M}_{1n}X^{-1^T} = M_{1n}$, $X^{-1}\hat{M}_{2n}X^{-1^T} = M_{2n}$, $X^{-1}\hat{M}_{3n}X^{-1^T} = M_{3n}$ and $X^{-1}\hat{M}_{4n}X^{-1^T} = M_{4n}$, $n = 1,\ldots,4$, we can obtain

$$\begin{bmatrix} \breve{\Pi} & \hat{\Pi}_1 & \hat{C}^T & \hat{\Omega}_k & \hat{\Lambda}_l \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\hat{S}_1 & 0 \\ * & * & * & * & -\hat{S}_2 \end{bmatrix} < 0, \quad k,l = 1,2, \qquad (37)$$

where $\breve{\Pi} = \left[\hat{\Pi}\right]_{6\times 6}$, $\hat{\Pi}_{1,1} = (AX^T + XA^T) + \hat{Q}_1\hat{N}_{21} + \hat{N}_{21}^T + XZX^T + Y^T\hat{G}^T R\hat{G}Y$ and the remaining parameters are defined in (36). Applying Schur complement, (37) becomes

$$\breve{\Xi}_{kl} = \begin{bmatrix} \hat{\Pi} & \hat{\Pi}_1 & \hat{C}^T & \hat{\Omega}_k & \hat{\Lambda}_l & X & Y^T\hat{G} \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -S_1 & 0 & 0 & 0 \\ * & * & * & * & -S_2 & 0 & 0 \\ * & * & * & * & * & -Z^{-1} & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \ k, l = 1, 2. \quad (38)$$

By using Eq (3), $\breve{\Xi}_{kl}$ in (38) can be written as

$$\hat{\Xi}_{kl} = \breve{\Xi}_{kl} + \tilde{\Im}_1 \hat{G} L \tilde{Y}_1 + \tilde{Y}_1^T L^T \hat{G}^T \tilde{\Im}_1^T + \tilde{\Im}_2 \hat{G} L \tilde{Y}_2 + \tilde{Y}_2^T L^T \hat{G}^T \tilde{\Im}_2^T + \tilde{\Im}_3 \hat{G} L \tilde{Y}_3 + \tilde{Y}_3^T L^T \hat{G}^T \tilde{\Im}_3^T + \tilde{\Im}_4 \hat{G} L \tilde{Y}_4 + \tilde{Y}_4^T L^T \hat{G}^T \tilde{\Im}_4^T.$$

Further, it follows from Lemma 2.2 that

$$\hat{\Xi}_{kl} \le \breve{\Xi}_{kl} + \epsilon_1 \tilde{\Im}_1 \hat{G}\hat{G}^T \tilde{\Im}_1^T + \epsilon_1^{-1} \tilde{Y}_1 \tilde{Y}_1^T + \epsilon_2 \tilde{\Im}_2 \hat{G}\hat{G}^T \tilde{\Im}_2^T + \epsilon_2^{-1} \tilde{Y}_2 \tilde{Y}_2^T + \epsilon_3 \tilde{\Im}_3 \hat{G}\hat{G}^T \tilde{\Im}_3^T + \epsilon_3^{-1} \tilde{Y}_3 \tilde{Y}_3^T + \epsilon_4 \tilde{\Im}_4 \hat{G}\hat{G}^T \tilde{\Im}_4^T + \epsilon_4^{-1} \tilde{Y}_4 \tilde{Y}_4^T. \quad (39)$$

Then, by using Lemma 2.1, it is easy to see that (39) is equivalent to (36). Thus, the system (5) is robustly asymptotically stabilized by using the sampled-data control law (7). This completes the proof. □

4 Numerical simulation

In this section, the flexible spacecraft with probabilistic delay is given to illustrate the effectiveness and superiority of the design scheme that is developed in previous section. More precisely, the proposed control scheme is applied to a flexible spacecraft with one flexible appendage which is proposed in [23]. Since low-frequency modes are generally dominant in a flexible system, only the lowest two bending modes have been considered for the implemented spacecraft model. Thus, we suppose that $\varpi_1 = 3.17$ rad/s, $\varpi_2 = 7.38$ rad/s with damping $\xi_1 = 0.001$ and $\xi_2 = 0.015$. Also, we assume that $F = [F_1 \ F_2]$, where the coupling coefficients of the first two bending modes are $F_1 = 1.27814$ and $F_2 = 0.91756$. Further, $\mathcal{J} = 35.72$ kg/m^2 is the nominal principal moment of inertia of pitch axis. The flexible spacecraft is supposed to move in a circular orbit with the altitude of 500 km, and then, the orbit rate is 0.0011 rad/s. Let the initial pitch attitude of the spacecraft is $\theta(0) = 0.08$ rad, $\dot{\theta}(0) = 0.04$ rad/s.

Table 1 Calculated upper bound τ_M for various values of α_0 under control effectiveness factor $\hat{G}$

α_0	0.80	0.85	0.90	0.95
τ_M	0.291	0.364	0.502	0.828

Next, we design a fault-tolerant sampled-data controller (7) such that the flexible spacecraft model (5) with random time delay is asymptotically stable and simultaneously show the relationship between the random value α_0, the maximum sampling period τ_M and the control effectiveness rate $\hat{G}$. For this, if we choose $\tau_0 = 0.1$, $\beta_1 = \beta_2 = \beta_3 = 0.01$, $Z = [0.25 \ 0.1; \ 0.1 \ 0.5]$, $R = 0.31$ and H_∞ performance index $\gamma = 2.5$, by solving the LMIs (36) in Theorem 3.2 using Matlab LMI toolbox, we can obtain the feasible solutions for the asymptotic stability of the closed-loop system. Moreover, the obtained maximum upper bound τ_M for various values of α_0 under control effectiveness factor $\hat{G} = 1, \ 0.7, \ 0.5, \ 0.2, \ 0.1$ is given in Table 1. In particular, the results that are proposed in this paper depend on both the upper and lower bounds of the sampling interval. Also, when $\alpha_0 = 0.85$ and time delay upper bound $\tau_M = 0.364$, the obtained output matrices X, Y and the designed controller gain matrices under different control effectiveness factors are listed in Table 2, respectively.

Finally, Fig. 1a, b presents the simulation result for the trajectories of pitch attitude angle and angle rate, respectively, under partial actuator failure and as well as normal operating case (without any failures), where the control effectiveness factors are chosen as $\hat{G} = 0.1$, $\hat{G} = 0.2$, $\hat{G} = 0.5$, $\hat{G} = 0.7$, $\hat{G} = 1$. From the simulation results, it is observed that for the case of partial actuator failure (other than $\hat{G} = 1$), the pitch attitude angle and angle rate are converging to the equilibrium point zero even though there exist some initial oscillations. It is concluded from the simulation result that the proposed controller makes the closed-loop system to exhibit a better performances even for the partial failure cases. Also, it is noted that the closed-loop system is robustly asymptotically stable with the proposed controller. Thus, the simulation result shows that the proposed fault-tolerant sampled-data control law is effective for the flexible spacecraft system with probabilistic time delay.

Table 2 The outputs X, Y, controller gain and J^* for various control effectiveness factor $\hat{G}$

$\hat{G}$	Output X	Output Y	$K = YX^{-T}$	J^*
1	$X = \begin{bmatrix} 0.0801 & -0.0045 \\ -0.0046 & 0.0010 \end{bmatrix}$	$Y = \begin{bmatrix} -0.0242 & -0.0071 \end{bmatrix}$	$\begin{bmatrix} -1.0002 & -12.1674 \end{bmatrix}$	11.7343
0.7	$X = \begin{bmatrix} 0.0799 & -0.0045 \\ -0.0046 & 0.0010 \end{bmatrix}$	$Y = \begin{bmatrix} -0.0347 & -0.0102 \end{bmatrix}$	$\begin{bmatrix} -1.4289 & -17.3865 \end{bmatrix}$	11.7462
0.5	$X = \begin{bmatrix} 0.0799 & -0.0045 \\ -0.0046 & 0.0010 \end{bmatrix}$	$Y = \begin{bmatrix} -0.0485 & -0.0142 \end{bmatrix}$	$\begin{bmatrix} -2.0005 & -24.3427 \end{bmatrix}$	11.7477
0.2	$X = \begin{bmatrix} 0.0801 & -0.0045 \\ -0.0046 & 0.0010 \end{bmatrix}$	$Y = \begin{bmatrix} -0.1211 & -0.0356 \end{bmatrix}$	$\begin{bmatrix} -5.0013 & -60.8405 \end{bmatrix}$	11.7352
0.1	$X = \begin{bmatrix} 0.0803 & -0.0046 \\ -0.0046 & 0.0010 \end{bmatrix}$	$Y = \begin{bmatrix} -0.2422 & -0.0713 \end{bmatrix}$	$\begin{bmatrix} -10.0071 & -121.6360 \end{bmatrix}$	11.7138

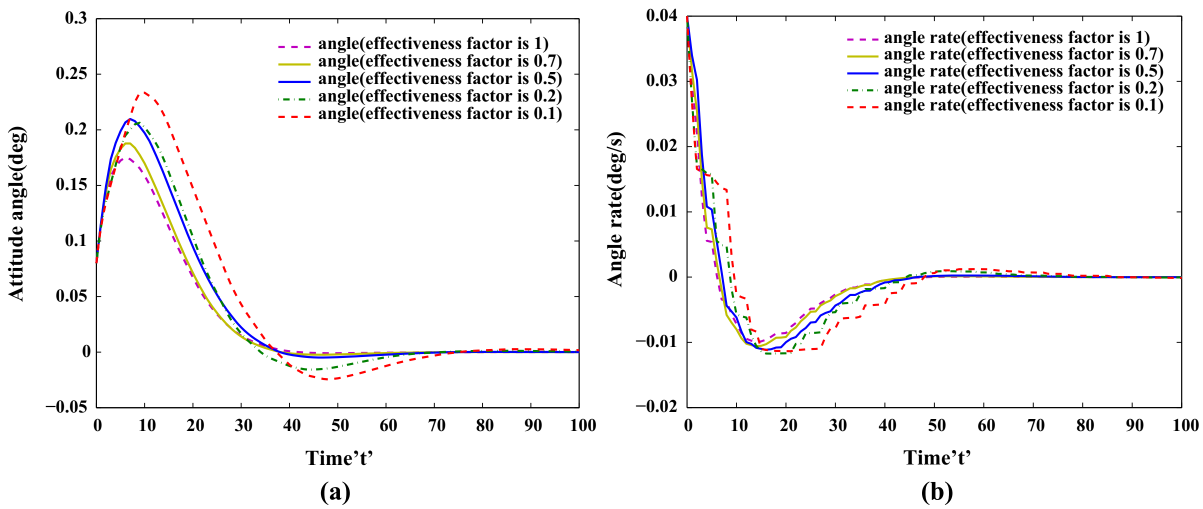

Fig. 1 State responses of system. **a** Pitch attitude angle **b** Angle rate

5 Conclusion

In this paper, fault-tolerant sampled-data control with random time-varying delays for flexible spacecraft model has been addressed. By constructing an appropriate Lyapunov–Krasovskii functional, a set of delay-distribution-dependent sufficient conditions for the existence of state feedback fault-tolerant sampled-data controller are derived. The proposed conditions ensure that the closed-loop system is asymptotically stable in the presence random delays in control input with a H_∞ disturbance attention level and partial actuator failures. More precisely, the stabilization conditions are expressed in terms of the solutions to a set of LMIs, which can be solved effectively by using standard LMI control tool box. Also, the desired state feedback fault-tolerant sampled-data control can be obtained when the given LMIs are feasible. Finally, the results are validated through a numerical example with simulation results, which reveal that the developed control scheme guarantees not only the asymptotic stability of the closed-loop system, but also yields better performance. Further, the output feedback guaranteed cost control and model predictive control strategies for flexible spacecrafts will be our future topics of research.

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