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ORIGINAL PAPER

Periodic solution of a pest management Gompertz model with impulsive state feedback control

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Abstract In this paper, a new model with two state impulses is proposed for pest management. According to different thresholds, an integrated strategy of pest management is considered, that is to say if the density of the pest population reaches the lower threshold h_1 at which pests cause slight damage to the forest, biological control (releasing natural enemy) will be taken to control pests; while if the density of the pest population reaches the higher threshold h_2 at which pests cause serious damage to the forest, both chemical control (spraying pesticide) and biological control (releasing natural enemy) will be taken at the same time. For the model, firstly, we qualitatively analyse its singularity. Then, we investigate the existence of periodic solution by successor functions and Poincaré-Bendixson theorem and the stability of periodic solution by the stability theorem for periodic solutions of impulsive differential equations. Lastly, we use numerical simulations to illustrate our theoretical results.

Keywords Gompertz growth rate · Integrated pest management (IPM) · State impulse · State feedback control · Periodic solution · Stability

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1 Introduction and model formulation

As complex ecosystems, forests play an important role in the environment for human to survive. But disease-causing organisms and insects have undesirable effects on the health of a forest [12]. For example, the mountain pine beetle, spruce budworm, gypsy moth and Dutch elm disease have led to substantial losses in Canada [10] and the Asian long-horned beetle and emerald ash borer are doing colossal damage to trees and forests in the United States [31]. On one hand, when the number of pests is very little, does not reach a certain value, as a natural part of the ecosystem, usually we do not have to worry; on the other hand, when the population and activity of pests become threatening so that they can spoil the health of a forest even kill many trees, we need to conduct interference artificially. Chemical control and biological control are two principal methods in practice. Although chemical pest control is still the main way of pest control in most of the place today, the biological control relying on predation, parasitism, herbivores or other biological mechanisms has received the welcome of people [4].

Usually, artificial interference may cause an abrupt change in pests and natural enemies populations, which very often results in difficulties in developing mathematical models to describe it. Fortunately, impulsive differential equation can accurately express this change. Driven by the desire in applications, theoretical study on impulsive differential equations attracted extra attention, please see [1–3,5–9,26,27] and the references therein. Since then, numerous mathematical models governed by impulse differential equations have been established in the area. Nevertheless, most of these models are with one time impulse [15,16,18–25,33–35,41,42] or two [32,39,40]. Recent research findings suggest that in the practice, according to the different density of the pest, a state feedback measure for controlling pest looks more businesslike and biological models concentrated on state impulse seem more reasonable. In this research direction, several such models have been investigated [11,14,28,36,38], where authors employed systems of impulsive differential equations with one state impulse. However, study of biological system including two state impulses is not many [28–30,43].

Motivated by the previous work, we propose a mathematical biological system including two state feedback control in this paper. For self-contained, we first list some relevant work in the following. In references [36] and [28] models

$$\begin{cases} \left.\begin{array}{l} \dot{x}(t) = x(t)(a - by(t)), \\ \dot{y}(t) = y(t)(cx(t) - d), \end{array}\right\} x \neq ET, \\ \left.\begin{array}{l} \Delta x(t) = -px(t), \\ \Delta y(t) = \tau, \end{array}\right\} x = ET, \end{cases} \tag{1}$$

and

$$\begin{cases} \left.\begin{array}{l} \dot{x}(t) = x(t)(a - by(t)), \\ \dot{y}(t) = y(t)\left(\frac{\lambda b x(t)}{1+bhx(t)} - d\right), \end{array}\right\} x \neq h_1, h_2, \\ \left.\begin{array}{l} \Delta x(t) = 0, \\ \Delta y(t) = \tau, \end{array}\right\} x = h_1, \\ \left.\begin{array}{l} \Delta x(t) = -px(t), \\ \Delta y(t) = -qy(t), \end{array}\right\} x = h_2, \end{cases} \tag{2}$$

are investigated. The authors studied the integrated pest management control strategies with the help of the Lambert W function and Poincaré map. In [37], Trzcinski and Reid investigated a mountain pine beetle population dynamics governed by the Gompertz population model, where the Gompertz equation [13] is given by

$$\dot{x}(t) = rx(t)\ln\frac{K}{x(t)}, \tag{3}$$

which describes the growth law of density dependence where the rate of increase declines linearly with the log e of population abundance.

Now we are in a position to give out our models as follows:

$$\begin{cases} \left.\begin{array}{l} \dot{x}(t) = rx(t)\ln\frac{K}{x(t)} - \beta x(t)y(t), \\ \dot{y}(t) = \lambda\beta x(t)y(t) - dy(t), \end{array}\right\} \\ x \neq h_1, h_2, \quad \text{or} \quad x = h_1, y > y^*, \\ \left.\begin{array}{l} \Delta x(t) = 0, \\ \Delta y(t) = \alpha, \end{array}\right\} x = h_1, y \leq y^*, \\ \left.\begin{array}{l} \Delta x(t) = -px(t), \\ \Delta y(t) = -qy(t) + \tau, \end{array}\right\} x = h_2, \end{cases} \tag{4}$$

where $\Delta x(t) = x(t^+) - x(t)$, $\Delta y(t) = y(t^+) - y(t)$, r is the Gompertz intrinsic growth rate of the prey in the absence of the predator, K is usually referred to the environment carrying capacity of saturation level, β represents the predation rate of natural enemies, λ represents the transformation rate at which ingested prey in excess of what is needed for maintenance is translated into predator population increase. d is the death rate of natural enemy. h_1 and h_2 are the thresholds with slight damage and serious damage to the forest, respectively. y^* is the intersection of the line $x = h_1$ and $r\ln\frac{K}{x(t)} - \beta y(t) = 0$; α, τ are release quantity of natural enemy $y(t)$. p represents the death rate of pests and q is the death rate of natural enemies due to pesticide. For biologically meaningful, we restrict our study in the region of $R_+^2 = \{(x, y)|x \geq 0, y \geq 0\}$.

Notice the fact that when pests population $x(t)$ is low enough such that the natural enemies of the natural world can control them, as a natural part of the ecosystem, there is no necessary to take any action. Our model reflects this fact and the pest control strategy mentioned above, namely we may increase the quantity of natural enemies by releasing natural enemies cultivated in the lab to control the pest instead of spraying pesticide only when the density of the pest population $x(t)$ reaches certain level, $x = h_1$, say. The procedure goes like this:

- Release natural enemies again if the density of pests remains at the same level after releasing the previous batch of natural enemies. This step may be repeated several times;
- Stop the release if the natural enemy population $y(t)$ increases and reaches level y^*.

Considering the cost of cultivating natural enemies and the loss caused by pests, controlling the pest only by natural enemies in natural law is not realistic. Thus, our newly developed model should also reflect the strategy that when the density of the pest population $x(t)$ reaches certain level $x = h_2$ (usually a higher level). Hence, we not only will release the natural enemies, but also spray less pesticide to kill the pests.

The rest of the paper is organised as follows. In Sect. 2, we briefly introduce some concepts and fundamental results, which are necessary in the later discussion. Section 3 focuses on the qualitative analysis of system (4) without impulsive effect. In the Sect. 4, we investigate the periodic solution of system (4) with impulsive state feedback control. Then, we carry out numerical simulations and discussions in Sect. 5, which show all simulations agree with the theoretic results well. We finally conclude our paper in Sect. 6.

2 Preliminaries

In this section, we briefly introduce some basic concepts and fundamental theories from [3,6,17,43,44]. Consider a system of impulsive differential equations

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= P(x, y), \\ \dot{y}(t) &= Q(x, y), \end{aligned}\right\} \text{ for } (x, y) \notin M\{x, y\}, \\ \left.\begin{aligned} \Delta x &= \alpha(x, y), \\ \Delta y &= \beta(x, y), \end{aligned}\right\} \text{ for } (x, y) \in M\{x, y\}, \end{cases} \tag{5}$$

where M is known as impulsive set, which can be a straight line or curve in R^2. Let $I : I(M) = N$, be a continuous mapping, I is called the impulse function from set M to set N and N is called the image set. Then, from [43,44], we have

Definition 2.1 For continuous function $f(x, t)$, if there exists a point P_0 and a period T such that $f(P_0, T) = Q_0 \in M\{x, y\}$ and $I(Q_0) = I(f(P_0, T)) = P_0 \in N$, then, we call $f(P_0, [0, T])$ a periodic solution of system (5).

Definition 2.2 Function

$$s(x) = s(x^+) - s(x)$$

is called a successor function of point x (Fig. 1).

Theorem 2.1 (Bendixson theorem for impulsive differential equations [6]) *Assume G is a Bendixson region of system (5). Then, if G does not contain any critical points of it, system (5) has a closed orbit in G.*

Theorem 2.2 *Assume that in continuous dynamic system (X, Π), there exist two points x_1, x_2 in the pulse phase concentration such that the successor function $s(x_1) > 0$ and $s(x_2) < 0$, then, there exists a point P falling in between points x_1 andx_2 such that $s(P) = 0$, then, the system has order one periodic solution.*

Theorem 2.3 (Analogue of Poincaré Criterion [3,17]) *The T-periodic solution $x = \phi(t)$, $y = \varphi(t)$ of model*

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= P(x, y), \\ \dot{y}(t) &= Q(x, y), \end{aligned}\right\}, \Phi(x, y) \neq 0, \\ \left.\begin{aligned} \Delta x &= \alpha(x, y), \\ \Delta y &= \beta(x, y), \end{aligned}\right\}, \Phi(x, y) = 0, \end{cases} \tag{6}$$

is orbitally asymptotically stable if $|\mu_2| < 1$, where μ_2 is the multiplier and calculated by

$$\mu_2 = \prod_{k=1}^{q} \Delta_k \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\phi(t), \varphi(t)) + \frac{\partial Q}{\partial y}(\phi(t), \varphi(t))\right) dt\right]$$

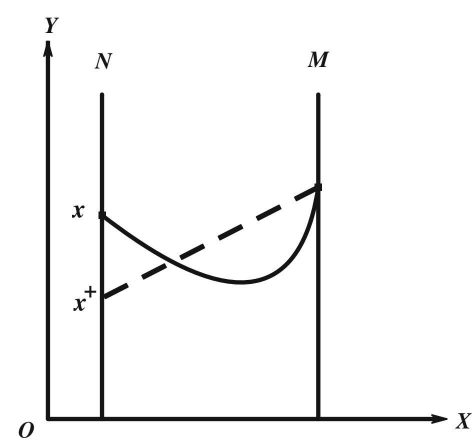

Fig. 1 Schematic diagram of the successor function of system (5)

with

$$\Delta_k = \frac{P_+(\frac{\partial\beta}{\partial y}\cdot\frac{\partial\Phi}{\partial x}-\frac{\partial\beta}{\partial x}\cdot\frac{\partial\Phi}{\partial y}+\frac{\partial\Phi}{\partial x})+Q_+(\frac{\partial\alpha}{\partial x}\cdot\frac{\partial\Phi}{\partial y}-\frac{\partial\alpha}{\partial y}\cdot\frac{\partial\Phi}{\partial x}+\frac{\partial\Phi}{\partial y})}{P\frac{\partial\Phi}{\partial x}+Q\frac{\partial\Phi}{\partial y}}$$

and $P, Q, \frac{\partial\alpha}{\partial x}, \frac{\partial\alpha}{\partial y}, \frac{\partial\beta}{\partial x}, \frac{\partial\beta}{\partial y}, \frac{\partial\Phi}{\partial x}, \frac{\partial\Phi}{\partial y}$ *are calculated at the point* $(\phi(t_k), \varphi(t_k))$ *and* $P_+ = P(\phi(t_k^+), \varphi(t_k^+))$, $Q_+ = Q(\phi(t_k^+), \varphi(t_k^+))$.

3 Qualitative analysis of system (4) without impulsive effect

We firstly consider system (4) without impulsive effect in this section. It implies that $\alpha, \tau, p, q = 0$. Then, we have a system in the form of

$$\begin{cases} x'(t) = rx\ln\frac{K}{x} - \beta xy, \\ y'(t) = \lambda\beta xy - dy. \end{cases} \tag{7}$$

Solving equations

$$\begin{cases} rx\ln\frac{K}{x} - \beta xy = 0, \\ \lambda\beta xy - dy = 0, \end{cases} \tag{8}$$

yields two equilibria: $A(K, 0)$ and $E(x^\iota, y^\iota)$ of system (7), where $x^\iota = \frac{d}{\lambda\beta}$, $y^\iota = \frac{r}{\beta}\ln\frac{\lambda\beta K}{d}$. Let $(H_1) : \lambda\beta K > d$, then, we have the following theorem.

Theorem 3.1 *System (7) has a positive equilibrium if and only if* (H_1) *holds.*

Next, we consider the stability of the equilibria. It is easy to see that system (7) has a Jacobian

$$J = \begin{pmatrix} r\ln\frac{K}{x} - r - \beta y & -\beta x \\ \lambda\beta y & \lambda\beta x - d \end{pmatrix}.$$

At $A(K, 0)$, we have

$$J(A) = \begin{pmatrix} -r & -\beta K \\ 0 & \lambda\beta K - d \end{pmatrix},$$

which implies that when (H_1) holds, $A(K, 0)$ is a saddle point. And at $E(x^\iota, y^\iota)$, we have

$$J(E) = \begin{pmatrix} -r & -\frac{d}{\lambda} \\ \lambda r\ln\frac{K\lambda\beta}{d} & 0 \end{pmatrix}.$$

The characteristic equation of $J(E)$ satisfies $f(\lambda) = \lambda^2 + a\lambda + b = 0$, where $a = r, b = rd\ln\frac{\lambda\beta K}{d}$. Obviously, $\Delta = a^2 - 4b = r^2 - 4rd\ln\frac{\lambda\beta K}{d}$, then we have the following conclusions:

(i) When $d < \lambda\beta K < de^{\frac{r}{4d}}$, $E(x^\iota, y^\iota)$ is a stable node;
(ii) When $\lambda\beta K = de^{\frac{r}{4d}}$, $E(x^\iota, y^\iota)$ is a stable critical node; and
(iii) when $\lambda\beta K > de^{\frac{r}{4d}}$, $E(x^\iota, y^\iota)$ is a stable focus.

Make an assumption $(H_2) : \lambda\beta K > de^{\frac{r}{4d}}$. Then, we have

Theorem 3.2 *If condition* (H_2) *holds true, the equilibrium E of system (7) is a stable focus.*

For system (7), we can prove the following.

Theorem 3.3 *The solution of system (7) is bounded.*

Proof Let initial conditions of system (7) be

$$\begin{cases} x(t_0) = x_0 > 0, \\ y(t_0) = y_0 > 0, \end{cases} \tag{9}$$

and $(x(t), y(t))$ be a solution of system (7) satisfied (9). Since point $A(K, 0)$ is a saddle point, for line $\Gamma_1 : x - K = 0$ passing through A, we have

$$\begin{aligned} \frac{d\Gamma_1}{dt}|_{\Gamma_1=0} &= rx(t)\ln\frac{K}{x(t)} - \beta x(t)y(t)|_{\Gamma_1=0} \\ &= -\beta Ky < 0. \end{aligned}$$

Thus, the line Γ_1 is a segment without contact and orbit of system (7) goes across it from the right. On the other hand, define in the first quadrant:

$$\Gamma_2 : y + \lambda x - M = 0, \Gamma_3 : y - M + \frac{d}{\beta} = 0.$$

Then, we have

$$\begin{aligned} \frac{d\Gamma_2}{dt}|_{\Gamma_2=0} &= y(\lambda\beta x - d) + \lambda x\left(r\ln\frac{K}{x} - \beta y\right)|_{\Gamma_2=0} \\ &= \lambda x\left(r\ln\frac{K}{x} + d\right) - Md, \\ \frac{d\Gamma_3}{dt}|_{\Gamma_3=0} &= y(\lambda\beta x - d). \end{aligned}$$

Thus, we have $\frac{d\Gamma_2}{dt} < 0$ for $\frac{d}{\lambda\beta} < x < K$ and $\frac{d\Gamma_3}{dt} < 0$ for $0 < x < \frac{d}{\lambda\beta}$, here M is large enough. So there exists an area Ω with a boundary being composed of $x = 0$, $y = 0$, $\Gamma_1\Gamma_2$ and Γ_3 such that $(x(t), y(t)) \in \Omega$ for initial point $(x(t_0), y(t_0))$ and $t > T$, where $T > 0$ is large. This completes the proof. □

Theorem 3.4 *If the condition* (H_2) *is true, the positive equilibrium E of system (7) is globally asymptotically stable.*

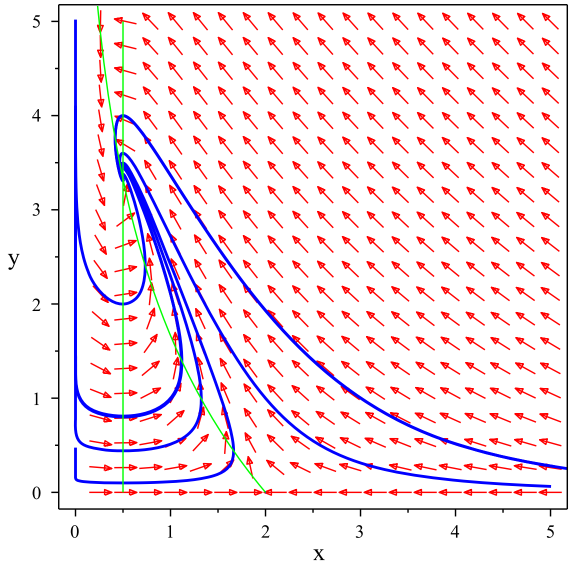

Fig. 2 Phase diagram of system (4) with $r = 1.2, K = 2, \beta = 0.5, \lambda = 1.6, d = 0.4$.

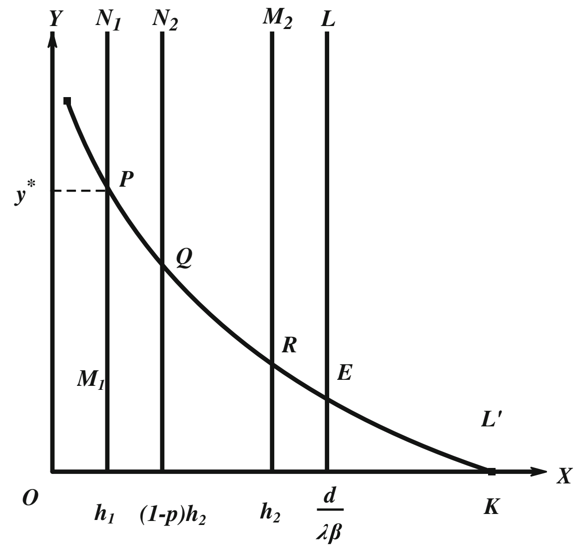

Fig. 3 The structure diagram of system (4)

Proof Let Dulac function $B = \frac{1}{xy}$, then, we have

$$D = \frac{\partial(PB)}{\partial x} + \frac{\partial(QB)}{\partial y} = -\frac{r}{xy} < 0.$$

According to the Bendixson-Dulac theorem, we confirm that there is no closed orbit around E. Furthermore, by theorems 3.2 and 3.3, we have that the solution of system (7) is bounded and the positive equilibrium point E of system (7) is locally stable. Thus, the positive equilibrium E of system (7) is globally asymptotically stable if the condition (H_2) is true (Fig. 2). □

Figure 7 shows that the positive equilibrium E of system (7) is a stable focus.

4 The order one periodic solutions of system (4) with impulsive state feedback control

4.1 The existence of order one periodic solutions of the system (4)

In this subsection, using the successor function, we study the existence of periodic solution of system (4). First setting $\dot{x} = 0, \dot{y} = 0$ yields: (1) the x-isolines, curve $L' : y = \frac{r}{\beta} \ln \frac{K}{x}$ and y-axis and (2) the y-isolines, curve $L : x = \frac{d}{\lambda\beta}$ and x-axis. Then, by Theorem 3.4, we know that $E(x^t, y^t)$ is a stable focus if (H_2) holds. For notation simplicity, we denote the first impulsive set by $M_1 = \{(x, y)|x = h_1, 0 \le y \le y^*\}$, the second by $M_2 = \{(x, y)|x = h_2, y \ge 0\}$ and the image sets corresponding to them by $N_1 = \{(x, y)|x = h_1, \alpha \le y \le y^* + \alpha\}$ and $N_2 = \{(x, y)|x = (1 - p)h_2, y \ge \tau\}$, respectively. In order to have physical meaningful, we restrict our study to the region lying in the left side of E, that is $h_1 < (1 - p)h_2 < h_2 < \frac{d}{\lambda\beta}$. Next, we investigate the trajectory of system (4), which passes the initial point $C_0(x_{c_0}, y_{c_0})$ (Fig. 3).

4.1.1 *The path curve beginning from C_0, a point above P on N_1*

Without loss of generality, we assume that C_0 satisfies $y^* < y_{C_0} \le y^* + \alpha$. In fact, if the path curve starting from C_0 intersects with M_1 at point $C_1(x_{c_1}, y_{c_1})$, and then produces pulse to point C_1^+ on M_1 such that $y_{C_1^+} < y^*$, then from (4), we have $y_{C_1^+} = y_{C_1} + \alpha$. Then after finite number of times impulse, point C_1^+ satisfies $y^* < y_{C_1^+} \le y^* + \alpha$. Thus, in what follows, we only consider the case of C_0 such that $y^* < y_{C_0} \le y^* + \alpha$. Then, we have three sub cases to be discussed.

Case I: the impulsive point C_1^+ overlaps with the initial point C_0. Then, the curve $C_0C_1C_1^+$ constitutes

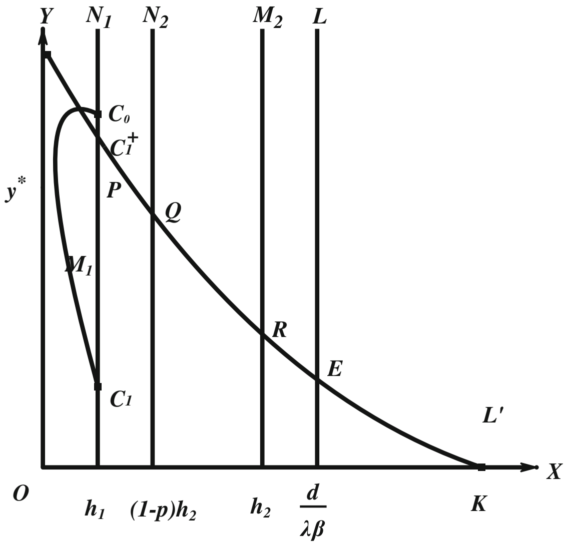

Fig. 4 The path curve beginning from C_0, a point above P on N_1 (Case I in Sect. 4.1.1)

a periodic path curve of system (4), please see Fig. 4.

Case II: the impulsive point C_1^+ is below C_0 on N_1. Then, we obtain $s(C_0) < 0$. Now, we choose a point D_0 next to P on N_1 (that is $|D_0P| < \varepsilon$). The path curve of system (4) beginning from point D_0 intersects with M_1 at point D_1, and then an impulse happens, D_1 jumps to point D_1^+ on the line N_1. Because D_0 is near to P, D_1 is near to P, and $y_{D_1^+} = y_{D_1} + \alpha$, we have $s(D_0) > 0$. By Theorem 2.2, there exists an order one period solution (see Fig. 5).

Case III: the impulsive point C_1^+ is above C_0 on N_1. In this case, we have $s(C_0) > 0$. Because the path curve does not produce pulse at $A(h_1, y^* + \alpha)$, we know $s(A) < 0$. Then, the Poincaré-Bendixson theorem implies the existence of a periodic path curve of system (4) (see Fig. 6).

4.1.2 *The path curve beginning from the point Q*

Now consider a path curve starting from a point Q on N_2. Then, it intersects with M_2 at point C_1 and produces pulse to point C_1^+ on N_2. According to system (4), the following is obtained

$$\begin{cases} x_{C_1^+} = (1-p)h_2, \\ y_{C_1^+} = (1-q)y_{C_1} + \tau. \end{cases}$$

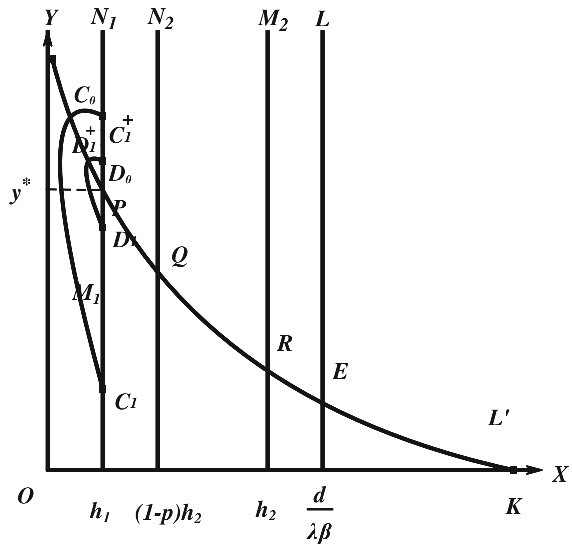

Fig. 5 The path curve beginning from the point C_0 (Case II in Sect. 4.1.1)

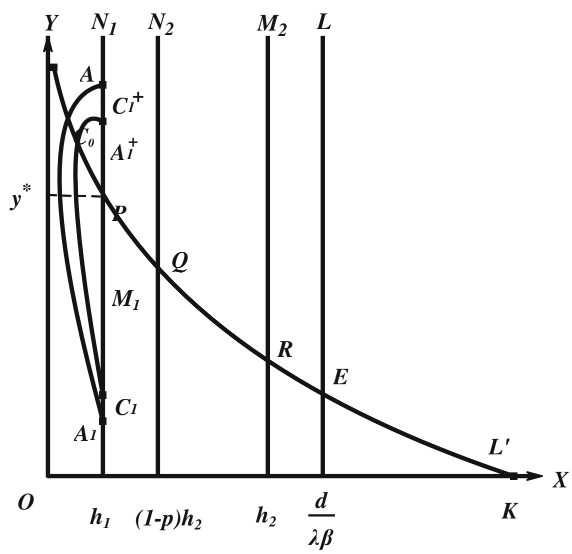

Fig. 6 The path curve beginning from the point C_0 (Case III in Sect. 4.1.1)

For different τ, three cases should be discussed.

Case I: the impulsive point C_1^+ is exactly Q. Here the curve $QC_1C_1^+$ constitutes a periodic path curve of system (4) (see Fig. 7).

Case II: the impulsive point C_1^+ is above Q on N_2. It implies that $s(Q) = y_{C_1^+} - y_Q > 0$. Now we choose D_0, a point above C_1^+ on N_2. The path curve starting from D_0 is vertical only when it intersects with N_1

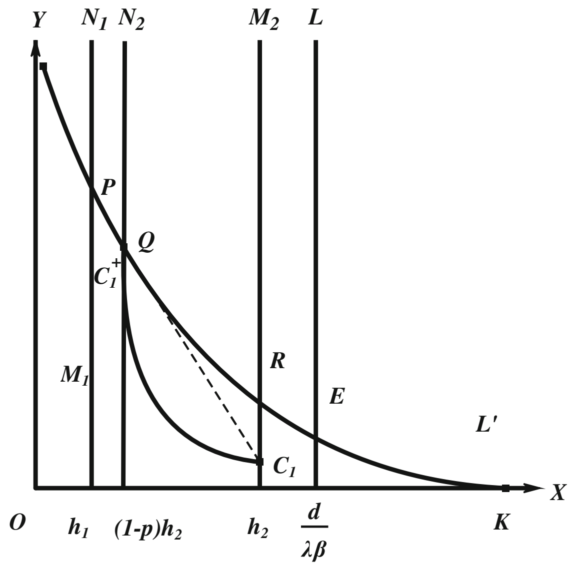

Fig. 7 The path curve beginning from the point Q (Case I in Sect. 4.1.2)

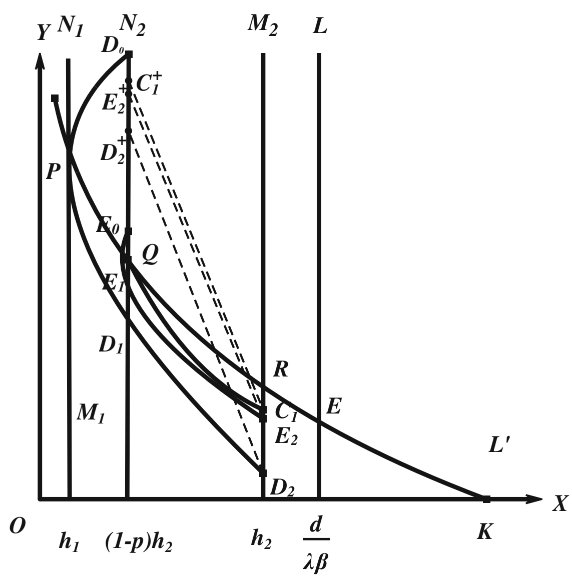

Fig. 8 The path curve beginning from the point Q (Case II in Sect. 4.1.2)

at P. Then it crosses N_2 from left to right, and then intersects with M_2 at point D_2 and pulses to point D_2^+ of the line N_2. By the existence and uniqueness theorem for impulsive differential equations, D_2 is below C_1 of M_2, and D_2^+ must below C_1^+ on N_2. Therefore, we have $s(D_0) = y_{D_2^+} - y_{D_0} < 0$. By Theorem 2.2, there exists order one periodic solution of system (4) in region $D_0 P D_1 D_2 C_1 C_1^+ D_0$ (see Fig. 8).

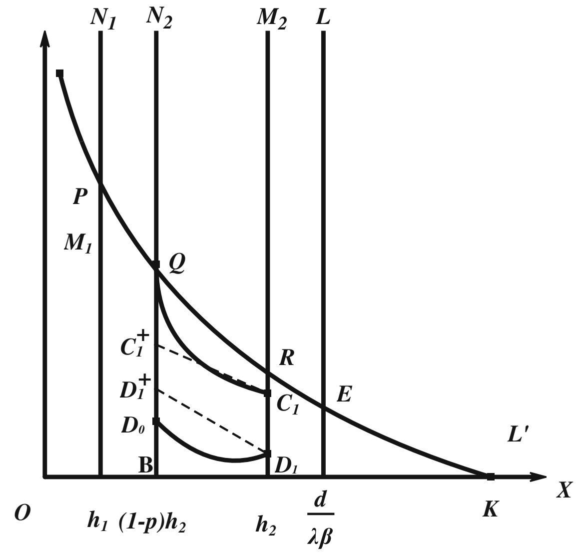

Fig. 9 The path curve beginning from the point Q (Case III in Sect. 4.1.2)

Case III: In this case, the impulsive point C_1^+ is below Q on N_2. Then we have $s(Q) = y_{C_1^+} - y_Q < 0$. Pick a point D_0 next to $B((1-p)h_2, 0)$ on N_2 ($|D_0 B| << \varepsilon$.). Starting from D_0, the path curve intersects with M_2 at point D_1, and then produces pulse to point D_1^+ on N_2. From system (4), we have

$$\begin{cases} x_{D_1^+} = (1-p)h_2, \\ y_{D_1^+} = (1-q)y_{D_1} + \tau. \end{cases}$$

Thus, D_1^+ has to be above D_0, and the successor function of D_0 satisfies $s(D_0) = y_{D_1^+} - y_{D_0} > 0$. As a result, the region κ surrounded by the closed curve $D_0 D_1 C_1 C_1^+$ involves a periodic solution of system (4) (see Fig. 9).

4.1.3 Initial point C_0 of path curve on N_1 falling in between the second impulsive set M_2 and its image set N_2.

In this section, we assume the path curve intersects with line M_2 at C_1, and then jumps onto point C_1^+ on N_2. According to values of τ, we have the following cases.

Case I: Point C_1^+ is point Q, and the path curve from point C_1^+ moves to point C_2 on M_2. For point C_2, we have three cases.

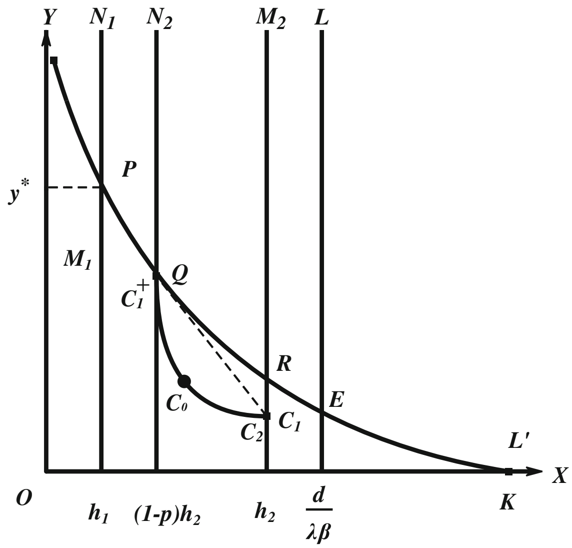

Fig. 10 The path curve beginning from the point C_0 (Case I(a) in Sect. 4.1.3)

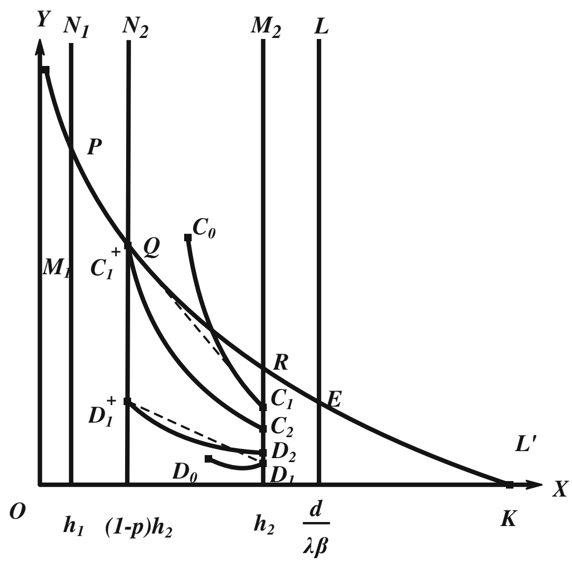

Fig. 11 The path curve beginning from the point C_0 (Case I(b) in Sect. 4.1.3)

Case I(a): Point C_2 is exactly point C_1. In this case, obviously the curve $C_1C_1^+C_2$ forms a periodic path curve of system (4) (see Fig.10).

Case I(b): Point C_2 is below point C_1. Then, it is easy to see the successor function of C_1 that is negative, namely $s(C_1) = y_{C_2} - y_{C_1} < 0$. Next, in the region between N_2 and M_2, we pick a point D_0 next to x-axis. Then, the path curve passing D_0 hits a point D_1

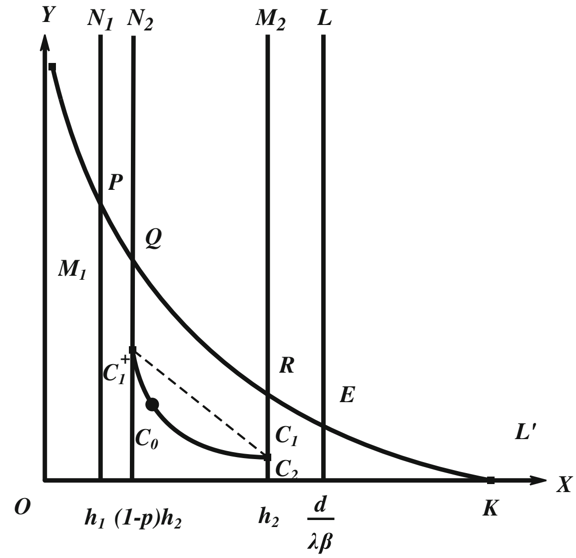

Fig. 12 The path curve beginning from the point C_0 (Case II in Sect. 4.1.3)

on M_2, and then jumps onto N_2 at a point D_1^+, from which jumps to a point D_2 on M_2. It is easy to verify that D_2 is above D_1, which implies the successor function, $s(D_1) = y_{D_2} - y_{D_1} > 0$. Then, there is a point C between C_1 and D_1 such that $s(C) = 0$, which implies the existence of a periodic path curve in the region enclosed by curve $C_1C_1^+D_1^+D_2$ (see Fig. 11).

Case I(c): Point C_2 is above C_1. In this case, the periodic path curve does not exist. Otherwise the curves $C_1^+C_2$ and C_0C_1 intersect each other, which conflict with the existence and uniqueness of solutions of impulsive differential equations.

Case II: The point C_1^+ is below Q. In this case, using the similar argument as above yields the same conclusion (see Figs. 12 and 13).

Case III: Point C_1^+ on N_2 is above Q. Then, we have two cases to be investigated.

Case III(a): the path curve starting from C_1^+ crosses N_1. Then, it becomes vertical only for going through the line L'. Using C_2 to denote the intersection of the path curve and N_1, then same conclusion can be made as what we did in Sect. 4.1.1 (see Fig. 14).

Case III(b): the path curve starting from C_1^+ does not touch N_1. Then, the path curve becomes vertical only when it crosses PQ. And then it goes back to M_2, and if we denote the intersection by C_2, then C_2 is one of the three points: point C_1, a point below C_1, or a point above C_1.

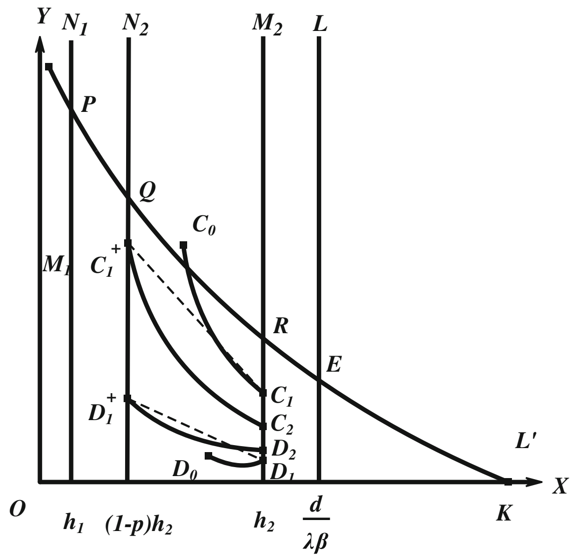

Fig. 13 The path curve beginning from the point C_0 (Case II in Sect. 4.1.3)

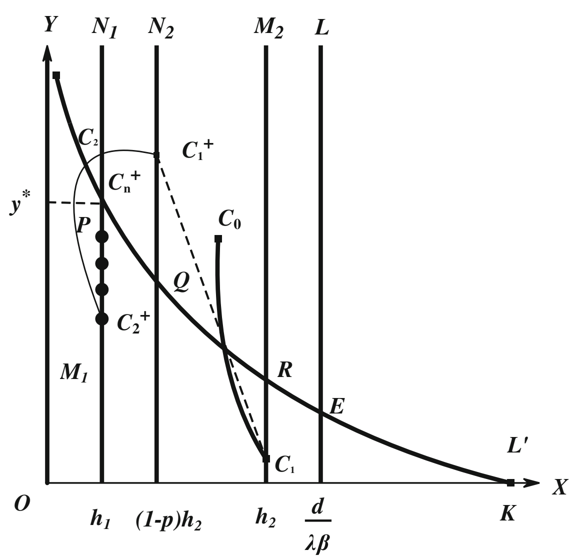

Fig. 14 The path curve beginning from the point Q (Case III(a) in Sect. 4.1.3)

If point C_2 is C_1, then the curve $C_1C_1^+C_2$ forms a periodic path curve (see Fig. 15).

If C_2 is below C_1, the successor function $s(C_1) = y_{C_2} - y_{C_1} < 0$. Then, we can select a point D_0 between N_2 and M_2 near to x-axis such that the path curve beginning from D_0 hits the point D_1 on M_2, and then jumps onto the point D_1^+ on N_2, and then returns to the point D_2 on M_2, and the successor function of D_1 is

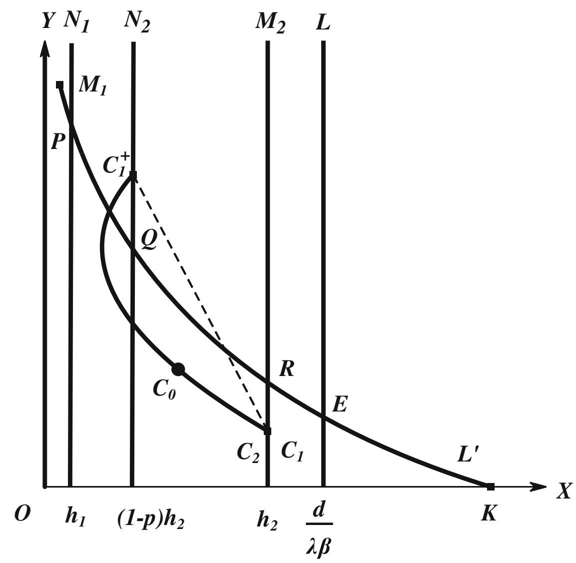

Fig. 15 The path curve beginning from the point C_0 (Case III(b) in Sect. 4.1.3)

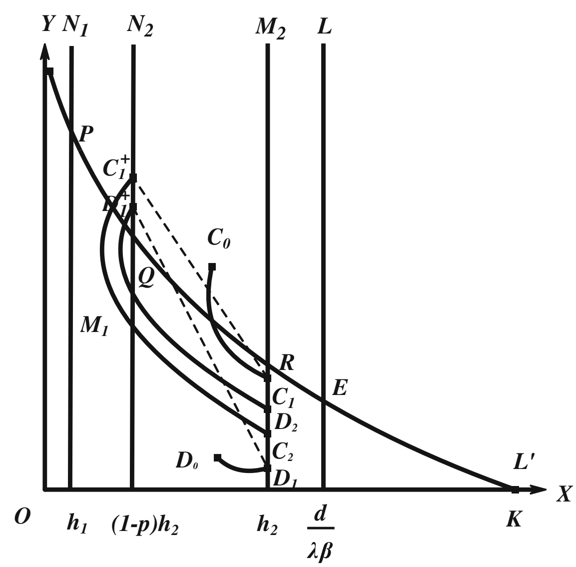

Fig. 16 The path curve beginning from the point C_0 (Case III(b) in Sect. 4.1.3)

$s(D_1) = y_{D_2} - y_{D_1} > 0$. It implies that there exists a periodic path curve in the region enclosed by curves $D_1C_1C_1^+C_2$ and $C_1^+D_1^+D_2C_2$ (see Fig. 16).

If C_2 is above C_1, the periodic path curve does not exist as the Case I(c) in Sect. 4.1.3.

Now, we have proved the existence of the periodic solution, next we study the stability of it.

4.2 The stability of order one periodic solutions of the system (4)

4.2.1 The stability of order one periodic solutions on impulsive set M_1

Theorem 4.1 *Let $(\phi(t), \varphi(t))$ be a periodic solution of system (4) with $\phi_0 = \phi(0) = h_1, \varphi_0 = \varphi(0), \phi_1 = \phi(T), \varphi_1 = \varphi(T), \phi_1^+ = \phi(T^+), \varphi_1^+ = \varphi(T^+)$. Then, the periodic solution is stable when $\varphi_0 < \frac{r}{\beta} \ln \frac{K}{h_1}$.*

Proof Let $(\phi(t), \varphi(t))$ be a periodic solution of system (4) with $\phi_0 = \phi(0) = h_1, \varphi_0 = \varphi(0), \phi_1 = \phi(T), \varphi_1 = \varphi(T), \phi_1^+ = \phi(T^+), \varphi_1^+ = \varphi(T^+)$. By system (4), we have $\phi_1 = \phi(T) = h_1, \varphi_1 = \varphi(T) = \varphi_0 - \alpha, \phi_1^+ = \phi(T^+) = h_1, \varphi_1^+ = \varphi(T^+) = \varphi_0$.

According to Theorem 2.3, let

$$\begin{cases} P(x, y) = x(r \ln \frac{K}{x} - \beta y), \\ Q(x, y) = y(\lambda\beta x - d), \\ \alpha(x, y) = 0, \\ \beta(x, y) = \alpha, \\ \Phi(x, y) = x - h_1, \end{cases} \tag{10}$$

and notice that

$$\frac{\partial\alpha}{\partial x} = \frac{\partial\alpha}{\partial y} = 0, \frac{\partial\beta}{\partial x} = \frac{\partial\beta}{\partial y} = 0, \frac{\partial\Phi}{\partial x} = 1, \frac{\partial\Phi}{\partial y} = 0, \tag{11}$$

we have

$$\begin{aligned} \int_0^T \frac{\partial P}{\partial x} \mathrm{d}t &= \int_0^T \left(r \ln \frac{K}{x} - \beta y - r\right) \mathrm{d}t, \\ &= \int_0^T \left(r \ln \frac{K}{x} - \beta y\right) \mathrm{d}t - rT, \\ &= \ln\left(\frac{\phi_1}{\phi_0}\right) - rT, \\ &= \ln \frac{\varphi_0 - \alpha}{\varphi_0} - rT, \\ \int_0^T \frac{\partial Q}{\partial y} \mathrm{d}t &= \int_0^T (\lambda\beta x - d)\, \mathrm{d}t, \\ &= \ln\left(\frac{\varphi_1}{\varphi_0}\right). \end{aligned}$$

Using Theorem 2.3 again and some algebraic manipulations, we reach the following

$$\begin{aligned} \Delta_1 &= \frac{P_+ \cdot \left(\frac{\partial\beta}{\partial y} \cdot \frac{\partial\Phi}{\partial x} - \frac{\partial\beta}{\partial x} \cdot \frac{\partial\Phi}{\partial y} + \frac{\partial\Phi}{\partial x}\right) + Q_+ \cdot \left(\frac{\partial\alpha}{\partial x} \cdot \frac{\partial\Phi}{\partial y} - \frac{\partial\alpha}{\partial y} \cdot \frac{\partial\Phi}{\partial x} + \frac{\partial\Phi}{\partial y}\right)}{P\frac{\partial\Phi}{\partial x} + Q\frac{\partial\Phi}{\partial y}} \\ &= \frac{P(\phi(T^+), \varphi(T^+))}{P(\phi(T), \varphi(T))} \\ &= \frac{P(h_1, \varphi_0)}{P(h_1, \varphi_0 - \alpha)} \\ &= \frac{r \ln \frac{K}{h_1} - \beta\varphi_0}{r \ln \frac{K}{h_1} - \beta(\varphi_0 - \alpha)} \end{aligned}$$

and

$$\begin{aligned} \mu_2 &= \Delta_1 \exp\left\{\int_0^T \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \mathrm{d}t\right\} \\ &= \frac{r \ln \frac{K}{h_1} - \beta\varphi_0}{r \ln \frac{K}{h_1} - \beta(\varphi_0 - \alpha)} \exp\left\{\ln\left(\frac{\phi_1}{\phi_0}\right) + \ln\left(\frac{\varphi_1}{\varphi_0}\right) - rT\right\} \\ &= \frac{r \ln \frac{K}{h_1} - \beta\varphi_0}{r \ln \frac{K}{h_1} - \beta(\varphi_0 - \alpha)} \frac{\varphi_0 - \alpha}{\varphi_0} \mathrm{e}^{-rT}. \end{aligned}$$

Obviously, if $\varphi_0 < \frac{r}{\beta} \ln \frac{K}{h_1}$, we have $r \ln \frac{K}{h_1} - \beta\varphi_0 > 0$, we have $|\mu_2| < 1$, which implies that the periodic solution is stable. □

4.2.2 The stability of order one periodic solutions with initial point C_0 on N_2

Again, let $\Pi(C_0, t)$ be the closed path curve beginning from $C_0((1 - p)h_2, \varphi_0)$. Then, by property of system (4), the path curve intersects with M_2 with the intersection to be denoted by $C_1(\phi(T), \varphi(T))$, which jumps to $C_1^+(\phi(T^+), \varphi(T^+))$. That is to say that $\Pi(C_0, T) = C_1, C_1^+ = I(C_1) = C_0$, where $\phi(T^+) = (1 - p)\phi(T), \varphi(T^+) = (1 - q)\varphi(T) + \tau$ and $\phi(T) = h_2, \varphi(T) = \frac{\varphi_0 - \tau}{1 - q}$. Let

$$\begin{cases} P(x, y) = x(r \ln \frac{K}{x} - \beta y), \\ Q(x, y) = y(\lambda\beta x - d), \\ \alpha(x, y) = -px, \\ \beta(x, y) = -qx + \tau, \\ \Phi(x, y) = x - h_2. \end{cases}$$

Calculating partial derivatives, one gets

$$\begin{cases} \frac{\partial\alpha}{\partial x} = -p, \frac{\partial\alpha}{\partial y} = 0, \\ \frac{\partial\beta}{\partial x} = 0, \frac{\partial\beta}{\partial y} = -q, \\ \frac{\partial\Phi}{\partial x} = 1, \frac{\partial\Phi}{\partial y} = 0. \end{cases}$$

It implies

$$\begin{aligned}\Delta_1 &= \frac{P_+\left(\frac{\partial\beta}{\partial y}\frac{\partial\Phi}{\partial x} - \frac{\partial\beta}{\partial x}\frac{\partial\Phi}{\partial y} + \frac{\partial\Phi}{\partial x}\right) + Q_+\left(\frac{\partial\alpha}{\partial x}\frac{\partial\Phi}{\partial y} - \frac{\partial\alpha}{\partial y}\frac{\partial\Phi}{\partial x} + \frac{\partial\Phi}{\partial y}\right)}{P\left(\frac{\partial\Phi}{\partial x}\right) + Q\left(\frac{\partial\Phi}{\partial y}\right)} \\ &= \frac{P(\phi(T^+), \varphi(T^+))(1-q)}{P(\phi(T), \varphi(T))} \\ &= \frac{P(\phi_0, \varphi_0)(1-q)}{P(\phi(T), \varphi(T))} \\ &= \frac{(1-p)h_2\left(r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0\right)(1-q)}{h_2(r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q})} \\ &= \frac{(1-p)(1-q)\left(r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0\right)}{r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q}}.\end{aligned}$$

Thus,

$$\begin{aligned}\mu_2 &= \Delta_1 \exp\left\{\int_0^T \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\mathrm{d}t\right\} \\ &= \frac{(1-p)(1-q)\left(r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0\right)}{r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q}} \\ &\quad\times \exp\left\{\ln(\frac{\phi_1}{\phi_0}) + \ln(\frac{\varphi_1}{\varphi_0}) - rT\right\} \\ &= \frac{(1-p)(1-q)\left(r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0\right)}{r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q}} \\ &\quad\times \frac{\frac{\phi_0}{1-p}}{\phi_0}\frac{\frac{\varphi_0-\tau}{1-q}}{\varphi_0}\mathrm{e}^{-rT} \\ &= \frac{r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0}{r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q}}\frac{\varphi_0-\tau}{\varphi_0}\mathrm{e}^{-rT}.\end{aligned}$$

Therefore, if $|\frac{r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0}{r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q}}\frac{\varphi_0-\tau}{\varphi_0}| < 1$, we have $|\mu_2| < 1$. Then, we can summarise our analysis in the following.

Theorem 4.2 *If* $|\frac{r\ln\frac{K}{(1-p)h_2} - \beta\varphi_0}{r\ln\frac{K}{h_2} - \beta\frac{\varphi_0-\tau}{1-q}}\frac{\varphi_0-\tau}{\varphi_0}| < 1$, *the periodic solution of system (4) is stable.*

5 An example and numerical simulations

In this section, we will give an example to verify the previous theoretical results. Let $r = 1.2, K = 2, \beta = 0.5, \lambda = 1.6, d = 0.4, p = 0.5, q = 0.2, \alpha = 0.5, \tau = 0.2, h_1 = 0.2, h_2 = 0.45$. By calculation, we obtain $y^* = 5.5262$, then we get $P(0.2, 5.5262)$, $Q(0.225, 5.2435)$ and $E(x^\iota, y^\iota) = (0.5, 3.3271)$. Then system (4) becomes

$$\begin{cases} \left.\begin{aligned} x'(t) &= 1.2x(t)\ln\frac{2}{x(t)} - 0.5x(t)y(t), \\ y'(t) &= 0.8x(t)y(t) - 0.4y(t), \end{aligned}\right\} \\ x \neq 0.2, \quad 0.45, \, or \;\; x = 0.2, y > 5.5262, \\ \left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= 0.5, \end{aligned}\right\} x = 0.2, y \leq 5.5262, \\ \left.\begin{aligned} \Delta x(t) &= -0.5x(t), \\ \Delta y(t) &= -0.2y(t) + 0.2, \end{aligned}\right\} x = 0.45. \end{cases} \tag{12}$$

Numerical analysis of system (12) is being done using Maple 14.0. We have the following cases.

5.1 The path curve beginning from C_0, a point above P on N_1 (corresponding to Sect. 4.1.1)

In this section, we set $C_0 = (0.2, 6)$ to guarantee that it is above $P(0.2, 5.5262)$.

Case I: The impulsive point C_1^+ corresponding to C_1 is exactly C_0, thus the curve $C_0C_1C_1^+$ shall constitute a periodic path curve (Fig. 17).

Case II: The impulsive point C_1^+ corresponding to C_1 is below C_0 (Fig. 18).

Case III: The impulsive point C_1^+ corresponding to C_1 is above C_0 (Fig. 19).

5.2 The path curve beginning from point Q (corresponding to Sect. 4.1.2)

In this case, we let C_0 be $Q(0.225, 5.2435)$.

Case I: The impulsive point C_1^+ corresponding to C_1 is exactly Q (Fig. 20).

Case II: The impulsive point C_1^+ corresponding to C_1 is above Q on N_2 (Fig. 21).

Case III: The impulsive point C_1^+ corresponding to C_1 is below Q on N_2 (Fig. 22).

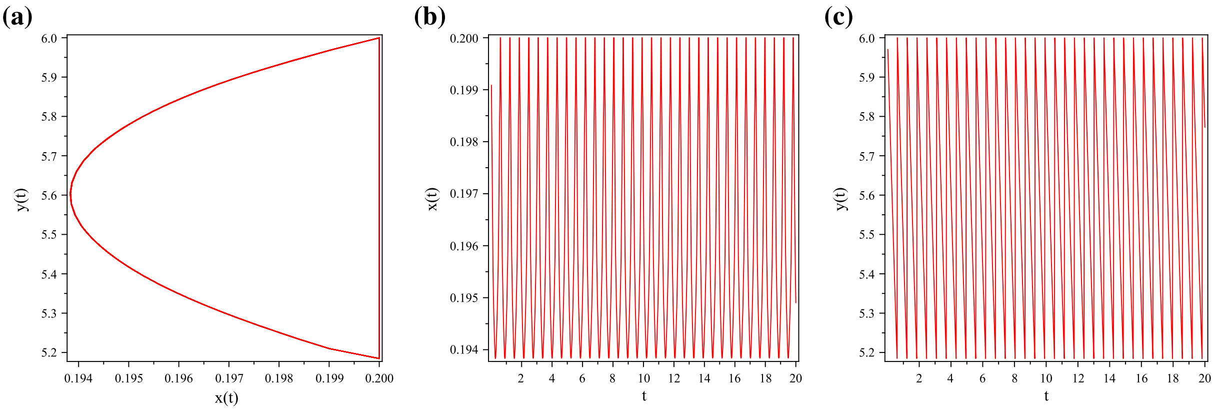

Fig. 17 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

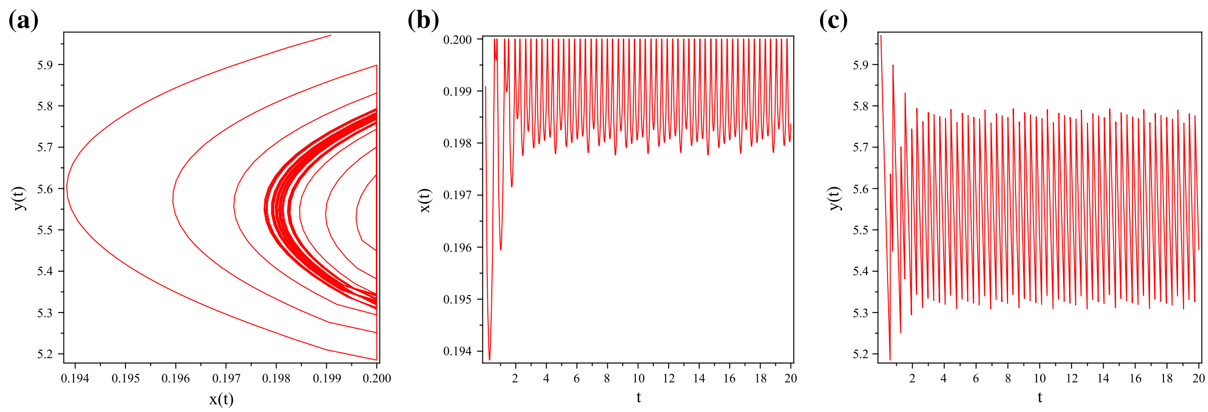

Fig. 18 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

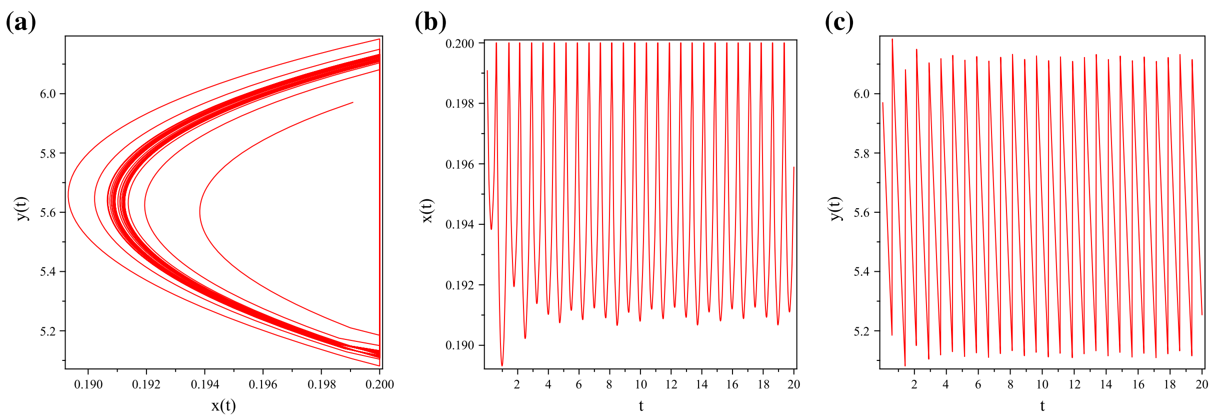

Fig. 19 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

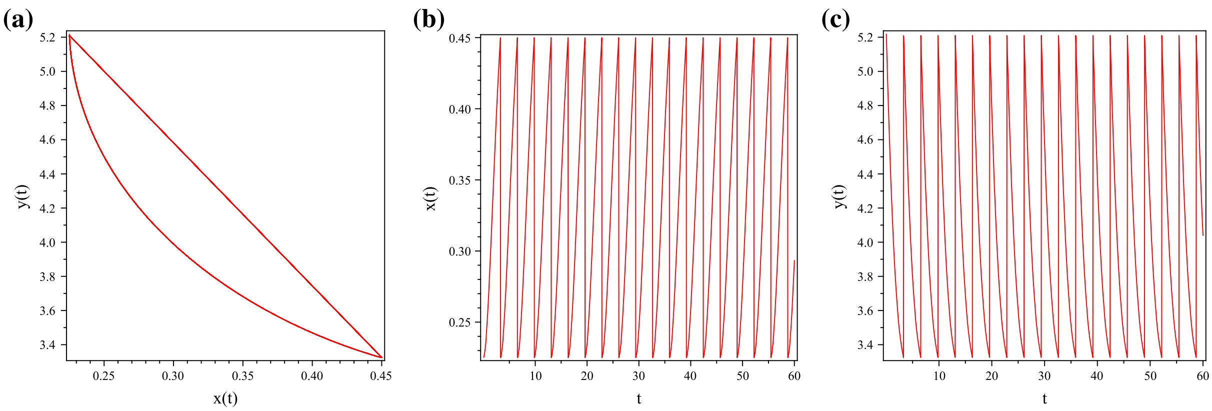

Fig. 20 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

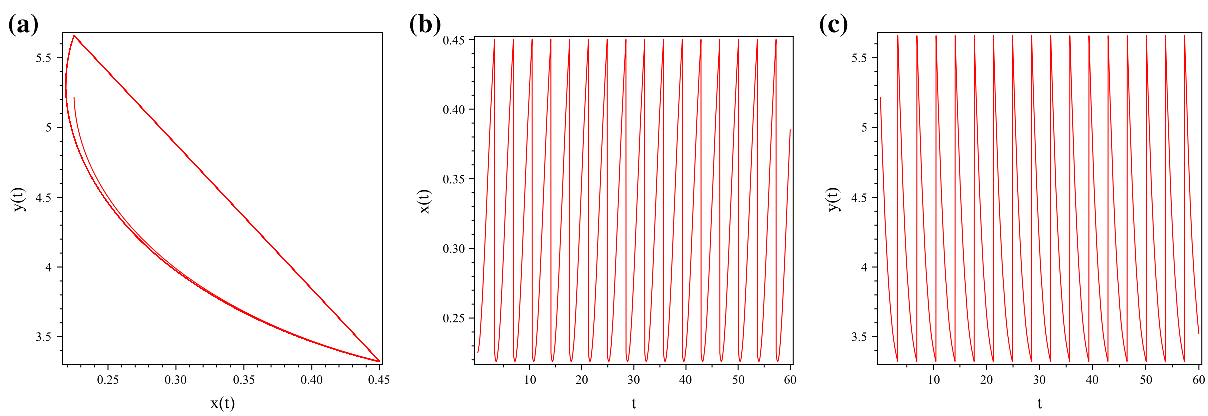

Fig. 21 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

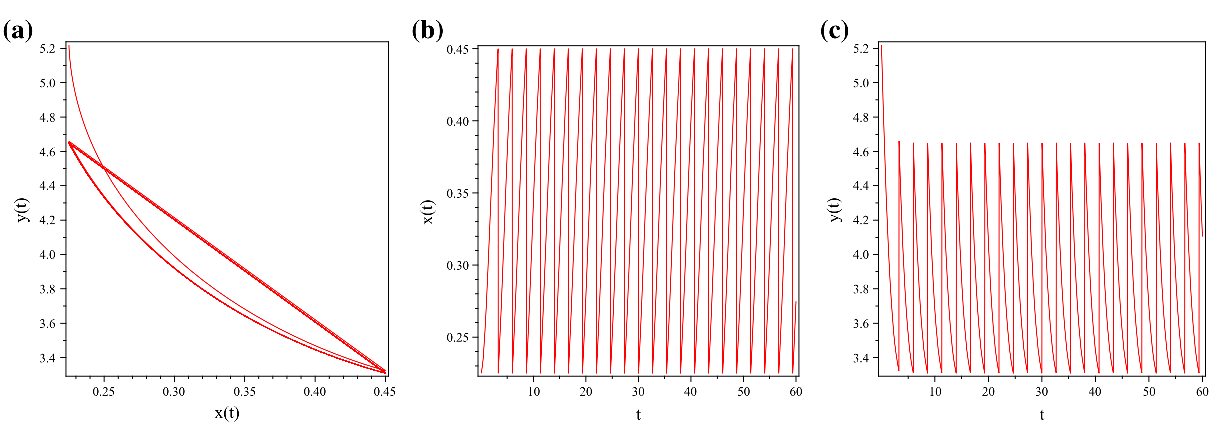

Fig. 22 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

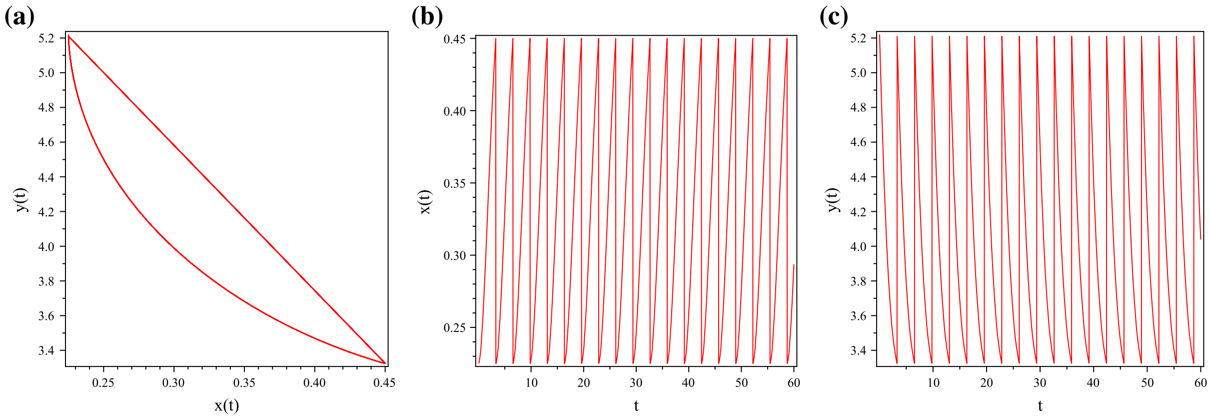

Fig. 23 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

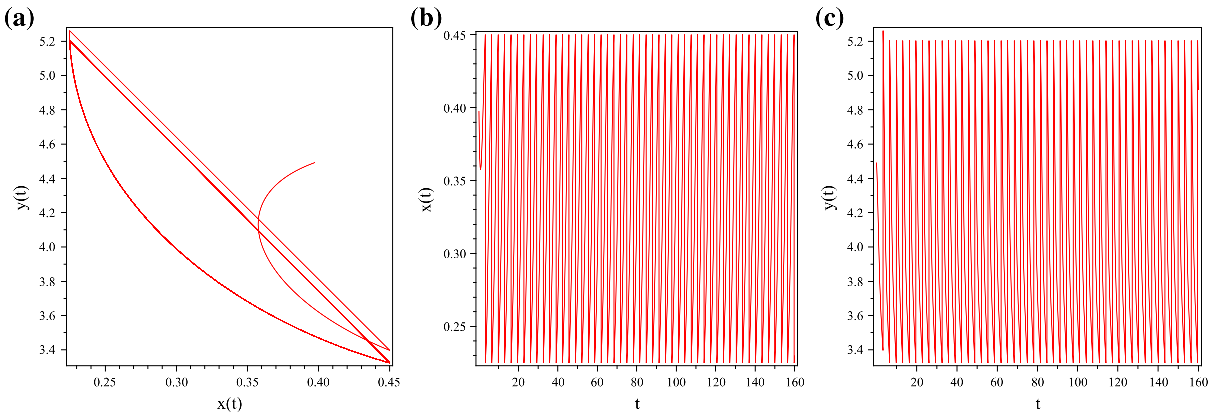

Fig. 24 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

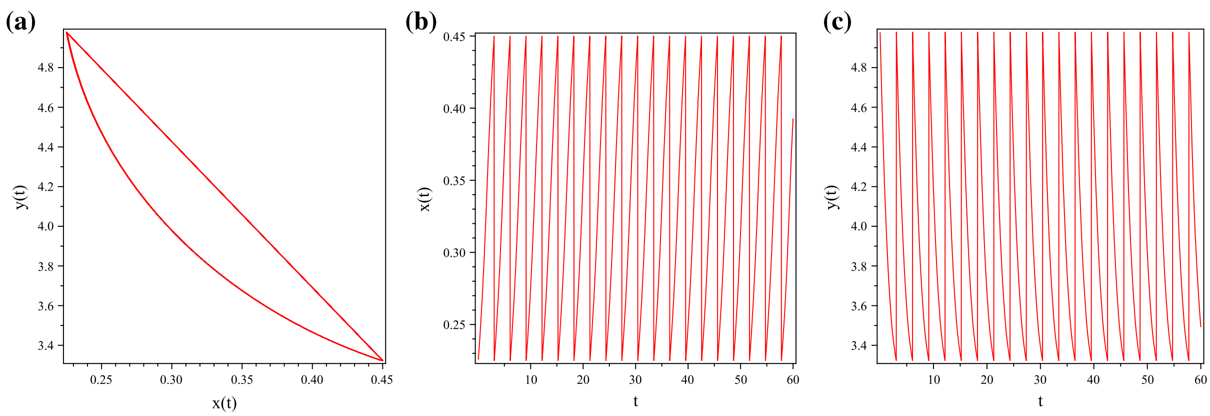

Fig. 25 The results of numerical simulation of system (12). **a** Phase diagram of system (**12**). **b** Time series of system (12). **c** Time series of system (12)

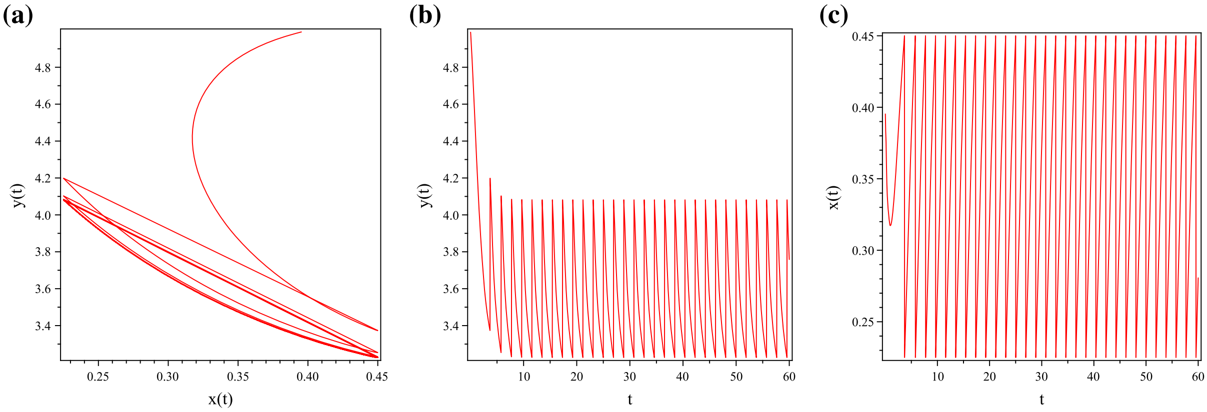

Fig. 26 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

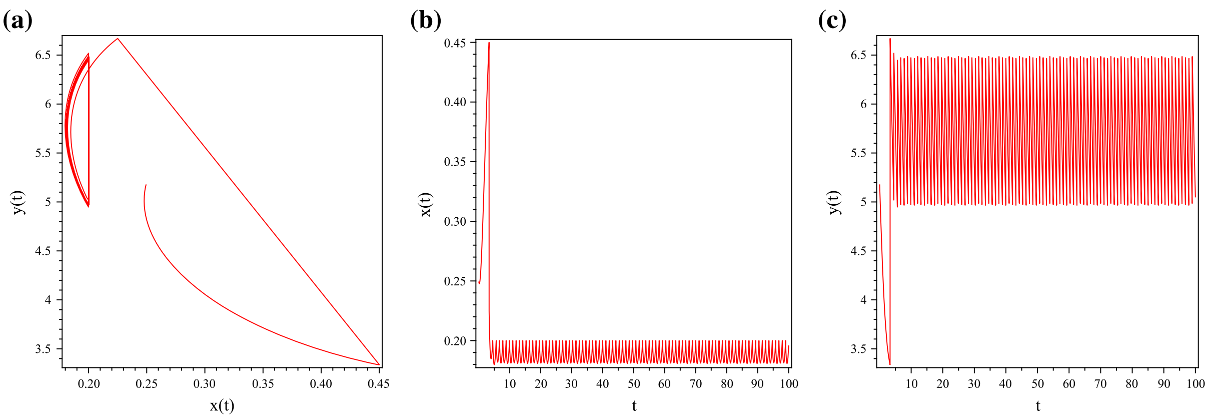

Fig. 27 The results of numerical simulation of system (12). (a) Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

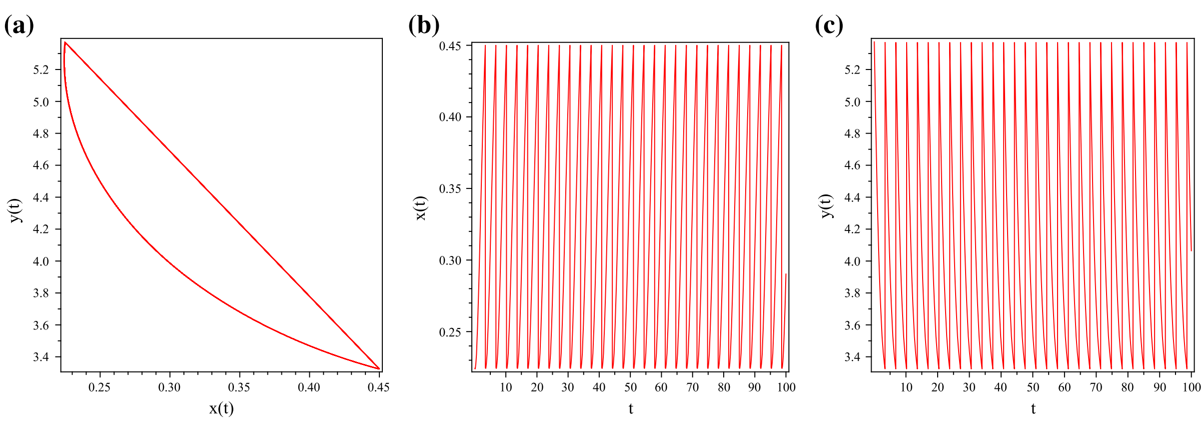

Fig. 28 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

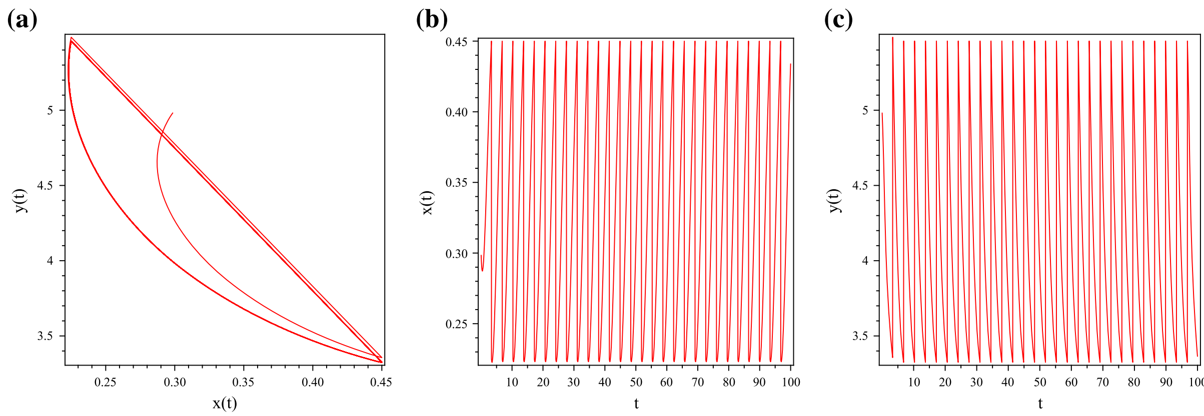

Fig. 29 The results of numerical simulation of system (12). **a** Phase diagram of system (12). **b** Time series of system (12). **c** Time series of system (12)

5.3 The path curve with initial point C_0, which is between the second impulsive set M_2 and its image set N_2 (corresponding to Sect. 4.1.3)

In this section, we choose $C_0 = Q(0.225, 0.9796)$.

Case I: If C_1^+ on N_2 is exactly Q, the path curve from point C_1^+ moves to the point C_2 on M_2.

Case I(a): C_2 is exactly C_1 (Fig. 23).

Case I(b): The impulsive point C_1^+ corresponding to C_1 is below Q on N_2 (Fig. 24).

Case I(c): The impulsive point C_1^+ corresponding to C_1 is above Q on N_2.

Case II: If C_1^+ on N_2 is below Q (Figs. 25 and 26).

Case III: If C_1^+ on N_2 is above Q.

Case III(a): The path curve beginning from C_1^+ crosses N_1 from the right to the left (Fig. 27).

Case III(b): The path curve beginning from C_1^+ moves on automatically to C_2 on M_2, which is exactly C_1 (28) or below C_1 (see Fig. 29).

All the numerical simulation above show agreement with our theoretical results.

6 Conclusion

In this paper, we formulated a mathematical model for pest management purpose, which is with impulsive state feedback control. By feedback information of pests' density from the monitor, we can control the pests with artificial disturbance. Firstly, we qualitatively investigated the dynamic behaviour of the system without impulsive effect, and obtained the sufficient condition for globally asymptotically stable of the positive equilibrium. And then the system with impulsive state feedback control was studied by geometric theory of impulsive differential equations. The existence and stability of order one periodic solution were proved. All the results suggested that the pest management model finally showed periodicity or steady state under impulsive state feedback control. We found that the density of the pest plays an important role in the periodic or stable state of system. When the density of the pest reaches an appropriate critical value, the state feedback measure including spraying pesticide and releasing natural enemy to control density of the pest would be taken. Theoretical derivation and numerical simulations show that the artificial intervention measures are effective. The reasonable selective feedback value can not only make the pest density in a controlled range but reduce the usage amount of pesticides for the ecological balance.

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