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ORIGINAL PAPER

Periodic solution of the system with impulsive state feedback control

Guoping Pang · Lansun Chen

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Abstract The order-1 periodic solution of the system with impulsive state feedback control is investigated. We get the sufficient condition for the existence of the order-1 periodic solution by differential equation geometry theory and successor function. Further, we obtain a new judgement method for the stability of the order-1 periodic solution of the semi-continuous systems by referencing the stability analysis for limit cycles of continuous systems, which is different from the previous method of analog of Poincarè criterion. Finally, we analyze numerically the theoretical results obtained.

Keywords Impulsive state feedback control · Order-1 periodic solution · Existence · Stability · Semi-continuous system

1 Introduction

In real life, there are many control methods such as feedback control with a time delay, sliding mode control, repetitive control, iterative learning control, adaptive control, coefficient control [1,2], but in pest management, the impulsive state feedback control, i.e., impulsively throwing natural enemies and spraying pesticides, is usually proposed [3–5] when pest density increases to the certain level called economic threshold (ET). ET is the index of pest density, crop output will not decrease much when pest density is lower than ET, we need not adopt any control measure, but once pest density rises to ET, some control measures should be adopted to prevent an increasing pest population from reaching the economic injury level.

Formerly, for the periodic impulsive system, many experts investigate the existence and stability of periodic solution by using Floquet theory and Poincarè criterion [6–8]. There is no doubt that is a well method, but in the paper, we will study the existence and stability of periodic solution for impulsive system with state feedback control by applying differential equation geometry theory which is different from Floquet theory.

A continuous delayed pest management system with Logistic growth and impulsive state feedback control is constructed as follows:

$$\begin{cases} \frac{dx}{dt} = rx\left[1 - cx - \omega \int\limits_{-\infty}^{t} \exp(a(t-s)x(s))ds\right], & x < h, \\ \Delta x = -\beta x, & x = h, \end{cases} \tag{1.1}$$

where $x(t)$ denotes the proportion of pest at time t, r denotes the intrinsic rate of increase, a, c, ω are positive constants, $0<\beta<1$ is the ratio of killing pests by spraying pesticides, h denotes ET. This model describes that in productive practice for controlling pests, people always take such a strategy that when the pests arrive

G. Pang (✉) · L. Chen
College of Mathematics and Information Science, Yulin Normal University, Yulin 537000, Guangxi, China
e-mail: g.p.pang@163.com

L. Chen
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, China

at a given ET h, they will begin to kill the pests with chemical pesticides.

The first equation of system (1.1) is continuous, but under impulsive effect and state feedback control, system (1.1) turns into a special discontinuous system, for which there are many properties of continuous dynamic system. The discontinuous dynamical system was called the switching system by Luo [9] and Chen [10] called the system as semi-continuous system (for details, see Definition 2.1).

At present, for the semi-continuous system, [11–15] have got some results. In [16–22] by using differential equation geometry theory, the method of successor function and analog of Poincarè criterion, the sufficient condition for the existence and the orbitally asymptotically stability of the periodic solutions have been obtained. In this paper, we try to reference the stability analysis of limit cycles for continuous systems to obtain a new judgement method which is different from the previous method of analog of Poincarè criterion for the stability of the periodic solutions of semi-continuous systems.

Taking the transform

$$y = \int_{-\infty}^{t} \exp(a(t-s)x(s))\mathrm{d}s, \quad y > 0,$$

system (1.1) can be written as

$$\begin{cases} \left.\begin{array}{l} \frac{\mathrm{d}x}{\mathrm{d}t} = rx(1-cx-\omega y) = P(x,y), \\ \frac{\mathrm{d}y}{\mathrm{d}t} = ax - ay = Q(x,y), \end{array}\right\} & x < h, \\ \Delta x = -\beta x, & x = h, \end{cases} \tag{1.2}$$

which is equivalent to system (1.1).

In the next section, we give some preliminaries. In Sect. 3, we get the sufficient condition for the existence of the periodic solution of system (1.2) by differential equation geometry theory and successor function. In Sect. 4, referencing the stability analysis of the limit cycles for continuous dynamic systems, we prove the periodic solution of system (1.2) is orbitally asymptotically stable under some conditions. In Sect. 5, we analyze numerically the theoretical results obtained.

2 Preliminaries

Definition 2.1 [10] Suppose impulsive state differential equation

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = P(x,y),\ \frac{\mathrm{d}y}{\mathrm{d}t} = Q(x,y),\ (x,y) \notin M\{x,y\}, \\ \Delta x = \alpha(x,y),\ \Delta y = \beta(x,y),\ (x,y) \in M\{x,y\}, \end{cases} \tag{2.1}$$

whose solution mapping composes the system called as semi-continuous dynamic system, denoted by (Ω, f, φ, M). Set initial point of mapping $p \in \Omega = R_2^+ \setminus M\{x,y\}$, φ is a continuous mapping, $\varphi(M) = N$, φ is called as impulse mapping, where $M(x,y)$ and $N(x,y)$ are straight lines or curves on the plane $R_2^+ = \{(x,y) \in R^2 : x \geq 0, y \geq 0\}$, $M\{x,y\}$ denotes impulse set, $N\{x,y\}$ denotes phase set.

In system (1.2), impulse set $M = \{(x,y) \in R_2^+ | x = h, y \geq 0\}$, impulse mapping φ: $(x,y) \in M \to ((1-\beta)h, y) \in R_2^+$, phase set $N = \varphi(M) = \{(x,y) \in R_2^+ | x = (1-\beta)h, y \geq 0\}$. Therefore, system (1.2) composes a semi-continuous dynamic system (Ω, f, φ, M).

Definition 2.2 Let $f(P,t)$ be the semi-continuous dynamical system mapping described by system (2.1) at $\Omega \to \Omega$, where $f(P,t)$ is a mapping in itself. If there are a point P_1 in phase set N and a t_1 such that $f(P_1,t_1) = Q_1 \in M\{x,y\}$, it also has $\varphi(Q_1) = \varphi(f(P_1,t_1)) = P_1 \in N$, then $f(P_1,t_1)$ is said to be the order-1 periodic solution.

Definition 2.3 [10] Suppose that N is the phase set of system (1.2), M is the impulse set of system (1.2), and both N and M are straight lines, see Fig. 1. The intersection point of N and x axis is Q, the distance between point A ($A \in N$) and point Q is noted by a, M_1 denotes the intersection point of trajectory passing trough point A and M, phase point of M_1 is A_1 ($A_1 \in N$), and the distance between A_1 and Q is noted by

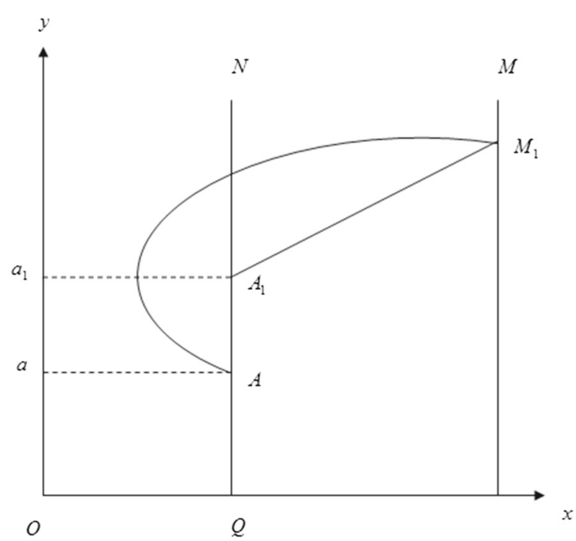

Fig. 1 Successor function $f(A) = a_1 - a$

a_1. We define that subsequent point of A is A_1, and successor function of A is $f(A) = a_1 - a$.

Remark 2.1 If $f(A) = 0$, the trajectory passing through point A is the order-1 periodic solution of the system.

Lemma 2.1 [10] *Successor function $f(A)$ is continuous.*

According to Lemma 2.1, we can get the following lemma.

Lemma 2.2 [10] *Assume continuous dynamical system (X, Ψ), if there exist two points A, B in the phase set such that successor function $f(A) > 0$, $f(B) < 0$, we can find a point C between A and B in the phase set satisfying $f(C) = 0$. So there must exist an order-1 periodic solution passing through point C.*

3 Existence of the order-1 periodic solution

In system (1.2), if $\beta = 0$, i.e., without impulse effects, there are two equilibrium points:

$$O(0,0), \quad P\left(\frac{1}{c+\omega}, \frac{1}{c+\omega}\right).$$

Obviously, $O(0,0)$ is the saddle point, $P\left(\frac{1}{c+\omega}, \frac{1}{c+\omega}\right)$ is globally asymptotically stable.

Theorem 3.1 *(1) If $\frac{1}{c+\omega} \geq h$, there exists a point $C \in N$ satisfying $f(C) = 0$, that is to say, there exists an order-1 periodic solution of system (1.2).*

(2) If $(1-\beta)h < \frac{1}{c+\omega} < h$, there exists a point $D \in N$ satisfying $f(D) = 0$, that is to say, there exists an order-1 periodic solution of system (1.2).

(3) If $\frac{1}{c+\omega} \leq (1-\beta)h$, there is no order-1 periodic solution of system (1.2).

Proof (1) If $\frac{1}{c+\omega} \geq h$ (see Fig. 2).

In fact, the impulse set of system (1.2) is line segment $\overline{M_1M_2}$, and the phase set of system (1.2) is line segment $\overline{N_1N_2}$, where M_1 is the intersections of impulse set and isoclinic line $\frac{dx}{dt} = 0$, $\overline{M_1M_2}$ intersects vertically x axis at M_2, and the phase set intersects vertically $\overline{M_1N_1}$, x axis at N_1, N_2, respectively.

Let $\frac{dx}{dt} = 0$ and N intersect at $B((1-\beta)h, b)$, according to qualitative theories of differential equation, there exists an unique trajectory L_1 passing

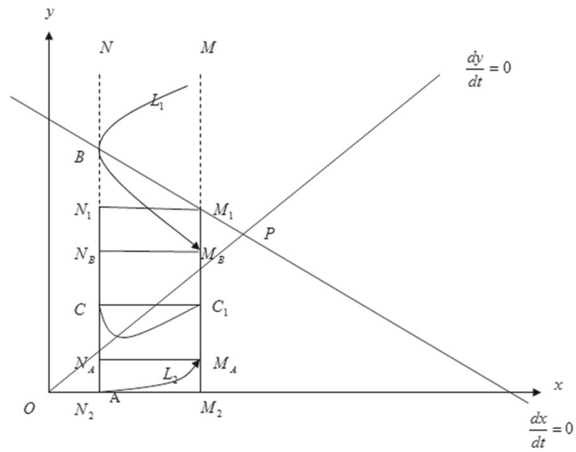

Fig. 2 The existence of order-1 periodic solution of system (1.2) when $\frac{1}{c+\omega} \geq h$

through B, being tangent with the straight line $x = (1-\beta)h$. Assume that L_1 and M intersect at $M_B(h, b_1)$, the phase point is $N_B((1-\beta)h, b_1)$ of M_B after generating impulse, and N_B must be under B. Then we have

$f(B) = b_1 - b < 0.$

Choose a point $A((1-\beta)h, a)$, $A \in N_1N_2$ sufficiently near x axis such that a is a small enough positive, the trajectory L_2 passing through A intersects M_1M_2 at $M_A(h, a_1)$, the phase point is $N_A((1-\beta)h, a_1)$ of M_A after generating impulse, N_A must be over A. Then we have

$f(A) = a_1 - a > 0.$

According to Lemma 2.2, there exists a point $C \in N$, satisfying $f(C) = 0$, that is to say, there exists an order-1 periodic solution of system (1.2).

(2) If $(1-\beta)h < \frac{1}{c+\omega} < h$ (see Fig. 3).

In the same way, it is easy to prove that there exists an order-1 periodic solution of system (1.2).

(3) If $\frac{1}{c+\omega} \leq (1-\beta)h$ (see Fig. 4).

For any point in phase set, there is no intersection of the trajectory passing through any point, and the impulsive set or the subsequent point is over itself, and then there is no order-1 periodic solution of system (1.2).

4 Stability of the order-1 periodic solution

Definition 4.1 [23] On the positive half-trajectory of semi-continuous dynamic system denoted by $f(P, I^+)$,

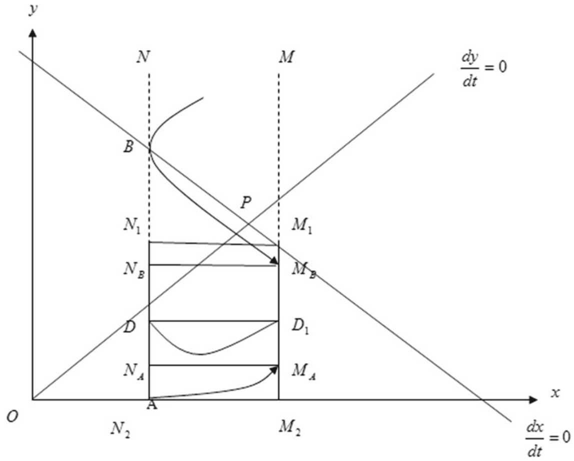

Fig. 3 The existence of order-1 periodic solution of system (1.2) when $(1-\beta)h < \frac{1}{c+\omega} < h$

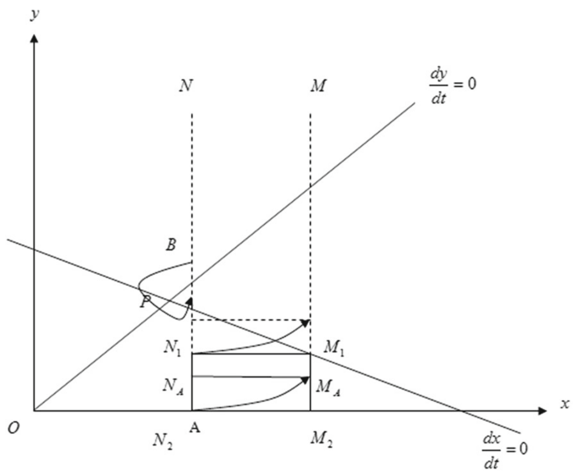

Fig. 4 There is no order-1 periodic solution of system (1.2) when $\frac{1}{c+\omega} \le (1-\beta)h$

$I^+ = (0, +\infty)$, choose any time series $\{0 \le t_1 < t_2 < \cdots < t_n < \cdots\}$ such that $\lim\limits_{t\to\infty} t_n = +\infty$. If Q is the limit point of point range $\{f(P, t_n)\}, n = 1, 2, \ldots$, we call Q as ω limit point of point range $\{f(P, t_n)\}, n = 1, 2, \ldots$. The set Ω made up of all limit points of point range $\{f(P, t_n)\}, n = 1, 2, \ldots$ is called as ω limit set.

Definition 4.2 Assume that Γ is the order-1 periodic solution of semi-continuous dynamic system. If there exists a neighborhood $U(\Gamma)$ sufficiently small such that ω limit set of trajectory starting from any point $P \in U(\Gamma)$ is always Γ, the order-1 periodic solution Γ is stable. Otherwise, the order-1 periodic solution Γ is unstable.

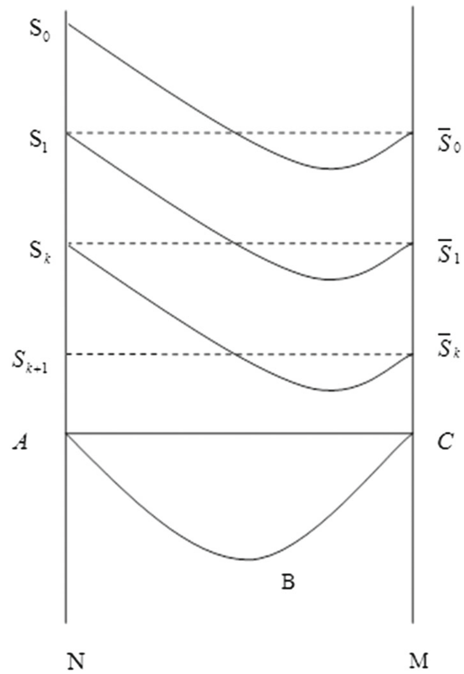

Fig. 5 $S_1, S_2, \ldots, S_k, S_{k+1}, \ldots$ are the subsequent points of $S_0, S_1, \ldots, S_{k-1}, S_k, \ldots$, respectively

In system (1.2), A is any point of the phase set N, see Fig. 5, assume the single-closed curve consisting of curve $\widehat{ABC}$ and line segment $\overline{CA}$ is an order-1 periodic solution of system (1.2), denoted by Γ. Get point S_0 near A, there exists a point range:

$\{S_1, S_2, \ldots, S_k, S_{k+1} \ldots\}$,

where

$S_1, S_2, \ldots, S_k, S_{k+1}, \ldots$

are the subsequent points of $S_0, S_1, \ldots, S_{k-1}, S_k, \ldots$, respectively.

Establish coordinates at phase set and near A, the coordinate of A is 0. Let

$s_0, s_1, \ldots, s_k, s_{k+1} \ldots$

denote the coordinates of points

$S_0, S_1, \ldots, S_k, S_{k+1}, \ldots,$

respectively.

Proposition 4.1 *For any point S_0 near A, when $k \to \infty$, the point range*

$S_0, S_1, \ldots, S_k, S_{k+1}, \ldots \to A,$

i.e.,

$s_0, s_1, \ldots, s_k, s_{k+1}, \ldots \to 0,$

then the order-1 periodic solution is stable (unidirectional).

Proposition 4.2 (Königs) *Assume that* $\overline{s} = f(s)$ *is a continuous transform from line segment N to itself,* $S = 0$ *is a fixed point under the transform. If the part near origin of curve* $\overline{s} = f(s)$ *on the plane* $(s, \overline{s})$ *lies in the interior of the domain*

$$\left|\frac{\overline{s}}{s}\right| \leq 1 - \varepsilon (\geq 1 + \varepsilon), \varepsilon > 0,$$

the fixed point $S = 0$ *is stable (unstable).*

Proof We prove firstly the fixed point $S = 0$ is stable. □

Choose $\eta > 0$ sufficiently small such that for any point S in noncentral neighborhood $U^0(0; \eta)$ of the fixed point $S = 0$, $|s| \leq \eta$.

Let

$$\left|\frac{\overline{s}}{s}\right| \leq 1 - \varepsilon = \delta < 1,$$

we have

$$|\overline{s}| \leq \delta |s| < |s|.$$

For any point range

$$\{S, S_1, S_2, \ldots, S_k, S_{k+1}, \ldots\},$$

where $S, S_k \in U^0(0; \eta)$, $k = 1, 2, \ldots, n, \ldots$, we get sequence

$$\{|s|, |s_1|, |s_2|, \ldots, |s_k|, |s_{k+1}|, \ldots\}.$$

Because of $|s_1| \leq \delta |s|$, $|s_2| \leq \delta |s_1|, \ldots$, it is easy to deduce that $|s_n| \leq \delta^n |s|$, hence $|s_n| \to 0$ when $n \to \infty$. Upon that, the fixed point $S = 0$ is stable.

In the same way, we prove that the fixed point $S = 0$ is unstable. The proof is completed.

Corollary 4.1 *Assume that function* $\overline{s} = f(s)$ *exists derivative at* $S = 0$*, then* $S = 0$ *is stable when* $\left|\frac{\mathrm{d}\overline{s}}{\mathrm{d}s}\right|_{s=0} < 1$.

From Fig. 6, assume that the closed orbit consisted of the curve $\widehat{ABC}$, and line segment $\overline{CA}$ is the order-1 periodic solution of system (4.1), denoted by Γ, where $A \in N$, $C \in M$, and N is the phase set, M is impulse set. Draw normal line n passing through $A \in \Gamma$ and establish coordinate system (s, n) on point A. Choose any point $D \in N$ in small enough neighborhood of A. The trajectory starting from D intersects vertically n axis at B_k and intersects impulse set M at $\overline{D}$. E denotes the phase point of $\overline{D}$, the trajectory passing trough point E intersects vertically n axis at B_{k+1} as t increases.

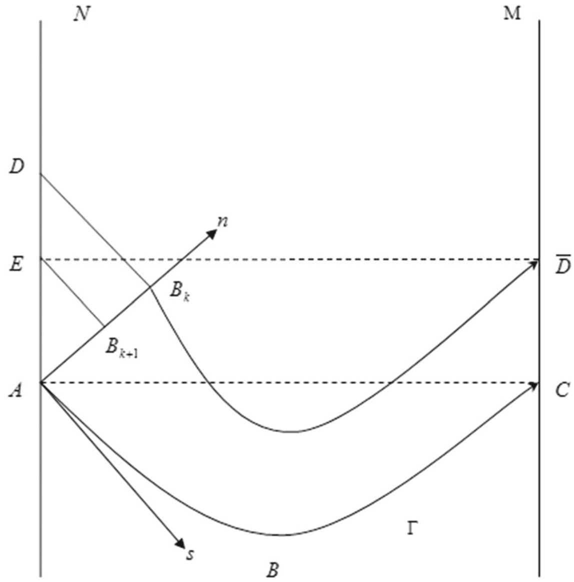

Fig. 6 Establish coordinate system (s, n) on point A

Assume that rectangular coordinate of A is $(\varphi(s), \psi(s))$, then for B_k, there is the relation between its rectangular coordinates (x, y) and curvilinear coordinates (s, n):

$$x = \varphi(s) - n\psi'(s), \quad y = \psi(s) + n\varphi'(s), \tag{4.1}$$

where

$$\varphi'(s) = \frac{\mathrm{d}x}{\mathrm{d}s}|_A = \frac{P_0}{\sqrt{P_0^2 + Q_0^2}},$$

$$\psi'(s) = \frac{\mathrm{d}y}{\mathrm{d}s}|_A = \frac{Q_0}{\sqrt{P_0^2 + Q_0^2}},$$

P_0, Q_0 denote the values P, Q lie in A, respectively, we have

$$P_0 = P(\varphi(s), \psi(s)), \quad Q_0 = Q(\varphi(s), \psi(s)).$$

From (4.1), it is easy that we have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\psi'(s) + \varphi'(s)\frac{\mathrm{d}n}{\mathrm{d}s} + n\varphi''(s)}{\varphi'(s) - \psi'(s)\frac{\mathrm{d}n}{\mathrm{d}s} - n\psi''(s)} = \frac{Q(\varphi(s) - n\psi'(s), \psi(s) + n\varphi'(s))}{P(\varphi(s) - n\psi'(s), \psi(s) + n\varphi'(s))},$$

hence

$$\frac{\mathrm{d}n}{\mathrm{d}s} = \frac{Q\varphi' - P\psi' - n(P\varphi'' + Q\psi'')}{P\varphi' + Q\psi'} = F(s, n). \tag{4.2}$$

Since there is a zero solution $n = 0$ for (4.2), when there exist continuous partial derivatives for functions P, Q, there does also on n for $F(s, n)$, (4.2) is written as

$$\frac{\mathrm{d}n}{\mathrm{d}s} = F_n'(s, n)\mid_{n=0} n + o(n). \tag{4.3}$$

In order to calculate

$$\frac{\mathrm{d}n}{\mathrm{d}s} = F_n'(s, n)\mid_{n=0},$$

we first get

$$\varphi''(s) = -\frac{Q_0}{P_0^2+Q_0^2}\left[P_0^2 Q_{x0} + P_0 Q_0 (Q_{y0} - P_{x0}) - Q_0^2 P_{y0}\right],$$
$$\psi''(s) = \frac{P_0}{P_0^2+Q_0^2}\left[P_0^2 Q_{x0} + P_0 Q_0 (Q_{y0} - P_{x0}) - Q_0^2 P_{y0}\right], \tag{4.4}$$

where P_{y0}, P_{x0}, Q_{y0}, Q_{x0} denote partial derivatives of P, Q when $n = 0$, respectively. Since $P = P_0$, $Q = Q_0$ when $n = 0$, it is easy to know $P_0\varphi'' + Q_0\psi'' = 0$. By (4.2) and (4.4), we have

$$F_n'(s, n)|_{n=0} = \frac{P_0^2 Q_{y0} - P_0 Q_0 (P_{y0} + Q_{x0}) + Q_0^2 P_{x0}}{(P_0^2+Q_0^2)^{3/2}} = H(s),$$

where $H(s)$ denotes the curvature of orthogonal trajectory at A for system (1.2). Therefore the approximate equation of (4.3) is

$$\frac{\mathrm{d}n}{\mathrm{d}s} = H(s)n,$$

whose solution is

$$n = n_0 \mathrm{e}^{\int_0^s H(s')\mathrm{d}s'}, \quad n_0 = n(0). \tag{4.5}$$

Theorem 4.1 *Assume that h is the length of curve $\widehat{ABC}$ which is a section of the order-1 periodic solution Γ of system (1.2). The order-1 periodic solution Γ is stable when*

$$\int_0^h H(s)\mathrm{d}s < 0. \tag{4.6}$$

Proof Let us investigate trajectory $B_k\overline{D}EB_{k+1}$ (see Fig. 6). In the coordinate system (s, n), the ordinate of B_k is denoted by n_0, and the ordinate of $\overline{D}$ is denoted by n. From (4.5), we have

$$|n(h)| < |n_0|$$

when $\int_0^h H(s)\mathrm{d}s < 0$, where h is the length of curve $\widehat{ABC}$. By Propositions 4.1 and 4.2, the order-1 periodic solution Γ is stable.

Corollary 4.2 (Diliberto) [24] *If the integral along the order-1 periodic solution Γ satisfies $H(s) < 0$, the order-1 periodic solution Γ is stable.*

Let $\mathrm{d}s = \sqrt{P_0^2+Q_0^2}\mathrm{d}t$, the left of (4.6) can be rewritten as

$$\begin{aligned}
&\int_0^h H(s)\mathrm{d}s \\
&= \int_0^T \frac{1}{P_0^2+Q_0^2}\left[P_0^2 Q_{y0} - P_0 Q_0 (P_{y0} + Q_{x0}) + Q_0^2 P_{x0}\right]\mathrm{d}t \\
&= \int_0^T \left[P_{x0}+Q_{y0}-\frac{P_0^2 P_{x0}+P_0 Q_0 (P_{y0}+Q_{x0})+Q_0^2 Q_{y0}}{P_0^2+Q_0^2}\right]\mathrm{d}t \\
&= \int_0^T (P_{x0}+Q_{y0})\mathrm{d}t - \int_0^T \frac{1}{2}\frac{1}{P_0^2+Q_0^2}\frac{\mathrm{d}}{\mathrm{d}t}(P_0^2+Q_0^2)\mathrm{d}t \\
&= \int_0^T (P_{x0}+Q_{y0})\mathrm{d}t - \int_0^T \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\ln(P_0^2+Q_0^2)\right]\mathrm{d}t,
\end{aligned}$$

i.e.,

$$\int_0^h H(s)\mathrm{d}s = \int_0^T (P_{x0}+Q_{y0})\mathrm{d}t - \int_0^T \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\ln(P_0^2+Q_0^2)\right]\mathrm{d}t < 0. \tag{4.7}$$

Consider the integral along the periodic solution Γ' of continuous system

$$J_{\Gamma'} = \int_0^T \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\ln(P_0^2+Q_0^2)\right]\mathrm{d}t = 0,$$

we suppose that the integral along the order-1 periodic solution Γ of semi-continuous system has the same result.

Denote $F(x, y) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\ln(P_0^2+Q_0^2)\right]$.

Lemma 4.1 *If function $F(x, y)$ is continuous and differentiable, the integral along the order-1 periodic solution of system (1.2) satisfies*

$$\int_0^T \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t = 0,$$

where period of the order-1 periodic solution is T.

Proof Let Γ be an order-1 periodic solution of system (1.2), see Fig. 7, $S_1(t)$ denotes the curve ofsystem (1.2)

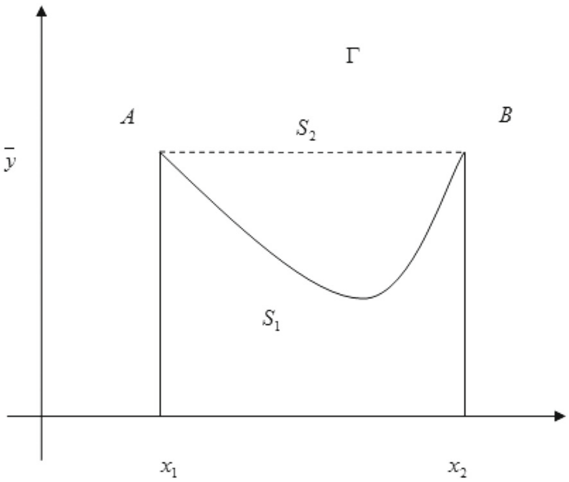

Fig. 7 Γ is the order-1 periodic solution of system (1.2)

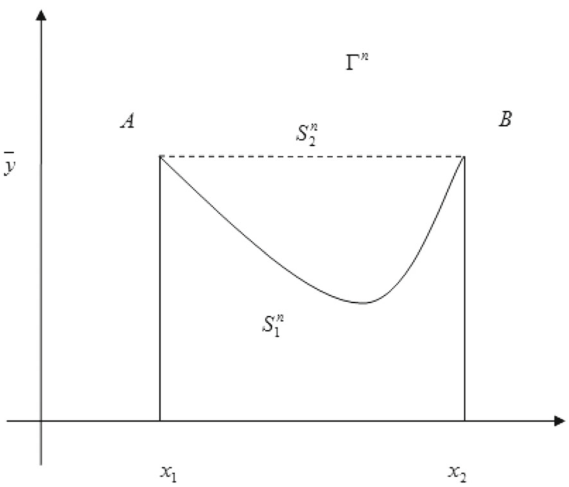

Fig. 8 Γ^n is the order-1 periodic solution of system (4.8)

from $A(x_1, \overline{y})$ to $B(x_2, \overline{y})$, $S_1(t) = A$ when $t = 0$, and $S_1(t) = B$ when $t = T$. $S_2(t)$ denotes line segment $\overline{BA}$.

Take the transform $\tau = \frac{n-1}{n}t$, system (1.2) can be written as

$$\begin{cases} \left.\begin{array}{l} \frac{dx}{d\tau} = rx(1 - cx - \omega y) = P(x, y), \\ \frac{dy}{d\tau} = ax - ay = Q(x, y), \end{array}\right\} & x < h, \\ \Delta x = -\beta x, & x = h, \end{cases} \tag{4.8}$$

where the trajectory Γ^n of system (4.8) is similar to system (1.2) except for time variable.

Let $S_1^n(\tau)$ denote the curve of system (4.8) from $A(x_1, \overline{y})$ to $B(x_2, \overline{y})$, see Fig. 8, $S_1^n(\tau) = A$ when $\tau = 0$, $S_1^n(\tau) = B$ when $\tau = \frac{n-1}{n}T$. $S_2^n(\tau)$ denotes line segment $\overline{BA}$, the parameter equation of $S_2^n(\tau)$ is

$$\begin{cases} x = \frac{(x_1 - x_2)n}{T}\tau + x_2, \\ y = \overline{y}, \end{cases}$$

$S_2^n(\tau) = B$ when $\tau = 0$, $S_2^n(\tau) = A$ when $\tau = \frac{T}{n}$.

Obviously, system (4.8) $\to$ system (1.2), i.e. $\Gamma^n \to \Gamma$, when $n \to \infty$, thus we have

$$\begin{aligned} \Gamma \oint_0^T \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t &= S_1 \oint_0^{\frac{n-1}{n}T} \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t \\ &\quad + S_2 \oint_{\frac{n-1}{n}T}^{T} \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t \\ &= S_1 \oint_0^{\frac{n-1}{n}T} \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t \\ &\quad + S_2 \oint_0^{\frac{T}{n}} \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t \to 0 (n \to \infty), \end{aligned}$$

i.e., the integral along the order-1 periodic solution Γ of system (1.2) satisfies

$$\int_0^T \frac{\mathrm{d}F(x, y)}{\mathrm{d}t}\mathrm{d}t = 0.$$

The proof is completed.

According to (4.7), we have the following theorem.

Theorem 4.2 *If the integral the along order-1 periodic solution Γ of system (1.2) satisfies*

$$\int_0^T (P_{x0} + Q_{y0})\mathrm{d}t < 0,$$

Γ is stable.

Theorem 4.3 *The order-1 periodic solution of system (1.2) is stable.*

Proof Structure Dulac function $u(x, y) = x^{-1}$ which is continuous, differentiable and positive on the domain $G(x > 0, y > 0)$, then system (1.2) can be written as

$$\begin{cases} \frac{dx}{d} = rx(1 - cx - \omega y) = p(x, y)u(x, y) \\ \quad = r(1 - cx - \omega y) = p_1(x, y), \\ \frac{dy}{dt} = ax - ay = q(x, y)u(x, y) = a - a\frac{y}{x} = q_1(x, y), \end{cases}$$

we have

$$\frac{\partial p_1(x, y)}{\partial x} + \frac{\partial q_1(x, y)}{\partial y} = -rc - a\frac{1}{x} < 0.$$

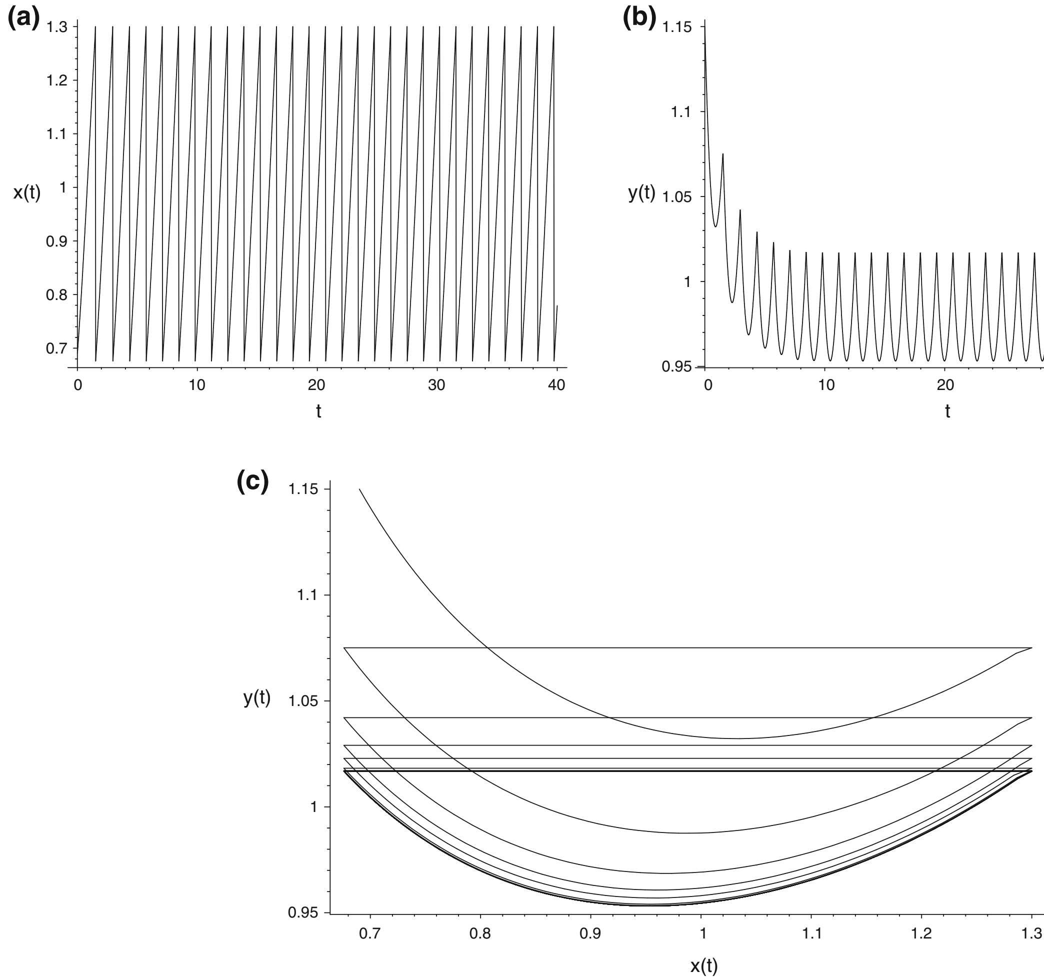

Fig. 9 Time series and phase portrait of system (1.2) with $h = 1.3$, $r = 1.53$, $c = 0.2$, $\omega = 0.5$, $a = 0.6$, $\beta = 0.48$

By Theorem 4.2, the order-1 periodic solution of system (1.2) is stable. The proof is completed.

5 Numerical analysis and discussion

For system (1.2), there is an equilibrium point $P(\frac{1}{c+\omega}, \frac{1}{c+\omega})$ which is globally asymptotically stable without impulse effects. According to Theorem 3.1, if $\frac{1}{c+\omega} \geq h$ or $(1-\beta)h < \frac{1}{c+\omega} < h$, there exist the order-1 periodic solutions; if $\frac{1}{c+\omega} \leq (1-\beta)h$, there is no order-1 periodic solution.

To verify the theoretical results obtained in this paper, we choose ET h as the parameter and analyze numerically the existence of order-1 periodic solution for system (1.2) under the different values of impulse set, there are the following three cases:

Case 1 Let $h = 1.3, r = 1.53, c = 0.2, \omega = 0.5, a = 0.6, \beta = 0.48$, we have impulse set $M = \{(x, y) \in R_2^+ | x = 1.3, y \geq 0\}$, phase set $N = \{(x, y) \in R_2^+ | x = (1-\beta)h = 0.676, y \geq 0\}$, and $\frac{1}{c+\omega} \doteq 1.43 \geq h$, that is to say, the equilibrium point $P\left(\frac{1}{c+\omega}, \frac{1}{c+\omega}\right)$ is on the right of impulse set (see Fig. 2). From Fig. 9, we can observe that there exists an order-1 periodic solution of system (1.2) which lies between phase set and impulse set (i.e. between 0.676 and 1.3). It indicates that the numerical simulation result is consistent with the first case of Theorem 3.1.

Case 2 Let $h = 1.45, r = 1.53, c = 0.2, \omega = 0.5, a = 0.6, \beta = 0.48$, we have impulse set $M = \{(x, y) \in R_2^+ | x = 1.45, y \geq 0\}$, phase set $N =$

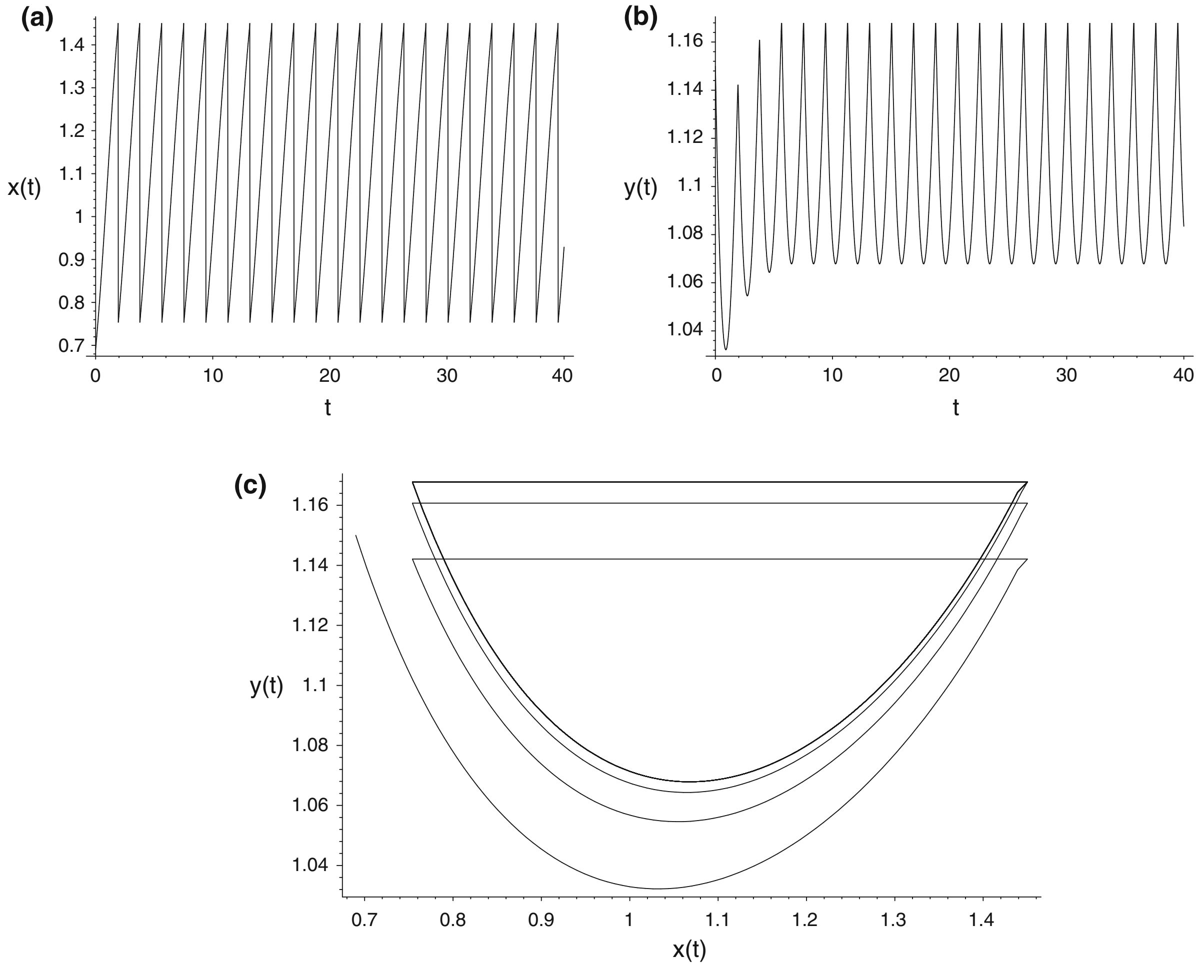

Fig. 10 Time series and phase portrait of system (1.2) with $h = 1.45, r = 1.53, c = 0.2, \omega = 0.5, a = 0.6, \beta = 0.48$

$\{(x, y) \in R_2^+ | x = (1-\beta)h = 0.754, y \geq 0\}$ and $(1-\beta)h < \frac{1}{c+\omega} \doteq 1.43 < h$, that is to say, the equilibrium point $P\left(\frac{1}{c+\omega}, \frac{1}{c+\omega}\right)$ lies between phase set and impulse set (see Fig. 3). From Fig. 10, we can observe that there exists an order-1 periodic solution of system (1.2) which lies between phase set and impulse set (i.e., between 0.754 and 1.45). It indicates that the numerical simulation result is consistent with the second case of Theorem 3.1.

Case 3 Let $h = 1.7, r = 1.53, c = 0.6, \omega = 0.8, a = 0.6, \beta = 0.38$, we have impulse set $M = \{(x, y) \in R_2^+ | x = 1.7, y \geq 0\}$, phase set $N = \{(x, y) \in R_2^+ | x = (1-\beta)h = 1.054, y \geq 0\}$ and $\frac{1}{c+\omega} \doteq 0.71 \leq (1-\beta)h$, that is to say, the equilibrium point $P\left(\frac{1}{c+\omega}, \frac{1}{c+\omega}\right)$ is on the left of phase set (see Fig. 4). From Fig. 11, we can observe there is no order-1 periodic solution of system (1.2). It indicates the numerical simulation result is consistent with the third case of Theorem 3.1.

According to Theorem 4.3, the order-1 periodic solution of system (1.2) is stable, it illustrates that we can achieve the aim of controlling pest by impulsively spraying pesticides when pest density increases to ET h.

The control strategy with impulsive state needs observing and recording the number of the pests. In theory, we can predict the cycle time without repeated measurements, which can save a lot of manpower and material resources. The model with impulsive state feedback control is closer to the reality than the periodic impulsive model that there is no density dependence.

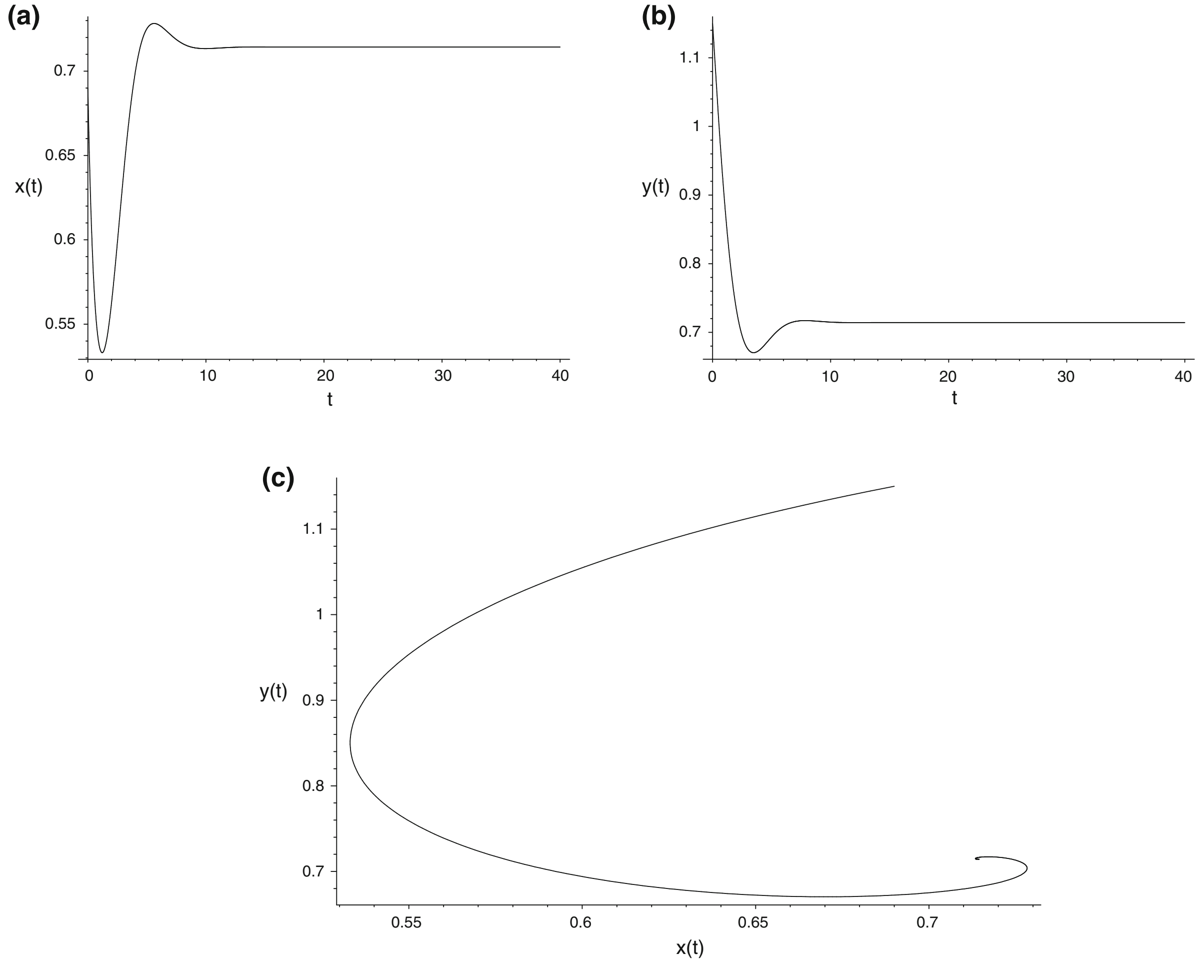

Fig. 11 Time series and phase portrait of system (1.2) with $h = 1.7$, $r = 1.53$, $c = 0.6$, $\omega = 0.8$, $a = 0.6$, $\beta = 0.38$

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References

1. Awrejcewicz, J., Tomczak, K., Lamarque, C.-H.: Controlling systems with impacts. Int. J. Bifurc. Chaos **9**(3), 547–553 (1999)
2. Awrejcewicz, J.: Numerical investigations of the constant and periodic motions of the human vocal cords including stability and bifurcation phenomena. Dyn. Stab. Syst. J. **5**(1), 11–28 (1990)
3. Pang, G., Chen, L.: Dynamic analysis of a pest-epidemic model with impulsive control. Math. Comput. Simul. **79**, 72–84 (2008)
4. Li, C., Tang, S.: The effects of timing of pulse spraying and releasing periods on dynamics of generalized predator-prey model. Int. J. Biomath. **5**, 157–183 (2012)
5. Wang, T., Chen, L.: Nonlinear analysis of a microbial pesticide model with impulsive state feedback control. Nonlinear Dyn. **65**(1–2), 1–10 (2011)
6. Seydel, R.: Practical Bifurcation and Stability Analysis, 3rd edn. Springer, New York (2009)
7. Zhou, Z., Yu, Y.: Poincarè mapping and periodic solution for the nonlinear differential system. J. Syst. Sci. Math. Sci. (Chin. Ser.) **26**(1), 59–68 (2006)
8. Jin, L., Lu, Q., Wang, Q.: Calculation methods of Floquet multipliers for non-smooth dynamic system. J. Appl. Mech. **21**(3), 21–26 (2004)
9. Luo, A.C.J.: Discontinuous Dynamical Systems. Higher Education Press and Springer, Beijing and Heidelberg (2012)

10. Chen, L.: Pest control and geometric theory of semi-continuous dynamical system. J. Beihua Univ. (Nat. Sci.) **12**(1), 1–9 (2011)
11. Clark, Colin W.: Mathematical Bioeconomics: The Optimal Management of Renewable Resources. Wiley, New York (1990)
12. Bonotto, E.M.: Flows of characteristic in impulsive semidynamical systems. J. Math. Anal. Appl. **332**(1), 81–96 (2007)
13. Bonotto, E.M.: LaSalle's theorems in impulsive semidynamical systems. Cad. Mat. **9**, 157–168 (2008)
14. Bonotto, E.M., Federson, M.: Limit sets and the Poincarè–Bendixson theorem in impulsive semidynamical systems. J. Differ. Equ. **244**, 2334–2349 (2008)
15. Bonotto, E.M., Federson, M.: Poisson stability for impulsive semidynamical systems. Nonlinear Anal. **71**(12), 6148–6156 (2009)
16. Huang, M., Duan, G., Song, X.: A predator–prey system with impulsive state feedback control. Math. Appl. **25**(3), 661–666 (2012)
17. Wei, C., Chen, L.: Periodic solution and heteroclinic bifurcation in a predator–prey system with Allee effect and impulsive harvesting. Nonlinear Dyn. **76**, 1109–1117 (2013). doi:10.1007/s11071-013-1194-z
18. Huang, M., Song, X., Guo, H., et al.: Study on species cooperative systems with impulsive state feedback control. J. Syst. Sci. Math. Sci. **32**(3), 265–276 (2012)
19. Wei, C., Chen, L.: A Leslie–Gower pest management model with impulsive state feedback control. J. Biomath. **27**(4), 621–628 (2012)
20. Bainov, D., Simeonov, P.: Impulsive Differential Equations: Periodic Solutions and Applications. Longman Scientific and Technical, New York (1993)
21. Fu J, Wang Y.: The mathematical study of pest management strategy. Discret. Dyn. Nat. Soc. (2012). doi:10.1155/2012/251942
22. Wei C, Zhang S, Chen L.: Impulsive state feedback control of cheese whey fermentation for single-cell protein production. J. Appl. Math. (2013). doi:10.1155/2013/354095
23. Chen L.: Theory and application of "semi-continuous dynamical system". J. Yulin Norm. Univ. (Nat. Sci.) 2013;**34**(2):1–10
24. Diliberto, S.P.: Contributions to the Theory of Nonlinear Oscillations, I. Princeton University Press, Princeton (1950)