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ORIGINAL PAPER

Conservation laws and Darboux transformation for the coupled cubic–quintic nonlinear Schrödinger equations with variable coefficients in nonlinear optics

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Abstract In this paper, by Darboux transformation and symbolic computation we investigate the coupled cubic–quintic nonlinear Schrödinger equations with variable coefficients, which come from twin-core nonlinear optical fibers and waveguides, describing the effects of quintic nonlinearity on the ultrashort optical pulse propagation in the non-Kerr media. Lax pair of the equations is obtained, and the corresponding Darboux transformation is constructed. One-soliton solutions are derived; some physical quantities such as the amplitude, velocity, width, initial phases, and energy are, respectively, analyzed; and finally an infinite number of conservation laws are also derived. These results might be of some value for the ultrashort optical pulse propagation in the non-Kerr media.

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1 Introduction

Optical solitons have the potential to become carriers in the telecommunication systems because of the capability of propagating long distances with high intensity and without attenuation [1–12]. Dynamics of light pulses are described by the nonlinear Schrödinger (NLS)-typed equations with cubic nonlinear terms [8,13], and the non-Kerr nonlinearity effect comes into play [9] when the intensity of the incident light field becomes stronger, which is described by the NLS-typed equations with higher-order nonlinear terms [10]. The NLS equation is a vital model to describe certain phenomena from Physics and Engineering to Biochemistry [14]. Certain interest has been focused on the NLS-typed equations since the experimental observation of the multi-stability of solitons in non-Kerr fibers [1,9, 15–27].

In this paper, the coupled cubic–quintic nonlinear NLS equations with variable coefficients [2,22] describing the effects of quintic nonlinearity on the ultrashort optical pulse propagation in the non-Kerr media are investigated [28],

$$\begin{aligned}
& i q_{1z} + r(z)\, q_{1tt} + m(z)\, \left(|q_1|^2 + |q_2|^2\right) q_1 \\
& \quad + n(z)\, \left(|q_1|^2 + |q_2|^2\right)^2 q_1 \\
& \quad - i\, p(z) \left[\left(|q_1|^2 + |q_2|^2\right) q_1\right]_t \\
& \quad + i\, s(z)\, \left(q_1^* q_{1t} + q_2^* q_{2t}\right) q_1 = 0, \\
& i q_{2z} + y(z)\, q_{2tt} + k(z)\, \left(|q_1|^2 + |q_2|^2\right) q_2 \\
& \quad + w(z)\, \left(|q_1|^2 + |q_2|^2\right)^2 q_2 \\
& \quad - i\, a(z) \left[(|q_1|^2 + |q_2|^2) q_2\right]_t \\
& \quad + i\, b(z) \left(q_1^* q_{1t} + q_2^* q_{2t}\right) q_2 = 0,
\end{aligned} \tag{1}$$

where the components q_1 and q_2 of the electromagnetic fields propagate along the coordinate z in the two cores of an optical waveguide, t is the local time [28,29], $r(z)$ and $y(z)$ represent the group velocity dispersions, $m(z)$ and $k(z)$ are the nonlinearity parameters, $n(z)$ and $w(z)$ are the saturation of the nonlinear refractive indexes, $p(z)$ and $a(z)$ are the self-steepening, and $s(z)$ and $b(z)$ are the delayed nonlinear response effects [30]. With a reduction of $q_1 = q$ and $q_2 = 0$ (or $q_1 = 0$ and $q_2 = q$), Eq. (1) turns to the integrable Kundu–Eckhaus equation [1,22] with variable coefficients, which possesses the applications in the nonlinear optics [27], quantum field theory [25], and weakly nonlinear dispersive matter waves [26]. Equation (1) with constant coefficients has been investigated in several respects [1]. But as far as we know, the Lax pair, Darboux transformation (DT), and conservation laws of Eq. (1) have not been presented as yet.

The outline of this paper is organized as follows: In Sect. 2, a Lax pair of Eq. (1) is presented and the corresponding DT constructed. In Sect. 3, one-soliton solutions of Eq. (1) is obtained and some physical quantities such as the amplitude, velocity, width, initial phases, and energy are, respectively, analyzed. In Sect. 4, an infinite number of conservation laws of Eq. (1) are derived by symbolic computation [2,31–37]. Section 5 contains our conclusions.

2 Lax pair and DT of Eq. (1)

In this section, we present a Lax pair of Eq. (1) [38]. Linear eigenvalue problem for Eq. (1) can be given as [1]

$$\Psi_t = U\Psi, \quad \Psi_z = V\Psi, \tag{2}$$

where $\Psi = [\psi_1(z,t), \psi_2(z,t), \psi_3(z,t)]^T$, T denotes the transpose of the vector, while U and V are respectively given by

$$\begin{aligned}
U &= \begin{pmatrix} iL\theta_t(z,t) - i\lambda & q_1\rho_1(z) & q_2\rho_2(z) \\ -q_1^*\rho_1^*(z) & i\lambda - iL\theta_t(z,t) & 0 \\ -q_2^*\rho_2^*(z) & 0 & i\lambda - iL\theta_t(z,t) \end{pmatrix}, \\
V &= \begin{pmatrix} a_1(z)\lambda^2 + a_2(z,t) & \lambda b_1(z,t) + b_2(z,t) & \lambda f_1(z,t) + f_2(z,t) \\ \lambda c_1(z,t) + c_2(z,t) & d_1(z)\lambda^2 + d_2(z,t) & k_1(z,t) \\ \lambda g_1(z,t) + g_2(z,t) & k_2(z,t) & d_1(z)\lambda(t)^2 + h_2(z,t) \end{pmatrix},
\end{aligned} \tag{3}$$

where L is a constant and

$$\begin{aligned}
& a(z) = b(z) = s(z) = p(z), \quad w(z) = n(z) = L\, b(z), \\
& r(z) = y(z) = \frac{b(z)}{4L}, \quad \theta(z,t) = \int \left(|q_1|^2 + |q_2|^2\right) \mathrm{d}t, \\
& m(z) = \frac{b(z)|\rho_2(z)|^2}{2L}, \quad k(z) = \frac{b(z)|\rho_1(z)|^2}{2L}, \\
& b_1(z,t) = \frac{1}{2} i \left[a_1(z) q_1 \rho_1(z) - d_1(z) q_1 \rho_1(z)\right], \\
& d_1(z) = 4ir(z) + a_1(z), \\
& f_1(z,t) = \frac{1}{2} i \left[a_1(z) q_2 \rho_2(z) - h_1(z) q_2 \rho_2(z)\right], \\
& c_1(z,t) = \frac{1}{2} i \left[d_1(z) q_1^* \rho_1^*(z) - a_1(z) q_1^* \rho_1^*(z)\right], \\
& g_1(z,t) = \frac{1}{2} i \left[h_1(z) q_2^* \rho_2^*(z) - a_1(z) q_2^* \rho_2^*(z)\right], \\
& k_1(z,t) = \frac{1}{4} q_2 q_1^* \rho_1^*(z) \left[a_1(z) - d_1(z)\right] \rho_2(z), \\
& k_2(z,t) = \frac{1}{4} q_1 q_2^* \rho_2^*(z) \left[a_1(z) - d_1(z)\right] \rho_1(z), \\
& b_2(z,t) = \frac{i\left[a_1(z) - d_1(z)\right]}{4} \\
& \qquad \times \left\{2L\rho_1(z)|q_1|^2 q_1 + \Delta_1 + i\rho_1(z) q_{1t}\right\}, \\
& f_2(z,t) = \frac{i\left[a_1(z) - h_1(z)\right]}{4} \\
& \qquad \times \left[\Delta_2 + i\rho_{2t}(z,t) q_2 + i\rho_2(z) q_{2t}\right], \\
& c_2(z,t) = \frac{i\left[d_1(z) - a_1(z)\right]}{4} \\
& \qquad \times \left(2L|q_1|^2 q_1^* + 2L|q_2|^2 q_1^* - i q_{1t}^*\right) \rho_1^*(z), \\
& g_2(z,t) = \frac{i\left[d_1(z) - a_1(z)\right]}{4} \\
& \qquad \times \left(2L|q_2|^2 + 2L|q_1|^2 q_2^* - i q_{2t}^*\right) \rho_2^*(z), \\
& a_2(z,t) = \frac{1}{4}\left[d_1(z) - a_1(z)\right] \\
& \qquad \times \left[|q_1|^2|\rho_1(z)|^2 + |q_2|^2|\rho_2(z)|^2\right]
\end{aligned}$$

$$+a_{22}(z) + iL\theta_z(z,t),$$
$$h_2(z,t) = h_{22}(z) + \frac{1}{4}|q_2|^2\rho_2^*(z) \times [a_1(z) - d_1(z)]\,\rho_2(z) - iL\theta_z(z,t),$$
$$d_2(z,t) = d_2(z) + \frac{1}{4}|q_1|^2\rho_1^*(z) \times [a_1(z) - d_1(z)]\,\rho_1(z) - iL\theta_z(z,t),$$
$$\Delta_1 = \left[2L|q_2|^2\rho_1(z) + i\rho_{1t}(z,t)\right]q_1,$$
$$\Delta_2 = 2L\rho_2(z)|q_2|^2q_2 + 2L|q_1|^2\rho_2(z)q_2,$$

the asterisk is the complex conjugate and λ denotes the spectral parameter. Equation (1) can be achieved from the compatibility condition

$$U_z - V_t + [U,\ V] = 0, \tag{4}$$

where $[U,\ V] = UV - VU$. Thus, the Lax pair of Eq. (1) has been derived.

As DT is composed of the eigenfunction and potential transformation, it can be used to construct a series of explicit solutions for the nonlinear evolution equations (NLEEs) from the initial ones in a recursive manner [39,40], and the procedure of the DT can be achieved by symbolic computation [41,42]. DT has been used to investigate many NLEEs [39,40] as a straightforward algorithm. Eigenfunction transformation for Lax Pair (2) can be taken as

$$\hat{\Psi} = D\Psi = \begin{pmatrix} A_n(z,t) & 0 & 0 \\ 0 & B_n(z,t) & 0 \\ 0 & 0 & C_n(z,t) \end{pmatrix}(\lambda I - S)\Psi, \tag{5}$$

where $n = 1, 2, 3$, and $A_n(z,t)$, $B_n(z,t)$, and $C_n(z,t)$ are the functions of z and t to be determined, I is the 3×3 identity matrix, S is a 3×3 matrix to be determined, and $\hat{\Psi}$ is required to satisfy

$$\hat{\Psi}_t = \hat{U}\hat{\Psi}, \quad \hat{\Psi}_z = \hat{V}\hat{\Psi}, \tag{6}$$

that is,

$$D_t + DU - \hat{U}D = 0, \tag{7}$$
$$D_z + DV - \hat{V}D = 0, \tag{8}$$

with

$$\hat{U} = \begin{pmatrix} iL\hat{\theta}_t(z,t) - i\lambda & \hat{q}_1\rho_1(z) & \hat{q}_2\rho_2(z) \\ -\hat{q}_1^*\rho_1^*(z) & i\lambda(t) - iL\hat{\theta}_t(z,t) & 0 \\ -\hat{q}_2^*\rho_2^*(z) & 0 & i\lambda - iL\hat{\theta}_t(z,t) \end{pmatrix},$$

$$\hat{V} = \begin{pmatrix} a_1(z)\lambda^2 + \hat{a}_2(z,t) & \lambda\hat{b}_1(z,t) + \hat{b}_2(z,t) & \lambda\hat{f}_1(z,t) + \hat{f}_2(z,t) \\ \lambda\hat{c}_1(z,t) + \hat{c}_2(z,t) & d_1(z)\lambda^2 + \hat{d}_2(z,t) & \hat{k}_1(z,t) \\ \lambda\hat{g}_1(z,t) + \hat{g}_2(z,t) & \hat{k}_2(z,t) & d_1(z)\lambda(t)^2 + \hat{h}_2(z,t) \end{pmatrix}, \tag{9}$$

and

$$\hat{\theta}(z,t) = \int \left(|\hat{q}_1|^2 + |\hat{q}_2|^2\right) \mathrm{d}t,$$
$$\hat{b}_1(z,t) = \frac{1}{2}i\left[a_1(z)\hat{q}_1\rho_1(z) - d_1(z)\hat{q}_1\rho_1(z)\right],$$
$$\hat{f}_1(z,t) = \frac{1}{2}i\left[a_1(z)\hat{q}_2\rho_2(z) - h_1(z)\hat{q}_2\rho_2(z)\right],$$
$$\hat{c}_1(z,t) = \frac{1}{2}i\left[d_1(z)\hat{q}_1{}^*\rho_1^*(z) - a_1(z)\hat{q}_1{}^*\rho_1^*(z)\right],$$
$$\hat{g}_1(z,t) = \frac{1}{2}i\left[h_1(z)\hat{q}_2{}^*\rho_2^*(z) - a_1(z)\hat{q}_2{}^*\rho_2^*(z)\right],$$
$$\hat{b}_2(z,t) = \frac{1}{4}i\left[a_1(z) - d_1(z)\right] \times \left\{2L\rho_1(z)|\hat{q}_1|^2\hat{q}_1 + \hat{\Delta}_1 + i\rho_1(z)\hat{q}_{1t}\right\},$$
$$\hat{f}_2(z,t) = \frac{1}{4}i\left[a_1(z) - h_1(z)\right] \times \left[\hat{\Delta}_2 + i\rho_{2t}(z,t)\hat{q}_2 + i\rho_2(z)\hat{q}_{2t}\right],$$
$$\hat{c}_2(z,t) = \frac{1}{4}i\left[d_1(z) - a_1(z)\right] \times \left(2L|\hat{q}_1|^2\hat{q}_1{}^* + 2L|\hat{q}_2|^2\hat{q}_1{}^* - i\hat{q}_{1t}^*\right)\rho_1^*(z),$$
$$\hat{g}_2(z,t) = \frac{1}{4}i\left[d_1(z) - a_1(z)\right] \times \left(2L|\hat{q}_2|^2 + 2L|\hat{q}_1|^2\hat{q}_2{}^* - i\hat{q}_{2t}^*\right)\rho_2^*(z),$$
$$\hat{k}_1(z,t) = \frac{1}{4}\hat{q}_2\hat{q}_1{}^*\rho_1^*(z)\left[a_1(z) - d_1(z)\right]\rho_2(z),$$
$$\hat{k}_2(z,t) = \frac{1}{4}\hat{q}_1\hat{q}_2{}^*\rho_2^*(z)\left[a_1(z) - d_1(z)\right]\rho_1(z),$$
$$\hat{a}_2(z,t) = \frac{1}{4}\left[d_1(z) - a_1(z)\right] \times \left[|\hat{q}_1|^2|\rho_1(z)|^2 + |\hat{q}_2|^2|\rho_2(z)|^2\right] + a_{22}(z) + iL\hat{\theta}_z(z,t),$$
$$\hat{h}_2(z,t) = \frac{1}{4}|\hat{q}_2|^2\rho_2^*(z)\left[a_1(z) - d_1(z)\right]\rho_2(z)_z(z,t) + h_{22}(z) - iL\hat{\theta},$$
$$\hat{d}_2(z,t) = \frac{1}{4}|\hat{q}_1|^2\rho_1^*(z)\left[a_1(z) - d_1(z)\right]\rho_1(z) - iL\hat{\theta}_z(z,t) + d_2(z),$$
$$\hat{\Delta}_1 = \left[2L|\hat{q}_2|^2\rho_1(z) + i\rho_{1t}(z,t)\right]\hat{q}_1,$$
$$\hat{\Delta}_2 = 2L\rho_2(z)|\hat{q}_2|^2\hat{q}_2 + 2L|\hat{q}_1|^2\rho_2(z)\hat{q}_2,$$

Then, the matrix S can be constructed as

$$S = H\Lambda H^{-1}, \tag{10}$$

with

$$H = \begin{pmatrix} \psi_1\ (\lambda_1) & \psi_2^*\ (\lambda_1) & \psi_3^*\ (\lambda_1) \\ \psi_2\ (\lambda_1) & -\psi_1^*\ (\lambda_1) & 0 \\ \psi_3\ (\lambda_1) & 0 & -\psi_1^*\ (\lambda_1) \end{pmatrix},$$

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1^* & 0 \\ 0 & 0 & \lambda_1^* \end{pmatrix}, \tag{11}$$

and

$$A(z,t) = \alpha_1(z)\ \exp\left[-\int \Delta_3\ \mathrm{d}t\right],$$
$$B(z,t) = \alpha_2(z)\ \exp\left[\int \Delta_3\ \mathrm{d}t\right],$$
$$C(z,t) = \alpha_3(z)\ \exp\left[\int \Delta_3\ \mathrm{d}t\right],$$
$$\Delta_3 = \frac{4iL\left(\lambda_1-\lambda_1^*\right)^2|\psi_1|^2\left(|\rho_2(z)|^2|\psi_2|^2+|\rho_1(z)|^2|\psi_3|^2\right)}{|\rho_1(z)|^2|\rho_2(z)|^2\left(|\psi_1|^2+|\psi_2|^2+|\psi_3|^2\right)^2}, \tag{12}$$

where $[\psi_1(\lambda_1), \psi_2(\lambda_1), \psi_3(\lambda_1)]^T$ is the solution of Lax Pair (2) with $\lambda = \lambda_1$, $\alpha_1(z)\,\alpha_2(z)$ and $\alpha_3(z)$ are functions of z. Transformations between the new potentials $\hat{q}_1$, $\hat{q}_2$ and the old ones q_1, q_2 can be presented as

$$\hat{q}_1 = A(z,t)B(z,t)^{-1}\rho_1(z)^{-1} \times \left(q_1\rho_1(z) + \frac{4\,\mathrm{Im}\,(\lambda_1)\ \psi_1\psi_2^*}{|\psi_1|^2+|\psi_2|^2+|\psi_3|^2}\right),$$
$$\hat{q}_2 = A(z,t)C(z,t)^{-1}\rho_2(z)^{-1} \times \left(q_2\rho_2(z) + \frac{4\,\mathrm{Im}\,(\lambda_1)\ \psi_1\psi_3^*}{|\psi_1|^2+|\psi_2|^2+|\psi_3|^2}\right). \tag{13}$$

3 One-soliton solutions of Eq. (1)

In this section, we will construct the one-soliton solutions of Eq. (1). Taking $q_1 = q_2 = 0$ as the seed solutions of Eq. (1), Lax Pair (2) with $\lambda = \lambda_1$ can be solved as

$$\psi_1\ (\lambda) = c_1 e^{\xi},\quad \psi_2\ (\lambda) = c_2 e^{-\xi},\qquad \psi_3\ (\lambda) = c_3 e^{-\xi}, \tag{14}$$

where c_1, c_2 and c_3 are arbitrary constants and $\xi = \int\left[a_1(z)\lambda_1^2 + a_{22}(z)\right]\mathrm{d}z - it\lambda_1$.

Substituting Eq. (14) into Eq. (13), we can get one-soliton solutions of Eq. (1) as follows:

$$|q_1| = \frac{4|\mathrm{Im}\,(\lambda_1)\,c_2|}{|\rho_1(z)|\sqrt{|c_2|^2+|c_3|^2}}\,\mathrm{sech}\left[\xi+\xi^*+\ln\Delta_4\right], \tag{15}$$

$$|q_2| = \frac{4|\mathrm{Im}\,(\lambda_1)\,c_3|}{|\rho_2(z)|\sqrt{|c_2|^2+|c_3|^2}}\,\mathrm{sech}\left[\xi+\xi^*+\ln\Delta_4\right], \tag{16}$$

$$\Delta_4 = \frac{|c_1|}{\sqrt{|c_2|^2+|c_3|^2}}.$$

Some physical quantities such as the amplitude A, velocity v, width W, initial phases I_p, and energy E are given to characterize the features of propagating solitons:

$$A_1 = \frac{4\,|\mathrm{Im}\,(\lambda_1)\,c_2|}{|\rho_1(z)|\sqrt{|c_2|^2+|c_3|^2}},\quad A_2 = \frac{4\,|\mathrm{Im}\,(\lambda_1)\,c_3|}{|\rho_2(z)|\sqrt{|c_2|^2+|c_3|^2}},$$
$$W_1 = W_2 = \frac{1}{2\,\mathrm{Im}\,(\lambda_1)},\quad I_{p1} = I_{p2} = \frac{1}{2\,\mathrm{Im}\,(\lambda_1)}\ln_4,$$
$$v_1 = v_2 = \frac{\mathrm{Re}\left[a_1(z)\lambda_1^2 + a_{22}(z)\right]}{-\mathrm{Im}\,(\lambda_1)},$$
$$E = \frac{32\,|\mathrm{Im}\,(\lambda_1)\,|^2}{|\rho_1(z)|^2}.$$

From above, we can see that the width and initial phases are both dependent on the imaginary part of λ_1, while the amplitude, velocity, and energy are determined by the imaginary part of λ_1 and variable coefficients.

Multi-soliton solutions can be achieved by the iterative algorithm. The dynamic features of the obtained soliton solutions are depicted in Fig. 1 using Eqs. (15)–(16).

4 An infinite number of conservation laws

In this section, according to Refs. [43,44] we present infinitely many independent conservation laws as a further support of the integrability for Eq. (1).

We will introduce two new variables,

$$\Gamma_1 = \frac{\psi_2}{\psi_1},\quad \Gamma_2 = \frac{\psi_3}{\psi_1}, \tag{17}$$

and take derivative of Γ_j $(j = 1, 2)$ with respect to t by the use of Eq. (2) to obtain the following two Riccati-type equations:

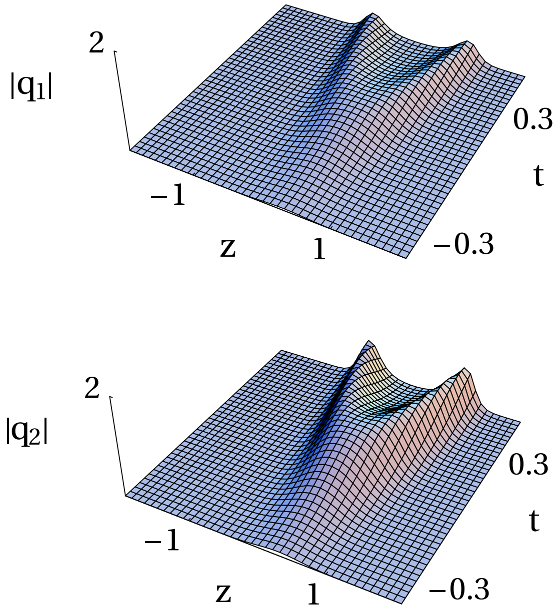

Fig. 1 The stable propagation of one-soliton solutions (15)–(16). Parameters are given as follows: $a_1(z) = z$, $a_{22}(z) = 1 + i$, $c_1 = c_2 = 1$, $c_3 = 1 + i$, $\rho_1(z) = 3 + 2i$, $\rho_2(z) = 2 - 3i$ and $\lambda_1 = 1 + 2i$

$$\Gamma_{1,t} = -q_1^*\rho_1^*(z) + 2\big[i\lambda_1 - iL\theta_t(z,t)\big]\Gamma_1 - \Gamma_1^2 q_1\rho_1(z) - q_2\rho_2(z)\Gamma_1\Gamma_2, \quad (18)$$

$$\Gamma_{2,t} = -q_2^*\rho_2^*(z) + 2\big[i\lambda_1 - iL\theta_t(z,t)\big]\Gamma_2 - \Gamma_1\Gamma_2 q_1\rho_1(z) - q_2\rho_2(z)\Gamma_2^2, \quad (19)$$

then multiply Eqs. (18) and (19), respectively, by q_1 and q_2, and expand $q_1\Gamma_1$ and $q_2\Gamma_2$ in power series of $1/\lambda$,

$$q_1\Gamma_1 = \sum_{m=1}^{\infty}\lambda^{-m}\Gamma_{1m}(z,t), \quad q_2\Gamma_2 = \sum_{m=1}^{\infty}\lambda^{-m}\Gamma_{2m}(z,t), \quad (20)$$

Γ_{1m} and Γ_{2m} $(m = 1, 2, \ldots)$ are determined by

$$\Gamma_{11} = -\frac{i}{2}|q_1|^2(\rho_1)^*(z), \quad \Gamma_{21} = -\frac{i}{2}|q_2|^2(\rho_2)^*(z),$$

$$\Gamma_{12} = -\frac{1}{4}q_1 q_{1t}^*(\rho_1)^*(z) - \frac{i}{2}|q_1|^2(\rho_1)^*(z)L\theta_t(z,t),$$

$$\Gamma_{22} = -\frac{1}{4}q_2 q_{2t}^*(\rho_2)^*(z) - \frac{i}{2}|q_2|^2(\rho_2)^*(z)L\theta_t(z,t),$$

$$\Gamma_{1m+1} = -\frac{i}{2}\Big[\rho_1(z)\sum_{k=1}^{m-1}\Gamma_{1m-1-k}\Gamma_{1k} + \rho_2(z)\sum_{k=1}^{m-1}\Gamma_{1m-k}\Gamma_{2k} + \left(\frac{\Gamma_{1m}}{q_1}\right)_t q_1 + 2i\,L\theta_t(z,t)\,\Gamma_{1m}\Big]\ (m>2),$$

$$\Gamma_{2m+1} = -\frac{i}{2}\Big[\rho_2(z)\sum_{k=1}^{m-1}\Gamma_{2m-1-k}\Gamma_{2k} + \rho_1(z)\sum_{k=1}^{m-1}\Gamma_{1m-k}\Gamma_{2k} + |!\left(\frac{\Gamma_{2m}}{q_2}\right)_t q_2 + 2i\,L\theta_t(z,t)\,\Gamma_{2m}\Big]\ (m>2).$$

By the compatibility condition $(\log\psi_1)_{zt} = (\log\psi_1)_{tz}$ yields the following equation in the form of conservation law:

$$\Big\{\big[-i\lambda_1 + iL\theta_t(z,t)\big] + q_1\,\Gamma_1\rho_1(z) + q_2\,\Gamma_2\rho_2(z)\Big\}_t = \Big\{a_1(z)\lambda^2 + a_2(z,t) + \big[\lambda b_1(z,t) + b_2(z,t)\big]\Gamma_1 + \big[\lambda f_1(z,t) + f_2(z,t)\big\}_z. \quad (21)$$

By substituting Eq. (20) into Eq. (21) and equating the terms with the same power of $1/\lambda$, we can obtain a sufficiently large number of conservation laws: $i\,\frac{\partial\rho_k}{\partial t} = \frac{\partial J_k}{\partial z}$ $(k = 1, 2, \ldots)$, where ρ_k and J_k $(k = 1, 2, \ldots)$ are the conserved densities and associated fluxes, respectively.

5 Conclusions

Twin-core nonlinear fibers and waveguides, i.e., couplers, have become a current interest in nonlinear optics [28]. In this paper, by virtue of DT (5) and symbolic computation, Eq. (1) describing the effects of quintic nonlinearity on the ultrashort optical pulse propagation in non-Kerr media has been investigated. Lax Pair (2) of Eq. (1) has been presented, and the corresponding DT (5) has been constructed. Moreover, one-soliton solutions, i.e., Solutions (15)–(16), have been obtained and an infinite number of conservation laws, i.e., Expressions (20)–(21), have also been derived. Using Solutions (15)–(16), the dynamic features of the soliton solutions have been displayed in Figure 1. Some physical quantities such as the amplitude, velocity, width, initial, phases, and energy are also, respectively, analyzed. These results might be of some value for the ultrashort optical pulse propagation in the non-Kerr media.

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