Nonlinear Dyn
DOI 10.1007/s11071-012-0445-8

ORIGINAL PAPER

A bus-following model with an on-line bus station

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Received: 15 April 2012 / Accepted: 28 April 2012

Abstract In this paper, we developed a bus-following model with consideration of an on-line bus station based on the properties of each bus's motion. The numerical results show that the proposed model can qualitatively describe the effects of an on-line bus station on each bus's motion.

Keywords Traffic flow · Bus-following model · On-line bus station

1 Introduction

Bus and bus station are important compositions of urban traffic system, which has attracted researchers to develop many models to study various complex traffic phenomena resulting by bus and bus station [1–18]. However, these models cannot completely reproduce the impacts of bus station on the bus-following behavior because many factors related to bus and bus station (e.g., the bus's dwelling time at the bus station, the bus's time headway at the origin, the passenger's arrival rate at the bus station, etc.) are not explicitly considered. In real traffic system, each bus has following behavior during its running process, so we in this paper use car-following theory to develop a bus-following model to study the impacts of an online bus station on the bus-following behavior.

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2 Model

The existing car-following models can be written as follows [19–29]:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = f(v_n, \Delta x_n, \Delta v_n, \ldots), \tag{1}$$

where v_n, Δx_n, Δv_n are, respectively, the nth vehicle's speed, headway and relative speed, $f(\cdot)$ is the nth vehicle's stimulus function determined by v_n, Δx_n, Δv_n and other factors. Equation (1) can describe many complex traffic phenomena from different perspectives, but it cannot be used to study the bus's motion in the traffic system with bus and bus station since the factors related to bus and bus station are not explicitly considered.

In fact, bus also has its following behavior in the traffic system with bus and bus station [1–5]; what's more, a special lane has been allocated to buses in many cities (e.g., Beijing), which further shows that bus has following behavior. So we can use the car-following theory to study the bus-following behavior. However, there are many factors that affect the bus-following behavior (e.g., the bus's dwelling time at the bus station, the number of bus stations, the passenger's

Published online: 31 May 2012

arrival rate at the bus station, etc.), so the bus's following behavior is much more complex than other vehicle's. In brief, the bus-following behavior in the traffic system with on-line bus stations has the following properties:

(1) Each bus station only affects the bus's behavior that is going to enter the bus station;
(2) Each bus may stay for some time at each bus station because some passengers should get on or off the bus.

In addition, we should here give the following assumptions:

(a) all the buses run only on the special lane and they have the same origin and destination; overtaking is not permitted on the special lane and there is an on-line bus station that only accommodates a bus;
(b) when a bus enters the bus station, the passengers on the platform can board the bus in queue and the bus can pick up all the passengers;
(c) when a bus arrives at the destination, it will immediately be removed and its following bus will become the leading bus.

Thus, we can develop a bus-following model with an on-line bus station, i.e.,

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \begin{cases} \kappa(V_n(\Delta x_n(t)) - v_n(t)) + \alpha \Delta v_n(t), \\ \quad \text{if } x_{n+1} < x_0 \text{ or } x_n > x_0, \\ \kappa(V_n(\Delta \bar{x}_n(t)) - v_n(t)) + \alpha \Delta \bar{v}_n(t), \\ \quad \text{if } x_n < x_0 \text{ and } x_{n+1} > x_0, \\ 0, \quad \text{if } x_n(t) = x_0 \text{ and } t_{n,0} + T_n > t \geq t_{n,0}, \end{cases} \tag{2}$$

where x_0 is the position of the bus station; $t_{n,0}$ is the time that the nth bus arrives the bus station; T_n is the time that the nth bus should station at the bus station; $\Delta \bar{x}_n = x_0 - x_n$; $\Delta \bar{v}_n = -v_n$; $V_n(\cdot)$ is the nth bus's optimal velocity; κ, α are two reaction coefficients. In this paper, we set κ as constant and define α as follows [24]:

$$\alpha = \begin{cases} a, & \Delta \bar{x}_n \leq s_c, \\ b, & \Delta \bar{x}_n > s_c, \end{cases} \tag{3}$$

where a, b, s_c are constant.

Before exploring what phenomena (2) can reproduce, we should first analyze the properties of the optimal speed $V_n(\cdot)$ since Nagatani [2] and Huijberts [4] found that on-line bus station will affect the optimal speed $V_n(\cdot)$ when the nth bus is going to enter the bus station. In fact, if there are passengers waiting at the platform when the nth bus is going to enter the bus station, the bus cannot achieve its ideal speed $\bar{V}_n(\Delta \bar{x}_n)$ (here $\bar{V}_n(\Delta \bar{x}_n)$ is the nth bus's optimal velocity for the distance $\Delta \bar{x}_n$ without on-line bus station). Although the bus's optimal velocity is complex and maybe related to many factors when it is going to enter the on-line bus station, the exact definition of $V_n(\Delta \bar{x}_n)$ has no qualitative effects on the numerical results. Therefore, we can for simplicity define $V_n(\Delta \bar{x}_n)$ of (2) as follows [2, 4]:

$$V_n\big(\Delta \bar{x}_n(t)\big) = \big(\beta + (1-\beta)\exp\big(-\lambda \Delta \bar{x}_n(t)\big)\big) \times \bar{V}\big(\Delta \bar{x}_n(t)\big), \tag{4}$$

where λ is the passenger's arrival rate, $0 \leq \beta \leq 1$ is a parameter that can reflect the properties of on-line bus station. As for the parameters λ, β, we should here give the following notes:

(1) $V_n(\Delta \bar{x}_n(t)) = \exp(-\lambda \Delta \bar{x}_n(t)) \bar{V}(\Delta \bar{x}_n(t))$ when $\beta = 0$, which means that passengers can board the nth bus everywhere;
(2) $V_n(\Delta \bar{x}_n(t)) = \bar{V}(\Delta \bar{x}_n(t))$ when $\beta = 1$, which show that there is no bus station in the traffic system;
(3) the passengers' arrival rate often has some stochastic properties, but we for simplicity assume λ is constant in this paper.

In fact, the optimal speeds $V_n(\Delta x_n)$, $\bar{V}_n(\Delta \bar{x}_n)$ are complex and related to many factors, but we can for simplicity define them as follows [24]:

$$\begin{aligned} V_n(\Delta x_n) &= V_1 + V_2 \tanh\big(C_1(\Delta x_n - l_c) - C_2\big), \\ \bar{V}_n(\Delta \bar{x}_n) &= V_1 + V_2 \tanh\big(C_1(\Delta \bar{x}_n) - C_2\big), \end{aligned} \tag{5}$$

where l_c is the bus length, V_1, V_2, C_1, C_2 are constant.

Next, we should define the nth bus's dwelling time at the bus station. In fact, each bus's dwelling time at the bus station depends on the number of the passengers who should board and leave the bus. For simplicity, we assume that the nth bus's dwelling time is only related to the number of the passengers who should board the bus. Thus, we define T_n as follows:

$$T_n = T_0 + \mu N_n, \tag{6}$$

where T_0 is the minimum dwelling time at the bus station, μ is the time it takes a person to board the bus, N_n is the number of the passengers who should board

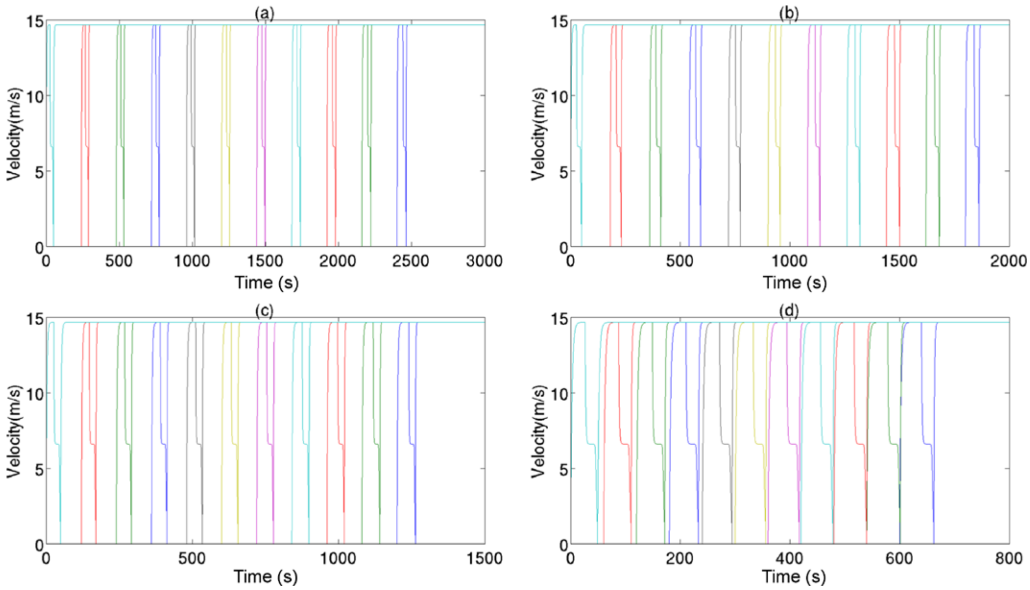

Fig. 1 The evolution of each bus's speed, where the bus's time headways at the origin in (**a**)–(**d**) are, respectively, 4 minutes, 3 minutes, 2 minutes and 1 minutes, and the parameter $\lambda = 0.2$

the nth bus at the bus station. Based on the passengers' arrival rate λ, N_n can be defined as follows:

$$N_n = \begin{cases} t_{1,0}\lambda, & \text{if } n = 1, \\ (t_{n,0} - (t_{n-1,0} + T_{n-1}))\lambda, & \text{if } n > 1. \end{cases} \tag{7}$$

Comparing with the existing models, the proposed bus-following model has explicitly considered some factors related to bus and on-line bus station (e.g., the bus's dwelling time, the passenger's arrival rate, etc.), so it can better describe the complex traffic phenomena of the traffic system with on-line bus station.

3 Numerical tests

It is difficult to obtain the analytical solution of (2), so we should use numerical scheme to study what phenomena the proposed model can describe. In fact, (2) has many numerical schemes, but the schemes have no qualitative effects on the numerical results, so we here use the Euler difference to discretize (2), i.e.

$$\begin{aligned} v_n(t+\Delta t) &= v_n(t) + \frac{\mathrm{d}v_n(t)}{\mathrm{d}t}\Delta t, \\ x_n(t+\Delta t) &= x_n(t) + v_n(t)\Delta t \\ &= x_n(t) + \frac{1}{2}\frac{\mathrm{d}v_n(t)}{\mathrm{d}t}(\Delta t)^2, \end{aligned} \tag{8}$$

where $x_n(t)$ is the nth bus's position, $\Delta t = 0.01$ s is the length of the time step.

Other parameters are defined as follows:

$$\begin{aligned} &\kappa = 0.41\ \mathrm{s}^{-1}, \quad a = 0.5, \quad b = 0, \\ &s_c = 150\ \mathrm{m}, \quad x_0 = 5\ \mathrm{km}, \quad L = 10\ \mathrm{km}, \\ &N = 20, \quad \beta = 0.8, \\ &V_1 = 6.75\ \mathrm{m/s}, \quad V_2 = 7.91\ \mathrm{m/s}, \\ &C_1 = 0.13\ \mathrm{m}^{-1}, \quad C_2 = 1.57, \\ &l_c = 10\ \mathrm{m}, \quad T_0 = 5\ \mathrm{s}, \quad \mu = 2, \end{aligned} \tag{9}$$

where L is the length of the bus route and N is the number of buses. In addition, we here set the bus's trail as the x-axis whose origin is the origin of the bus's route.

Using the proposed model and related parameters, we can obtain the evolution of each bus's speed and trail (see Figs. 1 and 2). From the two figures, we can obtain the following results:

(1) Each bus will immediately start at the origin, gradually stop when it enters the bus station and immediately start after it leaves the bus station for a couple of seconds (see Fig. 1).
(2) The bus's time headway has little effects on the evolution of each bus's velocity when it is greater than 1 minute (see Fig. 1).

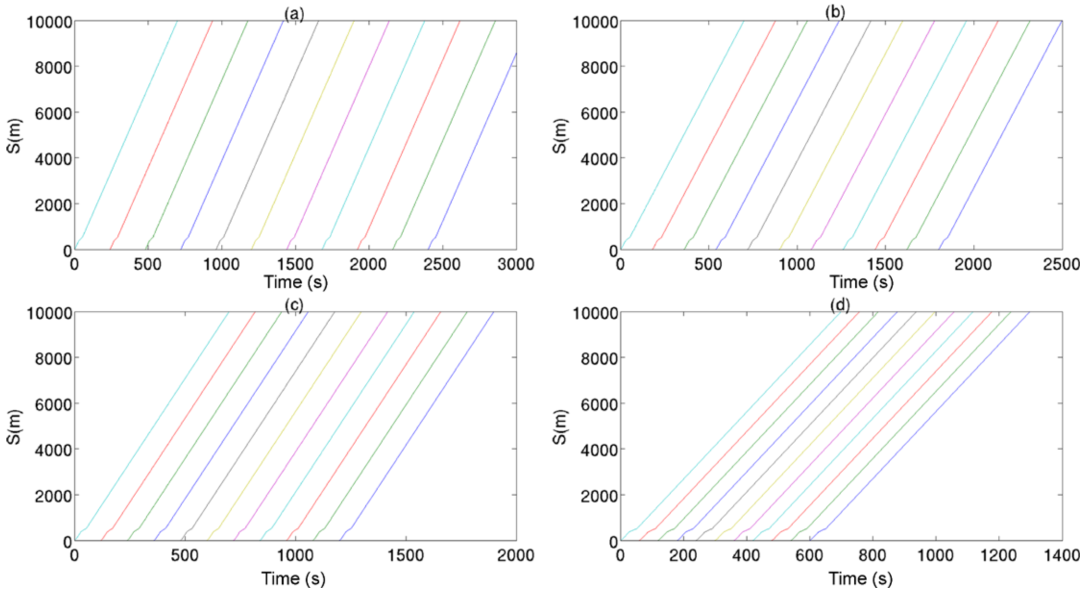

Fig. 2 The evolution of each bus's trail, where the bus's time headways at the origin in (**a**)–(**d**) are, respectively, 4 minutes, 3 minutes, 2 minutes and 1 minute, and the parameter $\lambda = 0.2$

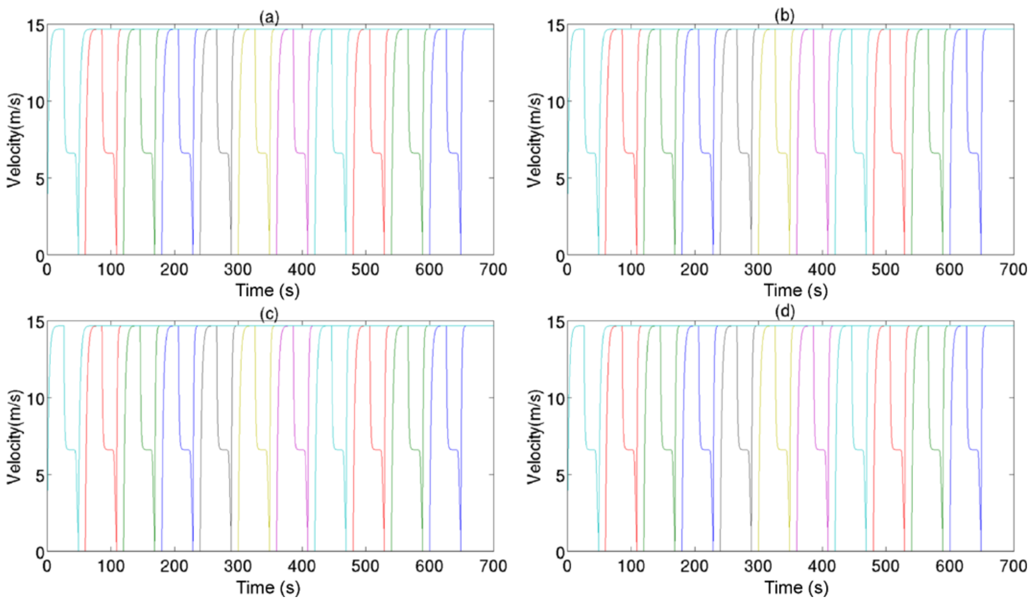

Fig. 3 The evolution of each bus's speed, where the parameter λ in (**a**)–(**d**) are, respectively, 0.2, 0.3, 0.4 and 0.5, and the bus's time headway is 1 minute

(3) When the bus's time headway is lower than or equal to 1 minute, it have great impacts on the evolution of each bus's velocity. At this time, the interactions between the two adjacent buses will slow down the starting process at the origin, the braking process at the bus station and the second starting process (see Fig. 1).

(4) A shock occurs at the bus station and its front-wave will turn more prominent when the bus's time headway increases (see Fig. 1).

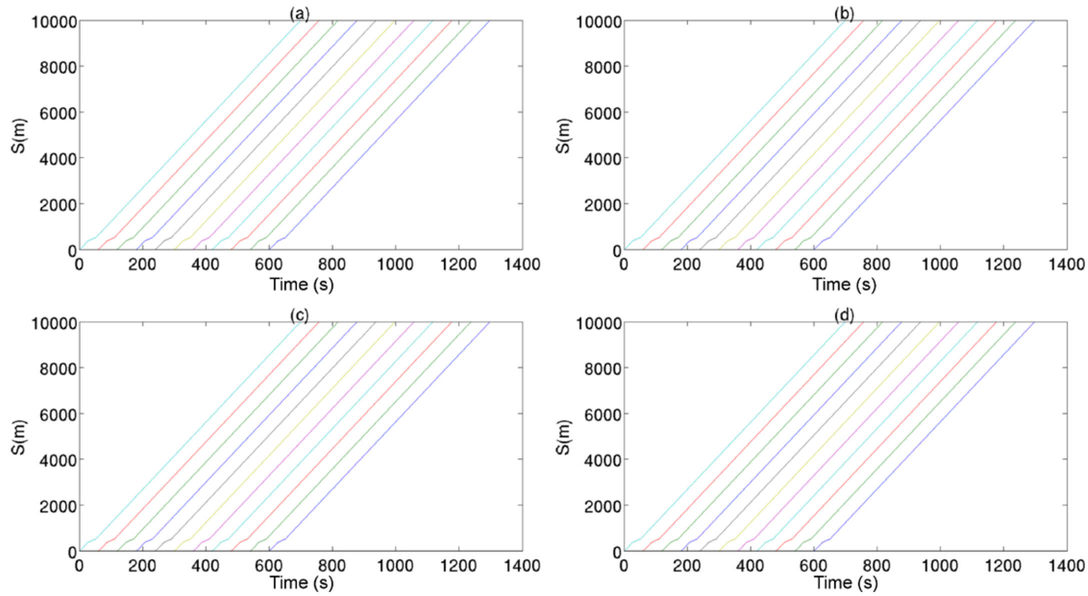

Fig. 4 The evolution of each bus's trail, where the parameter λ in (**a**)–(**d**) are, respectively, 0.2, 0.3, 0.4 and 0.5, and the bus's time headway is 1 minute

Using the proposed model and related parameters, we can obtain the evolution of each bus's speed and trail (see Figs. 3 and 4). From the two figures, we have:

(a) Qualitatively, the results are similar to those of Figs. 1 and 2, but the effects of the passengers' arrival rate on the evolution of each bus's velocity and the bus's movement trail are not as prominent as those of the bus's time headway (see Figs. 3 and 4).
(b) When the passengers' arrival rate increases, each bus's dwelling time will increase at the bus station, the second starting process will be postponed (see Fig. 3) and the shock that occurs at the bus stop will become more prominent (see Fig. 4).

Finally, we explore the impacts of on-line bus station on the starting process. Here, we set the bus's trail as the x-axis whose origin is the origin of the last bus's initial position. To compare with the car-following model [24], we define the initial conditions as follows: 11 buses distribute on the special lane with an on-line bus station, where the distance between the on-line bus station and the first bus is X and other buses' headways are equal to 15 meters; the 11 buses' initial speeds are equal to zero; once $t > 0$, the first bus will immediately start and other buses will gradually start. Using the proposed model and relevant parameters, we can obtain the evolutions of each bus's speed and acceleration (see Figs. 5 and 6). From the two figures, we can conclude the following results:

(i) Qualitatively, the effects of the bus station on the starting process are the same as those of one road-side memorial [29], but each bus should stop at the bus station (see Figs. 5 and 6).
(ii) When the distance between the leading bus and the bus station increases, the effects of the bus station on the starting process will become weaker (see Figs. 5 and 6).

4 Conclusions

In this paper, we develop a bus-following model based on the effects of a bus station on each bus's following behavior. The numerical results illustrate that the proposed model can describe some complex phenomena resulting by bus and a bus station (e.g., the starting and braking processes, etc.). However, we neither study the effects of bus station on the stability of traffic flow nor calibrate the related parameters. In addition, we do not consider the attributions of the bus's drivers [30]. Therefore, we should use empirical data to develop a more exact bus-following model to study

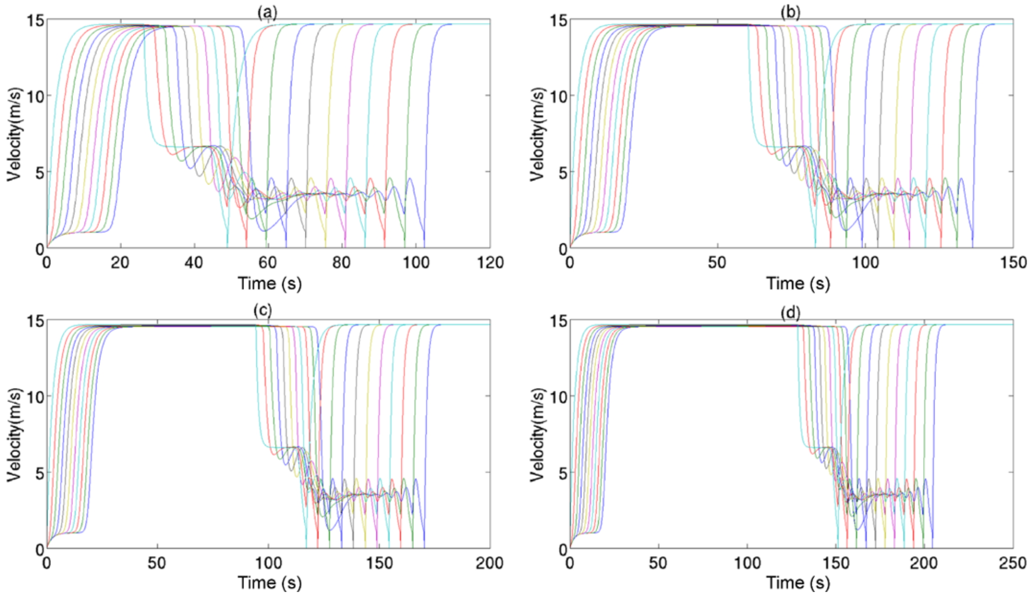

Fig. 5 The evolution of each bus's speed, where the parameter X in (**a**)–(**d**) are, respectively, equal to 500 meters, 1000 meters, 1500 meters and 2000 meters and the parameter $\lambda = 0.2$

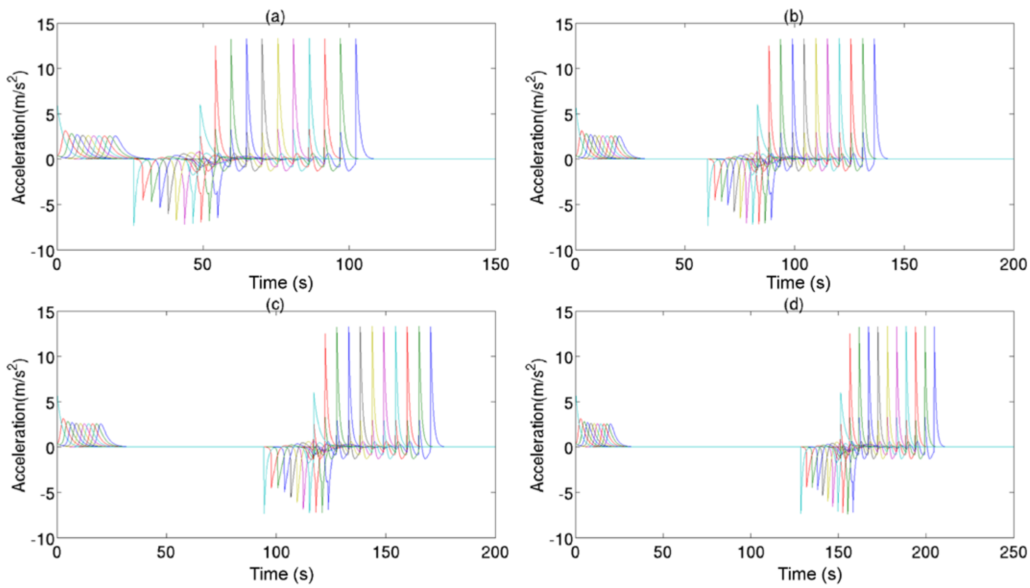

Fig. 6 The evolution of each bus's acceleration, where the parameter X in (**a**)–(**d**) are, respectively, equal to 500 meters, 1000 meters, 1500 meters and 2000 meters and the parameter $\lambda = 0.2$

the complex traffic phenomena resulting in the traffic system consisting of buses and bus station (including the complex traffic phenomena resulting by the attributions of the bus's drivers [30]).

Acknowledgements This study has been supported by the Program for New Century Excellent Talents in University, China (NCET-08-0038), the National Natural Science Foundation of China (70971007).

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