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Delayed-state-feedback exponential stabilization for uncertain Markovian jump systems with mode-dependent time-varying state delays

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Abstract This paper studies the problem of the robustly exponential stabilization for uncertain Markovian jump systems with mode-dependent time-varying state delays. The contribution of this paper is two-fold. Firstly, by constructing a modified Lyapunov functional and using free-weighting matrices technique, some delay-dependent robustly exponential stability criteria of such systems are obtained in terms of linear matrix inequalities (LMIs), which are less conservative than some existing ones. Secondly, a state feedback controller is designed, which can guarantee the robustly exponential stability of the uncertain closed-loop systems. Some illustrative numerical examples are given to demonstrate the reduced conservatism and applicability of the obtained results.

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1 Introduction

Markovian jump systems can model different types of dynamic ones subject to abrupt changes occurred in their structures, such as failure prone manufacturing systems, power systems and economics systems, and so on. This class of systems can be regarded as a special case of hybrid systems, since the state takes continuous values and the jumping parameters take discrete values in a system simultaneously. Since Markovian jump systems were firstly introduced by Krasovskii and Lidskii in [19], a great deal of attentions have been devoted to them in [10, 20, 27, 30, 31, 33] and the references therein.

Besides, the existence of time delays in systems can usually cause instability, oscillation and poor performance, and thus the study for the Markovian jump systems with time delay is of both theoretical and practical importance. The theory of stability analysis, feedback control and H_∞-control, as well as some other important applications for Markovian jump systems with time delay has been discussed in the past several decades. For example, the problems of stability analysis and H_∞ control of such systems via designing a state feedback controller have been addressed in [2, 6–8, 31]. The results about the stability analysis and H_∞ filtering for stochastic delayed Markovian jump systems have been obtained in [24, 32]. Lian et al. in [16, 17] and Wang et al. [28] have discussed the robust H_∞ sliding mode control, the adaptive variable structure control and the stabilization for a class of uncertain switched delay systems, respectively. The research on the controller design of Markovian jump systems with time delay has been conducted by Cao and Lam [6], and Wang et al. [26], but the results proposed in [6, 26] are delay-independent, which means that they are much more conservative than delay-dependent ones owing to failing to use of the information on the length of delays. Recently, Boukas et al. [2] have obtained some delay-dependent stability conditions for Markovian jump systems with time delay by employing the bounding technique and using the mode transformation. Chen et al. [5] have acquired some delay-dependent sufficient criteria for Markovian jump systems with time delay by using free-weighting matrices (see Refs. [14, 15, 25]). In [8], some less conservative criteria for Markovian jump systems with time delay were derived by constructing an appropriate Lyapunov–Krasovskii functional (LKF) and using Moon's inequality. Chen et al. [9] have also obtained some delay-dependent stability conditions for Markovian jump systems with time delay by exploiting a descriptor model transformation and employing the bounding technique. To obtain much less conservative criteria, Fei et al. [11] have achieved some delay-dependent conditions for Markovian jump systems with time delay by using the delay-partitioning approach (see Ref. [13]). But the results given in [2, 5, 8, 9, 11] are only suitable for the constant delay which is mode-independent. In [29], Xu et al. have obtained some delay-dependent sufficient conditions ensuring the stochastic stability for Markovian jump systems with time-varying delay by employing the free-weighting matrices (see Refs. [14, 15, 25]) and the time-varying delay is also mode-independent. As we know, in real systems, the transmission delays may occur randomly, which can be also modeled as a Markov process [21], and thus the mode-dependent time-varying delays are more natural and general than the mode-independent ones in Markovian jumping systems (see Refs. [19, 22, 27, 30, 31]).

On the other hand, typical stability analysis of Markovian jump systems with time delay mentioned above is involved with stochastic stability. But from the point of practical application, the exponential stability is of much significance, since the exponential stability can provide fast convergence and the desirable accuracy once the decay rate is determined. Although the exponential stability analysis and exponential stabilization of Markovian jump systems with time delay have been discussed in Shu et al. [23] and Wang et al. [26], the results are only suitable for constant delay which is mode-independent.

Inspired by the statements above, it is very necessary to discuss the exponential stability and exponential stabilization of the uncertain Markovian jump systems with mode-dependent time-varying state delays. In this paper, to reduce the conservatism of the stability conditions, a modified Lyapunov–Krasovskii functional combined with the Leibniz–Newton formula are introduced. The distributed delay free-weighting matrix functional coupled with the Leibniz–Newton formula can avoid the use of the mode transformation and the bounding technique. Moreover, the conditions obtained in this paper are formulated in terms of linear matrix inequalities (LMIs), which are effective methods to treat the problems and can be easily checked by resorting to available software packages. Compared with some existing reports, the use of this modified Lyapunov–Krasovskii functional coupled with the Leibniz–Newton formula in our paper can reduce the conservatism in searching for the upper bound of the time-varying delays such that our considered systems are robustly exponentially stable. And then based on the derived stability conditions, a memory state feedback controller is designed such that the closed-loop systems is also robustly exponentially stable. Finally, some illustrative examples are provided to demonstrate the effectiveness and applicability of our results, and also give a convincing comparison with some existing results.

Notations Unless otherwise specified, for a real square matrix A, the notation $A > 0$ ($A \geq 0$, $A < 0$, $A \leq 0$) means that A is a real symmetric and positive definite (positive semi-definite, negative definite, negative semi-definite, respectively); $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of the square matrix A, respectively. If A is a vector or matrix, its transpose is denoted by A^T; $|B| = \sqrt{\mathrm{trace}(B^T B)}$ denotes the Euclidean norm of a vector B or its induced norm of a matrix B. Unless explicitly stated, matrices in this paper are assumed to have real entries and compatible dimensions. Let $\tau > 0$ and $C([-\tau, 0]; R^n)$ be the family of all continuous R^n-valued functions ϕ on the interval $[-\tau, 0]$ with the norm $\|\phi\| = \sup\{|\phi(\theta)| : -\tau \le \theta \le 0\}$.

2 Problem formulation

Consider the following linear state-delay systems with Markovian jumping parameters:

$$\dot{x}(t) = A\big(t, r(t)\big)x(t) + A_d\big(t, r(t)\big)x\big(t - h\big(t, r(t)\big)\big) + B\big(t, r(t)\big)u(t), \quad t \ge 0, \tag{2.1}$$

$$x_0(\theta) = \varphi(\theta), \quad \theta \in [-h, 0], \ r(0) = r_0, \tag{2.2}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^p$ is the control input. $A(t, r(t))$, $A_d(t, r(t))$, $B(t, r(t))$ are the system matrices of the stochastic jumping process $\{r(t), t > 0\}$. Here, $\{r(t), t > 0\}$ is a continuous-time Markov process taking values in the finite discrete set $S = \{1, 2, \ldots, N\}$. $\Pi = \{\gamma_{ij} : i, j \in S\}$ be the density matrix of Markov chain $\{r(t)\}_{t \ge 0}$. Thus, $\gamma_{ij} \ge 0$ for $i \ne j$ and $\gamma_{ii} = -\sum_{j=1, j\ne i}^{N} \gamma_{ij}$. Furthermore, the transition probability of Markov chain $\{r(t)\}_{t\ge 0}$ can be described as

$$P\big\{r(t+\Delta) = j | r(t) = i\big\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & i \ne j, \\ 1 + \gamma_{ii}\Delta + o(\Delta), & i = j, \end{cases}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$. $h(t, r(t))$ denotes the mode-dependent time-varying delay when the model is in $r(t)$. When $r(t) = i, i \in S$, $h(t, r(t))$ is denoted by $h_i(t)$, which is satisfied $0 \le h_i(t) \le h_i$ and $\dot{h}_i(t) \le \mu_i$. The initial condition of the systems is specified as $(r_0, \varphi(\cdot))$ with $r_0 \in S$ being the initial mode and φ being the initial function such that $\varphi \in C([-h, 0]; R^n)$, where $h = \max\{h_i, i \in S\}$. For simplicity of notation, when the systems operate in the ith mode $(r(t) = i)$, $A(t, r(t))$, $A_d(t, r(t))$ and $B(t, r(t))$ are denoted as $A_i(t)$, $A_{id}(t)$ and $B_i(t)$, which are matrix functions, and for each $i \in S$,

$$\begin{aligned} A_i(t) &= A_i + \Delta A_i(t), \\ A_{id}(t) &= A_{id} + \Delta A_{id}(t), \\ B_i(t) &= B_i + \Delta B_i(t), \end{aligned} \tag{2.3}$$

where A_i, A_{id} and B_i are known real constant matrices representing the nominal systems for all $i \in S$, and $\Delta A_i(t)$, $\Delta A_{id}(t)$ and $\Delta B_i(t)$ $(i \in S)$ are unknown matrices representing time-varying parameter uncertainties, which can be described as

$$\begin{bmatrix} \Delta A_i(t) & \Delta A_{id}(t) & \Delta B_i(t) \end{bmatrix} = E_i F_i(t)[H_{1i} \quad H_{2i} \quad H_{3i}], \quad i \in S, \tag{2.4}$$

where $E_i, H_{1i}, H_{2i}, H_{3i}$ are known constant matrices with compatible dimensions for each $i \in S$, and $F_i(t)$ $(i \in S)$ are unknown Lebesgue measurable matrix functions satisfying

$$F_i^T(t)F_i(t) \le I, \quad \forall i \in S. \tag{2.5}$$

In particular, when $F_i(t) \equiv 0$ $(i \in S)$, systems (2.1) are referred to as their nominal systems:

$$\dot{x}(t) = A_i x(t) + A_{id}(t)x\big(t - h_i(t)\big) + B_i u(t). \tag{2.6}$$

The main purpose of this paper is to obtain sufficient conditions such that the following two requirements are satisfied:

(1) The uncertain Markovian jump systems (2.1) are robustly exponentially stable in mean square when $u(t) = 0$.
(2) Design a robust state feedback controller

$$u(t) = K_i x(t) + K_{id} x\big(t - h_i(t)\big), \tag{2.7}$$

which can exponentially stabilize systems (2.1), where $K_i \in R^{p\times n}, K_{id} \in R^{p\times n}$ $(i \in S)$ are the gain matrices of the state feedback controller. Thus, by applying this controller to the systems (2.1) for each $i \in S$, we obtain the following closed-loop systems of (2.1) with (2.7) which can be described as

$$\dot{x}(t) = \big(A_i(t) + B_i(t)K_i\big)x(t) + \big(A_{id}(t) + B_i(t)K_{id}\big)x\big(t - h_i(t)\big). \tag{2.8}$$

Definition 2.1 The nominal Markovian jump systems (2.6) (with $u(t) = 0$) are said to be exponentially stable in mean square if there exist two positive constant scalars $\alpha > 0$ and $\lambda > 0$ such that

$$E\big|x(t)\big|^2 \le \alpha \|\varphi\|^2 e^{-\lambda t},$$

for the initial value $\varphi \in C([-h, 0]; R^n)$ and $r_0 \in S$, where $x(t)$ denotes the trajectory of the systems state at time t from the initial system state φ and the initial mode r_0.

Definition 2.2 Consider the uncertain Markovian jump systems (2.1). If there exists a state feedback controller (2.7) such that the resulting closed-loop systems (2.8) are robustly exponentially stable, then the uncertain Markovian jump systems (2.1) are said to be robustly exponentially stabilizable and the corresponding state feedback controller (2.7) are said to robustly exponentially stabilize systems (2.1) for all admissible uncertainties (2.4)–(2.5).

Before ending this section, we introduce the following lemmas, which are the necessary tools for the development of our results.

Lemma 2.3 (Schur complement lemma [3]) *For a given matrix*

$$S=\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$$

with $S_{11}=S_{11}^T$, $S_{22}=S_{22}^T$, *the following conditions are equivalent*:

(1) $S<0$;
(2) $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$;
(3) $S_{11}<0$, $S_{22}-S_{12}^TS_{11}^{-1}S_{12}<0$.

Lemma 2.4 [3] *Let* U, V, W *and* M *be real matrices of appropriate dimensions with* M *satisfying* $M=M^T$, *then*

$$M+UVW+W^TV^TU^T<0, \quad \textit{for all } V^TV\le I,$$

if and only if there exist a scalar $\varepsilon>0$ *such that*

$$M+\varepsilon^{-1}UU^T+\varepsilon W^TW<0.$$

3 Main results

In this section, some LMIs-based sufficient conditions for the exponential stability for the nominal systems (2.6) when $u(t)=0$ are firstly given by using the free-weighting matrices (see Refs. [14, 15, 25]).

Theorem 3.1 *For a given scalar* $\kappa>0$, *and the time-varying delays* $h_i(t)$ *satisfies* $0\le h_i(t)\le h_i\le h$ *and* $\dot h_i(t)\le\mu_i$ $(i\in S)$. *If there exist some positive definite symmetric matrices* $P, Q_1, Q_2, Q_3, Q_4, Q_{1i}\in R^{n\times n}$ $(i\in S)$ *and some appropriately dimensional matrices* $N_i=[N_{1i}^T, N_{2i}^T, N_{3i}^T, N_{4i}^T, N_{5i}^T]^T$, $M_i=[M_{1i}^T, M_{2i}^T, M_{3i}^T, M_{4i}^T, M_{5i}^T]^T$ $(i\in S)$, *such that the following linear matrix inequalities* (LMIs) *hold*:

$$\begin{aligned}
&\Pi_i=\begin{bmatrix}\Omega_i & \Pi_{i12}\\ \Pi_{i12}^T & \Pi_{i22}\end{bmatrix}<0,\\
&\Psi_i=\begin{bmatrix}\Omega_i & \Psi_{i12}\\ \Psi_{i12}^T & \Psi_{i22}\end{bmatrix}<0,\\
&\Phi_i=\begin{bmatrix}\Omega_i & \Phi_{i12}\\ \Phi_{i12}^T & \Phi_{i22}\end{bmatrix}<0,\\
&\Upsilon_i=\begin{bmatrix}\Omega_i & \Upsilon_{i12}\\ \Upsilon_{i12}^T & \Upsilon_{i22}\end{bmatrix}<0,
\end{aligned}\tag{3.1}$$

$$|\gamma_{ii}|Q_1+\sum_{j=1}^N\gamma_{ij}Q_{1j}\le Q_2,\tag{3.2}$$

where

$$\Omega_i=\begin{bmatrix}\Omega_{i11} & \Omega_{i12} & \Omega_{i13}\\ * & \Omega_{i22} & \Omega_{i23}\\ * & * & \Omega_{i33}\end{bmatrix},$$

$$\Pi_{i12}=\Psi_{i12}=\begin{bmatrix} hN_{4i} & -hN_{1i}^T & A_i^TQ_4\\ -hN_{4i}+hM_{4i} & -hN_{2i}^T & A_{id}^TQ_4\\ -hM_{4i} & -hD_{3i}^T & 0\end{bmatrix},$$

$$\Pi_{i22}=\begin{bmatrix} -hQ_3 & -h^2N_{4i}^T & 0\\ * & -hQ_4 & 0\\ * & * & -\frac{\kappa}{e^{\kappa h}-1}Q_4\end{bmatrix},$$

$$\Psi_{i22}=\begin{bmatrix} -hQ_3 & -h^2N_{5i}^T & 0\\ * & -hQ_4 & 0\\ * & * & -\frac{\kappa}{e^{\kappa h}-1}Q_4\end{bmatrix},$$

$$\Phi_{i12}=\Upsilon_{i12}=\begin{bmatrix} hN_{5i} & -hM_{1i}^T & A_i^TQ_4\\ -hN_{5i}+hM_{5i} & -hM_{2i}^T & A_{id}^TQ_4\\ -hM_{5i} & -hM_{3i}^T & 0\end{bmatrix},$$

$$\Phi_{i22} = \begin{bmatrix} -hQ_3 & -h^2 M_{4i}^T & 0 \\ * & -hQ_4 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1} Q_4 \end{bmatrix},$$

$$\Upsilon_{i22} = \begin{bmatrix} -hQ_3 & -h^2 M_{5i}^T & 0 \\ * & -hQ_4 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1} Q_4 \end{bmatrix},$$

$$\begin{aligned}\Omega_{i11} &= \kappa P_i + P_i A_i + A_i^T P_i + \sum_{j=1}^N \gamma_{ij} P_j + e^{\kappa h} Q_1 \\ &\quad + e^{kh} Q_{1i} + \frac{e^{\kappa h}-1}{\kappa} Q_2 + \frac{e^{\kappa h}-1}{\kappa} Q_3 \\ &\quad + N_{1i}^T + N_{1i},\\ \Omega_{i12} &= P_i A_{id} + N_{2i} - N_{1i}^T + M_{1i}^T,\\ \Omega_{i13} &= N_{3i} - M_{1i}^T,\\ \Omega_{i22} &= \left[-(1-\mu_i)e^{\kappa h} Q_1\right] \vee \left[-(1-\mu_i) Q_1\right] \\ &\quad - N_{2i}^T - N_{2i} + M_{2i}^T + M_{2i},\\ \Omega_{i23} &= -N_{3i} - M_{2i}^T + M_{3i},\\ \Omega_{i33} &= -Q_{1i} - M_{3i}^T - M_{3i},\end{aligned}$$

and $*$ means symmetric terms, the nominal Markovian jump systems (2.6) when $u(t)=0$ are exponentially stable.

Proof Define a Lyapunov–Krasovskii functional candidate for systems (2.6) as

$$V(t,x_t,i) = V_1(t,x_t,i) + V_2(t,x_t,i) + V_3(t,x_t,i), \tag{3.3}$$

where

$$x_t = x(t+\theta), \quad -2h \le \theta \le 0,$$

and

$$\begin{aligned}V_1(t,x_t,i) &= e^{\kappa t} x^T(t) P_i x(t),\\ V_2(t,x_t,i) &= \int_{t-h_i(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s)\, ds \\ &\quad + \int_{t-h}^t e^{\kappa(s+h)} x^T(s) Q_{1i} x(s)\, ds,\\ V_3(t,x_t,i) &= \int_{-h}^0 \int_{t+\theta}^t e^{\kappa(s-\theta)} \\ &\quad \times x^T(s)(Q_2+Q_3)x(s)\, ds\, d\theta \\ &\quad + \int_{-h}^0 \int_{t+\theta}^t e^{\kappa(s-\theta)} \dot{x}^T(s) Q_4 \dot{x}(s)\, ds\, d\theta.\end{aligned}$$

Let $\mathfrak{L}$ be the weak infinitesimal generator of the random process $\{(x_t, r(t), t \ge 0\}$ (see Ref. [21]). For each $i \in S$ and $t > h$, we have

$$\mathfrak{L}V(t,x_t,i) = \mathfrak{L}V_1(t,x_t,i) + \mathfrak{L}V_2(t,x_t,i) + \mathfrak{L}V_3(t,x_t,i), \tag{3.4}$$

where

$$\begin{aligned}&\mathfrak{L}V_1(t,x_t,i)\\ &\quad = e^{\kappa t} x^T(t)\left[\kappa P + P_i A_i + A_i^T P_i + \sum_{j=1}^N \gamma_{ij} P_j\right] x(t) \\ &\qquad + 2e^{\kappa t} x^T(t) P_i A_{id} x\big(t-h_i(t)\big),\end{aligned} \tag{3.5}$$

$$\begin{aligned}&\mathfrak{L}V_2(t,x_t,i)\\ &\quad = e^{\kappa t}\big[x^T(t)\big[e^{\kappa h} Q_1 + e^{\kappa h} Q_{1i}\big] x(t) \\ &\qquad - \big(1-\dot{h}_i(t)\big) x^T\big(t-h_i(t)\big) \\ &\qquad \times e^{-\kappa(h_i(t)-h)} Q_1 x\big(t-h_i(t)\big) \\ &\qquad - x^T(t-h) Q_{1i} x(t-h)\big] \\ &\qquad + \sum_{j=1}^N \gamma_{ij} \int_{t-h_j(t)}^t e^{\kappa(s+h)} x^T(s) Q_1 x(s)\, ds \\ &\qquad + \int_{t-h}^t e^{\kappa(s+h)} x^T(s) \sum_{j=1}^N \gamma_{ij} Q_{1j} x(s)\, ds,\end{aligned} \tag{3.6}$$

$$\begin{aligned}&\mathfrak{L}V_3(t,x_t,i)\\ &\quad = e^{\kappa t}\left[x^T(t)\frac{e^{\kappa h}-1}{\kappa}(Q_2+Q_3)x(t) \right.\\ &\qquad \left. + \dot{x}^T(t)\frac{e^{\kappa h}-1}{\kappa} Q_4 \dot{x}(t)\right] \\ &\qquad - e^{\kappa t} \int_{t-h}^t e^{\kappa h} x^T(s) Q_2 x(s)\, ds \\ &\qquad - e^{\kappa t} \int_{t-h_i(t)}^t e^{\kappa h} x^T(s) Q_3 x(s)\, ds\end{aligned}$$

$$
\begin{aligned}
&- e^{\kappa t}\int_{t-h}^{t-h_i(t)} e^{\kappa h} x^T(s)Q_3 x(s)\,ds \\
&- e^{\kappa t}\int_{t-h_i(t)}^{t} e^{\kappa h} \dot{x}^T(s)Q_4 \dot{x}(s)\,ds \\
&- e^{\kappa t}\int_{t-h}^{t-h_i(t)} e^{\kappa h} \dot{x}^T(s)Q_4 \dot{x}(s)\,ds. \qquad (3.7)
\end{aligned}
$$

By calculation, as $\mu_i \le 1$ ($i \in S$),

$$
\begin{aligned}
&-\big(1-\dot{h}_i(t)\big)e^{\kappa h-\kappa h_i(t)} x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big) \\
&\quad \le -(1-\mu_i)e^{\kappa h-\kappa h_i(t)} x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big) \\
&\quad \le -(1-\mu_i)x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big), \qquad (3.8)
\end{aligned}
$$

and as $\mu_i > 1 (i \in S)$,

$$
\begin{aligned}
&-\big(1-\dot{h}_i(t)\big)e^{\kappa h-\kappa h_i(t)} x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big) \\
&\quad \le -(1-\mu_i)e^{\kappa h-\kappa h_i(t)} x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big) \\
&\quad \le -(1-\mu_i)e^{\kappa h} x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big). \qquad (3.9)
\end{aligned}
$$

Thus, from (3.8) and (3.9) it follows that

$$
\begin{aligned}
&-\big(1-\dot{h}_i(t)\big)e^{\kappa h-\kappa h_i(t)} x^T\big(t-h_i(t)\big)Q_1 x\big(t-h_i(t)\big) \\
&\quad \le x^T\big(t-h_i(t)\big)\big[-(1-\mu_i)e^{\kappa h}Q_1\big] \\
&\qquad \vee \big[-(1-\mu_i)Q_1\big]x\big(t-h_i(t)\big). \qquad (3.10)
\end{aligned}
$$

Noting that $\gamma_{ij} \ge 0$, for $j \ne i$ and $\gamma_{ii} \le 0$, it follows that

$$
\begin{aligned}
&\sum_{j=1}^{N}\gamma_{ij}\int_{t-h_j(t)}^{t} e^{\kappa(s+h)}x^T(s)Q_1 x(s)\,ds \\
&\quad \le \sum_{j\ne i}\gamma_{ij}\int_{t-h_j(t)}^{t} e^{\kappa(s+h)}x^T(s)Q_1 x(s)\,ds \\
&\qquad - |\gamma_{ii}|\int_{t-h_i(t)}^{t} e^{\kappa(s+h)}x^T(s)Q_1 x(s)\,ds \\
&\quad \le \sum_{j\ne i}\gamma_{ij}\int_{t-h}^{t} e^{\kappa(s+h)}x^T(s)Q_1 x(s)\,ds \\
&\quad = |\gamma_{ii}|\int_{t-h}^{t} e^{\kappa(s+h)}x^T(s)Q_1 x(s)\,ds. \qquad (3.11)
\end{aligned}
$$

On the other hand, from the Newton–Leibniz formula, it is clear that

$$
\begin{aligned}
a_{1i}(t) &:= 2e^{\kappa t}\zeta^T(t)N_i\bigg[x(t)-x\big(t-h_i(t)\big) \\
&\qquad - \int_{t-h_i(t)}^{t}\dot{x}(s)\,ds\bigg] = 0, \qquad (3.12) \\
a_{2i}(t) &:= 2e^{\kappa t}\zeta^T(t)M_i\bigg[x\big(t-h_i(t)\big)-x(t-h) \\
&\qquad - \int_{t-h}^{t-h_i(t)}\dot{x}(s)\,ds\bigg] = 0, \qquad (3.13)
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta(t) = \bigg[&x^T(t), x^T\big(t-h_i(t)\big), x^T(t-h), \\
&\bigg[\int_{t-h_i(t)}^{t} x(s)\,ds\bigg]^T, \bigg[\int_{t-h}^{t-h_i(t)} x(s)\,ds\bigg]^T\bigg]^T.
\end{aligned}
$$

Substituting (3.5)–(3.13) into (3.4) yields

$$
\begin{aligned}
&\mathfrak{L}V(t,x_t,i) \\
&\quad \le e^{\kappa t}\bigg\{x^T(t)\bigg[\kappa P_i + P_i A_i + A_i^T P_i \\
&\qquad + \sum_{j=1}^{N}\gamma_{ij}P_j + e^{\kappa h}Q_1 + e^{\kappa h}Q_{1i} \\
&\qquad + \frac{e^{\kappa h}-1}{\kappa}(Q_2+Q_3)\bigg]x(t) \\
&\qquad + 2x^T(t)P_i A_{id} x\big(t-h_i(t)\big) \\
&\qquad + x^T\big(t-h_i(t)\big)\big[-(1-\mu_i)e^{\kappa h}Q_1\big] \\
&\qquad \vee \big[-(1-\mu_i)Q_1\big]x\big(t-h_i(t)\big) \\
&\qquad - x^T(t-h)Q_{1i}x(t-h) \\
&\qquad + |\gamma_{ii}|\int_{t-h}^{t} e^{\kappa h}x^T(s)Q_1 x(s)\,ds \\
&\qquad + \int_{t-h}^{t} e^{\kappa h}x^T(s)\sum_{j=1}^{N}\gamma_{ij}Q_{1j}x(s)\,ds \\
&\qquad - e^{\kappa t}\int_{t-h}^{t} e^{\kappa h}x^T(s)Q_2 x(s)\,ds \\
&\qquad - e^{\kappa t}\int_{t-h_i(t)}^{t} e^{\kappa h}x^T(s)Q_3 x(s)\,ds \\
&\qquad - e^{\kappa t}\int_{t-h}^{t-h_i(t)} e^{\kappa h}x^T(s)Q_3 x(s) \\
&\qquad + \dot{x}^T(t)\frac{e^{\kappa h}-1}{\kappa}Q_4\dot{x}(t) \\
&\qquad - e^{\kappa t}\int_{t-h_i(t)}^{t} e^{\kappa h}\dot{x}^T(s)Q_4\dot{x}(s)\,ds
\end{aligned}
$$

$$
\begin{aligned}
& - e^{\kappa t} \int_{t-h}^{t-h_i(t)} e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s) \Big\} \\
& + a_{1i}(t) + a_{2i}(t) \\
\le\ & \frac{1}{h^2} \int_{t-h_i(t)}^{t} \int_{t-h_i(t)}^{t} e^{\kappa t} \eta^T(t,s,\alpha,i) \\
& \times \Pi_i' \eta(t,s,\alpha,i)\, ds\, d\alpha \\
& + \frac{1}{h^2} \int_{t-h_i}^{t-h_i(t)} \int_{t-h_i(t)}^{t} e^{\kappa t} \eta^T(t,s,\alpha,i) \\
& \times \Psi_i' \eta(t,s,\alpha,i)\, ds\, d\alpha \\
& + \frac{1}{h^2} \int_{t-h_i(t)}^{t} \int_{t-h}^{t-h_i(t)} e^{\kappa t} \eta^T(t,s,\alpha,i) \\
& \times \Phi_i' \eta(t,s,\alpha,i)\, ds\, d\alpha \\
& + \frac{1}{h^2} \int_{t-h}^{t-h_i(t)} \int_{t-h}^{t-h_i(t)} e^{\kappa t} \eta^T(t,s,\alpha,i) \\
& \times \Upsilon_i' \eta(t,s,\alpha,i)\, ds\, d\alpha \\
& + |\gamma_{ii}| e^{\kappa t} \int_{t-h}^{t} e^{\kappa h} x^T(s) Q_1 x(s)\, ds \\
& + \int_{t-h}^{t} e^{\kappa h} x^T(s) \sum_{j=1}^{N} \gamma_{ij} Q_{1j} x(s)\, ds \\
& - e^{\kappa t} \int_{t-h}^{t} e^{\kappa h} x^T(s) Q_2 x(s)\, ds,
\end{aligned} \tag{3.14}
$$

where $\xi(t,s,\alpha,i) = [x^T(t), x^T(t-h_i(t)), x^T(t-h), \dot{x}^T(s)]^T$, and

$$
\Pi_i' = \begin{bmatrix} \Omega_i & \Pi_{i12}' \\ \Pi_{i12}'^T & \Pi_{i22}' \end{bmatrix} + \frac{e^{\kappa h}-1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i,
$$

$$
\Psi_i' = \begin{bmatrix} \Omega_i & \Psi_{i12}' \\ \Psi_{i12}'^T & \Psi_{i22}' \end{bmatrix} + \frac{e^{\kappa h}-1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i,
$$

$$
\Phi_i' = \begin{bmatrix} \Omega_i & \Phi_{i12}' \\ \Phi_{i12}'^T & \Phi_{i22}' \end{bmatrix} + \frac{e^{\kappa h}-1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i,
$$

$$
\Upsilon_i' = \begin{bmatrix} \Omega_i & \Upsilon_{i12}' \\ \Upsilon_{i12}'^T & \Upsilon_{i22}' \end{bmatrix} + \frac{e^{\kappa h}-1}{\kappa} \tilde{A}_i^T Q_4^{-1} \tilde{A}_i,
$$

$$
\Pi_{i12}' = \Psi_{i12}' = \begin{bmatrix} hN_{4i} & -hN_{1i}^T \\ -hN_{4i} + hM_{4i} & -hN_{2i}^T \\ -hM_{4i} & -hN_{3i}^T \end{bmatrix},
$$

$$
\Phi_{i12}' = \Upsilon_{i12}' = \begin{bmatrix} hN_{5i} & -hM_{1i}^T \\ -hN_{5i} + hM_{5i} & -hM_{2i}^T \\ -hM_{5i} & -hM_{3i}^T \end{bmatrix},
$$

$$
\Pi_{i22}' = \begin{bmatrix} -hQ_3 & -h^2 N_{4i} \\ * & -hQ_4 \end{bmatrix},
$$

$$
\Psi_{i22}' = \begin{bmatrix} -hQ_3 & -h^2 N_{5i}^T \\ * & -hQ_4 \end{bmatrix},
$$

$$
\Psi_{i22}' = \begin{bmatrix} -hQ_3 & -h^2 M_{4i} \\ * & -hQ_4 \end{bmatrix},
$$

$$
\Upsilon_{i22}' = \begin{bmatrix} -hQ_3 & -h^2 M_{5i}^T \\ * & -hQ_4 \end{bmatrix},
$$

$$
\tilde{A}_i = [Q_4^T A_i, Q_4^T A_{id}, 0].
$$

It follows from (3.1), by applying Lemma 2.3, that for each $i \in S$

$$
\Pi_i' < 0, \qquad \Psi_i' < 0, \qquad \Phi_i' < 0, \qquad \Upsilon_i' < 0.
$$

And, by virtue of (3.2), when $t > h$, from (3.14) it implies that

$$
\mathfrak{L}V(t, x_t, i) \le 0. \tag{3.15}
$$

Let $\alpha_1 = \max_{i \in S}\{|A_i|\}$ and $\alpha_2 = \max_{i \in S}\{|A_{id}|\}$. Thus, it follows from $\dot{x}(t) = A_i x(t) + A_{id} x(t - h_i(t))$ that for any $t \ge 0$,

$$
\begin{aligned}
|x(t)| &= \left| x(0) + \int_0^t \big[A_i x(s) + A_{id} x\big(s - h_i(s)\big)\big] ds \right| \\
&\le |x(0)| + \int_0^t \alpha_2 \big|x\big(s - h_i(s)\big)\big| ds \\
&\quad + \int_0^t \alpha_1 |x(s)|\, ds.
\end{aligned}
$$

When $0 \le t \le h$, the above inequality implies that

$$
|x(t)| \le [\alpha_2 h + 1] \sup_{-h \le \theta \le 0} |\varphi(\theta)| + \int_0^t \alpha_1 |x(s)|\, ds.
$$

By applying the Gronwall inequality to this inequality, we obtain, when $0 \le t \le h$,

$$
|x(t)| \le [\alpha_2 h + 1] e^{\alpha_1 h} \sup_{\theta \in [-h, 0]} |\varphi(\theta)|. \tag{3.16}
$$

Hence, we have, when $0 \le t \le h$,

$$\left|x(t)\right|^2 \le \nu \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2, \tag{3.17}$$

where $\nu = [\alpha_2 h + 1]^2 e^{2\alpha_1 h}$. Now, by employing the Dynkin's formula, (3.15) and (3.17), it follows that, when $t > h$,

$$\begin{aligned}
&EV\big(t, x_t, r(t)\big) \\
&\quad = V\big(h, x_h, r(h)\big) + E\int_0^{t-h} \mathfrak{L}V\big(x_s, r(s), s\big)\,ds \\
&\quad \le V\big(h, x_h, r(h)\big) \\
&\quad = e^{\kappa h} x^T(h) P_i x(h) \\
&\qquad + \int_{h-h_i(h)}^{h} e^{\kappa(s+h)} x^T(s) Q_1 x(s)\,ds \\
&\qquad + \int_0^h e^{\kappa(s+h)} x^T(s) Q_{1i} x(s)\,ds \\
&\qquad + \int_{-h}^0 \int_{h+s}^h e^{\kappa h} x^T(s)(Q_2+Q_3)x(s)\,ds\,d\theta \\
&\qquad + \int_{-h}^0 \int_{h+s}^h e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s)\,ds\,d\theta \\
&\quad \le e^{\kappa h} \max_{i\in S}\big\{|P_i|\big\}\nu \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 \\
&\qquad + e^{\kappa h}|Q_1| \frac{e^{\kappa h}-1}{\kappa} \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 \\
&\qquad + \max_{i\in S}\big\{|Q_{1i}|\big\} e^{\kappa h} \frac{e^{\kappa h}-1}{\kappa} \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 \\
&\qquad + \big(|Q_2|+|Q_3|\big) \\
&\qquad \times e^{\kappa h} \frac{e^{\kappa h}-\kappa h-1}{\kappa^2} \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 \\
&\qquad + 2|Q_4|\big(\alpha_1^2\nu + \alpha_2^2\big) \\
&\qquad \times e^{\kappa h} \frac{e^{\kappa h}-\kappa h-1}{\kappa^2} \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2.
\end{aligned}$$

On the other hand,

$$EV(t, x_t, i) \ge \frac{1}{\max_{i\in S}\{|P_i^{-1}|\}} e^{\kappa t} E\left|x(t)\right|^2.$$

Hence, when $t > h$,

$$E\left|x(t)\right|^2 \triangleq \sigma(h) \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 e^{-\kappa t}, \tag{3.18}$$

where

$$\begin{aligned}
\sigma(h) = \max_{i\in S}\big\{\big|P_i^{-1}\big|\big\} \Big\{ & e^{\kappa h} \max_{i\in S}\big\{|P_i|\big\}\nu \\
& + e^{\kappa h}|Q_1| \frac{e^{\kappa h}-1}{\kappa} + \max_{i\in S}\big\{|Q_{1i}|\big\} e^{\kappa h} \frac{e^{\kappa h}-1}{\kappa} \\
& + \big(|Q_2|+|Q_3|\big) e^{\kappa h} \frac{e^{\kappa h}-\kappa h-1}{\kappa^2} \\
& + 2|Q_4|\big(\alpha_1^2\nu + \alpha_2^2\big) e^{\kappa h} \frac{e^{\kappa h}-\kappa h-1}{\kappa^2} \Big\}.
\end{aligned}$$

When $0 \le t \le h$, we obtain

$$\begin{aligned}
&\frac{1}{\max_{i\in S}\{|P_i^{-1}|\}} e^{\kappa t} E\left|x(t)\right|^2 \\
&\quad \le e^{\kappa t} E x^T(t) P_i x(t) \\
&\quad \le e^{\kappa h} \max_{i\in S}\big\{|P_i|\big\}\nu \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2.
\end{aligned}$$

That is,

$$\begin{aligned}
E\left|x(t)\right|^2 \le{} & \max_{i\in S}\big\{\big|P_i^{-1}\big|\big\} e^{\kappa h} \max_{i\in S}\big\{|P_i|\big\}\nu \\
& \times \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 e^{-\kappa t}.
\end{aligned} \tag{3.19}$$

It follows from (3.18) and (3.19) that when $t > 0$,

$$E\left|x(t)\right|^2 \le \sigma'(h) \sup_{\theta\in[-h,0]} \left|\varphi(\theta)\right|^2 e^{-\kappa t},$$

where $\sigma'(h) = \max\{\sigma(h), \max_{i\in S}\{|P_i^{-1}|\} e^{\kappa h} \times \max_{i\in S}\{|P_i|\}\nu\}$. Thus, the nominal systems (2.6) are exponentially stable with the decay rate κ. The proof of this theorem is completed. □

Remark 3.2 It is satisfactory that Theorem 3.1 holds without the restrictive condition: $\dot{h}_i(t) \le \mu_i < 1$ $(i \in S)$. Thus, Theorem 3.1 is more general than those given in [27, 30].

Remark 3.3 For the nominal Markovian jump systems (2.6) with mode-dependent time-varying state delays, the novelties of Theorem 3.1 are that the modified Lyapunov–Krasovskii functional (3.3) we introduced and the terms $-e^{\kappa t}\int_{t-h}^{t-h_i(t)} e^{\kappa h} x^T(s) Q_3 x(s)\,ds$ and $-e^{\kappa t}\int_{t-h}^{t-h_i(t)} e^{\kappa h} \dot{x}^T(s) Q_4 \dot{x}(s)\,ds$ are fully used, which can reduce the conservatism.

Remark 3.4 In considering Theorem 3.1, the derivative value of the Lyapunov–Krasovskii functional can ultimately be written as the sum (3.14) of four parts:

$$\frac{1}{h^2}\int_{t-h_i(t)}^{t}\int_{t-h_i(t)}^{t} e^{\kappa t}\eta^T(t,s,\alpha,i) \times \Pi_i'\eta(t,s,\alpha,i)\,ds\,d\alpha,$$

$$\frac{1}{h^2}\int_{t-h_i}^{t-h_i(t)}\int_{t-h_i(t)}^{t} e^{\kappa t}\eta^T(t,s,\alpha,i) \times \Psi_i'\eta(t,s,\alpha,i)\,ds\,d\alpha,$$

$$\frac{1}{h^2}\int_{t-h_i(t)}^{t}\int_{t-h}^{t-h_i(t)} e^{\kappa t}\eta^T(t,s,\alpha,i) \times \Phi_i'\eta(t,s,\alpha,i)\,ds\,d\alpha \quad \text{and}$$

$$\frac{1}{h^2}\int_{t-h}^{t-h_i(t)}\int_{t-h}^{t-h_i(t)} e^{\kappa t}\eta^T(t,s,\alpha,i) \times \Upsilon_i'\eta(t,s,\alpha,i)\,ds\,d\alpha.$$

This treatment is different from the ones in [19, 24, 27, 29, 32] and can effectively reduce the conservatism in [19, 24, 27, 29, 32], which will be illustrated by some numerical examples in Sect. 5.

Now, we generalize Theorem 3.1 to the corresponding uncertain case, and give the following sufficient conditions on the robustly exponential stability for the uncertain Markovian jump systems (2.1) when $u(t)=0$.

Theorem 3.5 *For a given scalar $\kappa>0$, and the time-varying delays $h_i(t)$ satisfies $0\le h_i(t)\le h_i\le h$ and $\dot h_i(t)\le\mu_i$ $(i\in S)$. If there exist some positive definite symmetric matrices*: $P, Q_1, Q_2, Q_3, Q_4, Q_{1i}\in R^{n\times n}$ $(i\in S)$ *and some appropriately dimensional matrices $N_i=[N_{1i}^T, N_{2i}^T, N_{3i}^T, N_{4i}^T, N_{5i}^T]^T$, $M_i=[M_{1i}^T, M_{2i}^T, M_{3i}^T, M_{4i}^T, M_{5i}^T]^T$ $(i\in S)$, and some positive scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}$ $(i\in S)$ such that (3.2) and the following linear matrix inequalities* (LMIs) *hold*:

$$\begin{aligned}
\Pi_i' &= \begin{bmatrix} \Omega_i^1 & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22}\end{bmatrix} < 0,\\
\Psi_i' &= \begin{bmatrix} \Omega_i^2 & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22}\end{bmatrix} < 0,\\
\Phi_i' &= \begin{bmatrix} \Omega_i^3 & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22}\end{bmatrix} < 0,\\
\Upsilon_i' &= \begin{bmatrix} \Omega_i^4 & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22}\end{bmatrix} < 0,
\end{aligned} \tag{3.20}$$

where

$$\Omega_i^k = \begin{bmatrix} \Omega_{i11}^k & \Omega_{i12}^k & \Omega_{i13} \\ * & \Omega_{i22} & \Omega_{i23} \\ * & * & \Omega_{i33}\end{bmatrix}, \quad k=1,2,3,4,$$

$$\Pi_{i12} = \Psi_{i12} = \begin{bmatrix} hN_{4i} & -hN_{1i}^T & A_i^TQ_4 & P_i^TE_i \\ -hN_{4i}+hM_{4i} & -hN_{2i}^T & A_{id}^TQ_4 & 0 \\ -hM_{4i} & -hN_{3i}^T & 0 & 0\end{bmatrix},$$

$$\Pi_{i22} = \begin{bmatrix} -hQ_3 & -h^2N_{4i}^T & 0 & 0 \\ * & -hQ_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}Q_4 & Q_4^TE_i \\ * & * & * & -\varepsilon_{1i}I\end{bmatrix},$$

$$\Psi_{i22} = \begin{bmatrix} -hQ_3 & -h^2N_{5i}^T & 0 & 0 \\ * & -hQ_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}Q_4 & Q_4^TE_i \\ * & * & * & -\varepsilon_{2i}I\end{bmatrix},$$

$$\Phi_{i12} = \Upsilon_{i12} = \begin{bmatrix} hN_{5i} & -hM_{1i}^T & A_i^TQ_4 & P_i^TE_i \\ -hN_{5i}+hM_{5i} & -hM_{2i}^T & A_{id}^TQ_4 & 0 \\ -hM_{5i} & -hM_{3i}^T & 0 & 0\end{bmatrix},$$

$$\Phi_{i22} = \begin{bmatrix} -hQ_3 & -h^2M_{4i}^T & 0 & 0 \\ * & -hQ_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}Q_4 & Q_4^TE_i \\ * & * & * & -\varepsilon_{3i}I\end{bmatrix},$$

$$\Upsilon_{i22} = \begin{bmatrix} -hQ_3 & -h^2M_{5i}^T & 0 & 0 \\ * & -hQ_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}Q_4 & Q_4^TE_i \\ * & * & * & -\varepsilon_{4i}I\end{bmatrix},$$

$$\begin{aligned}
\Omega_{i11}^k &= \kappa P_i + P_iA_i + A_i^TP_i + \sum_{j=1}^N \gamma_{ij}P_j + e^{\kappa h}Q_1 \\
&\quad + e^{kh}Q_{1i} + \frac{e^{\kappa h}-1}{\kappa}Q_2 + \frac{e^{\kappa h}-1}{\kappa}Q_3 \\
&\quad + N_{1i}^T + N_{1i} + \varepsilon_{ki}H_{1i}^TH_{1i},\\
\Omega_{i12}^k &= P_iA_{id} + N_{2i} - N_{1i}^T + M_{1i}^T + \varepsilon_{ki}H_{1i}^TH_{2i},\\
&\quad k=1,2,3,4,\ i\in S,
\end{aligned}$$

$\Omega_{i13}, \Omega_{i22}, \Omega_{i23}, \Omega_{i33}$ are given in Theorem 3.1 and $*$ means the symmetric terms, the uncertain Markovian jump systems (2.1) when $u(t)=0$ are robustly exponentially stable.

Proof Replacing A_i, A_{id} in (3.1) with $A_i+\Delta A_i(t)$, $A_{id}+\Delta A_{id}(t)$, $\Delta A_i(t), \Delta A_{id}(t)$ are described in (2.3) and (2.4), and we obtain

$$\Pi_i' = \Pi_i + \Gamma_i^T F_i(t)\Delta_i + \Delta_i^T F_i(t)\Gamma_i, \quad i\in S,$$

where

$$\Gamma_i = \begin{bmatrix} E_i^T P_i & 0 & 0 & 0 & 0 & E_i^T Q_4 \end{bmatrix},$$
$$\Delta_i = [H_{1i} \quad H_{2i} \quad 0 \quad 0 \quad 0 \quad 0].$$

From Lemma 2.4, we know $\Pi_i' < 0$ $(i\in S)$ are equivalent to

$$\Pi_i'' = \Pi_i + \varepsilon_{1i}^{-1}\Gamma_i^T\Gamma_i + \varepsilon_{1i}\Delta_i^T\Delta_i,$$

where $\varepsilon_{1i}>0$ $(i\in S)$. From $\Pi_i'<0$ $(i\in S)$, by using Lemma 2.3, we know that $\Pi_i''<0$ $(i\in S)$. Similarly, the other three cases can be proved. Hence, the proof is completed. □

4 A state feedback controller design

In this section, we consider the robust stabilization for the uncertain Markovian jump systems. When a control in law (2.6) is applied to systems (2.1), the closed-loop systems (2.8) become

$$\begin{aligned}\dot{x}(t) &= \big(\hat{A}_i + E_iF_i(t)\hat{H}_{1i}\big)x(t) \\ &\quad + \big(\hat{A}_{id} + E_iF_i(t)\hat{H}_{2i}\big)x\big(t-h_i(t)\big),\end{aligned} \tag{4.1}$$

where $\hat{A}_i = A_i + B_iK_i$, $\hat{A}_{id} = A_{id}+B_iK_{id}$, $\hat{H}_{1i} = H_{1i}+H_{3i}K_i$ and $\hat{H}_{2i} = H_{2i}+H_{3i}K_{id}$ $(i\in S)$.

Theorem 4.1 *For a given scalar $\kappa>0$, and time-varying delays $h_i(t)$ satisfies $0\le h_i(t)\le h_i\le h$ and $\dot{h}_i(t)\le\mu_i$ $(i\in S)$. If there exist some positive definite symmetric matrices $\tilde{Q}_3, \tilde{Q}_4, X_i, R_i, T_i, S_{1i}\in R^{n\times n}$ $(i\in S)$ and some appropriately dimensional matrices $\tilde{N}_i = [\tilde{N}_{1i}^T, \tilde{N}_{2i}^T, \tilde{N}_{3i}^T, \tilde{N}_{4i}^T, \tilde{N}_{5i}^T]^T$, $\tilde{M}_i = [\tilde{M}_{1i}^T, \tilde{M}_{2i}^T, \tilde{M}_{3i}^T, \tilde{M}_{4i}^T, \tilde{M}_{5i}^T]^T$ $(i\in S)$, such that the following linear matrix inequalities* (LMIs) *hold*:

$$\begin{aligned}
\Pi_i &= \begin{bmatrix} \Omega_i & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22}\end{bmatrix} < 0, \\
\Psi_i &= \begin{bmatrix} \Omega_i & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22}\end{bmatrix} < 0, \\
\Phi_i &= \begin{bmatrix} \Omega_i & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22}\end{bmatrix} < 0, \\
\Upsilon_i &= \begin{bmatrix} \Omega_i & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22}\end{bmatrix} < 0,
\end{aligned} \tag{4.2}$$

$$|\gamma_{ii}|R_i + \sum_{j=1}^{N}\gamma_{ij}S_{1j} \le T_i, \tag{4.3}$$

where

$$\Omega_i = \begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} \\ * & \Omega_{i22} & \Omega_{i23} \\ * & * & \Omega_{i33}\end{bmatrix},$$

$$\Pi_{i12} = \Psi_{i12} = \begin{bmatrix} hX_i^T\tilde{N}_{4i} & -h\tilde{N}_{1i}^T\tilde{Q}_4 & X_i^TA_i^T + \tilde{L}_i^TB_i^T & \Xi_i \\ -hX_i^T\tilde{N}_{4i} + hX_i^T\tilde{M}_{4i} & -h\tilde{N}_{2i}^T\tilde{Q}_4 & X_i^TA_{id}^T + \tilde{L}_{id}^TB_i^T & 0 \\ -hX_i^T\tilde{M}_{4i} & -h\tilde{N}_{3i}^T\tilde{Q}_4 & 0 & 0\end{bmatrix},$$

$$\Pi_{i22} = \begin{bmatrix} -h\tilde{Q}_3 & -h^2\tilde{N}_{4i}^T\tilde{Q}_4 & 0 & 0 \\ * & -h\tilde{Q}_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_4 & 0 \\ * & * & * & -\Gamma_i\end{bmatrix},$$

$$\Psi_{i22}=\begin{bmatrix} -h\tilde{Q}_3 & -h^2\tilde{N}_{5i}^T\tilde{Q}_4 & 0 & 0 \\ * & -h\tilde{Q}_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_4 & 0 \\ * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Phi_{i12}=\Upsilon_{i12}=\begin{bmatrix} hX_i^T\tilde{N}_{5i} & -h\tilde{M}_{1i}^T\tilde{Q}_4 & X_i^TA_i^T+\tilde{L}_i^TB_i^T & \Xi_i \\ -hX_i^T\tilde{N}_{5i}+hX_i^T\tilde{M}_{5i} & -h\tilde{M}_{2i}^T\tilde{Q}_4 & X_i^TA_{id}^T+\tilde{L}_{id}^TB_i^T & 0 \\ -hX_i^T\tilde{M}_{5i} & -h\tilde{M}_{3i}^T\tilde{Q}_4 & 0 & 0 \end{bmatrix},$$

$$\Phi_{i22}=\begin{bmatrix} -h\tilde{Q}_3 & -h^2\tilde{M}_{4i}^T\tilde{Q}_4 & 0 & 0 \\ * & -h\tilde{Q}_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_4 & 0 \\ * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Upsilon_{i22}=\begin{bmatrix} -h\tilde{Q}_3 & -h^2\tilde{M}_{5i}^T\tilde{Q}_4 & 0 & 0 \\ * & -h\tilde{Q}_4 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_4 & 0 \\ * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Omega_{i11}=\kappa X_i+A_iX_i+X_i^TA_i^T+B_i\tilde{L}_i+\tilde{L}_i^TB_i^T+\gamma_{ii}X_i+e^{\kappa h}R_i+e^{kh}S_{1i}+\frac{e^{\kappa h}-1}{\kappa}T_i+\tilde{N}_{1i}^TX_i+X_i^T\tilde{N}_{1i},$$

$$\Omega_{i12}=A_{id}X_i+B_i\tilde{L}_{id}+X_i^T\tilde{N}_{2i}-\tilde{N}_{1i}^TX_i+\tilde{M}_{1i}^TX_i,$$

$$\Omega_{i13}=X_i^T\tilde{N}_{3i}-\tilde{M}_{1i}^TX_i,$$

$$\Omega_{i22}=\left[-(1-\mu_i)e^{\kappa h}R_i\right]\vee\left[-(1-\mu_i)R_i\right]-\tilde{N}_{2i}^TX_i-X_i^T\tilde{N}_{2i}+\tilde{M}_{2i}^TX_i+X_i^T\tilde{M}_{2i},$$

$$\Omega_{i23}=-X_i^T\tilde{N}_{3i}-M_{2i}^TX_i+X_i^T\tilde{M}_{3i},$$

$$\Omega_{i33}=-\tilde{N}_{1i}-\tilde{M}_{3i}^TX_i-X_i^T\tilde{M}_{3i},$$

$$\Xi_i=\left[\sqrt{\gamma_{i1}}X_i^T,\ldots,\sqrt{\gamma_{i(i-1)}}X_i^T,\sqrt{\gamma_{i(i+1)}}X_i^T,\ldots,\sqrt{\gamma_{iN}}X_i^T,X_i^T\right],$$

$$\Gamma_i=\operatorname{diag}\left\{X_1,\ldots,X_{i-1},X_{i+1},\ldots,X_N,\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_3\right\},$$

and $*$ means the symmetric terms, then the nominal closed-loop systems (4.1) are exponentially stable. Moreover, the gain matrices K_i and K_{id} can be chosen as

$$K_i=\tilde{L}_iX_i^{-1}$$

and

$$K_{id}=\tilde{L}_{id}X_i^{-1},$$

for each $i\in S$.

Proof Setting

$$P_i=X_i^{-1},\qquad R_i=X_i^TQ_1X_i,$$
$$S_{1i}=X_i^{-1}Q_{1i}X_i,\qquad T_i=X_i^TQ_2X_i,$$
$$Q_3=\tilde{Q}_3^{-1},\qquad Q_4=\tilde{Q}_4^{-1},$$
$$N_{1i}=\tilde{N}_{1i}X_i^{-1},\qquad M_{1i}=\tilde{M}_{1i}X_i^{-1},$$
$$N_{2i}=\tilde{N}_{2i}X_i^{-1},\qquad M_{2i}=\tilde{M}_{2i}X_i^{-1},$$
$$N_{3i}=\tilde{N}_{3i}X_i^{-1},\qquad M_{3i}=\tilde{M}_{3i}X_i^{-1},$$

$$N_{4i}=\tilde N_{4i}\tilde Q_3^{-1},\qquad M_{4i}=\tilde M_{4i}\tilde Q_3^{-1},$$
$$N_{5i}=\tilde N_{5i}\tilde Q_3^{-1},\qquad M_{5i}=\tilde M_{5i}\tilde Q_3^{-1},\quad i\in S.$$

Then, pre-multiplying and post-multiplying (4.2) with $\operatorname{diag}\{X_i, X_i, X_i, \tilde Q_3, \tilde Q_4, \tilde Q_4\}$ $(i \in S)$ can result in (3.1) by using Lemma 2.3. By pre-multiplying and post-multiplying (4.3) with X_i $(i \in S)$, we obtain (3.2). Thus, by virtue of Theorem 3.1, the closed-loop systems (4.1) have exponential stability. The proof of this theorem is completed. □

Next, we also generalize Theorem 4.1 to the uncertain case, and the design theorem is given as follows:

Theorem 4.2 *For a given scalar $\kappa > 0$, and time-varying delays $h_i(t)$ satisfies $0 \le h_i(t) \le h_i \le h$ and $\dot h_i(t) \le \mu_i$ $(i \in S)$. If there exist some positive definite symmetric matrices $\tilde Q_3, \tilde Q_4, X_i, R_i, T_i, S_{1i} \in R^{n\times n}$ $(i \in S)$ and some appropriately dimensional matrices $\tilde N_i = [\tilde N_{1i}^T, \tilde N_{2i}^T, \tilde N_{3i}^T, \tilde N_{4i}^T, \tilde N_{5i}^T]^T$, $\tilde M_i = [\tilde M_{1i}^T, \tilde M_{2i}^T, \tilde M_{3i}^T, \tilde M_{4i}^T, \tilde M_{5i}^T]^T$ $(i \in S)$ and $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}$ $(i \in S)$ such that (4.3) and the following linear matrix inequalities* (LMIs) *hold*:

$$\begin{aligned}
\Pi_i &= \begin{bmatrix} \Omega_i^1 & \Pi_{i12} \\ \Pi_{i12}^T & \Pi_{i22} \end{bmatrix} < 0,\\
\Psi_i &= \begin{bmatrix} \Omega_i^2 & \Psi_{i12} \\ \Psi_{i12}^T & \Psi_{i22} \end{bmatrix} < 0,\\
\Phi_i &= \begin{bmatrix} \Omega_i^3 & \Phi_{i12} \\ \Phi_{i12}^T & \Phi_{i22} \end{bmatrix} < 0,\\
\Upsilon_i &= \begin{bmatrix} \Omega_i^4 & \Upsilon_{i12} \\ \Upsilon_{i12}^T & \Upsilon_{i22} \end{bmatrix} < 0,
\end{aligned}\tag{4.4}$$

where

$$\Omega_i^k = \begin{bmatrix} \Omega_{i11}^k & \Omega_{i12} & \Omega_{i13} \\ * & \Omega_{i22} & \Omega_{i23} \\ * & * & \Omega_{i33} \end{bmatrix},\quad k=1,2,3,4,$$

$$\Pi_{i12}=\Psi_{i12}=\begin{bmatrix} hX_i^T\tilde N_{4i} & -h\tilde N_{1i}^T\tilde Q_4 & X_i^TA_i^T+\tilde L_i^TB_i^T & X_i^TH_{1i}^T+\tilde L_i^TH_{3i}^T & \Xi_i \\ -hX_i^T\tilde N_{4i}+hX_i^T\tilde M_{4i} & -h\tilde N_{2i}^T\tilde Q_4 & X_i^TA_{id}^T+\tilde L_{id}^TB_i^T & X_i^TH_{2i}^T+\tilde L_{id}^TH_{3i}^T & 0 \\ -hX_i^T\tilde M_{4i} & -h\tilde N_{3i}^T\tilde Q_4 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{i22}=\begin{bmatrix} -h\tilde Q_3 & -h^2\tilde N_{4i}^T\tilde Q_4 & 0 & 0 & 0 \\ * & -h\tilde Q_4 & 0 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde Q_4+\varepsilon_{1i}E_iE_i' & 0 & 0 \\ * & * & * & -\varepsilon_{1i}I & 0 \\ * & * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Psi_{i22}=\begin{bmatrix} -h\tilde Q_3 & -h^2\tilde N_{5i}^T\tilde Q_4 & 0 & 0 & 0 \\ * & -h\tilde Q_4 & 0 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde Q_4+\varepsilon_{2i}E_iE_i' & 0 & 0 \\ * & * & * & -\varepsilon_{2i}I & 0 \\ * & * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Phi_{i12}=\Upsilon_{i12}=\begin{bmatrix} hX_i^T\tilde N_{5i} & -h\tilde M_{1i}^T\tilde Q_4 & X_i^TA_i^T+\tilde L_i^TB_i^T & X_i^TH_{1i}^T+\tilde L_i^TH_{3i}^T & \Xi_i \\ -hX_i^T\tilde N_{5i}+hX_i^T\tilde M_{5i} & -h\tilde M_{2i}^T\tilde Q_4 & X_i^TA_{id}^T+\tilde L_{id}^TB_i^T & X_i^TH_{2i}^T+\tilde L_{id}^TH_{3i}^T & 0 \\ -hX_i^T\tilde M_{5i} & -h\tilde M_{3i}^T\tilde Q_4 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{i22} = \begin{bmatrix} -h\tilde{Q}_3 & -h^2\tilde{M}_{4i}^T\tilde{Q}_4 & 0 & 0 & 0 \\ * & -h\tilde{Q}_4 & 0 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_4 + \varepsilon_{3i}E_iE_i' & 0 & 0 \\ * & * & * & -\varepsilon_{3i}I & 0 \\ * & * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Upsilon_{i22} = \begin{bmatrix} -h\tilde{Q}_3 & -h^2\tilde{M}_{5i}^T\tilde{Q}_4 & 0 & 0 & 0 \\ * & -h\tilde{Q}_4 & 0 & 0 & 0 \\ * & * & -\frac{\kappa}{e^{\kappa h}-1}\tilde{Q}_4 + \varepsilon_{4i}E_iE_i' & 0 & 0 \\ * & * & * & -\varepsilon_{4i}I & 0 \\ * & * & * & * & -\Gamma_i \end{bmatrix},$$

$$\Omega_{i11}^k = \kappa X_i + A_iX_i + X_i^TA_i^T + B_i\tilde{L}_i + \tilde{L}_i^TB_i^T + \gamma_{ii}X_i + e^{\kappa h}R_i + e^{kh}S_{1i}$$
$$+ \frac{e^{\kappa h}-1}{\kappa}T_i + \tilde{N}_{1i}^TX_i + X_i^T\tilde{N}_{1i} + \varepsilon_{ki}E_iE_i^T, \quad k = 1,2,3,4,\ i \in S,$$

the other quantities $\Omega_{i12}, \Omega_{i13}, \Omega_{i22}, \Omega_{i23}, \Omega_{i33}, \Xi_i$, Γ_i $(i \in S)$ are given in Theorem 4.1 and $*$ means the symmetric terms, then the uncertain closed-loop systems (4.1) are robustly exponential stable. Moreover, the gain matrices K_i and K_{id} can be chosen as

$$K_i = \tilde{L}_iX_i^{-1} \quad \text{and} \quad K_{id} = \tilde{L}_{id}X_i^{-1},$$

for each $i \in S$.

5 Some illustrative numerical examples

In this section, some illustrative examples are provided to show the feasibility of our obtained results.

Example 5.1 Consider the following nominal Markovian jump systems, which are considered in [18, 19, 24, 29, 30, 32]:

$$\dot{x}(t) = A_ix(t) + A_{id}x\big(t - h_i(t)\big), \quad i \in S = \{1,2\}, \tag{5.1}$$

with system matrices as follows:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_{1d} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \qquad A_{2d} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix},$$

Table 1 The upper bounds of h with different values of μ in Example 5.1

	μ		
	0.9	1.0	1.5
Yue and Han in [32]	0.9279	–	–
Shen et al. [24]	0.7529	0.7529	0.7255
Xu et al. [29]	0.9359	0.8886	0.8886
Ma et al. [19]	1.0428	1.0428	1.0428
Our result	1.2996	1.2996	1.2996

where $0 \le h_i(t) \le h_i \le h$, $\dot{h}_i(t) \le \mu_i$ $(i \in S)$. The parameter matrix $\Pi = \{\gamma_{ij}\}$ is given by $\Pi = \left[\begin{smallmatrix} -1 & 1 \\ 2 & -2 \end{smallmatrix}\right]$. The convergence rate $\kappa = 0.1$.

Case 1. Table 1 lists the comparison results with respect to [19, 24, 29, 32] when $\mu_1 = \mu_2 = \mu$. It is not hard to see from Table 1 that the upper bound h of the time-varying delay $h(t)$ is better than the ones obtained in [19, 24, 29, 32]. When $h = 0.8$ and $\mu = 1.5$, the upper bound of the decay rate k is 0.8065 in [19]. However, by using Theorem 3.1, it can be derived that the suboptimal upper bound of the decay rate k is 1.0101, which is larger than the one given in [19].

Case 2. Table 2 presents the comparison results with respect to [18, 30] with different values μ_1 and μ_2 when $k = 0.0$. From Table 2, it can be seen that Theorem 3.1 is much better than those obtained in [18, 30].

Table 2 The upper bounds of h with different values of μ_1 and μ_2 when $k=0.0$ in Example 5.1

	Authors		
	Xu et al. [30]	Ma et al. [18]	Theorem 3.1
$\mu_1=0.0, \mu_2=0.0$	0.9800	1.5370	1.5414
$\mu_1=0.0, \mu_2=0.5$	0.9800	1.3550	1.3984
$\mu_1=0.5, \mu_2=0.5$	0.4899	1.1240	1.3900
$\mu_1=1.0, \mu_2=1.5$	–	0.8870	1.3872

Table 3 The allowable upper bounds of h for different γ_{11} when $k=0.0$ in Example 5.2

	γ_{11}				
	-0.1	-0.3	-0.5	-0.7	-0.9
Xu et al. [29]	0.4021	0.4010	0.4001	0.3993	0.3987
Wu et al. [27]	0.4252	0.4250	0.4248	0.4246	0.4244
Our result	0.6186	0.6188	0.6191	0.6193	0.6195

Example 5.2 Consider the nominal Markovian jump system with two modes and the following parameters, which is considered in [2, 4, 6, 7, 9–11, 27, 29, 34]:

$$A_1=\begin{bmatrix}-3.4888 & 0.8057\\ -0.6451 & -3.2684\end{bmatrix},$$

$$A_{1d}=\begin{bmatrix}-0.8620 & -1.2919\\ -0.6841 & -2.0729\end{bmatrix},$$

$$A_2=\begin{bmatrix}-2.4898 & 0.2895\\ 1.3396 & -0.0211\end{bmatrix},$$

$$A_{2d}=\begin{bmatrix}-2.8306 & 0.4978\\ -0.8436 & -1.0115\end{bmatrix}.$$

Case 1. When $k=0.0$, Table 3 presents the maximal allowable delay h for different values γ_{11} compared with those in [27, 29] when $\gamma_{22}=-0.8$ and $\mu_1=\mu_2=0.9$. From Table 3, we can see that our result is also better than the ones given in [27, 29].

Case 2. When $k=0.0$, Table 4 gives the maximal allowable delay h for different values γ_{11} compared with those discussed in [2, 4, 6, 7, 9–11, 29, 34] when $\gamma_{22}=-0.8$ and $\mu_1=\mu_2=0.0$. It is easily seen that our result is less conservative than those derived in [2, 4, 6, 7, 9–11, 29, 34].

Example 5.3 Consider the uncertain Markovian jump systems in the form of (2.1) with two modes, that is

Table 4 The allowable upper bounds of h for different γ_{11} in Example 5.2

	γ_{11}			
	-0.1	-0.5	-0.8	-1.0
Fujisaki [10]	0.2224	0.2200	–	0.2174
Boukas et al. [2]	0.2224	0.2200	0.2184	0.2174
Chen et al. [9]	0.5012	0.4941	0.4915	0.4903
Cao et al. [6]	0.5012	0.4941	–	0.4903
Cao et al. [7]	0.5012	0.4941	–	0.4903
Xu et al. [29]	0.6797	0.5794	0.5562	0.5465
Fei et al. [11] ($m=1$)	0.6797	0.5794	0.5562	0.5465
Zhao et al. [34]	0.6797	0.5794	–	0.5465
Chen et al. [4] ($k=0.1$)	0.6814	0.5794	0.5563	0.5475
Chen et al. [4] ($k=0.3$)	0.6979	0.5898	0.5660	0.5568
Chen et al. [4] ($k=0.5$)	0.7053	0.5991	0.5739	0.5568
Our result	0.7773	0.6721	0.6579	0.6530

$S=\{1,2\}$. The mode switching is governed by the infinitesimal generator $\Pi=\left[\begin{smallmatrix}-3 & 3\\ 5 & -5\end{smallmatrix}\right]$. The systems matrices are shown as follows:

$$A_1=\begin{bmatrix}-1 & 0.5\\ -0.2 & -1\end{bmatrix},\qquad A_{1d}=\begin{bmatrix}-0.25 & 0\\ 0 & -0.2\end{bmatrix},$$

$$B_1=\begin{bmatrix}0\\ -0.1\end{bmatrix},\qquad E_1=\begin{bmatrix}0.2\\ 0.1\end{bmatrix},$$

$$H_{11}=\begin{bmatrix}0.1 & 0\\ 0 & 0.05\end{bmatrix},\qquad H_{21}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$

$$A_2=\begin{bmatrix}1.8 & 0.8\\ 0.15 & -2\end{bmatrix},\qquad A_{2d}=\begin{bmatrix}0.12 & -0.1\\ 0.1 & -0.11\end{bmatrix},$$

$$B_2=\begin{bmatrix}1\\ 0.1\end{bmatrix},\qquad E_2=\begin{bmatrix}0.21\\ 0.1\end{bmatrix},$$

$$H_{12}=\begin{bmatrix}0.2 & 0.0\\ 0.0 & 0.1\end{bmatrix},\qquad H_{22}=\begin{bmatrix}0.15 & 0\\ 0 & 0.05\end{bmatrix}.$$

Case 1. This uncertain closed-loop system with $h_1(t)=h_2(t)=h$ is considered in [23]. From Table 5, it can be seen that our results in this paper are less conservative than those in Shu et al. [23].

Case 2. The time-varying delays $h_1(t), h_2(t)$ satisfy $\mu_1=\mu_2=0.1$ and $h=1.0$. The purpose is to design a robust state feedback controller $u(t)$ satisfying (2.7), which can exponentially stabilize these systems. In

Table 5 The upper bound h of time delay for robust stabilization when the decay rate $\kappa = 1.5$ in Example 5.3 (Case 1)

Methods	h	Controller K_1	Controller K_2
Shu et al. [23]	0.8000	$[2.1264\ \ -57.4410]$	$[-4.3850\ \ -1.0087]$
Our result	1.5098	$1.0 \times e^6 \times [1.4441\ \ 0.2316]$	$1.0 \times e^4 \times [-1.9271\ \ -0.3091]$

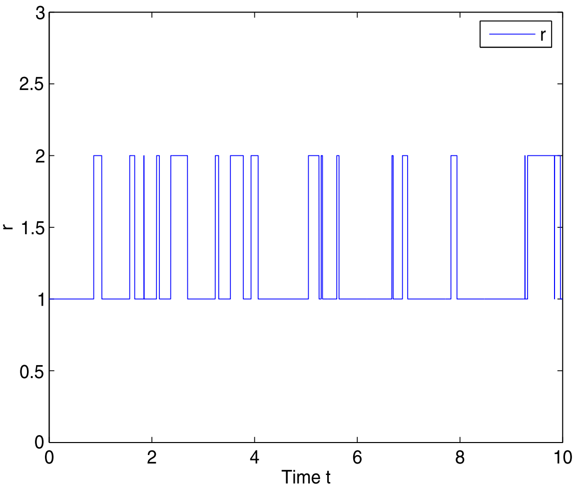

Fig. 1 The operation modes of Example 5.3

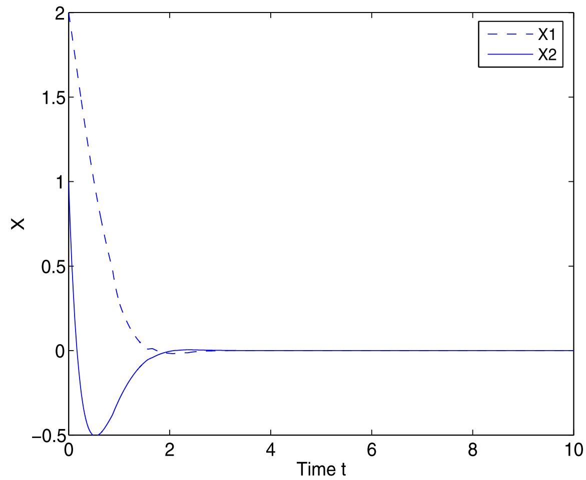

Fig. 2 The control input curve of Example 5.3 (Case 2)

this example, we assume the decay rate $\kappa = 0.5$. We choose

$$\tilde{N}_{11} = \tilde{N}_{21} = \tilde{N}_{31} = \tilde{M}_{11} = \tilde{M}_{21} = \tilde{M}_{31} = \begin{bmatrix} -0.05 & 0 \\ -0.01 & 0 \end{bmatrix},$$

$$\tilde{N}_{12} = \tilde{N}_{22} = \tilde{N}_{32} = \tilde{M}_{12} = \tilde{M}_{22} = \tilde{M}_{32} = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.05 \end{bmatrix},$$

$$\tilde{N}_{41} = \tilde{N}_{42} = \tilde{N}_{51} = \tilde{N}_{52} = \tilde{M}_{41} = \tilde{M}_{42} = \tilde{M}_{51} = \tilde{M}_{52} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

By resorting to the Matlab LMI Control Toolbox to solve the LMIs in (4.3) in Theorem 4.1 and (4.4) in Theorem 4.2, the gain matrices of a robustly exponentially stabilizing controller can be obtained as

$$K_1 = [\,20.9844 \quad 32.7289\,],$$

$$K_2 = [\,-7.5161 \quad -2.9073\,],$$

$$K_{1d} = [\,-0.1341 \quad -1.8274\,],$$

$$K_{2d} = [\,-0.1380 \quad 0.1188\,].$$

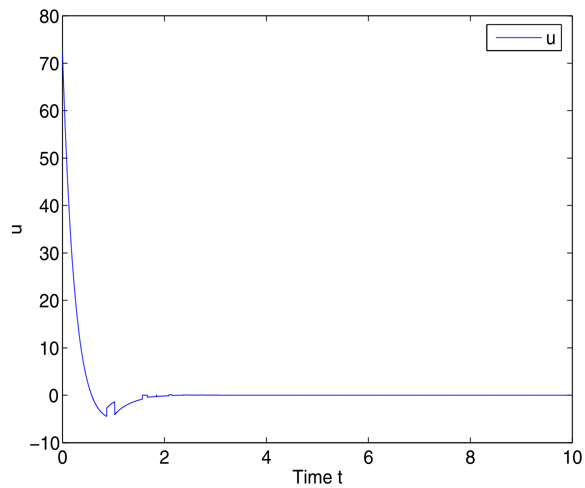

Fig. 3 The control input curve of Example 5.3 (Case 2)

The simulation is displayed for $h_1(t) = h_2(t) = \frac{1}{0.1t+1}$ under the initial condition $\varphi(t) = [2.0, 0.5]$ for $t \in [-1, 0]$ with $r(0) = 1$, $t \in [-1, 0]$. The simulation results are given in Figs. 1, 2 and 3. Figures 1, 2 and 3 shows the operation modes, the response and the control input curve, respectively. From these simulation results, it is easily seen that the uncertain closed-loop systems (4.1) are robustly exponentially stable.

6 Conclusion

In this paper, by constructing a modified Lyapunov–Krasovskii functional and employing the Newton–Leibniz formula, we obtain some sufficient conditions ensuring the robustly exponential stability for the uncertain Markovian jump systems. And then, a memory state feedback controller is designed such that the closed-loop systems is also robustly exponentially stale. The method used in this paper is different from some previous reports. And the conservatism can be effectively reduced, which can be shown by some illustrative numerical examples.

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