Nonlinear Dyn (2012) 69:231–245
DOI 10.1007/s11071-011-0260-7

ORIGINAL PAPER

Restoring force and dynamic loadings identification for a nonlinear chain-like structure with partially unknown excitations

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Received: 8 July 2011 / Accepted: 27 October 2011 / Published online: 19 November 2011

Abstract Most of the currently employed vibration-based identification approaches for structural damage detection are based on eigenvalues and/or eigenvectors extracted from dynamic response measurements, and strictly speaking, are only suitable for linear system. However, the inception and growth of damage in engineering structures under severe dynamic loadings are typical nonlinear procedures. Consequently, it is crucial to develop general structural restoring force and excitation identification approaches for nonlinear dynamic systems because the restoring force rather than equivalent stiffness can act as a direct indicator of the extent of the nonlinearity and be used to quantitatively evaluate the absorbed energy during vibration, and the dynamic loading is an important factor for structural remaining life forecast. In this study, based on the instantaneous state vectors and partially unknown excitation, a power series polynomial model (PSPM) was utilized to model the nonlinear restoring force (NRF) of a chain-like nonlinear multi-degree-of-freedom (MDOF) structure. To improve the efficiency and accuracy of the proposed approach, an iterative approach, namely weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME), where a weight coefficient and a learning coefficient were involved, was proposed to identify the restoring force of the structure as well as the unknown dynamic loadings simultaneously. The response measurements of the structure, i.e., the acceleration, velocity, and displacement, and partially known excitations were utilized for identification. The feasibility and robustness of the proposed approach was verified by numerical simulation with a 4 degree-of-freedom (DOF) numerical model incorporating a nonlinear structural member, and by experimental measurements with a four-story frame model equipped with two magneto-rheological (MR) dampers mimicking nonlinear behavior. The results show the proposed approach by combining the PSPM and WAILSE-IME algorithm is capable of effectively representing and identifying the NRF of the chain-like MDOF nonlinear system with partially unknown external excitations, and provide a potential way for damage prognosis and condition evaluation of engineering structures under dynamic loadings which should be regarded as a nonlinear system.

Keywords Nonlinear system identification · Power series polynomial model · Nonlinear restoring force ·

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Dynamic loadings · Weighted adaptive iterative least-squares estimation · Incomplete measured excitations · MR damper

Abbreviations

PSPM: Power Series Polynomial Model
NRF: Nonlinear Restoring Force
WAILSE-IME: Weighted Adaptive Iterative Least-Squares Estimation with Incomplete Measured Excitations.

1 Introduction

Civil infrastructure systems are prone to deterioration or damage due to several factors, e.g., material fatigue accumulation, strong dynamic loadings including earthquake, strong wind and impact, and crack initiation and growth. For damage detection of engineering structures under dynamic loadings, a number of vibration-based identification approaches have been proposed, which are based on the idea of extracting the eigenvalues, mode shapes, and/or their derivatives from the dynamic responses, to identify structural stiffness describing the location and severity of structural damage by solving an optimization problem. An investigation reported by Ghanem and Shinozuka [1] reviewed the application of a number of system identification techniques in earthquake engineering. Furthermore, the thorough reviews of vibration-based damage identification methods are available from the publications by Doebling et al. [2], Wu et al. [3], and Yan et al. [4]. Strictly speaking, most of the currently employed vibration-based damage detection approaches are only suitable for linear systems. But the eigenvalue-based methods are not necessarily wrongly used to detect possible damage evolution, provided that they are adopted to track the structural dynamics when damage is not growing under loading conditions which are less severe than those causing new damage inception and growth. By identifying possible drifts in eigenvalues and vibration mode shapes, the eigenvalue-based identification approaches are in principle able to understand whether damage is present in structure. If they are accurate enough, they can also detect where damage is growing. However, all engineering structures are nonlinear to some extent as the occurrence of a fault such as crack initiation and development, and the looseness and presence of friction characteristics of structural joints in structural members under dynamic loadings will result in nonlinear behavior. Consequently, the detection of nonlinearities is receiving increased attention and it's crucial to develop general nonlinear identification algorithms to evaluate the current reliability, performance, and condition of the structures for the prevention of potentially catastrophic events, as well as for the remaining life estimation as one of the main contents of structural damage prognosis (DP), which is the future of structural health monitoring [5].

Studies on the modeling and identification of nonlinear mechanical structures have been carried out for a relatively long time, and much progress has been made in this area. The first contribution to the identification of nonlinear structural models can be traced back to the work by Ibanez [6]. Since then, numerous methods have been developed due to the highly individualistic nature of nonlinear systems [7–9]. Because nonlinear restoring force (NRF) can be considered as a direct index describing the extent of nonlinearity, Masri et al. [10, 11] proposed a fruitful approach, namely the restoring force surface (RFS) method, to identify and analyze the nonlinear structural systems in terms of their internal RFSs by using Chebyshev polynomial for the expansion of NRFs. Based on the uses of power series expansions, a relatively simple nonparametric technique for the identification of nonlinear vibrating systems has been developed by Yang and Ibrahim [12]. Masri et al. [13, 14] presented a general data-based approach by utilizing power series fitting techniques to develop reduced-order nonparametric models in nonlinear multi-degree-of-freedom (MDOF) systems. Tasbihgoo et al. [15] discussed two broad classes of nonlinear identification approaches. One is based on the representation of the system restoring forces in a polynomial-basis format, and the other one uses artificial neural networks to map the complex transformations of the nonlinear MDOF systems with a nonparametric models. Haroon and Adams [16] presented a passive time and frequency domain method to exploit the changes in the nonlinear behavior of a mechanical system to identify damage. More recently, based on Wiener system and the available input and output measurements, Paduart et al. [17] proposed a Polynomial Nonlinear State Space (PNLSS) model to approximate multivariable nonlinear systems. Jamaali et al. [18] presented a novel modeling and identification approach for nonlinear systems using modal series state space model from the input and output measurements. All of these prior system identification methods require the measurements of input being completely available. However, in many practical situations, either sensors may not be installed to measure all the external excitations or some external excitations are unmeasurable. Moreover, the identification of unknown dynamic loadings is also a challenging issue for structural DP which is the future of structural health monitoring, because loading profile analysis plays key roles in the remaining service life estimation [5]. Consequently, it is highly desirable to develop identification algorithms for both structural nonlinear behavior and unknown external excitations in a simultaneous way utilizing structural vibration response measurements and incomplete measured external excitations.

Wang and Haldar [19] proposed an iterative least-squares technique with unknown inputs to identify large structural systems at element level. Li and Chen [20] proposed a method, called the statistic average algorithm, to identify the structural parameters under unknown ground motion. Moreover, Chen and Li [21, 22] proposed an iteration procedure under incomplete measured excitations to estimate structural parameters and the unmeasured excitations. The mentioned above methods proposed by Wang and Haldar [19], Li and Chen [20], as well as Chen and Li [21, 22] were employed to simultaneously identify the structural parameters and unknown inputs of linear system. Modeling and identification of nonlinear nonconservative dissipative system with incomplete inputs is a challenging problem that has been studied by many researchers in the past several decades. Mohammad et al. [23] proposed a direct parameter estimation method for identifying the physical parameters of linear and nonlinear MDOF structures with only excitation on a signal DOF. To eliminate the deficiency of the above approaches, many investigators extended the identification approaches with incomplete input to nonlinear domain by the use of extended Kalman filter (EKF) [24–28]. However, a major drawback of these approaches was that the identification results were easily unstable, especially for structures with a large number of unknowns. To improve the accuracy and stability of the results, an Unscented Kalman Filtering (UKF) procedure was proposed by Mariani and Ghisi [29] and employed in softening single degree-of-freedom structural systems for state tracking and model calibration.

In this study, a nonlinear performance identification approach with power series polynomial model (PSPM) proposed by the authors [30, 31] was extended to identify both the NRF and unknown dynamic loadings of a nonlinear MDOF chain-like structural system with partially unknown excitations. Moreover, in the proposed approach, in order to improve the efficiency and accuracy of the proposed approach, an iterative approach, referred to as weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME), was proposed to identify each coefficient of the polynomial as well as the unknown loadings by employing appropriate values of weight coefficient and learning coefficient, which was originally proposed for a linear structure [32]. The feasibility and robustness of the proposed approach was firstly validated numerically with a 4-DOF model incorporating a nonlinear component, and then experimentally on a 4-story steel frame structure equipped with two actively-controlled MR dampers which served to simulate nonlinear behavior. The results show that the proposed combination of the PSPM and WAILSE-IME approach is capable of effectively representing the NRF and identifying the unknown dynamic loadings of the nonlinear system with partially unknown external excitations, and provide a potential way for damage detection in the form of NRF and for DP of engineering structures where dynamic loading profile analysis plays key roles. In this study, the nonlinearity is mimicked with MR dampers, and further study on the performance of the proposed approach in tracking the evolution of damage of reinforced concrete and steel civil engineering structures under strong excitations such as earthquakes should be carried out accordingly in the future.

2 NRF identification with PSPM and WAILSE-IME

The starting point of the NRF identification for a general n-DOF lumped mass chain-like structural system is the equation of motion as specified by Newton's second law,

$$M\ddot{x}(t) + R(x, \dot{x}, p) = f(t), \tag{1}$$

where $\ddot{x}(t)$ = the acceleration vector of order n, M = the constant mass matrix that characterizes the inertia forces, $R(x, \dot{x}, p)$ = the nonlinear nonconservative restoring force vector, p = the vector of system-specific parameters, and $f(t)$ = the directly applied excitations, respectively.

When M is a known mass matrix, (1) can be rewritten as

$$R(x, \dot{x}, p) = f(t) - M\ddot{x}(t). \tag{2}$$

In this study, the NRF between the ith DOF and the $i-1$th DOF of the system is assumed to be expressed by the PSPM as shown in the following equation:

$$\begin{aligned} R_{i,i-1}(x, \dot{x}, p) &\approx R_{i,i-1}(v, s) \\ &\approx \sum_{a=0}^{k}\sum_{b=0}^{q} c^n_{i,i-1,a,b} v^a_{i,i-1} s^b_{i,i-1} \quad (a+b \neq 0), \end{aligned} \tag{3}$$

where $R_{i,i-1}(x, \dot{x}, p)$ is the NRF between the ith DOF and the $i-1$th DOF, $v_{i,i-1}$ and $s_{i,i-1}$ are relative velocity and relative displacement vectors (i.e., $v_{i,i-1} = \dot{x}_i - \dot{x}_{i-1}$, $s_{i,i-1} = x_i - x_{i-1}$), $c^n_{i,i-1,a,b}$ is the coefficient of the polynomial, and k and q are integers which depend on the nature and extent of the nonlinearity of the system, respectively. Consequently, the equation of motion corresponding to the ith DOF can be rearranged as follows:

$$\begin{aligned} &\sum_{a=0}^{k}\sum_{b=0}^{q} c^n_{i,i-1,a,b} v^a_{i,i-1} s^b_{i,i-1} \\ &\quad + \sum_{a=0}^{k}\sum_{b=0}^{q} c^n_{i+1,i,a,b} v^a_{i+1,i} s^b_{i+1,i} = f_i(t) - m_i\ddot{x}_i(t). \end{aligned} \tag{4}$$

Since the structural responses are measured for identification, the terms of $v^a_{i,i-1}s^b_{i,i-1}$ and $v^a_{i+1,i}s^b_{i+1,i}$ shown in (4) are known. Hence, (4) can be represented as

$$H^i_{h\times Li}\theta^i_{Li\times 1} = P^i_{h\times 1}, \tag{5}$$

where $H^i_{h\times Li}$ = response matrix composed of the system response vectors relating with the ith DOF (i.e., $v^a_{i,i-1}s^b_{i,i-1}$ and $v^a_{i+1,i}s^b_{i+1,i}$), $\theta^i_{Li\times 1}$ = the unknown coefficients of the NRF shown in (4) (i.e., $c^n_{i,i-1,a,b}$ and $c^n_{i+1,i,a,b}$), the subscript h = the number of sample points, the subscript Li = the number of the unknown coefficients relating with the ith DOF, and $P^i_{h\times 1}$ = input vector of the ith DOF composed of external excitations and inertia forces at time t.

Similar procedures can be implemented to the remaining DOFs. Consequently, for a complete n-DOF nonlinear system, (5) can be extended as follows:

$$H_{(h\times n)\times L}\theta_{L\times 1} = P_{(h\times n)\times 1}, \tag{6}$$

where H = the complete response matrix, θ = the totally unknown coefficients, the subscript L = the total number of the unknown coefficients, and P = the complete input vector which can be expressed as

$$P = [P(t_1) \quad P(t_2) \quad \cdots \quad P(t_h)]^T, \tag{7}$$

where $P(t_k) = [f_1(t_k) - m_1\ddot{x}_1(t_k) \quad f_2(t_k) - m_2\ddot{x}_2(t_k) \quad \cdots f_n(t_k) - m_n\ddot{x}_n(t_k)]^T$, $(k = 1, 2, \ldots, h)$.

Since the input and output information during the vibration are all employed for identification, the proposed approach cannot be used to track the nonlinear behavior in a real time format, but it can be employed for post-event damage detection. When all of the external excitations and the corresponding dynamic responses are available, the unknown coefficients shown in (6) can be obtained by means of any available optimization algorithms. For example, the structural parameters $\tilde{\theta}$ can be estimated by the least-squares estimation (LSE) algorithm as follows:

$$\tilde{\theta} = [H^T H]^{-1} H^T P. \tag{8}$$

A major drawback of the traditional LSE is that it requires all external excitations being available. However, from a practical point of view, it is not always possible to obtain the complete excitation measurements. When part of the external excitation is unavailable, the coefficients shown in (6) cannot be identified directly by the traditional LSE approach. In this case, the coefficients of PSPM and the unknown external loadings should be identified simultaneously by an iterative approach. In this study, an iterative approach referred to as weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME) was proposed to simultaneously identify the NRF which is modeled as a PSPM and the unknown external loadings.

When the external excitations are partially unknown, the total external excitations can be assumed to be composed of the known forces (f_K) and the unknown forces (f_U), which is unavailable

$$f = [f_K \quad f_U]^T. \tag{9}$$

Accordingly, the input vector shown in (7) can be rearranged as

$$P = [P_K \quad P_U]^T, \tag{10}$$

where $P_K = f_K - M_K\ddot{x}_K$, $P_U = f_U - M_U\ddot{x}_U$, the subscripts K and U = subset consisting of the DOFs on which the known excitations are applied, and the

DOFs on which the unknown excitations are applied, respectively.

Consequently, the estimated input vectors on the jth iteration are composed of two corresponding parts and shown below:

$$\tilde{P}^j = \big[\tilde{P}_K^j \quad \tilde{P}_U^j\big]^T, \tag{11}$$

where the symbol "$\sim$" indicates the estimated values, superscript j means the jth iteration, and subscript K and U are defined above. During the iteration process, the above estimated input matrix shown in (11) is firstly updated by replacing $\tilde{P}_K^j$ with known P_K as shown below:

$$\hat{P}^j = \big[P_K \quad \tilde{P}_U^j\big]^T, \tag{12}$$

where the symbol "$\wedge$" indicates updated values at the jth iteration.

Subsequently, in this study, in order to further accelerate the convergence behavior, the increment of the estimated unknown external excitations at the previous two iterations is employed to re-update the unknown part ($\tilde{P}_U^j$) as shown in (13) at the jth iteration,

$$\bar{P}^j = \big[P_K \quad \bar{P}_U^j\big]^T, \tag{13}$$

in which

$$\bar{P}_U^j = \tilde{P}_U^j + \gamma\big(\tilde{P}_U^{j-1} - \tilde{P}_U^{j-2}\big), \tag{14}$$

where the symbol "$-$" indicates the re-updated values, and γ is a learning coefficient taking a value within [0,1). Since the estimated values of the unknown external excitation at the previous two iterations are utilized, the re-updated procedure shown in (14) starts from the third iteration. To accelerate the convergence and to assure the stability of the iterative approach, the learning coefficient employed here can take variable values. At the beginning of the iteration, the learning coefficient can take a relatively larger value to accelerate the iteration and it can take a smaller value in the following iteration procedure to assure the stability of the approach because the estimated unknown excitation time series will be getting close to the actual values accompanying with the iteration procedure. For simplicity, in this paper, the learning coefficient takes the value of γ/j.

Furthermore, a weight positive definite matrix shown in (15) is employed to improve the efficiency and accuracy of the identified results,

$$W = \begin{bmatrix} \alpha I & 0 \\ 0 & \beta I \end{bmatrix}, \tag{15}$$

in which I = identity matrix, and α, β = weight coefficients ($\alpha \in [1, +\infty)$, $\beta \in (0, 1]$). The dimension of αI and βI depends on the dimension of P_K and P_U defined before, respectively. Because the weight coefficient of α is corresponding to the known excitation of P_K which is more reliable information than the unknown excitation to be identified, it is reasonable to take the value larger than 1. Accordingly, the weight coefficient of β corresponding to the unknown excitation of P_U, it takes the value smaller than 1. Based on the principle of least-squares, the objective function of the estimated system is improved by using the weight matrix, and thus the LSE algorithm shown in (8) can be rearranged as

$$\tilde{\theta} = \big[H^T W H\big]^{-1} H^T W \bar{P}. \tag{16}$$

The basic concept of the proposed approach can be described in the following steps in which the symbol "$\sim$," "$\wedge$," and "$-$," and the superscript j are defined before:

(a) Build the response matrix H, set the values of the weight coefficients and learning coefficient, arbitrarily assign the initial value of the unknown excitation force for all time steps, i.e., $f_U(t) = \eta$ ($\eta \in R$), and form the initial input matrix named $\bar{P}_0^j$.
(b) According to (16) and the re-updated inputs $\bar{P}_0^j$, estimate the system parameters $\tilde{\theta}^j$.
(c) Using the estimated system parameters $\tilde{\theta}^j$ found in step (b), solve for the estimated inputs $\tilde{P}_1^j = [\tilde{P}_{K,1}^j \tilde{P}_{U,1}^j]^T$ according to (6).
(d) According to (12), obtain the updated inputs $\hat{P}_1^j = [P_K \quad \tilde{P}_{U,1}^j]^T$.
(e) If $j > 2$, based on $\hat{P}_1^j$, obtain $\bar{P}_1^j$ through (13)–(14); if $j \le 2$, it is unnecessary to re-update the inputs, hence let $\bar{P}_1^j = \hat{P}_1^j$ directly.

Calculate the error between $\bar{P}_1^j$ and $\bar{P}_0^j$ as $e^j = \|\bar{P}_1^j - \bar{P}_0^j\|_1$ which $\|\bullet\|_1$ is the 1-norm. The 1-norm of a vector is defined as the summation of the absolute value of each component. If the error is lower than an acceptable threshold, i.e., $e^j \le \varepsilon$, the procedure is complete; otherwise let $\bar{P}_0^{j+1} = \bar{P}_1^j$, and repeat steps (b) through (e).

The flowchart of the approach is shown in Fig. 1 and helps to illustrate the procedure.

Since the coefficients of PSPM are determined by the proposed WAILSE-IME approach as discussed before, the NRFs of the system can be obtained according to (3). Moreover, the unknown external excitation is also identified simultaneously by the iteration. The feasibility and robustness of the proposed NRF and external excitation identification approach is validated numerically with a MDOF nonlinear model and experimentally with a 4-story frame structural model equipped with a nonlinear structural member in the following sections.

3 Numerical validation of the proposed approach with PSPM and WAILSE-IME

To illustrate the accuracy of the proposed approach under discussion, without loss of generality, a 4-DOF nonlinear lumped mass chain-like structure shown in Fig. 2 is considered as a numerical example. Each lumped mass of the model is associated with one horizontal DOF. The properties of the structure are assumed to be $m_i = 10$ kg, $k_i = 1 \times 10^5$ N/m, and $c_i = 100$ N·s/m ($i = 1, 2, 3, 4$). In order to mimic the nonlinearity, an MR damper, which is widely used as a typical energy dissipation device in civil engineering structures, is introduced and installed on the 2nd floor of the numerical model as shown in Fig. 2.

MR damper is a typical nonlinear member and many numerical models have been proposed to describe their mechanical behavior in parametric or nonparametric forms [33–35]. In this study a modified Dahl model, which can capture many commonly observed types of hysteretic behavior of MR dampers, is employed [36]. The modified Dahl model is given by the following equations:

$$F_n = K_0 y + C_0 \dot{y} + F_d Z + f_0', \tag{17}$$

$$\dot{Z} = \sigma \dot{y} \cdot (1 - Z \cdot \mathrm{sgn}(\dot{y})), \tag{18}$$

where $K_0 =$ the stiffness coefficient, $C_0 =$ the viscous damping coefficient, $F_d =$ the adjustable coulomb friction, $f_0' =$ the initial force, $\sigma =$ the coefficient used to control the shape of the hysteretic curve, $y =$ the displacement of the damper, and $Z =$ a dimensionless of hysteretic parameter which describes the coulomb friction. In this example, the following values for the MR damper model are used: $K_0 = 50$ N/m, $C_0 = 399$ N·s/m, $F_d = 34.85$ N, $f_0' = 0$ N, and $\sigma = 50000$ s/m.

To form the PSPM shown in (3), the following basis vectors, which represent the system responses, are selected,

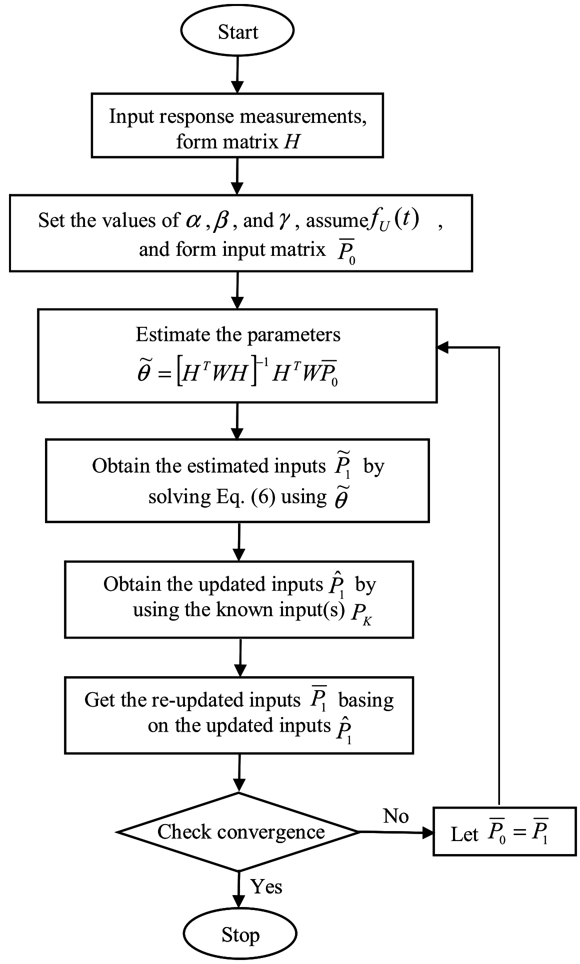


Fig. 1 Flowchart for iterative approach with incomplete measured excitation

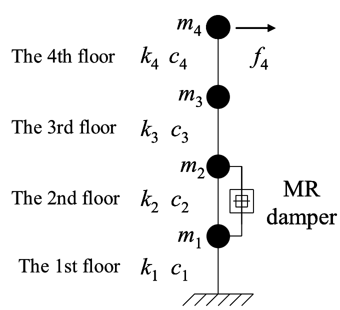


Fig. 2 4-DOF nonlinear numerical model with MR damper

Basis vectors

$$= \{v_{1,0}, v_{2,1}, v_{3,2}, v_{4,3}, s_{1,0}, s_{2,1}, s_{3,2}, s_{4,3}\}, \tag{19}$$

where $v_{i,i-1}$ and $s_{i,i-1}$ are relative velocity and relative displacement between DOF i and DOF $i - 1$.

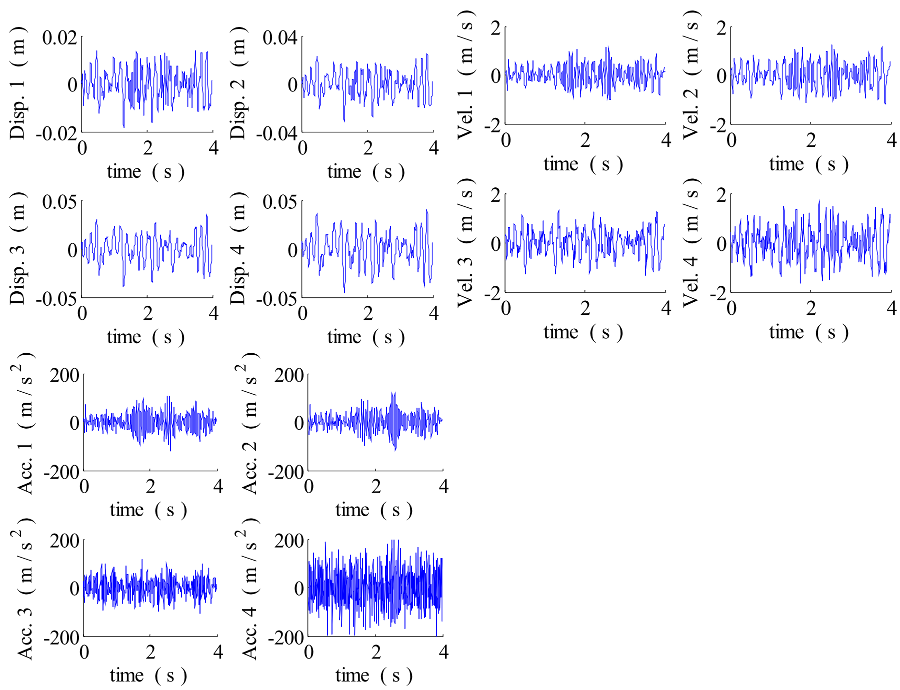


Fig. 3 The response measurements of the nonlinear system

It should be noted that $v_{1,0}$ and $s_{1,0}$ here respectively mean the relative velocity and relative displacement between the 1st DOF and the base. Selecting values for the order $k+q=3$ of the basis functions in (3) results in the following basis function including 9 power series for the nonlinear restoring force $\mathrm{R}_{i,i-1}$ between the ith DOF and the $i-1$th DOF:

$$\text{Power series}_{i,i-1} = \{v_{i,i-1}, s_{i,i-1}, v_{i,i-1}^2, v_{i,i-1}s_{i,i-1}, s_{i,i-1}^2, v_{i,i-1}^3, v_{i,i-1}^2 s_{i,i-1}, v_{i,i-1}s_{i,i-1}^2, s_{i,i-1}^3\}, \quad (i=1,2,3,4). \tag{20}$$

In this numerical study, the 4th DOF of the structure is excited horizontally by a normally distributed random force with a mean of 0 N and a standard deviation of 600 N. The corresponding response time histories of the model are calculated by the Newmark-β method and shown in Fig. 3.

The external excitation applied on the 4th floor is assumed to be unknown. Since there are no forces on the 1st, 2nd, and 3rd floor, the external excitations on these floors can be reasonably assumed to be zero and be considered as known information for identification. The weight coefficients of α and β, and learning coefficient γ are set to be 10, 0.1, and 0.8, respectively. The dynamic responses of the numerical model at 800 sampling points from 0s to 4s are used. The NRF of the system is expressed by PSPM of which coefficients are identified by the proposed WAILSE-IME approach. Since each coefficient is determined, the NRF of each floor can be easily obtained according to (3) and shown in the following equations:

$$R_{1,0} = 9.96\times10^4\times s_{1,0} + 101.57\times v_{1,0} - 1.44\times10^4\times s_{1,0}^2 - 248.18\times s_{1,0}v_{1,0} - 2.81\times v_{1,0}^2 + 2.49\times10^6\times s_{1,0}^3 - 3.21\times10^3\times s_{1,0}^2 v_{1,0} + 962.18\times s_{1,0}v_{1,0}^2 - 2.09\times v_{1,0}^3, \tag{21a}$$

$$R_{2,1} = 1.01\times10^5\times s_{2,1} + 1.01\times10^3\times v_{2,1} + 2.51\times10^4\times s_{2,1}^2 + 482.62\times s_{2,1}v_{2,1} - 32.59\times v_{2,1}^2 - 1.07\times10^6\times s_{2,1}^3 - 1.81\times10^4\times s_{2,1}^2 v_{2,1} + 1.13\times10^3\times s_{2,1}v_{2,1}^2 - 1.45\times10^3\times v_{2,1}^3, \tag{21b}$$

$$R_{3,2} = 1.00\times10^5\times s_{3,2} + 100.01\times v_{3,2} + 2.12\times10^{-10}\times s_{3,2}^2 - 1.37\times10^{-11}\times s_{3,2}v_{3,2} - 1.38\times10^{-13}\times v_{3,2}^2 - 7.71\times10^{-7}\times s_{3,2}^3 + 5.35\times10^{-9}\times s_{3,2}^2 v_{3,2} - 3.64\times10^{-10}$$

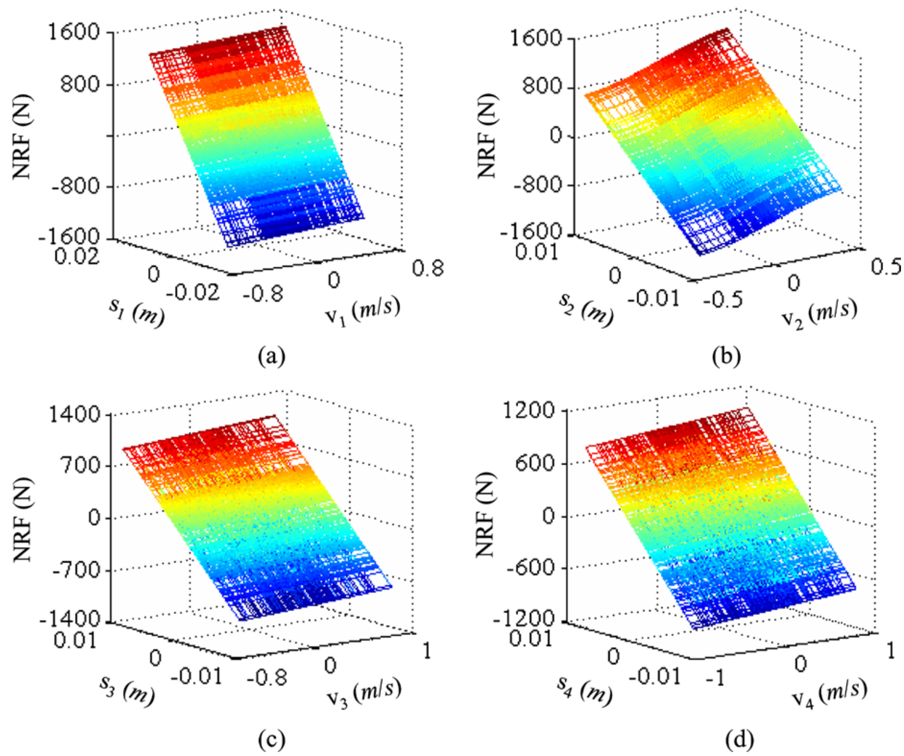


Fig. 4 The identified NRFs: (**a**) on the 1st floor, (**b**) on the 2nd floor, (**c**) on the 3rd floor, (**d**) on the 4th floor

$$\times s_{3,2}v_{3,2}^2 + 8.79 \times 10^{-13} \times v_{3,2}^3, \tag{21c}$$

$$\begin{aligned} R_{4,3} = {} & 1.00 \times 10^5 \times s_{4,3} + 100.00 \times v_{4,3} + 1.34 \\ & \times 10^{-9} \times s_{4,3}^2 - 1.87 \times 10^{-11} \times s_{4,3}v_{4,3} \\ & + 3.57 \times 10^{-14} \times v_{4,3}^2 + 2.22 \times 10^{-7} \times s_{4,3}^3 \\ & - 8.09 \times 10^{-9} \times s_{4,3}^2 v_{4,3} - 1.27 \times 10^{-11} \\ & \times s_{4,3}v_{4,3}^2 - 5.96 \times 10^{-13} \times v_{4,3}^3. \end{aligned} \tag{21d}$$

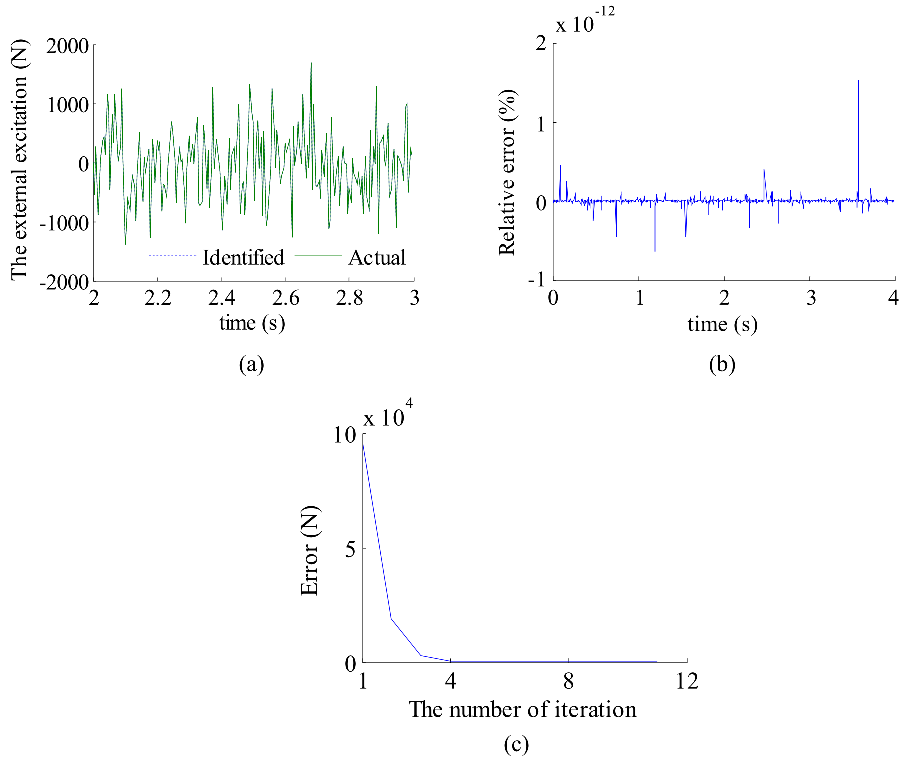
Based on the above established PSPM of NRF, the extent and characteristics of the nonlinearity can be represented more clearly in three-dimensional graphs. Figure 4 illustrates the relationship between the identified NRFs and relative displacement as well as relative velocity. From these graphs, it is obvious that the restoring force surface is obviously nonplanar on the 2nd floor and is very close to a plane on the remaining floors. The cornerstone of linear theory is linear superposition principle which means three-dimensional plot of the restoring force of linear system is a flat surface. If the system behaves nonlinearly, the restoring force surface will not be planar any more [9]. Consequently, it is easy to conclude that the nonlinear component (i.e., MR damper) is located on the 2nd floor.

Moreover, based on the proposed method, the unknown excitations can be simultaneously obtained for the example given. The identified excitation applied on the 4th DOF of the numerical model is plotted in Fig. 5(a) as dashed curves, whereas the solid curves are the corresponding actual one for comparison. For clarity of comparison, only a segment from 2 to 3 sec is presented, and the relative error between the identified excitation and the actual one in percentage is shown in Fig. 5(b). Here, the relative error is defined by the following equation:

$$e(t)(\%) = \frac{F_{\mathrm{id}}(t) - F_{\mathrm{ac}}(t)}{F_{\mathrm{ac}}(t)} \times 100, \tag{22}$$

where $F_{\mathrm{id}}(t)$ means the identified force at time instant t, and $F_{\mathrm{ac}}(t)$ means the actual force at time instant t. It's clear from Fig. 5(a–b) that the relative error is very close to zero and the identified excitation has a good agreement with the actual value. Furthermore, in order to show the convergence of the proposed approach, the summation of the absolute difference between the identified force and the measurement at the jth iteration (i.e., e^j previously defined) is also plotted

Fig. 5 The comparison of the external excitation: (**a**) partial comparison of the identified excitation with the corresponding actual one, (**b**) the relative error, (**c**) the convergence of the error of the identified force during the iteration



in Fig. 5(c). From Fig. 5(c), it can be seen that the error of the identified unknown force can be stably converged to the defined tolerance within a very limited iterations.

The NRF is provided by the elastic restoring force, the damping effects of the structure, and the nonlinear member force (e.g., MR damper force in this example). Consequently, in practice, it is usually difficult to validate the accuracy of the identified NRFs directly because it is impossible to measure the restoring force. However, it is possible to obtain the NRF provided by the nonlinear structural member such as the MR dampers in this simulation to validate the identified NRFs to some extent. The MR damper force in this example can be determined by subtracting the linear elastic restoring force and damping force provided by the structure itself from the identified total restoring force shown in (21a)–(21d). The identified hysteretic force provided by the MR damper for each floor of the structure is shown in Fig. 6. In order to evaluate the accuracy of the identified MR force, the simulated MR force determined by its numerical model (i.e., (17)–(18)) is also shown in Fig. 6. Note that identical amplitude scales are applied to all the plots.

From Fig. 6, it is easily observed that the MR damper is located on the 2nd floor due to the MR damper forces on the remaining floors being very close to zero. Moreover, it is obvious that the identified MR force has good agreement with the simulated one from Fig. 6(b). These findings mean the MR damper can be accurately located and the MR force can be identified efficiently by the proposed approach. Based on the identified results and the discussion mentioned before, it is obvious that the proposed PSPM and WAILSE-IME approach is capable of efficiently identifying the NRF of the nonlinear numerical model and accurately identifying the unknown external excitations simultaneously.

4 Experimental validation

To illustrate the application of the proposed method in conjunction with a real structure, a 4-story steel frame model structure shown in Fig. 7 is employed. The structure is 0.3 m × 0.4 m in plan and 1.2 m in height. The cross section of the columns is 30 mm × 5 mm, and the thickness of the floor plates is 10 mm. All joints are connected by bolts. The mass distribution of the structure is measured by electronic balance,

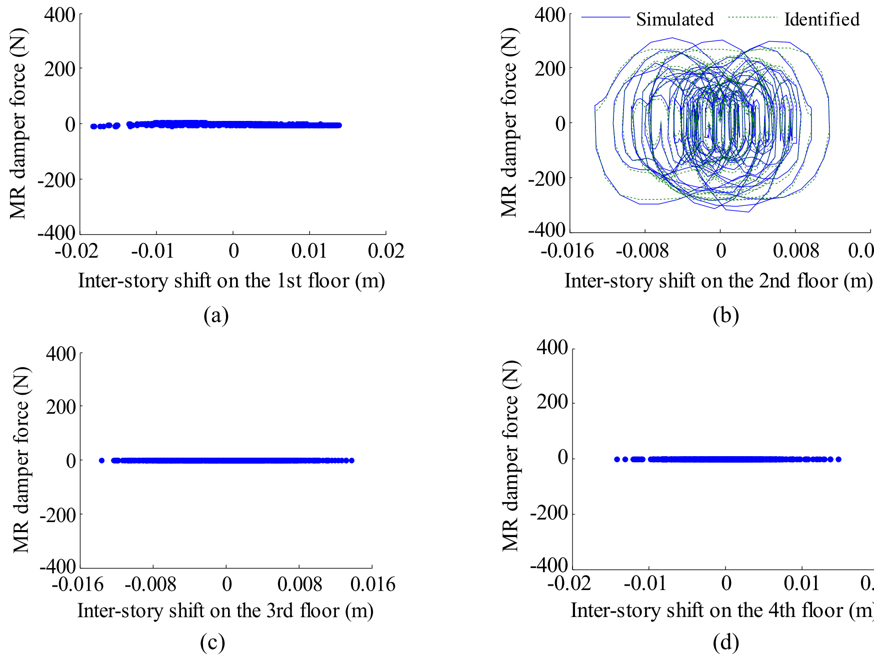


Fig. 6 The MR damper force: (**a**) on the 1st floor, (**b**) on the 2nd floor, (**c**) on the 3rd floor, (**d**) on the 4th floor

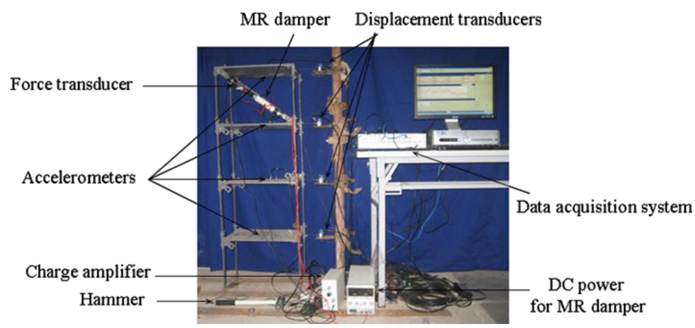


Fig. 7 Nonlinear multistory frame structure with MR dampers and vibration test setup

and can be approximately set to be $m_1 = m_2 = m_3 = 12.94$ kg and $m_4 = 12.59$ kg.

Two MR dampers with an input current of 0.2 A were installed on the 4th floor to induce a nonlinear hysteretic restoring force to the model structure. Different from the random excitation employed in the numerical simulation, an impact hammer was employed to excite the 3rd floor of the structure. The corresponding acceleration and displacement responses were directly measured with a sampling frequency of 1.024 Hz and shown in Fig. 8. The velocity of the structure was obtained by numerical integration from the measured acceleration. Even though the impact force was measured, it was assumed to be unknown during the NRF identification and was identified by the proposed approach. Moreover, the identified MR damper force was compared with the measurement.

Similar to the numerical example, let the sum of the integers k and q of the PSPM given in (3) be equal to 3. The weight coefficients of α and β, and learning coefficient γ are also set to be 10, 0.1, and 0.8, respectively. Based on the time-domain information of the system's responses, the NRFs of the nonlinear structure as well as the unknown impact force were determined. The identified NRFs are shown as follows:

$$\begin{aligned} R_{1,0} = {} & 1.32 \times 10^5 \times s_{1,0} + 29.78 \times v_{1,0} \\ & + 1.53 \times 10^6 \times s_{1,0}^2 - 1.86 \times 10^5 \\ & \times s_{1,0} v_{1,0} - 153.68 \times v_{1,0}^2 \\ & - 2.09 \times 10^8 \times s_{1,0}^3 + 2.46 \times 10^7 \\ & \times s_{1,0}^2 v_{1,0} - 3.55 \times 10^5 \\ & \times s_{1,0} v_{1,0}^2 + 354.45 \times v_{1,0}^3, \end{aligned} \tag{23a}$$

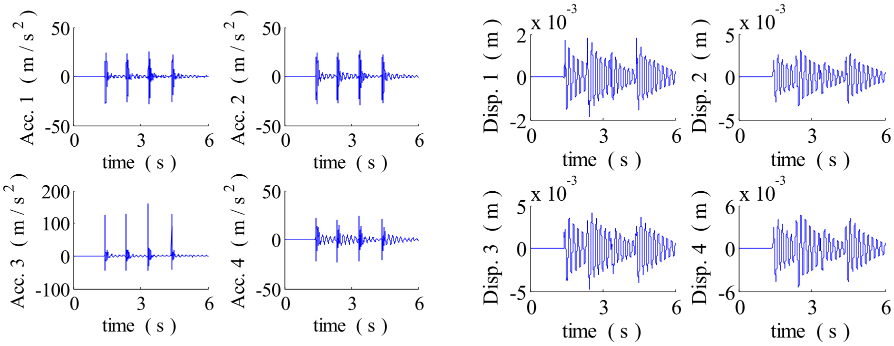


Fig. 8 The response measurements of the nonlinear structure

$$
\begin{aligned}
R_{2,1} = {} & 1.38 \times 10^5 \times s_{2,1} + 36.29 \times v_{2,1} \\
& + 2.85 \times 10^6 \times s_{2,1}^2 - 4.99 \times 10^3 \\
& \times s_{2,1} v_{2,1} + 98.29 \times v_{2,1}^2 \\
& - 3.62 \times 10^9 \times s_{2,1}^3 + 3.05 \times 10^7 \\
& \times s_{2,1}^2 v_{2,1} - 5.22 \times 10^5 \\
& \times s_{2,1} v_{2,1}^2 + 316.88 \times v_{2,1}^3, \qquad (23b)
\end{aligned}
$$

$$
\begin{aligned}
R_{3,2} = {} & 1.43 \times 10^5 \times s_{3,2} + 37.79 \times v_{3,2} \\
& - 7.28 \times 10^5 \times s_{3,2}^2 - 2.16 \times 10^4 \\
& \times s_{3,2} v_{3,2} - 469.44 \times v_{3,2}^2 \\
& + 6.62 \times 10^9 \times s_{3,2}^3 + 9.42 \times 10^6 \\
& \times s_{3,2}^2 v_{3,2} + 4.62 \times 10^5 \\
& \times s_{3,2} v_{3,2}^2 + 353.49 \times v_{3,2}^3, \qquad (23c)
\end{aligned}
$$

$$
\begin{aligned}
R_{4,3} = {} & 1.41 \times 10^5 \times s_{4,3} + 432.55 \times v_{4,3} \\
& - 9.14 \times 10^5 \times s_{4,3}^2 + 8.28 \times 10^4 \\
& \times s_{4,3} v_{4,3} + 139.93 \times v_{4,3}^2 \\
& + 2.49 \times 10^{10} \times s_{4,3}^3 - 6.31 \times 10^7 \\
& \times s_{4,3}^2 v_{4,3} - 5.67 \times 10^5 \\
& \times s_{4,3} v_{4,3}^2 + 3.55 \times 10^3 \times v_{4,3}^3. \qquad (23d)
\end{aligned}
$$

Figure 9 shows the relationship between the identified NRFs and the interstory displacement and interstory velocity of the nonlinear model structure. From Fig. 9(d), it is clear that the restoring force of the 4th floor is obvious nonlinear. Even the NRF surfaces of the remaining three floors are not an ideal plane, their nonlinearity is not obvious when compared with it of the 4th floor. The NRF surfaces of the remaining three floors are approximately planar implies that their behavior under excitation is close to a linear substructure. Consequently, it can get a conclusion that the MR damper should be placed on the 4th floor.

Moreover, the unknown external excitation applied on the 3rd floor was determined simultaneously by the proposed approach. The identified impact force and the corresponding errors by comparing with the measured value are both shown in Fig. 10. For comparison, the measurement of the excitation in the test is also shown in Fig. 10. Note that identical amplitude scales are employed for ease of comparison. Moreover, the power spectral density of the measured force is plotted in Fig. 10 as well. It can be seen from Fig. 10 that the identified excitation is close to the measured value and the corresponding error is relatively very small. It means that the proposed approach is capable of identifying the unknown inputs with acceptable accuracy in the real model structure.

As mentioned before, the NRF of a real structure is usually difficult to be measured directly. To investigate the reliability of the identified NRFs, the performance of the MR damper is separated from the total NRFs, and compared with the measurements of the MR damper force. To accomplish this, another dynamic test is carried out on the corresponding linear system, where the MR dampers are removed, to get the linear restoring force (LRF). Without loss of generality, the excitation is applied on the 2nd floor of the linear structure. Let $k + q = 1$, based on the PSPM and WAILSE-IME approach and the corresponding response measurements, the LRFs of the linear system are obtained as:

$$R_{1,0}{}^{l} = 1.31 \times 10^5 \times s_{1,0} + 21.11 \times v_{1,0}, \qquad (24a)$$

$$R_{2,1}{}^{l} = 1.45 \times 10^5 \times s_{2,1} + 25.81 \times v_{2,1}, \qquad (24b)$$

$$R_{3,2}{}^{l} = 1.38 \times 10^5 \times s_{3,2} + 34.19 \times v_{3,2}, \qquad (24c)$$

$$R_{4,3}{}^{l} = 1.41 \times 10^5 \times s_{4,3} + 32.96 \times v_{4,3}. \qquad (24d)$$

Noting that the identified coefficients of the $s_{i,i-1}$ and $v_{i,i-1}$ terms stand for the stiffness and damping coef-

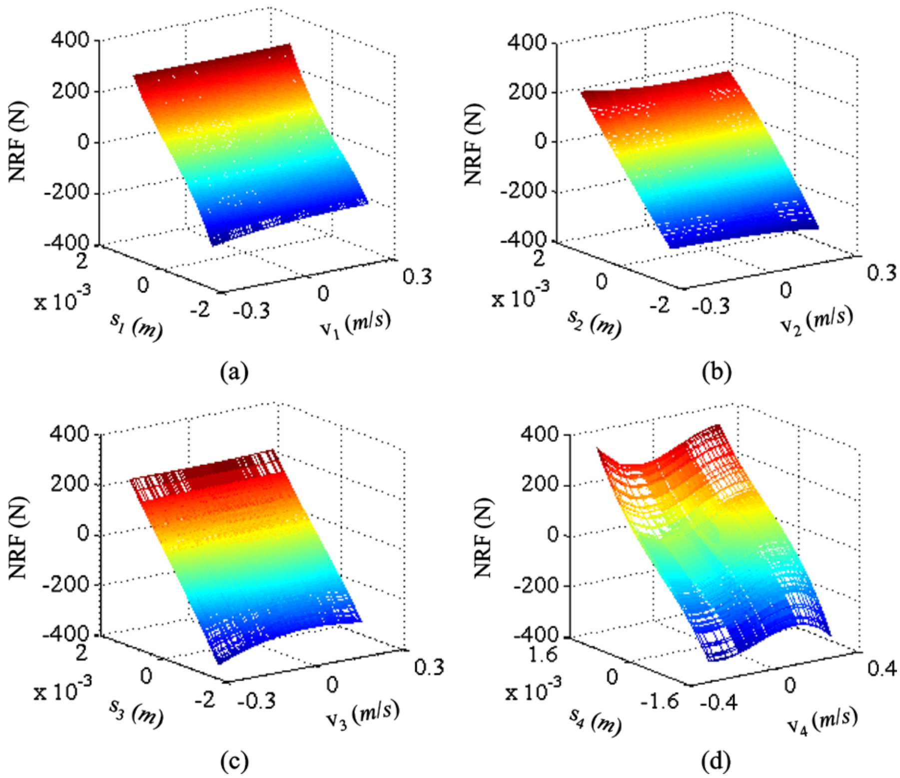


Fig. 9 The identified NRFs: (**a**) on the 1st floor, (**b**) on the 2nd floor, (**c**) on the 3rd floor, (**d**) on the 4th floor

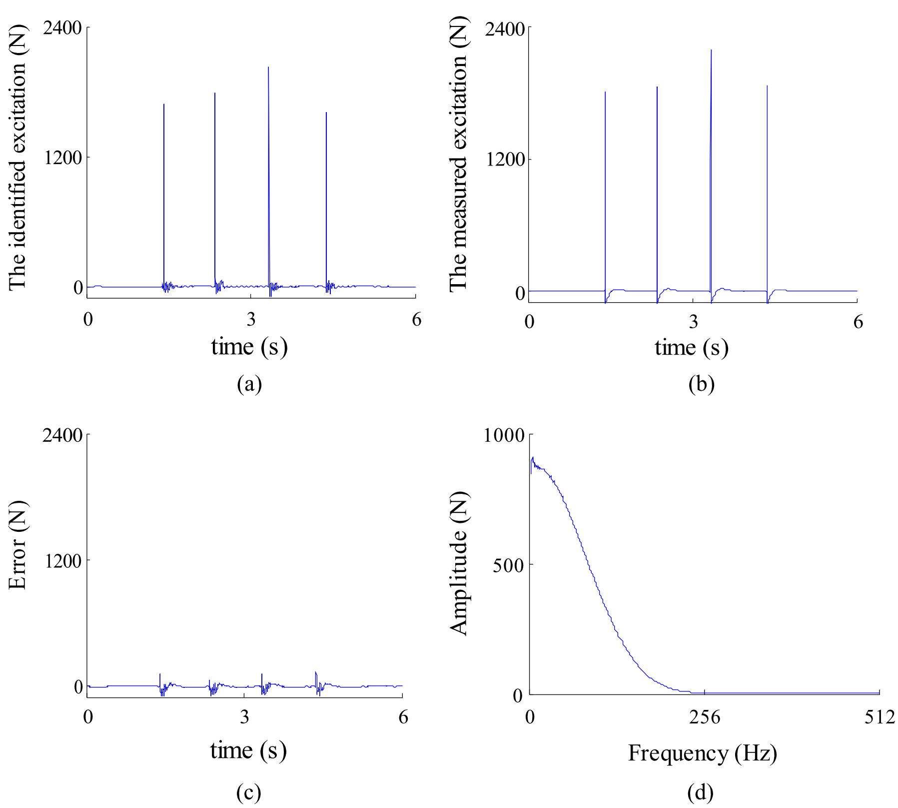


Fig. 10 The comparison of the excitation: (**a**) the identified excitation, (**b**) the measured excitation, (**c**) the errors, (**d**) the spectral of the measured force

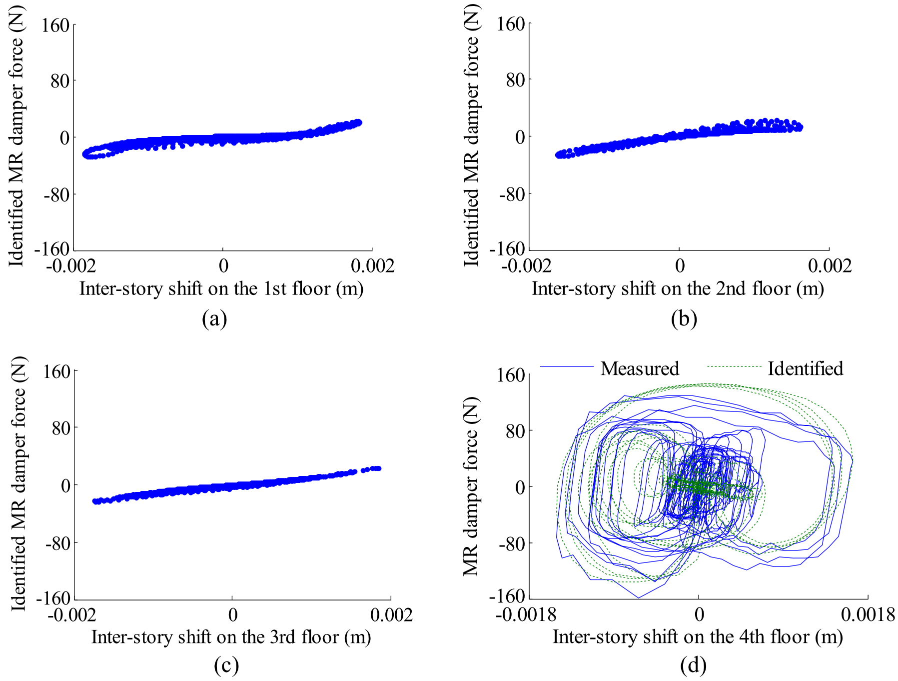


Fig. 11 The MR damper force: (**a**) on the 1st floor, (**b**) on the 2nd floor, (**c**) on the 3rd floor, (**d**) on the 4th floor

ficients of the linear system while $k+q=1$. By comparing the corresponding coefficients in (23a)–(23d) with them in (24a)–(24d), it can be found that there are no significant changes in these coefficients except the damping coefficient of the 4th floor. Based on the identified total NRFs and the LRFs provided by the structure itself, the nonlinear hysteretic force by the MR dampers can be given by the following equation:

$$F_{\mathrm{n}} = R_{i,i-1} - R^{l}_{i,i-1}, \quad (i = 1, 2, \ldots, n), \tag{25}$$

where F_{n} is the nonlinear hysteretic force provides by the MR dampers in this case. According to (25), the MR dampers force of each floor can be determined and plotted in Fig. 11. For comparison, the measured MR damper force during the vibration is also shown in Fig. 11. Note that the identified MR forces are in the horizontal direction. However, because the force gauges as shown in Fig. 7 is diagonal, the component in horizontal direction of the measured MR damper is the value to be compared with the identified results.

The purpose to carry out vibration test on the corresponding linear structure is to extract the MR damper force and to compare it with the measurements. In a real situation, a newly built structure can be regarded as the linear system whose parameters can be identified by any available modal analysis and identification approaches.

It is obvious that the MR dampers are not placed on the 1st, 2nd , and 3rd floor because the identified MR forces on these floors are relatively small. Figure 11(d) shows the comparison between the identified MR damping force with the measured value. It can be concluded that the proposed method provides a reasonably accurate identification of the NRF and LRF in the real model structure, both qualitatively and quantitatively, even the part of the external excitations are unknown.

5 Concluding remarks

In this study, an improved method by combining the PSPM and WAILSE-IME approach involving a learning coefficient and a weight matrix is proposed to identify the NRF as well as the unknown dynamic loadings of nonlinear chain-like MDOF structural system. The feasibility and robustness of the proposed method is validated via numerical simulation with a 4-DOF system incorporating a parametric MR model, and via an experiment with a 4-story steel frame structure equipped with two actively-controlled MR dampers. Results show that the proposed approach is capable of simultaneously identifying the NRF and the unknown inputs, and provides a promising way for the analysis of nonlinearity and evaluation of damage severity of engineering structures under.

An attractive aspect of the proposed approach is that other than the traditional least-square estimation requiring the completely known input information for identification, it does not need the complete external excitation to be available as shown in the numerical example and dynamic test. More importantly, the combination approach can be employed to identify the NRF of nonlinear system even the external excitations are partially unknown. Moreover, the proposed approach is also applicable for linear system. Consequently, it provides a potential way for analysis of system including nonlinear system and linear system, and for diagnosis of damage in civil engineering structures. Furthermore, the identified unknown external excitations can be utilized for the remaining service life estimation and loading profile analysis for structural damage diagnosis.

Acknowledgements The authors gratefully acknowledge the support provided through the National Natural Science Foundation of China (NSFC) under Grant No. 50978092, the Hunan Provincial Natural Science Foundation of China under Grant No. 08JJ1009, and the Fundamental Research Funds for the Central Universities at Hunan University. Partial support by the Program for New Century Excellent Talents in University (NCET-08-118) is also greatly appreciated.

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