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RESPONSE

Response to the comments on "Adaptive synchronization of fractional-order chaotic systems via a single driving variable"

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Abstract This paper is a Response to the comment by M.P. Aghababa (Nonlinear Dyn. 2001, doi:10.1007/s11071-011-0216-y). Some ideas are proposed to rebut the Comments.

Keywords Fractional-order chaotic system · Adaptive synchronization · Lyapunov stability theory

First of all, the authors are most grateful to the authors of comments for pointing out the mistakes in our published paper in Nonlinear Dynamics. The authors acknowledge their contribution.

1. As pointed out in [1], the Lyapunov stability theory used in [2] cannot be used for an integer-order system mixed with a fractional-order system. In fact, we did not apply the traditional Lyapunov stability theory to prove the asymptotic stability of an integer-order system mixed with a fractional-order system.

Actually, according to $s = D_t^{\alpha-1} z + \int_0^t cz(\tau)\,d\tau$ and our paper [2], we find that s is a function about z, and

$$\dot{s} = f_2(x, z) + cz - k|z|\text{sign}(s) \tag{1}$$

and k is adapted according to the following update law:

$$\dot{k} = \theta|z||s| \quad (\theta > 0). \tag{2}$$

Let the system (1) and (2) be the augment system. Consider the following Lyapunov function candidate

$$V(s, k) = \frac{1}{2}s^2 + \frac{1}{2\theta}(k - k^*)^2 \quad (k^* > \lambda_2 + c).$$

Taking the derivative of $V(s, k)$ with respect to time, one has

$$\begin{aligned}\dot{V}(s, k) &= s\dot{s} + (k - k^*)\dot{k}/\theta \\ &= s\big(D_t^\alpha z + cz\big) + (k - k^*)\dot{k}/\theta \\ &= s\big(f_2(x, z) - k|z|\text{sign}(s) + cz\big) \\ &\quad + (k - k^*)|z||s| \\ &\le (\lambda_2 + c)|z||s| - k|z||s| + (k - k^*)|z||s| \\ &= -(k^* - \lambda_2 - c)|z||s| \le 0.\end{aligned}$$

So, a Lyapunov function has been found that satisfies the conditions of the Lyapunov theorem ($V > 0$; $\dot{V} < 0$). Thus, the closed-loop system in the presence of the controller (1) is globally asymptotically stable.

Actually, if $x = 0$ be an equilibrium point of a nonlinear system $\dot{x} = f(x)$, $V(x) > 0$ and $\dot{V}(x) > 0$, then

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$x = 0$ is stable even if x includes the fractional term. See [3–7].

Therefore, the main results of our paper [2] are correct.

2. In the proof approach of the closed-loop system in [2], the condition $k^* > \lambda_2$ of (10) in [2] should be replaced by $k^* > \lambda_2 + c$. Remark 2 presented in the Comment is acknowledged here.

3. The controller, which is selected as (6) or (13) in [1], contains large enough but unknown constants k_1 and k_2. The method to find the suitable feedback constants k_1 and k_2 is to test again and again. However, our paper investigates the adaptive control for a class of three-dimensional fractional-order chaotic systems. In our paper [2], the feedback gain k is unknown in advance, but it automatically converges to a suitable constant k^*. So, we think the critical arguments stated in Theorem 3 and 4 in the Comment are irrelevant or incorrect.

4. We have first proposed an appropriate fractional-order sliding surface as $s = D_t^{\alpha-1} z + \int_0^t cz(\tau)\,d\tau$ in [2]. Recently, we found that the author of [1] proposed the same sliding surface in [8]. The author of [1] claimed that he proposed the fractional-order sliding surface, see (9) in [8]. We cannot understand that.

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