Nonlinear Dyn (2012) 67:2889–2897
DOI 10.1007/s11071-011-0196-y

ORIGINAL PAPER

Fuzzy neural adaptive tracking control of unknown chaotic systems with input saturation

Da Lin · Xingyuan Wang · Yi Yao

Received: 7 May 2011 / Accepted: 8 August 2011 / Published online: 11 September 2011

Abstract The contribution of this work is to study the control of unknown chaotic systems with input saturation, and the backstepping-based an adaptive fuzzy neural controller (AFNC) is proposed. In many practical dynamic systems, physical input saturation on hardware dictates that the magnitude of the control signal is always constrained. Saturation is a potential problem for actuators of control systems. It often severely limits system performance, giving rise to undesirable inaccuracy or leading instability. To deal with saturation, we construct a new system with the same order as that of the plant. With the error between the control input and saturation input as the input of the constructed system, a number of signals are generated to compensate the effect of saturation. Finally, simulation results show that the AFNC can achieve favorable tracking performances.

Keywords Chaos · Adaptive control · System identification · Fuzzy neural network · Input saturation

D. Lin (✉) · Y. Yao
School of Automatic and Electronic Information, Sichuan University of Science and Engineering, Zigong 643000, China
e-mail: Linda_740609@yahoo.cn

X. Wang
Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian 116024, China

1 Introduction

Chaos control is an important topic in the nonlinear science. In essence, chaos control is guiding a chaotic system to reach a desired goal dynamics via various controllers. Chaotic signals are typically broadband, noise-like, difficult to predict, and can hide information efficiently and securely. Since chaos control problem was firstly considered by Ott et al. [1], it has been investigated extensively by lots of authors, and many control methods have been employed to control chaos [2–7].

In most control engineering applications the performance of the controller is directly related to the accuracy of the mathematical model obtained for the controlled system. However, there are many situations in control systems when the control engineer faces the difficulty of incomplete or insufficient information [8]. Fuzzy control methodologies have emerged in recent years as promising ways to approach nonlinear control problems. Fuzzy control, in particular, has an impact in the control community, because it can provide a simple approach to the control of plants that are complex, uncertain, ill-defined, and have available heuristic knowledge from domain experts for their controllers design [9–15]. However, neural network (NN) adaptive control algorithms have attracted attention due to their inherently parallel and highly redundant processing architecture that makes it possible to develop parallel weight update laws. This parallelism makes it possible to effectively update an NN online.

Consequently, the use of the NNs for system identification and control of complex highly uncertain dynamical systems has become an active research field [16–19].

Fuzzy model did not seem good under testing conditions for the sake of inadequate experimental data. The production rules cannot produce a good precision in case of lack of information. The structure (number of fuzzy rule) also influences identification capability and accuracy [20]. Recent results show that the fusion procedure of neural networks and fuzzy systems seems to be very effective for nonlinear system modeling [21]. A general combination method is to use neural training for a fuzzy model [22]. From prior knowledge, the structure of fuzzy system can be determined. After neural training, the membership functions of fuzzy systems are changed to match the training data. Fuzzy neural network (FNN) possesses the merits of the low-level learning and the computational power of the neural network (NN), and the high-level human knowledge representation and the thinking of fuzzy theory. FNN has been proven to be universal approximators [23]. Recently model-free, computationally intelligent techniques using either fuzzy logic or neural networks have been investigated in order to circumvent existing difficulties. The concept of incorporating fuzzy logic into neural network has emerged and has become a popular research area [24–29].

In many practical dynamic systems, physical input saturation on hardware dictates that the magnitude of the control signal is always constrained. Saturation is a potential problem for actuators of control systems. It often severely limits system performance, giving rise to undesirable inaccuracy or leading instability. The development of adaptive control schemes for systems with input saturation has been a task of major practical interest as well as theoretical significance. However, the number of available results by taking saturation into account in the design and analysis is still limited due to the difficulty of the problem. For linear stable systems with known parameters and input saturation, a few control schemes have been proposed in [30–34]. When the system parameters are unknown, adaptive control schemes have been proposed in [35–37], where uncertain parameters must be inside a known compact set. An adaptive force-balancing control scheme with actuator limits for a MEMS gyroscope was also presented in [38], where the plant is a stable second-order uncertain linear system. In [39], controlling a class of uncertain nonlinear systems with input saturation was proposed.

Backstepping approach is a Lyapunov-based recursive design procedure. With this technique, transient performance can be established and improved with explicit tuning of design parameters. A great deal of attention has been paid to tackle both linear and nonlinear systems with unknown parameters. A number of results have been obtained as summarized in [40].

In this paper, we will address the problem of controlling a class of unknown chaotic systems with input saturation. To deal with saturation, we construct a new system with the same order as that of the plant similar to [38, 39]. With the error between the control input and saturation input as the input of the constructed system, a number of signals are generated to compensate the effect of saturation. With the proposed fuzzy neural adaptive backstepping controller, the tracking error is shown to approach a signal generated by the constructed system. The tracking error is also adjustable by an explicit choice of design parameters. Thus, our designed backstepping scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way. Finally, simulation results are presented to demonstrate the effectiveness of the proposed control scheme.

2 System description and problem statement

Consider a class of chaotic systems as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(\boldsymbol{X}) + u(v) \\ y = x_1 \end{cases}, \tag{1}$$

where $X = [x_1, \dot{x}_1, \ldots, x_1^{(n-1)}]^{\mathrm{T}} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \boldsymbol{R}^n$ is the state vector, $f(\boldsymbol{X})$ is unknown continuous bounded function. $y \in R$ is the output, v is the control input, and $u(v) \in \boldsymbol{R}$ denotes the plant input subject to saturation described by

$$u(v) = \mathrm{sat}(v) = \begin{cases} \mathrm{sign}(v) u_M & |v| \geq u_M \\ v & |v| < u_M \end{cases}, \tag{2}$$

where u_M is the saturation bound of u.

The control objectives are to design backstepping adaptive control law v such that the closed loop system is globally stable in sense that all the signals in the system are uniformly ultimately bounded, and the tracking error $y - y_r$ is adjustable by an explicit choice of design parameters.

3 Description of FNN

The configuration of the FNN shown in Fig. 1 consists of fuzzy logic and neural network. The fuzzy logic system can be divided into two parts: some fuzzy IF-THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping form an input linguistic vector $\boldsymbol{X}^{\mathrm{T}} = [x_1, x_2, \ldots, x_n] \in \boldsymbol{R}^n$ to an output linguistic variable $y \in \boldsymbol{R}$. The ith fuzzy IF-THEN rule is written as

$$\begin{aligned} R^{(i)} &: \text{if } x_1 \text{ is } A_1^i \text{ and} \ldots \text{and } x_n \text{ is } A_n^i \\ &\quad \text{then } y \text{ is } B^i \end{aligned} \tag{3}$$

where $A_1^i, A_2^i, \ldots, A_n^i$ and B^i are fuzzy sets [20, 41]. Let h be the number of the fuzzy IF-THEN rules. By using product inference, center-average and singleton fuzzifier, the output of the fuzzy logic system can be expressed as

$$y(\boldsymbol{X}) = \frac{\sum_{i=1}^{h} \bar{y}^i (\prod_{j=1}^{n} \mu A_j^i(x_j))}{\sum_{i=1}^{h} (\prod_{j=1}^{n} \mu A_j^i(x_j))} = \theta^{\mathrm{T}} \xi(\boldsymbol{X}), \tag{4}$$

where $\mu A_j^i(x_j)$ is the membership function value of the fuzzy variable, h is the total number of the IF-THEN rules, $\bar{y}^i$ is the point at which $\mu B^i(\bar{y}^i) = 1$, $\theta^{\mathrm{T}} = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^h]$ is an adjustable parameter vector, and $\xi^{\mathrm{T}} = [\xi^1, \xi^2, \ldots, \xi^h]$ is a fuzzy basis vector, where ξ^i is defined as

$$\xi^i(\boldsymbol{X}) = \frac{\prod_{j=1}^{n} \mu A_j^i(x_j)}{\sum_{i=1}^{h} (\prod_{j=1}^{n} \mu A_j^i(x_j))}. \tag{5}$$

When the inputs are given into the FNN shown in Fig. 1, the truth value ξ^i (layer 3) of the antecedent part of the ith implication is calculated by (5). Among the commonly used defuzzification strategies, the outputs (layer 4) of the fuzzy neural system are expressed as (4). The fuzzy logic approximator based on NN

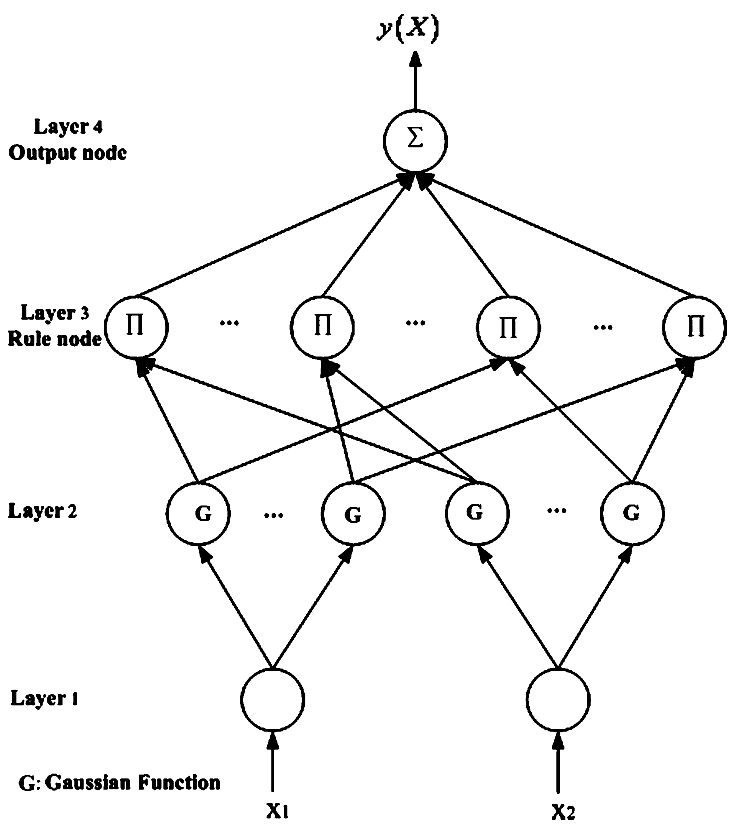


Fig. 1 Schematic diagram of fuzzy neural network

can be established [42, 43]. Figure 1 shows the configuration of the fuzzy neural function approximator. The approximator has four layers. At layer 1, nodes which are input ones stand for input linguistic vector $\boldsymbol{X}^{\mathrm{T}} = [x_1, x_2, \ldots, x_n]$. At layer 2, nodes represent the values of the membership function of total linguistic variables. Each node of layer 2 performs a membership function value. At layer 3, nodes are the values of the fuzzy basis vector ξ. Each node of layer 3 performs a fuzzy rule. The links between layer 3 and layer 4 are full connected by the weighting factors, $\theta^{\mathrm{T}} = [\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^h]$, i.e., the adjusted parameters. At layer 4, the outputs stand for the values of the output $y(\boldsymbol{X})$.

4 Design of adaptive fuzzy neural controller

In order to compensate the effect of the saturation, the following system is constructed to generate signals $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_n]^{\mathrm{T}}$

$$\begin{cases} \dot{\lambda}_1 = \lambda_2 - c_1\lambda_1 \\ \dot{\lambda}_i = \lambda_{i+1} - c_i\lambda_i, \qquad i = 2, 3, \ldots, n-1 \\ \dot{\lambda}_n = -c_n\lambda_n + \Delta u \end{cases} \tag{6}$$

where c_i are positive constants and $\Delta u = u(v) - v$.

The following change of coordinates is made.

$$z_1 = y - y_r - \lambda_1. \tag{7}$$

$$z_i = x_i - \alpha_{i-1} - y_r^{(i-1)} - \lambda_i, \quad i = 2, 3, \ldots, n, \tag{8}$$

where α_{i-1} is the virtual control at the ith step to be determined.

Remark 1 With the error Δu as the input of the constructed system, it has no effect on z_i. Thus, it will not affect the design of controllers. Then by following the standard backstepping approach, the adaptive law will ensure the boundedness of parameter estimates regardless of Δu. On the other hand, such estimates will depend on Δu when standard backstepping is used without using the transformed systems.

In the following, backstepping control scheme is proposed. To illustrate the design procedures, only the first and the last step are elaborated in details.

Step 1: Starting from the equations for the tracking error obtained from (1) and (6)–(8), we get

$$\dot{z}_1 = x_2 - \lambda_2 + c_1\lambda_1 - \dot{y}_r$$
$$= z_2 + \alpha_1 + c_1\lambda_1. \tag{9}$$

We design the virtual control law α_1 as

$$\alpha_1 = -c_1(x_1 - y_r), \tag{10}$$

where $c_1 > 1/2$ is a positive design parameter. A positive Lyapunov function V_1 is defined as

$$V_1 = \frac{1}{2}z_1^2. \tag{11}$$

Then the derivative of V_1 along with (9) and (10) is given as

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 + z_1 z_2 \\ &\le -c_1 z_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \\ &= -\bar{c}_1 z_1^2 + \frac{1}{2}z_2^2, \end{aligned} \tag{12}$$

where $\bar{c}_1 = c_1 - \frac{1}{2} > 0$.

Step i ($i = 2, \ldots, n-1$): For $z_i = x_i - \alpha_{i-1} - y_r^{(i-1)} - \lambda_i$, we choose virtual control law α_i as

$$\begin{aligned} \alpha_i &= -c_i(x_i - \alpha_{i-1} - y_r^{(i-1)}) \\ &\quad + \dot{\alpha}_{i-1}(x_1, \ldots, x_{i-1}), \end{aligned} \tag{13}$$

where $c_i, i = 2, \ldots, n-1$ are positive design parameters satisfying $c_i > 1$. From (8) and (13), we obtain

$$z_i\dot{z}_i = -c_i z_i^2 + z_i z_{i+1}. \tag{14}$$

We choose Lyapunov function as

$$V_i = \sum_{k=1}^{i} \frac{1}{2}z_k^2. \tag{15}$$

Then the derivative of V_i along with (13) and (14) is given by

$$\begin{aligned} \dot{V}_i &\le -c_i z_i^2 + z_i z_{i+1} + \frac{1}{2}z_i^2 \\ &\le -\sum_{k=1}^{i} \bar{c}_i z_i^2 + \frac{1}{2}z_{i+1}^2, \end{aligned} \tag{16}$$

where $\bar{c}_i = c_i - 1 > 0$.

Step n: From (1) and (8) for $i = n$, we get

$$\dot{z}_n = v + f(\boldsymbol{X}) - \dot{\alpha}_{n-1} + c_n\lambda_n - y_r^{(n)}. \tag{17}$$

In this paper, we use FNN to identify the unknown function $f(\boldsymbol{X})$, and let $f(\boldsymbol{X}) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{X})$. We design the adaptive fuzzy neural controller v_c as follows:

$$v_c = -c_n\big(x_n - \alpha_{n-1} - y_r^{(n-1)}\big) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{X}) + \dot{\alpha}_{n-1} + y_r^{(n)}, \tag{18}$$

where $c_n > 1/2$, $\hat{\boldsymbol{\theta}}^{\mathrm{T}}$ is an estimate of $\boldsymbol{\theta}^{\mathrm{T}}$.

Define the model error as

$$\begin{aligned} \varepsilon &= f(\boldsymbol{X}) - \hat{f}(\boldsymbol{X}) \\ &= \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{X}) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{X}), \end{aligned} \tag{19}$$

and $|\varepsilon| \le \bar{\varepsilon}$, where $\bar{\varepsilon}$ is positive constant.

The robust controller u_s is designed as

$$v_s = -\mathrm{sgn}(z_n)\bar{\varepsilon}. \tag{20}$$

The synthesis controller is

$$v = v_c + v_s. \tag{21}$$

The following adaptive law for adjusting the link weight of FNN between layer 3 and layer 4 is given:

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\xi}(\boldsymbol{X})z_n. \tag{22}$$

We define a positive Lyapunov function V as

$$V = \sum_{i=1}^{n} \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}, \tag{23}$$

where $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$. Then the derivative of V along with (17)–(22) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} z_i\dot{z}_i + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\theta}}}^{\mathrm{T}} \\ &\le -\sum_{i=1}^{n} \bar{c}_i z_i^2 + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}[\boldsymbol{\xi}(\boldsymbol{X})z_n - \dot{\hat{\boldsymbol{\theta}}}] \\ &\quad - z_n\mathrm{sgn}(z_n)\bar{\varepsilon} + z_n\varepsilon \\ &\le -\sum_{i=1}^{n} \bar{c}_i z_i^2 + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}[\boldsymbol{\xi}(\boldsymbol{X})z_n - \dot{\hat{\boldsymbol{\theta}}}] \\ &\quad - |z_n|\bar{\varepsilon} + |z_n|\bar{\varepsilon} \\ &\le -\sum_{i=1}^{n} \bar{c}_i z_i^2, \end{aligned} \tag{24}$$

where $\bar{c}_n = c_n - \frac{1}{2}$.

This shows that V is uniformly bounded. Thus, $z_i, i = 1, \ldots, n$ and $\hat{\boldsymbol{\theta}}$ are bounded. For chaotic systems, $x_i, i = 1, \ldots, n$ are bounded and its input is bounded. So, that the boundedness of $\alpha_1, \ldots, \alpha_{n-1}$ and control signal v can be obtained from (10), (13), and (21). Thus, $\Delta u = u(v) - v$ is also bounded. Therefore, boundedness of all signals in the closed loop system is ensured as stated in the following theorem.

Theorem 1 *Consider the unknown chaotic system (1) in the presence of input saturation. With the application of controller (21) and the link weight of FNN update law (22), the following statements hold*:

The steady state tracking error satisfies

$$\lim_{t\to\infty}[y(t) - y_r(t) - \lambda_1(t)] = 0. \tag{25}$$

A bound of the transient tracking error will be given by

$$\|y(t) - y_r(t)\|_2 \le \frac{1}{\sqrt{\bar{c}_1}}\left[\frac{1}{2}\tilde{\boldsymbol{\theta}}(0)^{\mathrm{T}}\tilde{\boldsymbol{\theta}}(0)\right]^{1/2} + \frac{1}{\sqrt{c_0}}\|\Delta u\|_2. \tag{26}$$

Proof From (24), we established that V is nonincreasing. Hence, $z_i, i = 1, \ldots, n, \hat{\boldsymbol{\theta}}$ are bounded. By applying the LaSalle–Yoshizawa theorem to (24), it further follows that $z_i(t) \to 0, i = 1, \ldots, n$ as $t \to \infty$, which implies that $\lim_{t\to\infty}[y(t) - y_r(t) - \lambda_1(t)] = 0$.

From (24), we also have that

$$\begin{aligned} \|z_1\|_2^2 = \|y - y_r - \lambda_1\|_2^2 &= \int_0^{\infty} |z_1(\tau)|^2\, d\tau \\ &\le \frac{1}{\bar{c}_1}[V(0) - V(\infty)] \le \frac{1}{\bar{c}_1}V(0). \end{aligned} \tag{27}$$

Thus, by setting $z_i(0) = 0, i = 1, \ldots, n$, we have

$$V(0) = \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(0)\tilde{\boldsymbol{\theta}}(0). \tag{28}$$

This means that the bound resulting from (27) and (28) is

$$\|y(t) - y_r(t) - \lambda_1(t)\|_2 \le \frac{1}{\sqrt{\bar{c}_1}}\left[\frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(0)\tilde{\boldsymbol{\theta}}(0)\right]^{1/2}. \tag{29}$$

Now we derive the bound of λ_1.

We construct the positive Lyapunov function $V_\lambda = \sum_{i=1}^{n} \frac{1}{2}\lambda_i^2$. Then the derivative of V_λ is given as

$$\begin{aligned}\dot{V}_\lambda &= -c_1\lambda_1^2 + \lambda_1\lambda_2 - c_2\lambda_2^2 + \lambda_2\lambda_3 + \cdots \\ &\quad + \lambda_{n-1}\lambda_n - c_n\lambda_n^2 + \lambda_n \Delta u \\ &\le \sum_{i=1}^{n} -\bar{c}_i\lambda_i^2 + \Delta u^2 \\ &\le -c_0\|\lambda\|^2 + \Delta u^2, \end{aligned} \tag{30}$$

where $\bar{c}_1 = c_1 - \frac{1}{2}$, $\bar{c}_i = c_i - 1$ $(i = 1, \dots, n-1)$, $\bar{c}_n = c_n - \frac{3}{4}$, $c_0 = \min(\bar{c}_i, i = 1, \dots, n)$. Integrating both sides of (30), we have

$$\begin{aligned}\|\lambda\|_2^2 &= \int_0^\infty \|\lambda\|^2 \, d\tau \\ &\le \frac{1}{c_0}\left[(V_\lambda(0) - V_\lambda(\infty)) + \int_0^\infty (\Delta u)^2 \, d\tau\right]. \end{aligned} \tag{31}$$

By setting $\lambda_i(0) = 0$, the initial value of the Lyapunov function is $V_\lambda(0) = 0$. Then a bound on the state $\|\lambda\|_2$ is established as follows:

$$\|\lambda\|_2 \le \frac{1}{\sqrt{c_0}}\|\Delta u\|_2. \tag{32}$$

Thus, from (29) and (32), it is obtained

$$\|y - y_r\|_2 \le \frac{1}{\sqrt{\bar{c}_1}}\left[\frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(0)\tilde{\boldsymbol{\theta}}(0)\right]^{1/2} + \frac{1}{\sqrt{c_0}}\|\Delta u\|_2. \tag{33}$$

This proof completed. □

Remark 2 The transient performance depends on the initial estimate error $\tilde{\boldsymbol{\theta}}(0)$ and the explicit design parameters. The closer the initial estimate $\hat{\boldsymbol{\theta}}(0)$ to the true value $\boldsymbol{\theta}$, the better the transient performance.

Remark 3 The bound for $\|y(t) - y_r(t)\|_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimate on the transient performance by increasing parameter c_1.

Remark 4 The bound of $\|y(t) - y_r(t)\|_2$ depends on the bound of Δu, the effects of which on system performance can be decreased by increasing parameter c_0. If $\Delta u \to 0$ as $t \to \infty$, we have $\lambda_1 \to 0$. Then $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$. This implies that if the system has no saturation or the control signal is not saturated as $t \to \infty$, then perfect tracking is ensured.

Now we give the update laws of the Gaussian membership functions. Based on [44], the BP algorithm for online tuning the means and variances of the Gaussian membership functions is discussed. Consider the structure of FNN is 2-6-9-1, and the Gaussian membership function is defined by

$$\mu A_{jk} = -\frac{(x_j - m_{jk})^2}{(\sigma_{jk})^2}, \quad j = 1, 2; k = 1, 2, 3, \tag{34}$$

where m_{jk} and σ_{jk} are respectively, the mean and the variance of the Gaussian function in the kth term of the jth input linguistic variable x_j. Let $e_1(t) = y(t) - y_r(t)$, the cost function to be minimized is defined as

$$J = J(t) = \frac{1}{2}(e_1(t))^2. \tag{35}$$

$$\begin{aligned} d_l^3 &= \frac{-\partial J}{\partial net_l^3} = \frac{-\partial J}{\partial net_1^4}\frac{\partial net_1^4}{\partial net_l^3} \\ &= e_1 \cdot \boldsymbol{\theta}_l \quad (l = 1, 2, \dots, 9). \end{aligned} \tag{36}$$

$$\begin{aligned} d_i^2 &= \frac{-\partial J}{\partial net_i^2} = \frac{-\partial J}{\partial y_i^2}\frac{\partial y_i^2}{\partial net_i^2} \\ &= -\left(\sum_p \frac{\partial J}{\partial net_p^3}\frac{\partial net_p^3}{\partial y_i^2}\right) \cdot \frac{\partial y_i^2}{\partial net_i^2} \\ &= \left(\sum_p d_p^3 \cdot y_{i'}^2\right) \cdot y_i^2, \quad (i = 1, 2, \dots, 6), \end{aligned} \tag{37}$$

where the subscript p denotes the rule node in connection with the ith node in Layer 2, and i' denotes the other node in Layer 2 which connection with the pth node in Layer 3. Then, the adaptive rule of m_{jk} is

$$\begin{aligned} \Delta m_{jk} &= -\frac{\partial J}{\partial m_{jk}} = -\frac{\partial J}{\partial net_k^2}\frac{\partial net_k^2}{\partial m_{jk}} \\ &= \delta_k^2 \cdot \frac{2(y_j^1 - m_{jk})}{\sigma_{jk}^2}, \end{aligned} \tag{38}$$

and the adaptive rule of σ_{jk} is

$$\begin{aligned} \Delta \sigma_{jk} &= -\frac{\partial J}{\partial \sigma_{jk}} = -\frac{\partial J}{\partial net_k^2}\frac{\partial net_k^2}{\partial \sigma_{jk}} \\ &= \delta_k^2 \cdot \frac{2(y_j^1 - m_{jk})^2}{\sigma_{jk}^3}, \end{aligned} \tag{39}$$

where $j = 1, 2$; $k = 1, 2, 3$. The parameters of the Gaussian membership functions can be modified as

$$m_{jk}(t+1) = m_{jk}(t) + \eta \Delta m_{jk}(t), \quad (40)$$

and

$$\sigma_{jk}(t+1) = \sigma_{jk}(t) + \eta \Delta \sigma_{jk}(t), \quad (41)$$

where η is learning rate.

Note that net_q^L and y_q^L denote the summed net input of qth node and the output of qth node, respectively, and the superscript denotes the layer number.

5 Computer simulations

In this section, we apply the proposed AFNC to control the Duffing chaotic system [45]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t)\,. \\ y = x_1 \end{cases} \quad (42)$$

It can be shown that without control, the system is chaotic. The chaotic motion of the Duffing system is shown in Fig. 2.

Let $f(\boldsymbol{X}) = -0.1x_2 - x_1^3 + 12\cos(t)$, we now use the AFNC to control the output y to track the reference trajectory $y_r = \sin(t)$. We choose $c_1 = c_2 = 10$, and $\bar{\varepsilon} = 0.5$. In this paper, we use a FNN to represent the approximation of unknown function $f(\boldsymbol{X})$. The structure of FNN is 2-6-9-1. The initial FNN parameters are selected randomly, i.e., $\boldsymbol{\theta} \in [-12, 12]$, $\boldsymbol{m} \in [-2, 2]$, $\boldsymbol{\sigma} \in [0, 1]$. Here, $\boldsymbol{\theta}$ represent the link weigh vector, and $\boldsymbol{m}$ represents the mean vector of the Gaussian membership functions, and $\boldsymbol{\sigma}$ represents the variance vectors of the Gaussian membership functions. We choose the initial system states $\boldsymbol{X}(0) = [-2, -2]$. The simulation results are shown in Figs. 3–4.

6 Conclusions

In this paper, we have developed an adaptive backstepping fuzzy neural control scheme for a class of unknown chaotic systems in the presence of input saturation. To design the AFNC, no exact knowledge of system is needed. In addition, the online tuning parameters include the weighting factors in the consequent part, and the means and variances of the Gaussian membership functions in the antecedent part of fuzzy implications. Besides showing global stability, we also give an explicit bound on the performance of the tracking error in terms of design parameters. Finally, this method has been applied to control the Duffing chaotic system to track a reference trajectory. Next, we will study the control of uncertain chaotic systems with input saturation, and the total states of the chaotic systems are not assumed to be available for measurement.

The computer simulation results show that the AFNC can perform successful control and achieved desired performance.

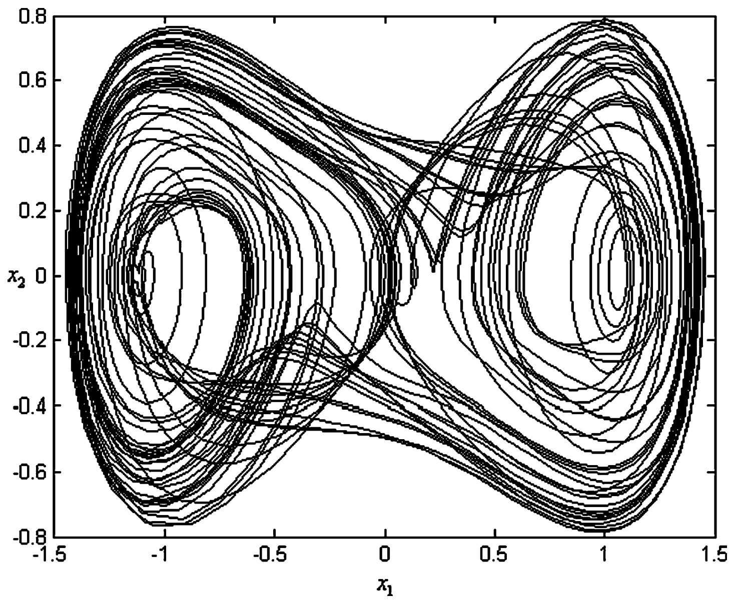


Fig. 2 The chaotic attractor of Duffing system

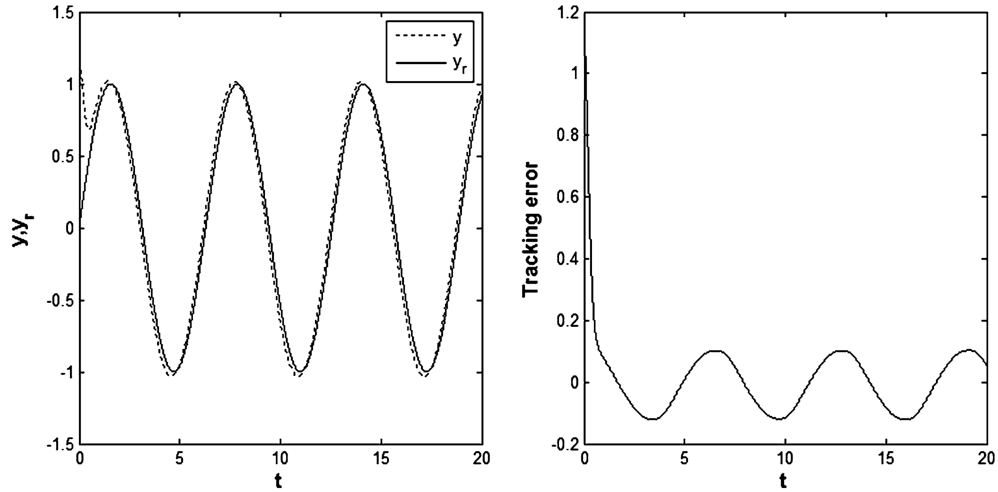
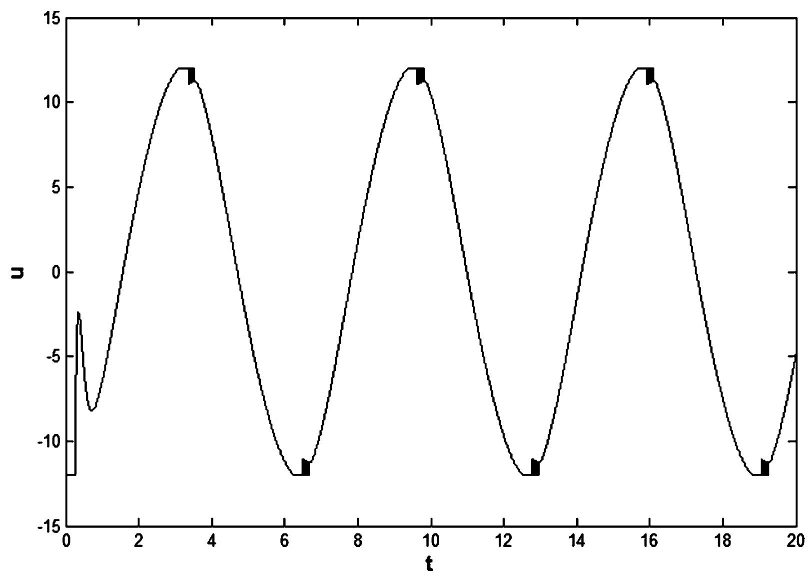


Fig. 3 Trajectories of y, y_r and the tracking error

Fig. 4 The control input



Acknowledgements This research is supported by the National Natural Science Foundation of China (Nos. 60573172, 60973152), the Superior University Doctor Subject Special Scientific Research Foundation of China (No. 20070141014) and the Natural Science Foundation of Liaoning province (No. 20082165).

References

1. Ott, E., Grebogi, C., Yorke, J.A.: Controlling chaos. Phys. Rev. Lett. **64**, 1196–1199 (1990)
2. Raj, S.P., Rajasekar, S.: Migration control in two couple Duffing oscillators. Phys. Rev. E **55**, 6237–6240 (1997)
3. Myneni, K., Barr, T.: New method for the control of fast chaotic oscillations. Phys. Rev. Lett. **83**, 2175–2178 (1999)
4. Yu, Y., Zhang, S.: Controlling uncertain Lü system using backstepping design. Chaos Solitons Fractals **15**, 897–902 (2003)
5. Hua, C., Guan, X., Shi, P.: Adaptive feedback control for a class of chaotic systems. Chaos Solitons Fractals **23**, 757–765 (2005)
6. Soong, C.Y., Huang, W.T., Lin, F.P.: Chaos control on autonomous and non-autonomous systems with various types of genetic algorithm-optimized weak perturbations. Chaos Solitons Fractals **34**, 1519–1537 (2007)
7. Liu, Y.J., Zheng, Y.Q.: Adaptive robust fuzzy control for a class of uncertain chaotic systems. Nonlinear Dyn. **57**, 431–439 (2009)

8. Zhang, H.G., Li, M., Yang, J., Yang, D.D.: Fuzzy model-based robust networked control for a class of nonlinear systems. IEEE Trans. Syst. Man Cybern., Part A, Syst. Hum. **39**, 437–447 (2009)
9. Tong, S.C., Li, Y.M.: Observer-based fuzzy adaptive control for strict-feedback nonlinear systems. Fuzzy Sets Syst. **160**, 1749–1764 (2009)
10. Tong, S.C., Li, C.Y., Li, Y.M.: Fuzzy adaptive observer backstepping control for MIMO nonlinear systems. Fuzzy Sets Syst. **160**, 2755–2775 (2009)
11. Liu, Y.J., Wang, W.: Adaptive fuzzy control for a class of uncertain nonaffine nonlinear systems. Inf. Sci. **177**, 3901–3917 (2007)
12. Liu, Y.J., Tong, S.C., Wang, W.: Adaptive fuzzy output tracking control for a class of uncertain nonlinear systems. Fuzzy Sets Syst. **160**, 2727–2754 (2009)
13. Liu, Y.J., Tong, S.C., Wang, W., Li, Y.M.: Observer-based direct adaptive fuzzy control of uncertain nonlinear systems and its applications. Int. J. Control. Autom. Syst. **7**, 681–690 (2009)
14. Zhou, N., Liu, Y.J., Tong, S.C.: Adaptive fuzzy output feedback control of uncertain nonlinear systems with nonsymmetric dead-zone input. Nonlinear Dyn. **63**, 771–778 (2011)
15. Liu, Y.J., Tong, S.C., Li, T.S.: Observer-based adaptive fuzzy tracking control for a class of uncertain nonlinear MIMO systems. Fuzzy Sets Syst. **164**, 25–44 (2011)
16. Narendra, K.S., Parthasarathy, K.: Identification and control of dynamical systems using neural networks. IEEE Trans. Neural Netw. **1**, 4–27 (1990)
17. Hunt, K.J., Sbarbaro, D., Zbikowski, R., Gawthrop, P.J.: Neural networks for control: a survey. Automatica **28**, 1083–1112 (1992)
18. Liu, Y.J., Chen, C.L.P., Wen, G.X., Tong, S.C.: Adaptive neural output feedback tracking control for a class of uncertain discrete-time nonlinear systems. IEEE Trans. Neural Netw. **22**, 1162–1167 (2011)
19. Liu, Y.J., Tong, S.C., Wang, D., Li, T.S., Chen, C.L.P.: Adaptive neural output feedback controller design with reduced-order observer for a class of uncertain nonlinear siso systems. IEEE Trans. Neural Netw. (2011). doi:10.1109/TNN.2011.2159865
20. Wang, L.X.: Adaptive Fuzzy Systems and Control: Design and Stability Analysis. Prentice-Hall, Englewood Cliffs (1994)
21. Yu, W., Li, X.: Fuzzy identification using fuzzy neural networks with stable learning algorithms. IEEE Trans. Fuzzy Syst. **12**, 411–420 (2004)
22. Jang, J.S.: ANFIS: adaptive-network-based fuzzy inference system. IEEE Trans. Syst. Man Cybern. **23**, 665–685 (1993)
23. Lin, C.T., Lee, C.S.G.: Neural Fuzzy Systems. Prentice-Hall, Englewood Cliffs (1996)
24. Chen, B., Liu, X., Tong, S.: Adaptive fuzzy approach to control unified chaotic systems. Chaos Solitons Fractals **34**, 1180–1187 (2007)
25. Kim, J.H., Hyun, C.H., Kim, E., Park, M.: Adaptive synchronization of uncertain chaotic systems based on T-S fuzzy model. IEEE Trans. Fuzzy Syst. **15**, 359–369 (2007)
26. Wang, X., Meng, J.: Observer-based adaptive fuzzy synchronization for hyperchaotic systems. Chaos **18**, 033102-5 (2008)
27. Roopaei, M., Jahromi, M.Z., Jafari, S.: Adaptive gain fuzzy sliding mode control for the synchronization of nonlinear chaotic gyros. Chaos **19**, 013125-9 (2009)
28. Liu, Y.J., Wang, W., Tong, S.C., Liu, Y.S.: Robust adaptive tracking control for nonlinear systems based on bounds of fuzzy approximation parameters. IEEE Trans. Syst. Man Cybern., Part A, Syst. Hum. **40**, 170–184 (2010)
29. Lin, D., Wang, X.: Observer-based decentralized fuzzy neural sliding mode control for interconnected unknown chaotic systems via network structure adaptation. Fuzzy Sets Syst. **161**, 2066–2080 (2010)
30. Kapoor, N., Teel, A.R., Daoutidis, P.: An anti-windup design for linear systems with input saturation. Automatica **34**, 559–574 (1998)
31. Bemporad, A., Teel, A.R., Zaccarian, L.: Anti-windup synthesis via sampled-data piecewise affine optimal control. Automatica **40**, 549–562 (2004)
32. Fliegner, T., Logemann, H., Ryan, E.P.: Low-gain integral control of continuous time linear systems subject to input and output nonlinearities. Automatica **39**, 455–462 (2003)
33. Grognard, F., Sepulchre, R., Bastin, G.: Improving the performance of low-gain designs for bounded control of linear systems. Automatica **38**, 1777–1782 (2002)
34. Chaoui, F.Z., Giri, F., Msaad, M.: Asymptotic stabilization of linear plants in the presence of input and output saturations. Automatica **37**, 37–42 (2001)
35. Karason, S.P., Annaswamy, A.M.: Adaptive control in the presence of input constraints. IEEE Trans. Autom. Control **39**, 2325–2330 (1994)
36. Nicolao, G.D., Scattolini, R., Sala, G.: An adaptive predictive regulator with input saturations. Automatica **32**, 597–601 (1996)
37. Chaoui, F.Z., Giri, F., Msaad, M.: Adaotive control of input-constrained tyoe-1 plants stabilization and tracking. Automatica **37**, 197–203 (2001)
38. Jagannathan, S., Hameed, M.: Adaptive force-banlancing control of MEMS gyroscope with actuator limits. In: Proceedings of American Control Conference, Boston, Massachusetts, pp. 1862–1867 (2004)
39. Zhou, J., Wen, C.: Adaptive backstepping control of uncertain systems. In: LNCS, vol. 372, pp. 189–197. Springer, Berlin (2008)
40. Krstic, M., Kanellakopoulos, I., Kokotovic, P.V.: Nonlinear and Adaptive Control Design. Wiley, New York (1995)
41. Jamshidi, M., Vadiee, N., Ress, T.J.: Fuzzy Logic and Control. Prentice-Hall, Englewood Cliffs (1993)
42. Leu, Y.G., Lee, T.T., Wang, W.Y.: On-line tuning of fuzzy-neural network for adaptive control of nonlinear dynamical systems. IEEE Trans. Syst. Man Cybern., Part B, Cybern. **27**, 1034–1043 (1997)
43. Lin, C.T., Lee, C.S.G.: Neural-network-based fuzzy logic control and decision system. IEEE Trans. Comput. **40**, 1320–1336 (1991)
44. Chen, Y.C., Teng, C.C.: A model reference control structure using a fuzzy neural network. Fuzzy Sets Syst. **73**, 291–312 (1995)
45. Wang, L.X.: Stable adaptive fuzzy control of nonlinear systems. IEEE Trans. Fuzzy Syst. **1**, 146–155 (1993)