Nonlinear Dyn (2012) 67:2747–2757
DOI 10.1007/s11071-011-0186-0

ORIGINAL PAPER

Adaptive fuzzy formation control for a swarm of nonholonomic differentially driven vehicles

An H^∞-based robust control design

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Received: 20 March 2011 / Accepted: 27 July 2011 / Published online: 7 September 2011

Abstract In this paper, an adaptive fuzzy robust H^∞ controller is proposed for formation control of a swarm of differential driven vehicles with nonholonomic dynamic models. Artificial potential functions are used to design the formation control input for kinematic model of the robots and matrix manipulations are used to transform the nonholonomic model of each differentially driven vehicle into equivalent holonomic one. The main advantage of the proposed controller is the robustness to input nonlinearity, external disturbances, model uncertainties, and measurement noises, in a formation control of a nonholonomic robotic swarm. Moreover, robust stability proof is given using Lyapunov functions. Finally, simulation results are demonstrated for a swarm formation problem of a group of six unicycles, illustrating the effective attenuation of approximation error and external disturbances, even in the case of robot failure.

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Keywords Formation control · Swarm robots · Differentially driven vehicle · Robust adaptive control

1 Introduction

Each Multi-Agent System (MAS) is comprised of a group of intelligent agents, which are capable of solving particular problems (e.g., Disaster Management [1] and Social Structures Modeling [2]) that are not easily solvable by individuals. MAS have many advantages in comparison with individual agents such as flexibility which is a result of decentralized behavior of these systems. In addition, these systems can be a good solution for large scale problems.

Although, MAS emerged as a software concept, it found its way to the practical robotic problems soon. In fact, the early works on robots motion control has considered motion of single robots [3–7]. However, in the recent years, by using multiagent system theory, control of a robotic swarm has been interested by control communities. Some possible applications of a multirobot system include underwater or outer space exploration, factory transportation services, hazardous inspiration, guarding, escorting, and patrolling missions [8–12].

Recently, H^∞ optimal control techniques has been found to be an effective solution to treat robust stabilization and tracking problems, in the presence of external disturbances and system uncertainties [13–18]. In an H^∞ control technique, the main design goal is to force the gain from unmodeled dynamics, external disturbances, and approximation errors to be equal or less than a prescribed disturbance attenuation level (H^∞ attenuation constraint) [13]. This goal is generally represented as a Linear Matrix Inequality (LMI) problem.

In the traditional H^∞ control, the exact model of the system must be known. However, in order to propose a robust control method, an integration between this robust scheme with fuzzy logic approximators can propose effective controllers for uncertain dynamic models [19–21].

In addition, although, control of nonholonomic mobile robots has been a subject of intense research in recent years [22–24], most of the works in this field do not consider dynamic characteristics of mobile robots. In fact, the main idea behind most of control techniques is to define velocity control inputs, considering only the kinematic model of robot. However, describing a mechanical system contains both kinematic and dynamic equations, where most dynamical systems can be considered as a cascade system in which the kinematic and dynamic equations are two separate subsystems which are related by a direct interconnection.

In this research, a matrix conversion is used to translate the nonholonomic model of a differentially driven vehicle to the dynamic model containing the forces and angular torque applied to each robot. Coordinates of a holonomic point on the robot can be the key-idea for this translation. In addition, to satisfy the geometric formation, which is considered as the goal of this article, a simple artificial potential field is defined to guide the mobile robots through this formation. Then, an unknown nonlinear dynamic model is adopted to each 3-DOF mobile robot. Therefore, an adaptive fuzzy approximator is combined with H^∞ control technique to propose a novel decentralized adaptive fuzzy formation control methodology, with robust characteristics. The main advantage of this control strategy is insensitivity to robot dynamic uncertainties, external disturbances and input nonlinearities, where control laws are applicable to nonholonomic robots (e.g., a unicycle).

The rest of this paper is organized as follows: Sect. 2 presents the potential function evaluation. Translation of nonholonomic robot models to dynamic ones is described in Sect. 3. Design of the proposed controller, comparison with an existing method and stability analysis are discussed in Sect. 4. Simulation results are included in Sect. 5, and Sect. 6 provides the concluding remarks.

2 Potential function design

The major goal in this study is to solve a swarm formation control problem (i.e., controlling the relative position of the robots to create a desirable formation). One of the effective solutions for this problem is using an electrostatic-like potential function design which guides the robots through continuous smooth paths and avoids robot collisions. Such a potential function design has been discussed in various papers (e.g., [8, 11, 12, 25]).

Therefore, in this section, we will explain a simple potential function design, in order to solve the formation control of a group of N point massless robots, where the kinematic of the ith robot is considered as

$$\dot{z}_i = u_i \quad i \in \{1, 2, \dots, N\}, \tag{1}$$

in which, $z_i \in \mathbb{R}^2$ is the coordinate matrix (for a robot with 2-DOF) and $u_i \in \mathbb{R}^2$ denotes the control inputs.

However, one of the main shortcomings of this kinematic model is that it does not correspond to the nonholonomic constraints and dynamics of realistic robots. To overcome this shortcoming, more general dynamic models like unicycle models [26] or other wheeled vehicle models can be discussed.

To propose a control law, an artificial potential function is designed. This potential function can be comprised of interrobot interactions, environmental effects (e.g., obstacles, goals, etc.) or other exceptional terms.

Consider the pair-wise potential fields, which are defined between robots as

$$F_{ij} = L_{ij}\big(|z_i - z_j|\big), \quad \forall i, j \in \{1, 2, \dots, N\}, \tag{2}$$

where L_{ij} denotes a proper inter-robot potential function. It is assumed that each robot senses the resultant potential of all other robots.

The overall potential function is proposed to be in the form of

$$F = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} L_{ij}\big(|z_i - z_j|\big) + \sum_{i=1}^{N} Q_i\big(|z_i|\big), \tag{3}$$

where Q_i denotes the global potential of each robot.

Three assumptions for potential function in (3) is considered [25, 27]:

Assumption 1 *F is continuously differentiable.*

Assumption 2 *F is strictly convex.*

Assumption 3 *F is positive definite.*

For example, the following potential function can be chosen for a desired polygonal formation in a 2D space:

$$F = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(|z_i - z_j|^2 - d_{ij}\right)^2 + \sum_{i=1}^{N} \left(|z_i|^2 - c_i\right)^2, \tag{4}$$

where d_{ij} denotes the distance between ith and jth robot and c_i denotes the desirable coordinate of ith robot.

At the first step, to propose a solution for multi-robot formation control, the ith steepest descent direction ([8, 25, 27]) is chosen as

$$f_i = \nabla_{z_i} F \tag{5}$$

where $f_i \in \mathbb{R}^2$ and the control law for the ith robot is chosen as

$$u_i = -f_i. \tag{6}$$

By substituting (6) in (1), the kinematic model is obtained as

$$\dot{z}_i = -f_i = -\nabla_{z_i} F, \quad \forall i \in \{1, 2, \ldots, N\}. \tag{7}$$

In the next section, a transformation procedure will be used to transform the nonholonomic model of differentially driven vehicles to a holonomic model. Then, in Sect. 4, the dynamic model will be used to force satisfaction of (7).

3 Translating nonholonomic model to holonomic one

Consider the model of ith differentially driven robot (Fig. 1a) in polar coordinates as

$$\dot{r}_i = \begin{pmatrix} v_i \cos\theta_i \\ v_i \sin\theta_i \end{pmatrix} \quad \dot{\theta}_i = w_i \tag{8}$$

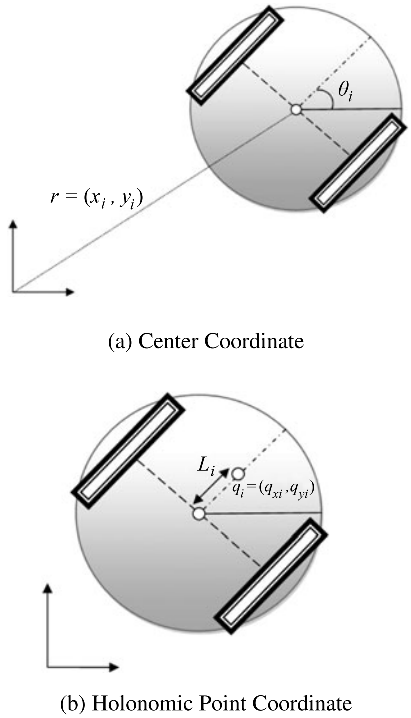


Fig. 1 Differentially driven vehicle

where the overdot denotes differential with respect to time. $r_i = (x_i, y_i)^{\mathrm{T}}$ is the coordinate of ith robot and θ_i denotes the robot direction. v_i and w_i represent the linear and angular velocities. In order to include the dynamic model of unicycle, equations

$$\dot{v}_i = \frac{1}{m_i} F_i, \qquad \dot{w}_i = \frac{1}{J_i} \tau_i \tag{9}$$

should be added to (8), where m_i is the ith vehicle mass, J_i is the ith vehicle moment of inertia, and F_i and τ_i are the force and angular torque applied to the ith unicycle, respectively.

As shown in Fig. 1b, based on [28, 29], we can define a holonomic point (q_i) in distance L_i perpendicular to the axis which connects center of wheels as

$$q_i = r_i + L_i \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix} \tag{10}$$

and

$$\dot{q}_i = \dot{r}_i + L_i \frac{\partial}{\partial t} \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta_i) & -L_i \sin(\theta_i) \\ \sin(\theta_i) & L_i \cos(\theta_i) \end{pmatrix} \begin{pmatrix} v_i \\ \omega_i \end{pmatrix}. \tag{11}$$

After simply multiplying the right-hand side of (11), differentiating this equation with respect to time, and using (9) and (8) we will have

$$\ddot{q}_i = \begin{pmatrix} -v_i\omega_i \sin(\theta_i) - L_i\omega_i^2 \cos(\theta_i) \\ v_i\omega_i \cos(\theta_i) - L_i{\omega_i}^2 \sin(\theta_i) \end{pmatrix} + \begin{pmatrix} \frac{1}{m_i}\cos(\theta_i) & -\frac{L_i}{J_i}\sin(\theta_i) \\ \frac{1}{m_i}\sin(\theta_i) & -\frac{L_i}{J_i}\cos(\theta_i) \end{pmatrix} \begin{pmatrix} F_i \\ \tau_i \end{pmatrix}. \tag{12}$$

Matrix Γ_i is defined as

$$\Gamma_i = \begin{pmatrix} \frac{1}{m_i}\cos(\theta_i) & -\frac{L_i}{J_i}\sin(\theta_i) \\ \frac{1}{m_i}\sin(\theta_i) & -\frac{L_i}{J_i}\cos(\theta_i) \end{pmatrix}, \tag{13}$$

then its determinant can be easily calculated as

$$\det \Gamma_i \neq 0.$$

Therefore, matrix Γ_i is nonsingular and invertible.

By multiplying Γ_i^{-1} to both sides of (12), we can write

$$M_i(\theta_i)\ddot{q}_i + g_i(v_i, \omega_i, \theta_i) = u_i \tag{14}$$

where

$$M_i(\theta_i) = \Gamma_i^{-1}, \tag{15}$$

$$g_i(v_i, \omega_i, \theta_i) = -\Gamma_i^{-1} \begin{pmatrix} -v_i\omega_i \sin(\theta_i) - L_i\omega_i^2 \cos(\theta_i) \\ v_i\omega_i \cos(\theta_i) - L_i{\omega_i}^2 \sin(\theta_i) \end{pmatrix} \tag{16}$$

and

$$u_i = \begin{pmatrix} F_i \\ \tau_i \end{pmatrix}. \tag{17}$$

To find a direct relation between holonomic point and mobile robot center, based on (10) and (11), mobile robot position (r_i), linear velocity (v_i) and angular velocity (ω_i) can be described as

$$\begin{pmatrix} r_{xi} \\ r_{yi} \\ v_i \\ w_i \end{pmatrix} = \begin{pmatrix} q_{xi} - L_i \cos(\theta_i) \\ q_{yi} - L_i \sin(\theta_i) \\ \dot{q}_{xi} \cos(\theta_i) + \dot{q}_{yi} \sin(\theta_i) \\ -\frac{1}{L_i}\dot{q}_{xi} \cos(\theta_i) + \frac{1}{L_i}\dot{q}_{yi} \cos(\theta_i) \end{pmatrix}. \tag{18}$$

Therefore, (14) can be rewritten as

$$M_i(\theta_i)\ddot{q}_i + g_i(q_i, \dot{q}_i, \theta_i) = u_i. \tag{19}$$

Equation (19) is a holonomic dynamic model which needs the exact position of holonomic point and its velocity to be known. However, in practice just measuring the coordinate and velocity of robot center is applicable. Therefore, let us define the coordinate and velocity measurement error as Δz_i and $\Delta \dot{z}_i$, respectively. Now we can consider $z_i = q_i + \Delta z_i$ and $\dot{z}_i = \dot{q}_i + \Delta z_i$, then it is straight forward to write (19) as

$$M_i(\theta_i)(\ddot{z}_i - \Delta\ddot{z}_i) + g_i(z_i - \Delta z_i, \dot{z}_i - \Delta\dot{z}_i, \theta_i) = u_i \tag{20}$$

and if we define δ_{ai} as the overall additive uncertainty, then the simple form of (20) is

$$M_i(\theta_i)\ddot{z}_i + g_i(z_i, \dot{z}_i, \theta_i) = u_i + \delta_{ai}, \tag{21}$$

It can be seen that (21) is similar to the general dynamic model of holonomic robots $M(z)\ddot{z} + C(z, \dot{z})\dot{z} + g(z) = u + d$ introduced in [30]. Therefore, the robust control method proposed in following section is designed based on the previous work of authors on swarm formation control of multi-robot systems in [31].

4 Problem formulation and control design

In this section, a novel formation error based on the integral of formation gradient (5) will be proposed and a robust H^∞ controller will be designed. In addition, fuzzy logic system will be utilized to approximate the unknown parts of dynamic models.

Consider a group of N fully autonomous nonholonomic dynamically driven mobile robots. The dynamics of the ith simple mobile robot is nonlinear and can be written by using (21) in the general form as

$$M_i(\theta_i)\ddot{z}_i + g_i(z_i, \dot{z}_i, \theta_i) = u_i + d_i \tag{22}$$

where $z_i \in \mathbb{R}^2$ is the coordinate matrix (for the holonomic point of robot with 2-DOF), $u_i \in \mathbb{R}^2$ is the control input and $d_i \in \mathbb{R}^2$ include the measurement errors (δ_{ai}) and the external disturbances.

Using the definition of M_i in (15), as this matrix is nonsingular, its inverse can be used to write

$$\ddot{z}_i = -G_i(\theta_i)g_i(z_i,\dot{z}_i,\theta_i) + G_i(\theta_i)u_i + G_i(\theta_i)d_i, \tag{23}$$

where $G_i(\theta_i) = M_i^{-1}(\theta_i)$.

Consider, the kinematic formation error for the ith robot as

$$\underline{e}_i(t) = z_i(t) + \int_0^t f_i(\tau)\,d\tau, \tag{24}$$

where $\underline{e}_i \in \mathbb{R}^2$ and f_i is the gradient of potential function defined in (5). It is straightforward to write the first and second derivatives of (24) as

$$\dot{\underline{e}}_i(t) = \dot{z}_i(t) + f_i, \tag{25}$$

$$\ddot{\underline{e}}_i(t) = \ddot{z}_i(t) + \dot{f}_i. \tag{26}$$

Our design goal is to propose an adaptive fuzzy controller so that

$$\ddot{\underline{e}}_i + k_1\dot{\underline{e}}_i + k_2\underline{e}_i = 0 \tag{27}$$

is achieved, where k_1 and k_2 are chosen to make (27) asymptotically stable.

To design the controller, consider the control law proposed as

$$u_i = G_i^{-1}(\theta_i)\big(H_i(z_i,\dot{z}_i,\theta_i) - \dot{f}_i - k_1\dot{\underline{e}}_i - k_2\underline{e}_i\big), \tag{28}$$

where $H_i(z_i,\dot{z}_i,\theta_i) = G_i(\theta_i)g_i(z_i,\dot{z}_i,\theta_i)$.

In order to use the control law (28), which is designed based on the feedback linearization control method, the functions $H_i(.)$ and $G_i(.)$ must be known and no disturbance is allowed (i.e., $d_i = 0$). However, in practice these matrices may be unknown for most of real dynamical robots, external disturbances, and measurement noises emerge easily. To overcome this, we make use of two adaptive fuzzy logic systems $\hat{H}_i(.)$ and $\hat{G}_i(.)$ to approximate $H_i(.)$ and $G_i(.)$, respectively.

4.1 Adaptive fuzzy approximator design

By designing two simple fuzzy logic systems as fuzzy models and using the singleton fuzzifier, product inference, and weighted average defuzzifier [32], the output of our fuzzy models can be expressed as

$$\hat{H}_i(z_i,\dot{z}_i,\theta_i \mid \underline{\phi}_{Hi}) = \underline{\zeta}_{Hi}^{\mathrm{T}}(z_i,\dot{z}_i,\theta_i)\underline{\phi}_{Hi}, \tag{29}$$

$$\hat{G}_i(\theta_i \mid \underline{\phi}_{Gi}) = \underline{\zeta}_{Gi}^{\mathrm{T}}(\theta_i)\underline{\phi}_{Gi}. \tag{30}$$

Therefore, we can rewrite the overall control law (28) as

$$u_i = \hat{G}^{-1}(\theta_i \mid \underline{\phi}_{Gi})\big[\hat{H}_i(z_i,\dot{z}_i,\theta_i \mid \underline{\phi}_{Hi}) - \dot{f}_i - k_1\dot{\underline{e}}_i - k_2\underline{e}_i + u_{ai}\big] \tag{31}$$

where u_{ai} is engaged to attenuate the fuzzy logic approximation error, external disturbances and measurement noises.

To derive the adaptive law for adjusting $\underline{\phi}_{Hi}$ and $\underline{\phi}_{Mi}$, we first define the optimal parameter vectors $\underline{\phi}_{Hi}^*$ and $\underline{\phi}_{Mi}^*$ as

$$\underline{\phi}_{Hi}^* = \arg\min_{\underline{\phi}_{Hi}\in\Omega_H}\big[\sup\big\|\hat{H}_i(z_i,\dot{z}_i,\underline{\theta}_i \mid \underline{\phi}_{Hi}) - H_i(z_i,\dot{z}_i,\underline{\theta}_{Hi})\big\|\big] \tag{32}$$

and

$$\underline{\phi}_{Gi}^* = \arg\min_{\underline{\phi}_{Gi}\in\Omega_G}\big[\sup\big\|\hat{G}_i(\underline{\theta}_i \mid \underline{\phi}_{Gi}) - G_i(\underline{\theta}_{Gi})\big\|\big] \tag{33}$$

where Ω_H and Ω_G are proper compact sets defined as

$$\Omega_H = \big\{\underline{\phi}_{Hi} \in \mathbb{R}^n \mid \|\underline{\phi}_{Hi}\| \le D_H\big\}$$

and

$$\Omega_G = \big\{\underline{\phi}_{Gi} \in \mathbb{R}^n \mid \|\underline{\phi}_{Gi}\| \le D_G\big\}.$$

The minimum approximation error is defined as

$$\underline{w}_i = \big(\hat{H}_i\big(z_i,\dot{z}_i,\underline{\theta}_i \mid \underline{\phi}_{Hi}^\star\big) - H_i(z_i,\dot{z}_i,\underline{\theta}_i)\big) + \big(G_i(\underline{\theta}_i) - \hat{G}_i\big(\underline{\theta}_i \mid \underline{\phi}_{Gi}^\star\big)\big)u_i \tag{34}$$

where it can be assumed that $\underline{w}_i \in L_\infty$ [32].

By choosing the control input as (31) after some manipulations, (23) can be rewritten as

$$\ddot{z}_i = (\hat{H}_i - H_i) + (G_i - \hat{G}_i)u_i - \dot{f}_i - k_1\dot{\underline{e}}_i - k_2\underline{e}_i + \underline{u}_{ai} + G_i d_i \tag{35}$$

and the formation error dynamic can be expressed as

$$\ddot{\underline{e}}_i = (\hat{H}_i - H_i) + (G_i - \hat{G}_i)u_i - k_1\dot{\underline{e}}_i - k_2\underline{e}_i + \underline{u}_{ai} + G_i d_i. \tag{36}$$

Moreover, by defining $\underline{E}_i = [e_{1i}, \dot{e}_{1i}, e_{2i}, \dot{e}_{2i}]$, it is straightforward to write

$$\dot{\underline{E}}_i = A\underline{E}_i + B\underline{u}_{ai} + B\big[(\hat{H}_i - H_i) + (G_i - \hat{G}_i)u_i\big] + BG_i d_i \tag{37}$$

where

$$A = I_{2\times 2} \otimes \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}_{2\times 2},$$

$$B = I_{2\times 2} \otimes [0\ 1]^{\mathrm{T}}$$

in which $\otimes$ denotes the Kronecker matrix product. Based on (29), (30), and (34), the matrix form of formation error in (37) can be rewritten as

$$\begin{aligned}\dot{\underline{E}}_i &= A\underline{E}_i + B\underline{u}_{ai} \\ &\quad + B\big(\underline{\zeta}^{\mathrm{T}}_{Hi}(z_i, \dot{z}_i, \theta_i)\tilde{\underline{\phi}}_{Hi} - \underline{\zeta}^{\mathrm{T}}_{Gi}(\theta_i)\tilde{\underline{\phi}}_{Gi} u_i\big) \\ &\quad + B\underline{w}_i + BG_i d_i \\ &= A\underline{E}_i + B\underline{u}_{ai} \\ &\quad + B\big(\underline{\zeta}^{\mathrm{T}}_{Hi}(z_i, \dot{z}_i, \theta_i)\tilde{\underline{\phi}}_{Hi} - \underline{\zeta}^{\mathrm{T}}_{Gi}(\theta_i)\tilde{\underline{\phi}}_{Gi} u_i\big) \\ &\quad + B\underline{w}'_i \end{aligned} \tag{38}$$

where

$$\tilde{\underline{\phi}}_{Hi} = \underline{\phi}_{Hi} - \underline{\phi}^{\star}_{Hi}, \tag{39}$$

$$\tilde{\underline{\phi}}_{Gi} = \underline{\phi}_{Gi} - \underline{\phi}^{\star}_{Gi}, \tag{40}$$

$$\underline{w}'_i = B\underline{w}_i + BG_i d_i. \tag{41}$$

In the following theorem, it will be shown that the proposed control law (31) guarantees the robust stability of formation problem.

Theorem 1 *Consider a group of N fully autonomous nonholonomic differentially driven vehicles with the dynamic represented in (23), and with the control law in (31). The robust compensator of ith robot $\underline{u}_{ai}$ and the fuzzy adaptation laws are chosen as*

$$\underline{u}_{ai} = -\frac{1}{r} B^{\mathrm{T}} P \underline{E}_i, \tag{42}$$

$$\dot{\underline{\phi}}_H = -\gamma_1 \underline{\zeta}_{Hi}(z_i, \dot{z}_i, \theta_i) B^{\mathrm{T}} P E_i, \tag{43}$$

$$\dot{\underline{\phi}}_G = +\gamma_2 \underline{\zeta}_{Gi}(\theta_i) B^{\mathrm{T}} P E_i u_i^{\mathrm{T}}, \tag{44}$$

where r, γ_1 and γ_2 are positive constants and P is the positive semidefinite solution of following Riccati-like equation:

$$PA + A^{\mathrm{T}}P + Q - \frac{2}{r}PBB^{\mathrm{T}}P + \frac{1}{\rho^2}PBB^{\mathrm{T}}P = 0 \tag{45}$$

in which, Q is a positive semidefinite matrix and $2\rho^2 \geq r$.

Therefore, the H^∞ criterion

$$\begin{aligned}&\sum_{i=1}^{N}\bigg[\int_0^{\mathrm{T}} \underline{E}_i^{\mathrm{T}} Q \underline{E}_i\, dt\bigg] \\ &\quad \leq \sum_{i=1}^{N}\bigg[\underline{E}_i(0)^{\mathrm{T}} P \underline{E}_i(0) + \frac{1}{\gamma_1}\tilde{\underline{\phi}}^{\mathrm{T}}_{Hi}(0)\tilde{\underline{\phi}}_{Hi}(0) \\ &\qquad + \frac{1}{\gamma_2}\operatorname{tr}\big(\tilde{\underline{\phi}}^{\mathrm{T}}_{Gi}(0)\tilde{\underline{\phi}}_{Gi}(0)\operatorname{tr}\big)\bigg] \\ &\qquad + \rho^2 \sum_{i=1}^{N}\bigg[\int_0^{\mathrm{T}} \underline{w}'^{\mathrm{T}}_i \underline{w}'_i\, dt\bigg]\end{aligned} \tag{46}$$

can be achieved for a prescribed attenuation level ρ and all the variables of closed loop system are bounded.

4.2 Comparison with the existing methods

In recent years, some methods based on potential fields are integrated with some nonlinear control schemes such as feedback linearization method and Sliding Mode Control (SMC), which concludes in formation control design of dynamic robots [25, 27, 33, 34]. Cortes et al. [33] suggested the use of decentralized coverage algorithms as formation control algorithms, and they presented various density functions that lead the multivehicle network to predetermined geometric patterns. In particular, they presented simple density functions that lead to segments, ellipses, polygons, or uniform distributions inside convex environments. Cheaha et al. [27] presented a region-based shape controller for a swarm of fully actuated robots, where a linear approximator was used to approximate the unknown dynamic model and an SMC controller integrated with artificial potential functions was used to satisfy a predetermined geometric 2D formation. Ranjbarsahraei et al. [34] proposed an adaptive control scheme for multi-robot formation control. Their control method was based on artificial potential functions integrated with adaptive fuzzy SMC technique. They

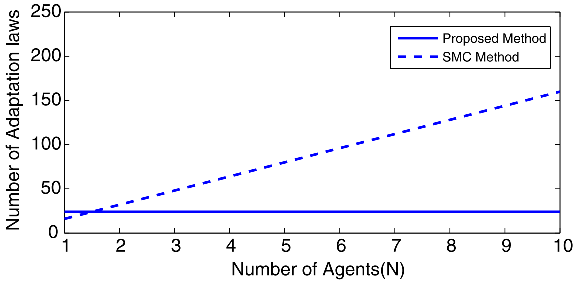


Fig. 2 Computational complexity comparison of SMC-based method [34] with the proposed control method

considered fully actuated mobile robots with completely unknown dynamics. An adaptive fuzzy logic system was used to approximate the unknown system dynamics.

To compare the computational complexity of our proposed method with the method proposed in [34], consider N nonholonomic robots, each one with 3-DOF. In the proposed method, each robot fuzzy approximator uses the position and velocity of itself. However in [34] when two membership functions are defined for each variable (i.e., z_x, z_y, $\dot{z}_x$, and $\dot{z}_y$) of each robot, then a collection of $8N$ adaptation rules are needed. Figure 2 illustrates the computational complexity comparison for different swarm populations (N).

From Fig. 2, it is completely clear that the computational complexity of proposed method is independent of swarm population N, while in the existing method [34] (and many similar methods such as [27]). This computation complexity increases with the swarm population (or at least number of neighboring robots), which is undesirable and impractical.

4.3 Stability proof

In order to prove the multirobot robust stability, a Lyapunov candidate is chosen as

$$V=\sum_{i=1}^{N}\bigg[\frac{1}{2}\underline{E}_i^{\mathrm{T}}P\underline{E}_i+\frac{1}{2\gamma_1}\tilde{\underline{\phi}}_{Hi}^{\mathrm{T}}\tilde{\underline{\phi}}_{Hi} + \frac{1}{2\gamma_1}\operatorname{tr}\big(\tilde{\underline{\phi}}_{Gi}^{\mathrm{T}}\tilde{\underline{\phi}}_{Gi}\big)\bigg]. \tag{47}$$

Based on (38) and (42), and using the fact that $\dot{\tilde{\underline{\phi}}}_{Hi}=\dot{\underline{\phi}}_{Hi}$ and $\dot{\tilde{\underline{\phi}}}_{Gi}=\dot{\underline{\phi}}_{Gi}$, the time derivative of V is

$$\begin{aligned}
\dot{V}&=\frac{1}{2}\sum_{i=1}^{N}\bigg[\dot{\underline{E}}_i^{\mathrm{T}}P\underline{E}_i+\underline{E}_i^{\mathrm{T}}P\dot{\underline{E}}_i+\frac{1}{\gamma_1}\dot{\tilde{\underline{\phi}}}_{Hi}^{\mathrm{T}}\tilde{\underline{\phi}}_{Hi} + \frac{1}{\gamma_2}\operatorname{tr}\big(\dot{\tilde{\underline{\phi}}}_{Gi}^{\mathrm{T}}\tilde{\underline{\phi}}_{Gi}\big)\bigg]\\
&=\frac{1}{2}\sum_{i=1}^{N}\bigg[\underline{E}_i^{\mathrm{T}}\Big(A^{\mathrm{T}}P+PA-\frac{2}{r}\underline{P}BB^{\mathrm{T}}P\Big)\underline{E}_i\bigg]\\
&\quad+\frac{1}{2}\sum_{i=1}^{N}\bigg[\Big(\underline{E}_i^{\mathrm{T}}PB\underline{\zeta}_{Hi}^{\mathrm{T}}(z_i,\dot{z}_i.\theta_i)+\frac{1}{\gamma_1}\dot{\underline{\phi}}_{Hi}^{\mathrm{T}}\Big)\tilde{\underline{\phi}}_{Hi}\bigg]\\
&\quad-\frac{1}{2}\sum_{i=1}^{N}\bigg[\Big(\underline{E}_i^{\mathrm{T}}PB\underline{\zeta}_{Gi}^{\mathrm{T}}(\theta_i)\tilde{\underline{\phi}}_{Gi}u_i - \frac{1}{\gamma_2}\operatorname{tr}\big(\dot{\underline{\phi}}_{Gi}^{\mathrm{T}}\tilde{\underline{\phi}}_{Gi}\big)\Big)\bigg]\\
&\quad+\frac{1}{2}\sum_{i=1}^{N}\big[\underline{w}_i'^{\mathrm{T}}B^{\mathrm{T}}P\underline{E}_i+\underline{E}_i^{\mathrm{T}}PB\underline{w}'_i\big].
\end{aligned}\tag{48}$$

Using adaptation law (43) and (44), and the Riccati-like equation (45), the above equation becomes

$$\begin{aligned}
\dot{V}&=\frac{1}{2}\sum_{i=1}^{N}\bigg[-\underline{E}_i^{\mathrm{T}}Q\underline{E}_i-\frac{1}{\rho^2}\underline{E}_i^{\mathrm{T}}PBB^{\mathrm{T}}P\underline{E}_i\bigg]\\
&\quad+\frac{1}{2}\sum_{i=1}^{N}\big[\underline{w}_i'^{\mathrm{T}}B^{\mathrm{T}}P\underline{E}_i+\underline{E}_i^{\mathrm{T}}PB\underline{w}_i'\big]\\
&\le\frac{1}{2}\sum_{i=1}^{N}\big[-\underline{E}_i^{\mathrm{T}}Q\underline{E}_i+\rho^2\underline{w}_i'^{\mathrm{T}}\underline{w}_i'\big].
\end{aligned}\tag{49}$$

Integrating the above inequality from $t=0$ to T yields to

$$V(T)-V(0)\le\frac{1}{2}\sum_{i=1}^{N}\bigg[-\int_0^{\mathrm{T}}\underline{E}_i^{\mathrm{T}}Q\underline{E}_i\,dt + \rho^2\int_0^{\mathrm{T}}\underline{w}_i'^{\mathrm{T}}\underline{w}_i'\,dt\bigg]. \tag{50}$$

Using the fact that $V(T)\ge 0$ and from (47), the H^∞ criterion (46) can be achieved and the proof is completed.

Table 1 Parameter specifications of hexagonal formation

	$\|i-j\|=1$	$\|i-j\|=2$	$\|i-j\|=3$	$\|i-j\|=4$	$\|i-j\|=5$
d_{ij}	1.0	1.7	2.0	1.7	1.0

5 Simulation results

This section presents three simulation examples to illustrate the effectiveness of the proposed control scheme. In the first example, a group of six unicycles with known dynamics is considered. The second example presents the hexagonal formation of six unknown unicycles and an adaptive fuzzy logic system is used to approximate the unknown dynamics. This example shows the system stability under the proposed novel controller. In order to prove the controller robustness, in the third example a white Gaussian noise is applied to all measured data and one of the robots is forced to be stationary and still the formation maintains its stabilizing performance. All the simulation results are implemented in MATLAB with 0.01 s as the step-size.

The unique formation problem used in all three simulation examples, is a 2D hexagon with unit radius defined by

$$F=\sum_{i=1}^{5}\sum_{j=i+1}^{6}\left(|z_i-z_j|^2-d_{ij}\right)^2, \tag{51}$$

where d_{ij} is specified in Table 1.

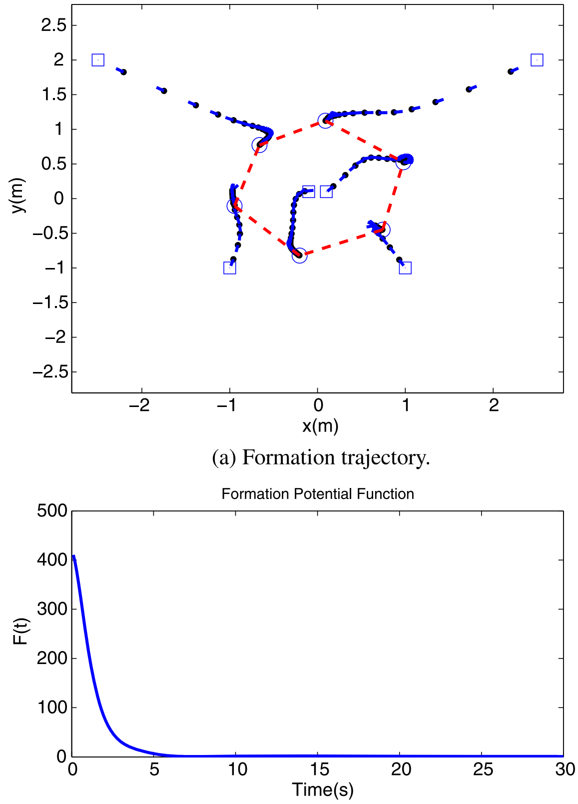
In addition, six random points in the 2D space are chosen to be the initial positions for six unicycles with random initial orientations. These points are assumed to be fixed in all three numerical simulations.

5.1 Example I. Six unicycles with known dynamics

Consider a group of six mobile unicycles with dynamic models. The nonlinear dynamic of the ith robot is considered as the model in (8)–(9), while $m_i=0.2$ and $J_i=1$.

To give a solution for the formation problem (51), formation error is defined as (24) and the control law is designed based on (28), where $k_1=15$ and $k_2=4$.

Figure 3a shows the formation trajectory of these six unicycles starting from initial coordinates to the final unit hexagon (51) in 30 s and Fig. 3b shows the potential function decrement defined in (51).



(a) Formation trajectory.

(b) Formation potential.

Fig. 3 Hexagonal formation of six unicycles with known dynamics

The first subfigure (Fig. 3a) shows how smooth the controller guides all the unicycles to form the desired hexagon. This geometric formation doesn't have any fixed position or direction, and it will only be determined by the unicycles initial position.

The second subfigure (Fig. 3b) illustrates the potential decrement through the time. It is shown that the potential is forced to get stabilized in less than 20 s.

5.2 Example II. Six unicycles with unknown dynamics

To verify the effectiveness of proposed method the same formation potential and the novel formation error are chosen as (51) and (24); respectively. Consider a group of six unicycles with the same dynamic models in (8)–(9), while $m_i=0.2$ and $J_i=1$. However, to design the control law, the dynamic model of unicycles is assumed to be unknown (i.e., m_i and J_i for $i=1,2,\dots,6$ are unknown).

Table 2 Gaussian membership functions

$\mu_{F_1^1}(z_x) = \frac{1}{1+\exp(3(z_x+0.5))}$	$\mu_{F_1^3}(z_y) = \frac{1}{1+\exp(3(z_y+0.5))}$
$\mu_{F_1^2}(z_x) = \frac{1}{1+\exp(-3(z_x-0.5))}$	$\mu_{F_1^4}(z_y) = \frac{1}{1+\exp(-3(z_y-0.5))}$
$\mu_{F_2^1}(\dot{z}_x) = \frac{1}{1+\exp(30(\dot{z}_x+0.15))}$	$\mu_{F_2^4}(\dot{z}_y) = \frac{1}{1+\exp(30(\dot{z}_y+0.15))}$
$\mu_{F_2^2}(\dot{z}_x) = \exp(-30 \times \dot{z}_x^2)$	$\mu_{F_2^5}(\dot{z}_y) = \exp(-30 \times \dot{z}_y^2)$
$\mu_{F_2^3}(\dot{z}_x) = \frac{1}{1+\exp(-30(\dot{z}_x-0.15))}$	$\mu_{F_2^6}(\dot{z}_y) = \frac{1}{1+\exp(-30(\dot{z}_y-0.15))}$
$\mu_{F_3^1}(\theta) = \frac{1}{1+\exp(6(\theta+0.6\pi))}$	$\mu_{F_3^3}(\theta) = \frac{1}{1+\exp(-6(\theta-0.6\pi))}$
$\mu_{F_3^2}(\theta) = \exp(-0.9 \times \theta^2)$	

Table 3 Fuzzy rule-base

a. Rule-base for approximation of $\hat{H}(z, \dot{z}, \theta)$:

R_l: IF z_x is F_1^i AND z_y is F_1^j AND θ is F_3^k THEN y is $\mathbb{G}_1^{ij}$,
$i = 1, 2$ $j = 3, 4$ $k = 1, 2, 3$

R_l: IF $\dot{z}_x$ is F_2^i AND $\dot{z}_y$ is F_2^j THEN y is $\mathbb{G}_2^{ij}$,
$i = 1, 2, 3$ $j = 4, 5, 6$

b. Rule-base for approximation of $\hat{G}(\theta)$:

R_l : IF θ is F_3^k THEN y is $\mathbb{G}_3^{ij}$,
$k = 1, 2, 3$

Therefore, twelve fuzzy logic approximators (two for each robot) are designed to approximate the unknown dynamic, where each robot approximator just needs the current position and velocity of itself. Gaussian membership functions are listed in Table 2.

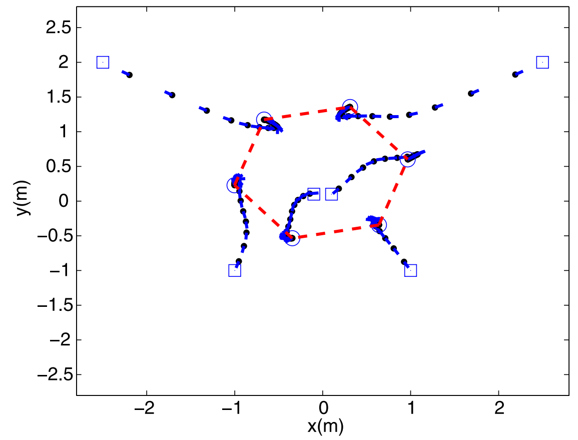
Using the aforementioned 13 membership functions, 2 fuzzy rule-bases are designed as shown in Table 3, where z_x and z_y are the position coordinate and θ is the robot angle of an individual robot and y is the output of each rule.

The output of the fuzzy system is achieved by choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base, and product inference engine [32] as

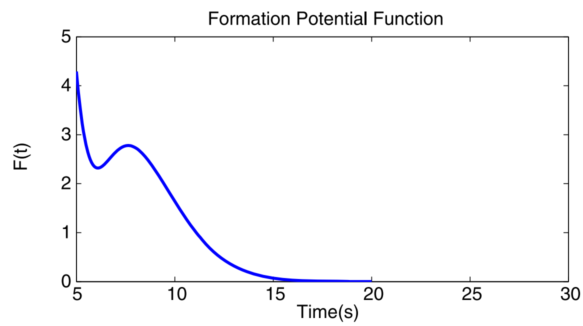
$$\hat{H}(z_x, z_y, \dot{z}_x, \dot{z}_y, \theta \mid \phi_H) = \begin{bmatrix} \underline{\phi}_{H1}^{\mathrm{T}} \underline{\zeta}_H(z_x, z_y, \dot{z}_x, \dot{z}_y, \theta) \\ \underline{\phi}_{H2}^{\mathrm{T}} \underline{\zeta}_H(z_x, z_y, \dot{z}_x, \dot{z}_y, \theta) \end{bmatrix}$$

and

$$\hat{G}(\theta \mid \phi_G) = \begin{bmatrix} \underline{\phi}_{G11}^{\mathrm{T}} \underline{\zeta}_G(\theta) & \underline{\phi}_{G12}^{\mathrm{T}} \underline{\zeta}_G(\theta) \\ \underline{\phi}_{G21}^{\mathrm{T}} \underline{\zeta}_G(\theta) & \underline{\phi}_{G22}^{\mathrm{T}} \underline{\zeta}_G(\theta) \end{bmatrix}$$



(a) Formation trajectory.



(b) Formation potential.

Fig. 4 Hexagonal formation of six unicycles with partially unknown dynamics

where

$$\underline{\phi}_{H1} = [\phi_{H1_1}, \phi_{H1_2}, \ldots, \phi_{H1_{21}}]^{\mathrm{T}},$$

$$\underline{\phi}_{H2} = [\phi_{H2_1}, \phi_{H2_2}, \ldots, \phi_{H2_{21}}]^{\mathrm{T}}$$

and

$$\underline{\phi}_{G11} = [\phi_{G11_1}, \phi_{G11_2}, \phi_{G11_3}]^{\mathrm{T}},$$

$$\underline{\phi}_{G12} = [\phi_{G12_1}, \phi_{G12_2}, \phi_{G12_3}]^{\mathrm{T}},$$

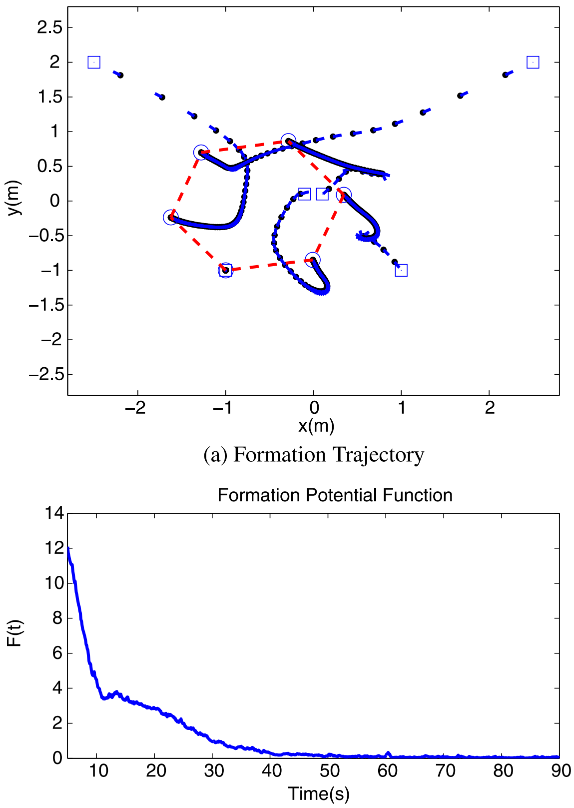
$$\underline{\phi}_{G21} = [\phi_{G21_1}, \phi_{G21_2}, \phi_{G21_3}]^{\mathrm{T}},$$

$$\underline{\phi}_{G22} = [\phi_{G22_1}, \phi_{G22_2}, \phi_{G22_3}]^{\mathrm{T}}$$

are adjustable parameters. $\underline{\zeta}_H(z_x, z_y, \dot{z}_x, \dot{z}_y, \theta)$ and $\zeta_G(\theta)$ are the set of fuzzy basis functions.

All $\underline{\phi}$s are initialized from zero vectors and the learning rate in (43), (44) are set to $\gamma_1 = \gamma_2 = 15$.

Simulation results of the proposed adaptive fuzzy H^∞ technique are shown in Figs. 4a, b. The motion trajectory in the first 30 s is illustrated in Fig. 4a and



(a) Formation Trajectory

(b) Formation Potential

Fig. 5 Hexagonal formation in presence of 20 dB noise and one unicycle failure

the formation potential (51) is shown to be stabilized in Fig. 4b.

5.3 Example III. Formation problem in presence of measurement noise and robot failure

In this example, the robustness of proposed controller in presence of measurement noise and robot failure, will be proved. The proposed potential function (3) and gradient based method proposed in Sect. 2 are able to obtain the exact formation even in the case of one unicycle failure. Therefore, in this example it will be shown that when unicycle #3 ($x_3(0) = -1$, $y_3(0) = +1$) is forced to be stationary with zero velocity, other unicycles move toward this unicycle to achieve the hexagon formation. In addition, a white Gaussian noise with SNR $= 20$ dB is applied to all the measured data. All of the model characteristics and controller designs are the same as previous example in Sect. 5.2. Motion trajectory and formation potential (51) of the first 90 s of simulation are shown in Figs. 5a, b respectively.

6 Conclusion

In this paper, the formation control problem of a class of multirobot systems with unknown nonholonomic dynamics was investigated. On the basis of the Lyapunov stability theory, a novel decentralized adaptive fuzzy controller with corresponding parameter update law was developed and the stability of the system was proved even in the case of external disturbances and input nonlinearities. All the theoretical results were verified by simulation examples and good performance of the proposed controller was shown even in the case of robot failure and presence of measurement noise.

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