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Direct adaptive neural control for strict-feedback stochastic nonlinear systems

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Abstract This note considers the problem of direct adaptive neural control for a class of nonlinear single-input/single-output (SISO) strict-feedback stochastic systems. The variable separation technique is introduced to decompose the coefficient functions of the diffusion term. Radical basis function (RBF) neural networks are used to approximate unknown and desired control signals, then a novel direct adaptive neural controller is constructed via backstepping. The proposed adaptive neural controller guarantees that all the signals in the closed-loop system remain bounded in probability. A main advantage of the proposed controller is that it contains only one adaptive parameter needed to be updated online. Simulation results demonstrate the effectiveness of the proposed approach.

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1 Introduction

During the past decades, the research on stability analysis and control synthesis of nonlinear systems with uncertainties has obtained much progress and many remarkable results have been reported in [1–12]. In addition, stochastic disturbance often exist in practical systems and is usually a source of instability of control systems. Therefore, investigation on stability analysis and control design for stochastic nonlinear systems is a meaningful issue and has attracted increasing attention in the control community in recent years [13–25], Pan and Basar [14] were the first to solve the stabilization problem for a class of stochastic nonlinear strict-feedback systems based on a risk-sensitive cost criterion, and their result guarantees global asymptotic stability in the sense of probability. By employing the quartic Lyapunov function, Deng and Krstić [15–18] proposed a backstepping design for stochastic strict-feedback systems and then the results were extended to inverse optimal control of the stochastic case. Furthermore, this design idea was generalized to several different cases, such as tracking control [19] and decentralized control [22, 23]. Then it is noticed that fewer results have been obtained for the control problem of stochastic systems with unknown nonlinear functions [24, 25]. A common weakness of these control methods developed in [24, 25] is that the number of adaptation laws depends on the number of the neural network nodes or the number of the fuzzy rule bases. With an increase of neural network nodes or fuzzy rules to improve approximation accuracy, the number of parameters to be estimated will increase significantly. As a result, the on-line learning time will become prohibitively large. To solve this problem, Yang et al. [4, 5] considered the norm of the ideal weighting vector in fuzzy logic systems as the estimation parameter instead of the elements of weighting vector. Therefore, the number of adaptation laws is reduced considerably. Inspired by [4, 5], Chen et al. [7] developed a new direct adaptive fuzzy control method for a class of nonlinear system. A main advantage of this method is that only one parameter is needed to be estimated online regardless of the number of fuzzy rule bases used and the order of systems. But how to generalize this method of direct adaptive control to the stochastic case is a challenging and meaningful issue.

Motivated by the aforementioned discussion, we will consider the problem of direct adaptive neural control for a class of strict-feedback stochastic nonlinear systems. The main contributions of this paper are given as follows. A novel direct adaptive neural controller is proposed to control a class of stochastic systems with completely unknown nonlinear functions. To develop a novel control design procedure, the separation technique is used to decompose unknown functions in the diffusion terms into a series of the product of continuous functions and state variables. The upper bound of the norm of weight vector rather than the weight vector elements themselves is used as the estimated parameter. In this way, the presented adaptive law contains only one adaptive parameter. Therefore, the computational burden is significantly alleviated and the control scheme is more implemented in practical applications. As shown later, all the signals in the closed-loop system are proved to be bounded in probability.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Sect. 2. A novel adaptive neural control scheme is presented in Sect. 3. The simulation examples are given in Sect. 4, followed by Sect. 5 which concludes the work.

For the clarity of notations, throughout this paper R^+ denotes the set of all nonnegative real numbers; R^n indicates the real n-dimensional space; $R^{n\times r}$ denotes the real $n \times r$ matrix space. For a given vector or matrix X, X^T denotes its transpose; $Tr\{X\}$ is its trace when X is square; and $\|X\|$ denotes the Euclidean norm of a vector X. C^i denotes the set of all functions with continuous ith partial derivative. K denotes the set of all functions: $R^+ \to R^+$, which are continuous, strictly increasing and vanishing at zero; K_∞ refers to the set of all functions which are of class K and unbounded.

2 Preliminaries and problem formulation

Consider the following stochastic nonlinear strict-feedback system:

$$\begin{cases} dx_i = (g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i))dt + \psi_i(\bar{x}_i)dw, \\ \quad 1 \le i \le n-1, \\ dx_n = (g_n(\bar{x}_n)u + f_n(\bar{x}_n))dt + \psi_n(\bar{x}_n)dw, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $u \in R$, and $y \in R$ are the state variable, the control input, and the system output, respectively, $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$, w is an r-dimensional standard Brownian motion defined on the complete probability space (Ω, F, P) with Ω being a sample space, F being a σ-field, $\{F_t\}_{t\ge 0}$ being a filtration, and P being a probability measure. $f_i(.)$, $g_i(.) : R^i \to R$, $\psi_i^T(.) : R^i \to R^r$, $(i = 1, 2, \dots, n)$ are unknown smooth nonlinear functions with $f_i(0) = 0$, $\psi_i^T(0) = 0 (1 \le i \le n)$.

To introduce some useful conceptions and lemmas, consider the following stochastic system:

$$dx = f(x,t)dt + h(x,t)dw, \tag{2}$$

where x and w have the same definition as in (1), and $f : R^n \times R^+ \to R^n$, $h : R^n \times R^+ \to R^{n\times r}$ are locally Lipschitz functions in $x \in R^n$, with $f(0,t) = 0$, $h(0,t) = 0$, $\forall t \ge 0$.

Definition 1 For any given $V(x,t) \in C^{2,1}(R^n \times R^+; R^+)$, associated with the stochastic differential equation (2) we define the differential operator L as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\left\{ h^T \frac{\partial^2 V}{\partial x^2} h \right\}. \tag{3}$$

Remark 1 The term $\frac{1}{2}Tr\{h^T \frac{\partial^2 V}{\partial x^2} h\}$ is called Itô correction term, in which the second-order differential $\frac{\partial^2 V}{\partial x^2}$ makes the controller design much more difficult than that of the deterministic case.

Definition 2 [26] The solution process $\{x(t), t \geq 0\}$ of stochastic system (2) is said to be bounded in probability, if

$$\lim_{c\to\infty} \sup_{0\leq t<\infty} P\{\|x(t)\| > c\} = 0.$$

Lemma 1 [23] *Consider the stochastic system* (2) *and assume that* $f(x,t), h(x,t)$ *are* C^1 *in their arguments and* $f(0,t)$, $h(0,t)$ *are bounded uniformly in* t. *If there exist functions* $V(x,t) \in C^{1,2}(R^n \times R^+, R^+)$, $\mu_1(\cdot), \mu_2(\cdot) \in K_\infty$, *constants* $a_0 > 0$, $b_0 \geq 0$, *such that*

$$\mu_1(|x|) \leq V(x,t) \leq \mu_2(|x|),$$

$$LV \leq -a_0 V(x,t) + b_0,$$

then the solution process of (2) *is bounded in probability.*

The following lemmas will be used in this note.

Lemma 2 (Young's inequality [15]) *For* $\forall(x,y) \in R^2$, *the following inequality holds*:

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\epsilon^q}|y|^q,$$

where $\varepsilon > 0$, $p > 1$, $q > 1$, *and* $(p-1)(q-1) = 1$.

Lemma 3 *For any continuous function* $f(x): R^n \to R$ *with* $f(0) = 0$, $x = [x_1, x_2, \ldots, x_n]^T$, *then there exist positive smooth functions* $h_j(x_j): R \to R^+$, $j = 1, 2, \ldots, n$, *such that*

$$|f(x)| \leq \sum_{j=1}^{n} |x_j| h_j(x_j). \tag{4}$$

Proof Inequality (4) can be proved directly by Lemma 2.1 in [6] and Lemma 2 of [9]. Hence, the details are omitted. □

Remark 2 Lemma 3 implies that, for a continuous function $\psi_i(\cdot)$ in (1) there exists non-negative unknown smooth functions $\phi_{il}: R \to R^+$ $(l = 1, \ldots, i)$, such that for $i = 1, 2, \ldots, n$,

$$\|\psi_i(\bar{x}_i)\| \leq \sum_{l=1}^{i} |x_l| \phi_{il}(x_l). \tag{5}$$

To develop a novel adaptive neural control approach, the following assumptions are imposed on the system (1).

Assumption 1 There exist constants b_m and b_M such that for $1 \leq i \leq n$,

$$0 < b_m \leq |g_i(\bar{x}_i)| \leq b_M < \infty, \quad \forall \bar{x}_i \in R^i. \tag{6}$$

Remark 3 Assumption 1 means that the unknown functions $g_i(\bar{x}_i)$ are strictly either positive or negative. Without loss generality, it is further assumed that $0 < b_m \leq g_i(\bar{x}_i)$. In addition, because the constants b_m and b_M are not used for the controller design, their true values are unnecessary to be known. This assumption relaxes the ones in [24] and [25].

In this research, the following RBF neural networks will be used to approximate any continuous function $f(Z): R^n \to R$,

$$f_{nn}(Z) = W^T S(Z), \tag{7}$$

where $Z \in \Omega_Z \subset R^q$ is the input vector with q being the neural networks input dimension, weight vector $W = [w_1, w_2, \ldots, w_l]^T \in R^l$, $l > 1$ is the neural networks node number, and $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T$ means the basis function vector with $s_i(Z)$ being chosen as the commonly used Gaussian function of the form

$$s_i(Z) = \exp\left[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{\eta_i^2}\right],$$
$$i = 1, 2, \ldots, l, \tag{8}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. In [27], it has been indicated that with sufficiently large node number l the RBF neural networks (7) can approximate any continuous function $f(Z)$ over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy $\varepsilon > 0$ as

$$f(Z) = W^{*T} S(Z) + \delta(Z), \quad \forall z \in \Omega_z \in R^q, \tag{9}$$

where W^* is the ideal constant weight vector and is defined as

$$W^* := \arg \min_{W \in R^l} \left\{ \sup_{Z \in \Omega_Z} |f(Z) - W^T S(Z)| \right\},$$

and $\delta(Z)$ denotes the approximation error and satisfies $|\delta(Z)| \leq \varepsilon$.

Lemma 4 [10] *Consider the Gaussian RBF networks (7) and (8). Let $\rho := \frac{1}{2} \min_{i \neq j} \|\mu_i - \mu_j\|$, then an upper bound of $\|S(Z)\|$ is taken as*

$$\|S(Z)\| \leq \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{-2\rho^2 k^2/\eta^2} := s. \tag{10}$$

In [10], the constant s has been proved to be a limited value. Moreover, it is independent of Z (the neural networks input) and l (the dimension of neural weights W).

3 Main results

In this section, a backstepping-based design procedure will be proposed to construct the adaptive neural controller. The main idea lies in that the RBF neural networks are used to approximate the unknown nonlinear functions, the conventional adaptive technique is used to estimate the upper bound of the norms of neural networks weight vectors, and then backstepping is utilized to construct the control Lyapunov function. For simplicity, we first introduce the unknown constant θ which is specified as

$$\theta = \max \left\{ \frac{1}{b_m} \|W_i^*\|^2; i = 1, 2, \ldots, n \right\}, \tag{11}$$

where b_m is defined in Assumption 1, and $\| W_i^* \|$ denotes the norm of the ideal weight vector of the neural network, which will be specified at the ith design step.

3.1 Adaptive neural control design

In the following part, for the purpose of simplicity, the time variable t and the state vector $\bar{x}_i$ will be omitted from the corresponding functions.

Step 1: Let $z_1 = x_1$, the first stochastic differential equation of system (1) gives

$$dz_1 = (g_1 x_2 + f_1)dt + \psi_1 dw. \tag{12}$$

To stabilize this subsystem, take a stochastic Lyapunov function candidate as

$$V_1 = \frac{1}{4} z_1^4 + \frac{1}{2\lambda} b_m \tilde{\theta}^2, \tag{13}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ with $\hat{\theta}$ being the estimation of θ and λ being a positive design parameter. By (3) and (12), one has

$$LV_1 \leq z_1^3(g_1 x_2 + f_1) + \frac{3}{2} z_1^2 \psi_1 \psi_1^T - \lambda^{-1} b_m \tilde{\theta} \dot{\hat{\theta}}. \tag{14}$$

Notice that $\psi_1(0) = 0$, thus there exists a function $\phi_{11}(.)$ such that $\psi_1(z_1) = z_1 \phi_{11}(z_1)$. Furthermore, the following holds:

$$\frac{3}{2} z_1^2 \psi_1 \psi_1^T = \frac{3}{2} z_1^2 \|\psi_1\|^2 = \frac{3}{2} z_1^4 \phi_{11}^2.$$

Substituting this inequality into (14) yields

$$\begin{aligned} LV_1 &\leq z_1^3 \left(g_1 x_2 + f_1 + \frac{3}{2} z_1 \phi_{11}^2 \right) - \lambda^{-1} b_m \tilde{\theta} \dot{\hat{\theta}} \\ &\leq z_1^3 (g_1 x_2 + \bar{f}_1) - \frac{3}{4} g_1^{\frac{4}{3}} z_1^4 - \lambda^{-1} b_m \tilde{\theta} \dot{\hat{\theta}}, \end{aligned} \tag{15}$$

where $\bar{f}_1 = f_1 + \frac{3}{2} z_1 \phi_{11}^2 + \frac{3}{4} g_1^{\frac{4}{3}} z_1$. To stabilize this subsystem, view x_2 as the control signal. Then a desired feedback control signal is

$$\hat{\alpha}_1 = -k_1 z_1 - g_1^{-1} \bar{f}_1$$

with k_1 being a positive design constant. Further, add and subtract $g_1 \hat{\alpha}_1$ in the last bracket in (15) produces the following inequality.

$$LV_1 \leq -k_1 g_1 z_1^4 + z_1^3 g_1 (x_2 - \hat{\alpha}_1) - \frac{3}{4} g_1^{\frac{4}{3}} z_1^4 - \frac{b_m}{\lambda} \tilde{\theta} \dot{\hat{\theta}}. \tag{16}$$

Remark 4 Apparently, since $g_1^{-1} \bar{f}_1$ is an unknown smooth function, $\hat{\alpha}_1$ cannot be implemented in practice. To solve this problem, RBF neural network $W_1^T S_1(Z_1)$ can be utilized to model the unknown $g_1^{-1} \bar{f}_1$. This technique based on neural networks approximation will be repeated in later design steps to deal with the unknown nonlinearities.

Based on the neural networks universal approximation capability, for any given $\varepsilon_1 > 0$, the function

$g_1^{-1}\bar{f}_1$ can be expressed as

$$h_1(Z_1) \triangleq g_1^{-1}\bar{f}_1 = W_1^{*T}S_1(Z_1) + \delta_1(Z_1),$$
$$|\delta_1(Z_1)| \le \varepsilon_1, \tag{17}$$

where $Z_1 = x_1$, and $\delta_1(Z_1)$ refers to the approximation error. By using (17), Lemma 2, Assumption 1 and (11), one has

$$\begin{aligned} z_1^3 g_1 h_1(Z_1) &= z_1^3 g_1 \frac{W_1^{*T}}{\|W_1^*\|}\|W_1^*\|S_1(Z_1) + z_1^3 g_1\delta_1(Z_1) \\ &\le \frac{b_m}{2a_1^2}z_1^6\frac{\|W_1^*\|^2}{b_m}S_1^T(Z_1)S_1(Z_1) \\ &\quad + \frac{1}{2}a_1^2 b_M^2 + \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 + \frac{1}{4}\varepsilon_1^4 \\ &\le \frac{b_m}{2a_1^2}z_1^6\theta S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 b_M^2 \\ &\quad + \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 + \frac{1}{4}\varepsilon_1^4, \end{aligned} \tag{18}$$

where a_1 is a design parameter. Now choose the feasible virtual control law as

$$\alpha_1 = -k_1 z_1 - \frac{1}{2a_1^2}z_1^3\hat{\theta}S_1^T(Z_1)S_1(Z_1). \tag{19}$$

Thus, the following inequality can be obtained by using (18) and (19):

$$\begin{aligned} &z_1^3 g_1(\alpha_1 - \hat{\alpha}_1) \\ &\quad = z_1^3 g_1\left(-\frac{1}{2a_1^2}z_1^3\hat{\theta}S_1^T(Z_1)S_1(Z_1) + h_1(Z_1)\right) \\ &\quad \le -\frac{b_m}{2a_1^2}z_1^6\hat{\theta}S_1^T(Z_1)S_1(Z_1) \\ &\qquad + \frac{b_m}{2a_1^2}z_1^6\theta S_1^T(Z_1)S_1(Z_1) \\ &\qquad + \frac{b_M^2}{2}a_1^2 + \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 + \frac{1}{4}\varepsilon_1^4 \\ &\quad = \frac{b_m}{2a_1^2}z_1^6\tilde{\theta}S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}b_M^2 a_1^2 \\ &\qquad + \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 + \frac{1}{4}\varepsilon_1^4. \end{aligned} \tag{20}$$

Furthermore, from (16), (20), and Lemma 2, one has

$$\begin{aligned} LV_1 &\le -k_1 g_1 z_1^4 + z_1^3 g_1(x_2 - \hat{\alpha}_1) - \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 - \frac{b_m}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \\ &= -k_1 g_1 z_1^4 + z_1^3 g_1(x_2 - \alpha_1 + \alpha_1 - \hat{\alpha}_1) \\ &\quad - \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 - \frac{b_m}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \\ &\le -k_1 g_1 z_1^4 + z_1^3 g_1(x_2 - \alpha_1) - \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 - \frac{b_m}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \\ &\quad + \frac{b_m}{2a_1^2}z_1^6\tilde{\theta}S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}b_M^2 a_1^2 \\ &\quad + \frac{3}{4}g_1^{\frac{4}{3}}z_1^4 + \frac{1}{4}\varepsilon_1^4 \\ &= -k_1 g_1 z_1^4 + z_1^3 g_1 z_2 + \frac{1}{2}b_M^2 a_1^2 + \frac{1}{4}\varepsilon_1^4 \\ &\quad + \frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6 S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right) \\ &\le -\left(k_1 - \frac{3}{4}\right)g_1 z_1^4 + \frac{1}{2}b_M^2 a_1^2 + \frac{1}{4}\varepsilon_1^4 \\ &\quad + \frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6 S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right) + \frac{1}{4}g_1 z_2^4 \\ &\le -c_1 z_1^4 + \frac{1}{2}b_M^2 a_1^2 + \frac{1}{4}\varepsilon_1^4 \\ &\quad + \frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6 S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right) + \frac{1}{4}g_1 z_2^4, \end{aligned} \tag{21}$$

where $z_2 = x_2 - \alpha_1$ and $c_1 = (k_1 - \frac{3}{4})b_m > 0$.

Remark 5 In Step 1, to stabilize the first subsystem, the desired control signal $\hat{\alpha}_1$ is first found. Because it contains the unknown nonlinearities, it cannot be implemented in practice. The RBF neural networks are thus utilized to model the unknown nonlinear part in $\hat{\alpha}_1$. Furthermore, a feasible control signal α_1 is constructed. Such a design approach will be used in the posterior design steps.

Step 2: This step focuses on finding a feasible control signal, i.e., α_2, such that x_2 follows α_1. To this end, choose the following stochastic Lyapunov function as

$$V_2 = V_1 + \frac{1}{4}z_2^4. \tag{22}$$

By Itô formula,

$$\begin{aligned} LV_2 &= LV_1 + z_2^3(g_2 x_3 + f_2 - L\alpha_1) \\ &\quad + \frac{3}{2}z_2^2\left(\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right)\left(\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right)^T, \end{aligned} \tag{23}$$

where

$$L\alpha_1 = \frac{\partial \alpha_1}{\partial x_1}(g_1 x_2 + f_1) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{1}{2}\frac{\partial^2 \alpha_1}{\partial x_1^2}\psi_1\psi_1^T. \quad (24)$$

Remark 6 The main difficulty for control design in this step is that the term $\psi_2 - \frac{\partial \alpha_1}{\partial x_1}\psi_1$ is unknown and cannot be acted by the virtual control x_3 directly. In the following, the variable separation approach will be applied to solve this problem.

Using (5), it follows:

$$\begin{aligned}\left\|\psi_2 - \frac{\partial \alpha_1}{\partial x_1}\psi_1\right\| &\le \|\psi_2\| + \left\|\frac{\partial \alpha_1}{\partial x_1}\psi_1\right\| \\ &\le \sum_{l=1}^{2}|x_l|\phi_{2l} + |x_1|\left|\frac{\partial \alpha_1}{\partial x_1}\right|\phi_{11} \\ &\le |z_1|\phi_{21} + |z_2|\phi_{22} + |\alpha_1|\phi_{22} \\ &\quad + |z_1|\left|\frac{\partial \alpha_1}{\partial x_1}\right|\phi_{11} \le \sum_{l=1}^{2}|z_l|\varphi_{2l}, \quad (25)\end{aligned}$$

where $\varphi_{21} = |\frac{\partial \alpha_1}{\partial x_1}|\phi_{11} + \phi_{21} + (k_1 + \frac{z_1^2}{2a_1^2}\hat{\theta} \times S_1^T(Z_1)S_1(Z_1))\phi_{22}$ and $\varphi_{22} = \phi_{22}$. Consequently, by (25), the inequality $(a+b)^2 \le 2a^2 + 2b^2$ and Lemma 2, one has

$$\begin{aligned}\frac{3}{2}z_2^2\left\|\psi_2 - \frac{\partial \alpha_1}{\partial x_1}\psi_1\right\|^2 &\le \frac{3}{2}z_2^2\left(\sum_{l=1}^{2}|z_l|\varphi_{2l}\right)^2 \\ &\le 3z_2^4\varphi_{22}^2 + \frac{3}{4}r_2^2 + \frac{3}{r_2^2}z_1^4 z_2^4\varphi_{21}^4\end{aligned} \quad (26)$$

with $r_2 > 0$ is a constant. Then, substituting (21), (24)–(26) into (23) becomes

$$\begin{aligned}LV_2 \le& -c_1 z_1^4 + \frac{1}{2}a_1^2 b_M^2 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}r_2^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6 S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right) \\ &- z_2^3\frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} + z_2^3\Bigg(g_2 x_3 + f_2 \\ &- \frac{\partial \alpha_1}{\partial x_1}(g_1 x_2 + f_1) - \frac{1}{2}\frac{\partial^2 \alpha_1}{\partial x_1^2}\psi_1\psi_1^T + 3z_2\varphi_{22}^2 \\ &+ \frac{3}{r_2^2}z_1^4 z_2\varphi_{21}^4 + \frac{1}{4}g_1 z_2\Bigg) \\ \le& -c_1 z_1^4 + \frac{1}{2}a_1^2 b_M^2 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}r_2^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6 S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right) \\ &+ z_2^3(g_2 x_3 + \bar{f}_2) \\ &- \frac{3}{4}g_2^{\frac{4}{3}}z_2^4 + z_2^3\left(\varphi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}\right),\end{aligned} \quad (27)$$

where

$$\begin{aligned}\bar{f}_2 =& f_2 - \frac{\partial \alpha_1}{\partial x_1}(g_1 x_2 + f_1) - \frac{1}{2}\frac{\partial^2 \alpha_1}{\partial x_1^2}\psi_1\psi_1^T \\ &+ 3z_2\varphi_{22}^2 + \frac{3}{r_2^2}z_1^4 z_2\varphi_{21}^4 + \frac{1}{4}g_1 z_2 \\ &+ \frac{3}{4}g_2^{\frac{4}{3}}z_2 - \varphi_2(Z_2)\end{aligned} \quad (28)$$

and

$$\begin{aligned}\varphi_2(Z_2) =& -k_0\hat{\theta}\frac{\partial \alpha_1}{\partial \hat{\theta}} - \frac{\lambda s^2}{2a_2^2}z_2^3\left|z_2^3\frac{\partial \alpha_1}{\partial \hat{\theta}}\right| \\ &+ \frac{\partial \alpha_1}{\partial \hat{\theta}}\frac{\lambda}{2a_1^2}z_1^6 S_1^T(Z_1)S_1(Z_1)\end{aligned}$$

with $Z_2 = [x_1, x_2, \hat{\theta}]^T$, k_0 and a_2 being design parameters. The term $z_2^3(\varphi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}})$ will be considered in Sect. 3.2.

Remark 7 The adaptive law $\dot{\hat{\theta}}$, which will be given in (67), contains not only the current error variables z_1 and z_2, but also the latter ones, namely z_i, $i = 3, \dots, n$. Therefore, the term $\frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}$ in (24) can't be utilized directly to construct the packaged uncertain function $\bar{f}_2$ in (28) as the previous backstepping-based adaptive neural control approaches. Here, a function $\varphi_2(Z_2)$ is introduced to simplify the design procedure and is mainly used to compensate for $\frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}$. This method will be repeated at the following design step by introducing a function $\varphi_i(Z_i)$, $i = 3, 4, \dots, n$.

To control the first two subsystems, view x_3 as a virtual control input and the desired control signal is

$$\hat{\alpha}_2 = -k_2 z_2 - g_2^{-1}\bar{f}_2,$$

where k_2 is a positive design parameter. Obviously, under the action of $\hat{\alpha}_2$, one has

$$z_2^3(g_2x_3+\bar{f}_2)=z_2^3\big(g_2(x_3-\hat{\alpha}_2+\hat{\alpha}_2)+\bar{f}_2\big)$$
$$=-k_2g_2z_2^4+z_2^3g_2(x_3-\hat{\alpha}_2). \quad (29)$$

Further, the following inequality can be obtained:

$$LV_2\le -c_1z_1^4+\frac{1}{2}a_1^2b_M^2+\frac{1}{4}\varepsilon_1^4+\frac{3}{4}r_2^2$$
$$+\frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6S_1^T(Z_1)S_1(Z_1)-\dot{\hat{\theta}}\right)$$
$$-k_2g_2z_2^4+z_2^3g_2(x_3-\hat{\alpha}_2)$$
$$-\frac{3}{4}g_2^{\frac{4}{3}}z_2^4+z_2^3\left(\varphi_2(Z_2)-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right). \quad (30)$$

Since $h_2(Z_2)=g_2^{-1}\bar{f}_2$ is the unknown part of $\hat{\alpha}_2$, by repeating the way used in Step 1 an RBF neural network $W_2^TS_2(Z_2)$ is currently employed to approximate $h_2(Z_2)$ such that for the given precise $\varepsilon_2>0$,

$$h_2(Z_2)=W_2^{*T}S_2(Z_2)+\delta_2(Z_2),\quad |\delta_2(Z_2)|\le\varepsilon_2, \quad (31)$$

where $\delta(Z_2)$ denotes the approximation error. At the present stage, by exploring the method utilized in (17), one has

$$z_2^3g_2h_2(Z_2)\le\frac{b_m}{2a_2^2}z_2^6\theta S_2^T(Z_2)S_2(Z_2)$$
$$+\frac{1}{2}b_M^2a_2^2+\frac{3}{4}g_2^{4/3}z_2^4+\frac{1}{4}\varepsilon_2^4. \quad (32)$$

Choose the feasible virtual control signal as

$$\alpha_2=-k_2z_2-\frac{1}{2a_2^2}z_2^3\hat{\theta}S_2^T(Z_2)S_2(Z_2). \quad (33)$$

With the similar method used in (20), the following inequality can be obtained:

$$z_2^3g_2(\alpha_2-\hat{\alpha}_2)\le\frac{b_m}{2a_2^2}z_2^6\tilde{\theta}S_2^T(Z_2)S_2(Z_2)$$
$$+\frac{1}{2}b_M^2a_2^2+\frac{3}{4}g_2^{\frac{4}{3}}z_2^4+\frac{1}{4}\varepsilon_2^4. \quad (34)$$

By using (31)–(34), (30) can be rewritten as

$$LV_2\le -c_1z_1^4+\frac{1}{2}a_1^2b_M^2+\frac{1}{4}\varepsilon_1^4+\frac{3}{4}r_2^2$$
$$+\frac{b_m}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6S_1^T(Z_1)S_1(Z_1)-\dot{\hat{\theta}}\right)$$
$$-k_2g_2z_2^4+z_2^3g_2(x_3-\alpha_2+\alpha_2-\hat{\alpha}_2)$$
$$-\frac{3}{4}g_2^{\frac{4}{3}}z_2^4+z_2^3\left(\varphi_2(Z_2)-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right)$$
$$\le-\sum_{j=1}^{2}c_jz_j^4+\frac{1}{2}b_M^2\sum_{j=1}^{2}a_j^2+\frac{1}{4}\sum_{j=1}^{2}\varepsilon_j^4$$
$$+\frac{3}{4}r_2^2+\frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{2}\frac{\lambda}{2a_j^2}z_j^6S_j^T(Z_j)S_j(Z_j)-\dot{\hat{\theta}}\right)$$
$$+z_2^3\left(\varphi_2(Z_2)-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\right)+\frac{1}{4}g_2z_3^4, \quad (35)$$

where $z_3=x_3-\alpha_2$, $c_j=(k_j-\frac{3}{4})b_m>0$, $j=1,2$.

Step i $(3\le i\le n-1)$**:** The ith feasible virtual control law α_i will be constructed at this step. Define a variable $z_i=x_i-\alpha_{i-1}$, one has

$$dz_i=(g_ix_{i+1}+f_i-L\alpha_{i-1})dt$$
$$+\left(\psi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right)dw, \quad (36)$$

where

$$L\alpha_{i-1}=\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(g_jx_{j+1}+f_j)$$
$$+\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}+\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}\psi_p\psi_q^T. \quad (37)$$

Consider a stochastic Lyapunov function as

$$V_i=V_{i-1}+\frac{1}{4}z_i^4. \quad (38)$$

It follows immediately from (3) that

$$LV_i=LV_{i-1}+z_i^3(g_ix_{i+1}+f_i-L\alpha_{i-1})$$
$$+\frac{3}{2}z_i^2\left(\psi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right)$$
$$\times\left(\psi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right)^T, \quad (39)$$

where the term LV_{i-1} in (39) can be obtained in the following form by repeating the same process as former steps:

$$LV_{i-1} \leq -\sum_{j=1}^{i-1} c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^{i-1} a_j^2 + \frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{i-1} r_j^2 + \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) + \sum_{j=2}^{i-1} z_j^3\left(\varphi_j(Z_j) - \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right) + \frac{1}{4} g_{i-1} z_i^4. \tag{40}$$

Subsequently, we will deal with the last term in (39). Following the same procedure as (25), we can obtain

$$\left\|\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right\| \leq \sum_{j=1}^{i}|x_j|\phi_{ij} + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\sum_{k=1}^{j}|x_k|\phi_{jk}. \tag{41}$$

For the last term in (41), by rearranging the sequence, it follows

$$\begin{aligned}\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\sum_{k=1}^{j}|x_k|\phi_{jk} &= \left|\frac{\partial\alpha_{i-1}}{\partial x_1}\right||x_1|\phi_{11} + \left|\frac{\partial\alpha_{i-1}}{\partial x_2}\right|(|x_1|\phi_{21} + |x_2|\phi_{22}) + \cdots \\ &\quad + \left|\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\right|(|x_1|\phi_{(i-1)1} + |x_2|\phi_{(i-1)2} + \cdots + |x_{i-1}|\phi_{(i-1)(i-1)}) \\ &= \sum_{j=1}^{i-1}|x_j|\sum_{k=j}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\phi_{kj},\end{aligned} \tag{42}$$

substituting the above equation into (41) yields

$$\begin{aligned}\left\|\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right\| &\leq \sum_{j=1}^{i}|x_j|\phi_{ij} + \sum_{j=1}^{i-1}|x_j|\sum_{k=j}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\phi_{kj} \\ &\leq \sum_{j=1}^{i}|x_j|\varphi_{ij}^* \\ &\leq \sum_{j=1}^{i}|z_j + \alpha_{j-1}|\varphi_{ij}^* \leq \sum_{j=1}^{i}|z_j|\varphi_{ij},\end{aligned} \tag{43}$$

where $\varphi_{ii} = \varphi_{ii}^* = \phi_{ii}$, $\varphi_{ij} = \varphi_{ij}^*(1 + k_{j-1} + \frac{1}{2a_{j-1}^2}z_{j-1}^2 \times \hat{\theta}S_{j-1}^T(Z_{j-1})S_{j-1}(Z_{j-1})) = (\phi_{ij} + \sum_{k=j}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_k}|\phi_{kj}) \times (1 + k_{j-1} + \frac{1}{2a_{j-1}^2}z_{j-1}^2\hat{\theta}S_{j-1}^T(Z_{j-1})S_{j-1}(Z_{j-1}))$, $j = 2, 3, \ldots, i-1$, $\varphi_{i1} = \varphi_{i1}^* = \phi_{i1} + \sum_{k=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_k}|\phi_{k1}$.

Take the same procedures as (26), the last term in (39) can be rewritten as

$$\begin{aligned}\frac{3}{2}z_i^2\left\|\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right\|^2 &\leq \frac{3}{2}z_i^2\left(\sum_{j=1}^{i}|z_j|\varphi_{ij}\right)^2 \\ &\leq \frac{3}{2}iz_i^4\varphi_{ii}^2 + \frac{3}{4}r_i^2 + \frac{3}{4}r_i^{-2}i^2z_i^4\left(\sum_{j=1}^{i-1}z_j^2\varphi_{ij}^2\right)^2.\end{aligned} \tag{44}$$

Combining (39) with (40) and (44), the inequality below holds.

$$LV_i \leq -\sum_{j=1}^{i-1} c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^{i-1} a_j^2 + \frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{i} r_j^2 + \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) + \sum_{j=2}^{i} z_j^3\left(\varphi_j(Z_j) - \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right) + z_i^3(g_i x_{i+1} + \bar{f}_i) - \frac{3}{4}g_i^{\frac{4}{3}} z_i^4, \tag{45}$$

where

$$\bar{f}_i = f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(g_j x_{j+1} + f_j) - \frac{1}{2}\sum_{p,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q}\psi_p \psi_q^T + \frac{3}{2} i z_i \varphi_{ii}^2 + \frac{3}{4} r_i^{-2} i^2 z_i \left(\sum_{j=1}^{i-1} z_j^2 \varphi_{ij}^2\right)^2 + \frac{1}{4} g_{i-1} z_i + \frac{3}{4} g_i^{\frac{4}{3}} z_i - \varphi_i(Z_i), \tag{46}$$

and $\varphi_i(Z_i)$ is introduced as

$$\varphi_i(Z_i) = -k_0 \hat{\theta}\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} - z_i^3 \frac{\lambda s^2}{2a_i^2}\sum_{j=2}^{i}\left| z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\right| + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) \tag{47}$$

with $Z_i = [\bar{x}_i, \hat{\theta}]^T$. Taking the intermediate control signal as

$$\hat{\alpha}_i = -k_i z_i - g_i^{-1}\bar{f}_i \tag{48}$$

with k_i being a positive design constant. Similar to (29), the equation below can be produced.

$$z_i^3(g_i x_{i+1} + \bar{f}_i) = -k_i g_i z_i^4 + z_i^3 g_i(x_{i+1} - \hat{\alpha}_i). \tag{49}$$

Then (45) can be rewritten as

$$LV_i \le -\sum_{j=1}^{i-1} c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^{i-1} a_j^2 + \frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{i} r_j^2 + \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) + \sum_{j=2}^{i} z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) - k_i g_i z_i^4 + z_i^3 g_i(x_{i+1} - \hat{\alpha}_i) - \frac{3}{4} g_i^{\frac{4}{3}} z_i^4. \tag{50}$$

By following the same line used in procedure from (31) to (32), an RBF neural network $W_i^T S_i(Z_i)$ is applied to approximate the unknown function $h_i(Z_i) = g_i^{-1}\bar{f}_i$ in (48) such that for any given positive constant ε_i, the following holds:

$$z_i^3 g_i h_i(Z_i) \le \frac{b_m}{2a_i^2} z_i^6 \theta S_i^T(Z_i)S_i(Z_i) + \frac{1}{2} b_M^2 a_i^2 + \frac{3}{4} g_i^{4/3} z_i^4 + \frac{1}{4}\varepsilon_i^4. \tag{51}$$

Choose the virtual control α_i as

$$\alpha_i = -k_i z_i - \frac{1}{2a_i^2} z_i^3 \hat{\theta} S_i^T(Z_i)S_i(Z_i). \tag{52}$$

Similar to (34), the following inequality can be obtained.

$$z_i^3 g_i(\alpha_i - \hat{\alpha}_i) \le \frac{b_m}{2a_i^2} z_i^6 \tilde{\theta} S_i^T(Z_i)S_i(Z_i) + \frac{1}{2} b_M^2 a_i^2 + \frac{3}{4} g_i^{\frac{4}{3}} z_i^4 + \frac{1}{4}\varepsilon_i^4. \tag{53}$$

Substituting (51)–(53) into (45) gives

$$\begin{aligned} LV_i \le{}& -\sum_{j=1}^{i-1} c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^{i-1} a_j^2 + \frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{i} r_j^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) + \sum_{j=2}^{i} z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) - k_i g_i z_i^4 \\ &+ z_i^3 g_i(x_{i+1} - \alpha_i + \alpha_i - \hat{\alpha}_i) - \frac{3}{4} g_i^{\frac{4}{3}} z_i^4 \\ \le{}& -\sum_{j=1}^{i} c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^{i} a_j^2 + \frac{1}{4}\sum_{j=1}^{i}\varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{i} r_j^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i}\frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) + \sum_{j=2}^{i} z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) + \frac{1}{4} g_i z_{i+1}^4, \end{aligned} \tag{54}$$

where $z_{i+1} = x_{i+1} - \alpha_i$ and $c_j = (k_j - \frac{3}{4})b_m > 0$, $j = 1, 2, \ldots, i$.

Step n: This is the final step. The actual control input u will be constructed to stabilize the system (1). By $z_n = x_n - \alpha_{n-1}$ and Itô formula, we have

$$dz_n = (g_n u + f_n - L\alpha_{n-1})dt + \left(\psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\psi_j\right)dw, \tag{55}$$

where

$$L\alpha_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}(g_j x_{j+1} + f_j) + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{1}{2}\sum_{p,q=1}^{n-1} \frac{\partial^2 \alpha_{n-1}}{\partial x_p \partial x_q}\psi_p \psi_q^T. \tag{56}$$

Take the stochastic Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{4}z_n^4. \tag{57}$$

Repeating the method used in the process from (39) to (46) yields

$$\begin{aligned} LV_n \le& -\sum_{j=1}^{n-1} c_j z_j^4 + \frac{1}{2}b_M^2 \sum_{j=1}^{n-1} a_j^2 \\ &+ \frac{1}{4}\sum_{j=1}^{n-1} \varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{n} r_j^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{n-1} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) \\ &+ \sum_{j=2}^{n} z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) \\ &+ z_n^3(g_n u + \bar{f}_n) - \frac{3}{4}g_n^{\frac{4}{3}} z_n^4, \end{aligned} \tag{58}$$

where $\bar{f}_n$ and $\varphi_n(Z_n)$ are defined in (46) and (47), respectively, with $i = n$. Apparently, to stabilize this system, a desired control law is

$$\hat{\alpha}_n = -k_n z_n - g_n^{-1}\bar{f}_n. \tag{59}$$

By adding and subtracting $\hat{\alpha}_n$, (58) can be rewritten as

$$\begin{aligned} LV_n \le& -\sum_{j=1}^{n-1} c_j z_j^4 + \frac{1}{2}b_M^2 \sum_{j=1}^{n-1} a_j^2 \\ &+ \frac{1}{4}\sum_{j=1}^{n-1} \varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{n} r_j^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{n-1} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) \\ &+ \sum_{j=2}^{n} z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) \\ &+ z_n^3 g_n\left[(u - \hat{\alpha}_n + \hat{\alpha}_n) + \bar{f}_n\right] - \frac{3}{4}g_n^{\frac{4}{3}} z_n^4 \\ \le& -\sum_{j=1}^{n-1} c_j z_j^4 + \frac{1}{2}b_M^2 \sum_{j=1}^{n-1} a_j^2 \\ &+ \frac{1}{4}\sum_{j=1}^{n-1} \varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{n} r_j^2 \\ &+ \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{n-1} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) \\ &+ \sum_{j=2}^{n} z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) \\ &- k_n g_n z_n^4 + z_n^3 g_n(u - \hat{\alpha}_n) - \frac{3}{4}g_n^{\frac{4}{3}} z_n^4. \end{aligned} \tag{60}$$

Again, an RBF neural network $W_n^T S_n(Z_n)$ is used to model $h_n(Z_n) \triangleq g_n^{-1}\bar{f}_n$ which is the unknown part of $\hat{\alpha}_n$ in (59). $\hat{\alpha}_n$ can be rewritten as

$$\hat{\alpha}_n = -k_n z_n - W_n^{*T} S_n(Z_n) - \delta_n(Z_n), \tag{61}$$

where $\delta_n(Z_n)$ denotes the approximation error and satisfies $|\delta_n(Z_n)| \le \varepsilon_n$.

Next, the actual control input u can be constructed as

$$u = -k_n z_n - \frac{1}{2a_n^2} z_n^3 \hat{\theta} S_n^T(Z_n)S_n(Z_n). \tag{62}$$

Notice that the following inequalities are true:

$$z_n^3 g_n h_n(Z_n) \le \frac{b_m}{2a_n^2} z_n^6 \theta S_n^T(Z_n) S_n(Z_n) + \frac{1}{2} b_M^2 a_n^2 + \frac{3}{4} g_n^{4/3} z_n^4 + \frac{1}{4}\varepsilon_n^4, \tag{63}$$

$$z_n^3 g_n u \le -k_n b_m z_n^4 - \frac{b_m}{2a_n^2} z_n^6 \hat{\theta} S_n^T(Z_n) S_n(Z_n). \tag{64}$$

By using the formulas (59)–(64), we have

$$\begin{aligned} z_n^3 g_n(u - \hat{\alpha}_n) &= z_n^3 g_n u - z_n^3 g_n \hat{\alpha}_n \\ &\le -k_n b_m z_n^4 - \frac{b_m}{2a_n^2} z_n^6 \hat{\theta} S_n^T(Z_n) S_n(Z_n) \\ &\quad + k_n g_n z_n^4 + z_n^3 g_n h_n(Z_n) \\ &\le -k_n b_m z_n^4 + k_n g_n z_n^4 \\ &\quad + \frac{b_m}{2a_n^2} z_n^6 \tilde{\theta} S_n^T(Z_n) S_n(Z_n) + \frac{1}{2} b_M^2 a_n^2 \\ &\quad + \frac{3}{4} g_n^{4/3} z_n^4 + \frac{1}{4}\varepsilon_n^4. \end{aligned} \tag{65}$$

Substituting the inequality (65) into (60) gives

$$\begin{aligned} LV_n \le & -\sum_{j=1}^n c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^n a_j^2 \\ & + \frac{1}{4}\sum_{j=1}^n \varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^n r_j^2 \\ & + \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^n \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) - \dot{\hat{\theta}}\right) \\ & + \sum_{j=2}^n z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right), \end{aligned} \tag{66}$$

where $c_j = (k_j - \frac{3}{4})b_m > 0$, $j = 1, 2, \ldots, n-1$, $c_n = k_n b_m > 0$.

So far, the real controller u has been constructed. At the present stage, we summarize the main result in the following theorem.

Theorem 1 *Consider the stochastic nonlinear system (1) with Assumption 1. Suppose that for $1 \le i \le n$, the packaged unknown functions in $\hat{\alpha}_i (1 \le i \le n)$ can be approximated by the RBF neural networks in the sense that the approximating errors are bounded. If a control law is constructed in (62) with the virtual control signals α_i being defined in (19), (33) and (52), and the adaptive law*

$$\dot{\hat{\theta}} = \sum_{i=1}^n \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) - k_0 \hat{\theta}, \tag{67}$$

where the design parameters $\lambda > 0$, $k_0 > 0$ and $a_i > 0$ $(i = 1, 2, \ldots, n)$, then all the signals in the closed-loop system remain bounded in probability.

The proof of Theorem 1 will be obtained in the following subsection.

3.2 Analysis of stability

For the stability analysis of the closed-loop system, choose the stochastic Lyapunov function as $V = V_n$, from (66), it follows

$$\begin{aligned} LV \le & -\sum_{j=1}^N c_j z_j^4 + \frac{1}{2} b_M^2 \sum_{j=1}^n a_j^2 \\ & + \frac{1}{4}\sum_{j=1}^n \varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^n r_j^2 \\ & + \frac{b_m}{\lambda}\tilde{\theta}\left(\sum_{j=1}^n \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) - \dot{\hat{\theta}}\right) \\ & + \sum_{j=2}^n z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right). \end{aligned} \tag{68}$$

Substituting the adaption law $\dot{\hat{\theta}}$ in (67) into the penultimate term in (68) results in

$$\begin{aligned} LV \le & -\sum_{j=1}^n c_j z_j^4 + \frac{b_M^2}{2}\sum_{j=1}^n a_j^2 + \frac{1}{4}\sum_{j=1}^n \varepsilon_j^4 \\ & + \frac{3}{4}\sum_{j=2}^n r_j^2 + \frac{k_0 b_m}{\lambda}\tilde{\theta}\hat{\theta} \\ & + \sum_{j=2}^n z_j^3\left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right). \end{aligned} \tag{69}$$

In the following, it will be proved that the last term in (69) is negative. By using the fact of $0 < S_j^T S_j \le s^2$ in

Lemma 4 and the definition of $\dot{\hat{\theta}}$ in (67), we have

$$
\begin{aligned}
&-\sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\
&= \sum_{j=2}^{n} k_0 z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \hat{\theta} \\
&\quad - \sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=1}^{n} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right) \\
&= \sum_{j=2}^{n} k_0 z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \hat{\theta} \\
&\quad - \sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=1}^{j-1} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right. \\
&\quad \left. + \sum_{i=j}^{n} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right) \\
&= \sum_{j=2}^{n} k_0 z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \hat{\theta} \\
&\quad - \sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=1}^{j-1} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right) \\
&\quad - \sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=j}^{n} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right).
\end{aligned} \tag{70}
$$

For the last term of (70), by rearranging the sequence and using the fact $0 < S_i^T S_i \leq s^2$ for all i, the following inequality holds:

$$
\begin{aligned}
&-\sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=j}^{n} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right) \\
&\leq \sum_{j=2}^{n} \left| z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right| \left(\sum_{i=j}^{n} \frac{\lambda s^2}{2a_i^2} z_i^6 \right) = \left| z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \right| \frac{\lambda s^2}{2a_2^2} z_2^6 \\
&\quad + \left| z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \right| \frac{\lambda s^2}{2a_3^2} z_3^6 + \cdots + \left| z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \right| \frac{\lambda s^2}{2a_n^2} z_n^6 + \cdots \\
&\quad + \left| z_{n-1}^3 \frac{\partial \alpha_{n-2}}{\partial \hat{\theta}} \right| \frac{\lambda s^2}{2a_{n-1}^2} z_{n-1}^6 \\
&\quad + \left| z_{n-1}^3 \frac{\partial \alpha_{n-2}}{\partial \hat{\theta}} \right| \frac{\lambda s^2}{2a_n^2} z_n^6 + \left| z_n^3 \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \right| \frac{\lambda s^2}{2a_n^2} z_n^6 \\
&= \sum_{j=2}^{n} \frac{\lambda s^2}{2a_j^2} z_j^6 \left(\sum_{i=2}^{j} \left| z_i^3 \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right| \right).
\end{aligned} \tag{71}
$$

Combining (70) with (71) and using the definition of $\varphi_i(Z_i)$ in (47) with $j = i$ shows

$$
\begin{aligned}
&-\sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\
&\leq \sum_{j=2}^{n} k_0 z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \hat{\theta} \\
&\quad - \sum_{j=2}^{n} z_j^3 \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=1}^{j-1} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right) \\
&\quad + \sum_{j=2}^{n} \frac{\lambda s^2}{2a_j^2} z_j^6 \left(\sum_{i=2}^{j} \left| z_i^3 \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right| \right) \\
&= \sum_{j=2}^{n} z_j^3 \left(k_0 \hat{\theta} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right. \\
&\quad - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \left(\sum_{i=1}^{j-1} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i) S_i(Z_i) \right) \\
&\quad \left. + \frac{\lambda s^2}{2a_j^2} z_j^3 \left(\sum_{i=2}^{j} \left| z_i^3 \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right| \right) \right) \\
&= -\sum_{j=2}^{n} z_j^3 \varphi_j(Z_j),
\end{aligned} \tag{72}
$$

which implies

$$
\sum_{j=2}^{n} z_j^3 \left(\varphi_j(Z_j) - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) \leq 0. \tag{73}
$$

As for the term $\frac{k_0 b_m}{\lambda} \tilde{\theta} \hat{\theta}$ in (69), the following result can be obtained.

$$
\begin{aligned}
\frac{k_0 b_m}{\lambda} \tilde{\theta} \hat{\theta} &= -\frac{k_0 b_m}{\lambda} \tilde{\theta}^2 + \frac{k_0 b_m}{\lambda} \tilde{\theta} \theta \\
&\leq -\frac{b_m}{2\lambda} k_0 \tilde{\theta}^2 + \frac{b_m}{2\lambda} k_0 \theta^2.
\end{aligned} \tag{74}
$$

Substituting (73) and (74) into (69) results in

$$
LV \leq -\sum_{j=1}^{n} c_j z_j^4 - \frac{b_m}{2\lambda} k_0 \tilde{\theta}^2 + \frac{b_m}{2\lambda} k_0 \theta^2
$$

$$+\frac{b_M^2}{2}\sum_{j=1}^{n}a_j^2+\frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4+\frac{3}{4}\sum_{j=2}^{n}r_j^2. \quad (75)$$

Furthermore, let $a_0 = \min\{4c_j, k_0, j = 1, 2, \ldots, n\}$ and $b_0 = \frac{b_m k_0}{2\lambda}\theta^2 + \frac{1}{2}b_M^2\sum_{j=1}^{n}a_j^2 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \frac{3}{4}\sum_{j=2}^{n}r_j^2$, it follows

$$LV \le -a_0 V + b_0, \quad t \ge 0. \quad (76)$$

Therefore, according to Lemma 1, z_j, $j = 1, 2, \ldots, n$, and $\tilde{\theta}$ are bounded in probability. Since θ is a constant, $\hat{\theta}$ is bounded in probability. Based on the result of $S_j^T(Z_j)S_j(Z_j) \le s^2$, it can be seen that α_j is a function of z_j and $\hat{\theta}$. So, α_j is also bounded in probability. Hence, we conclude that all the signals x_j in the closed-loop system (1) remain bounded in the sense of probability.

Remark 8 In this note, we have proposed a direct adaptive neural control approach for stochastic nonlinear systems. The primary difference between the indirect adaptive neural control and the direct one lies in that the indirect one utilizes the neural networks to approximate the unknown nonlinear functions in the system dynamics while the latter one applies neural networks to model the unknown dynamics in the desired control signals. Thus, the direct adaptive neural controller has a simpler structure.

4 Simulation example

Example 1 In order to demonstrate the effectiveness of our result, we consider the following second-order stochastic nonlinear system:

$$dx_1 = \big((1+x_1^2)x_2 + x_1\sin(x_1)\big)dt + x_1^3 dw,$$

$$dx_2 = \left(\left(2+\frac{x_2^2}{1+x_1^2}\right)u + x_1x_2^2\right)dt + (1+\sin x_1)x_2 dw,$$

$$y = x_1,$$

where x_1 and x_2 denote the state variables and u is the system control input. It is obvious that the system satisfies Assumption 1. Now, by using Theorem 1, the virtual control law,the actual control law, and the adaptive laws are chosen as

$$\alpha_1 = -k_1z_1 - \frac{1}{2a_1^2}z_1^3\hat{\theta}S_1^T(Z_1)S_1(Z_1),$$

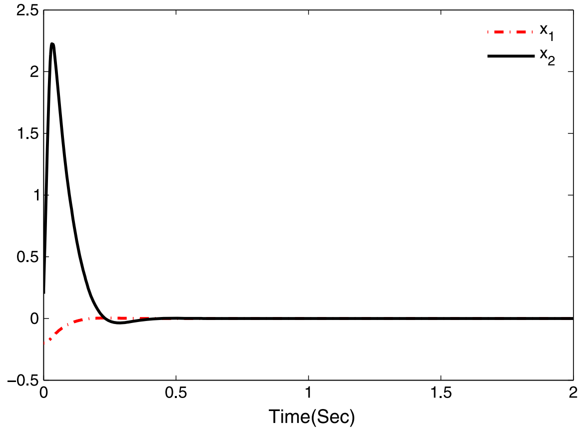

Fig. 1 States of closed-loop system (*dash-dot line*) x_1 and x_2 (*solid line*)

$$u = -k_2z_2 - \frac{1}{2a_2^2}z_2^3\hat{\theta}S_2^T(Z_2)S_2(Z_2),$$

$$\dot{\hat{\theta}} = \sum_{k=1}^{2}\frac{\lambda}{2a_k^2}z_k^6S_k^T(Z_k)S_k(Z_k) - k_0\hat{\theta},$$

where $z_1 = x_1, z_2 = x_2 - \alpha_1, Z_1 = z_1$ and $Z_2 = [z_1, z_2, \hat{\theta}]^T$. The simulation is run under the initial conditions $[x_1(0), x_2(2)]^T = [-0.2, 0.2]^T$, and $\hat{\theta}(0) = 0.3$. In the simulation,design parameters are taken as follows: $k_1 = k_2 = 15$, $a_1 = a_2 = 8, k_0 = 10$, and $\lambda = 0.1$, and RBF neural networks are chosen in the following way. Neural network $W_1^T S_1(Z_1)$ contains eleven nodes with centers spaced evenly in the interval $[-5, 5]$ and widths equal to two. Neural network $W_2^T S_2(Z_2)$ also contains eleven nodes with centers spaced evenly in the interval $[-5, 5] \times [-5, 5] \times [-5, 5]$ and widths still equal to two. The simulation results are shown by Figs. 1–3. Figure 1 shows that x_1 and x_2 converge to zero within less than one second. Figure 2 shows the control input signal u, and Fig. 3 shows the response curve of the adaptive parameter $\hat{\theta}$. Apparently, simulation results show that good convergence performances are achieved and all the signals in the closed-loop system are bounded.

Example 2 To further show the control capability of the proposed scheme, we consider the third-order nonlinear system below:

$$dx_1 = \big((0.3+x_1^2)x_2 - 0.8\sin x_1\big)dt + x_1\sin x_1 dw,$$

$$dx_2 = \big((1+x_2^2)x_3 - x_2 - 0.5x_2^3 - x_1^3 - \sqrt{x_1}\big)dt$$

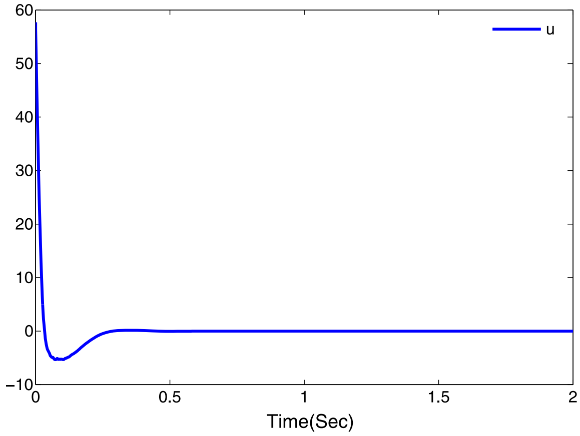

Fig. 2 The control input u

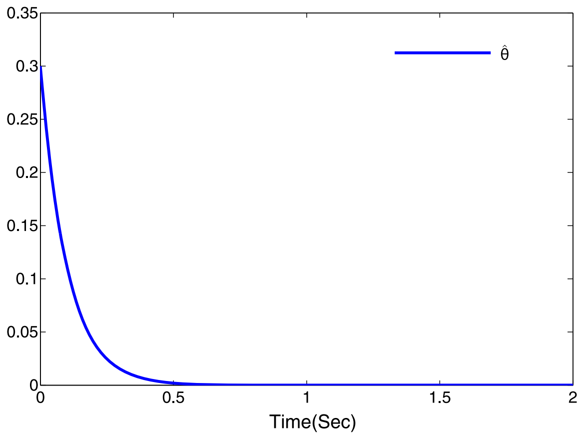

Fig. 3 Adaptive laws $\hat{\theta}$

$$
\begin{aligned}
&\quad + x_1 \cos x_2 dw, \\
dx_3 &= \left(\big(1.5 + \sin(x_1 x_2)\big) u - 0.5 x_3 - \frac{1}{3} x_3^2 \right. \\
&\quad \left. - x_2^2 x_3 - \frac{x_1}{1 + x_1^2} \right) dt + 3 x_1 e^{-x_2^2} dw, \\
y &= x_1.
\end{aligned}
$$

Similarly, Theorem 1 is used to design the direct adaptive neural controller for this system. Therefore, the virtual control laws and the true control law are chosen as

$$\alpha_1 = -k_1 z_1 - \frac{1}{2a_1^2} z_1^3 \hat{\theta} S_1^T(Z_1) S_1(Z_1),$$

$$\alpha_2 = -k_2 z_2 - \frac{1}{2a_2^2} z_2^3 \hat{\theta} S_2^T(Z_2) S_2(Z_2),$$

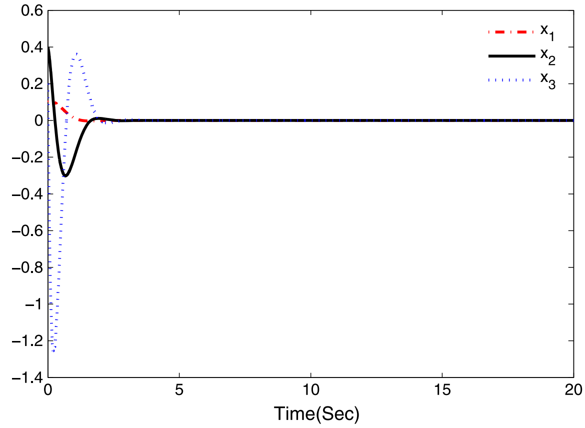

Fig. 4 States of x_1 (*dash-dot line*), x_2 (*solid line*), and x_3 (*dot line*)

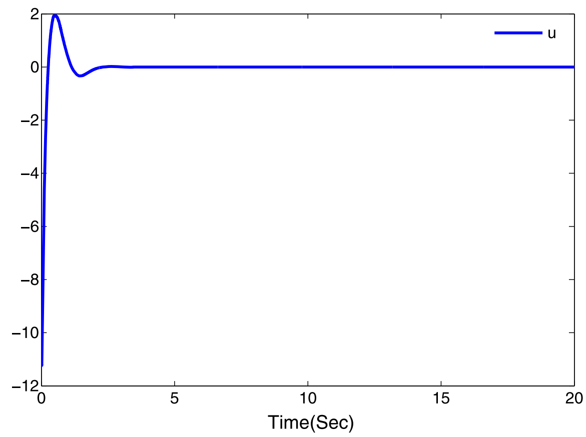

Fig. 5 The control input u

$$u = -k_3 z_3 - \frac{1}{2a_3^2} z_3^3 \hat{\theta} S_3^T(Z_3) S_3(Z_3),$$

with the adaption law as follows:

$$\dot{\hat{\theta}} = \sum_{k=1}^{3} \frac{\lambda}{2a_k^2} z_k^6 S_k^T(Z_k) S_k(Z_k) - k_0 \hat{\theta},$$

where $z_1 = x_1$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$, $Z_1 = z_1$ and $Z_2 = [z_1, z_2, \hat{\theta}]^T$, $Z_3 = [z_1, z_2, z_3, \hat{\theta}]^T$. and the design parameters are adopted respectively as $k_1 = 4$, $k_2 = 3$, $k_3 = 4$, $a_1 = 12$, $a_2 = 14$, $a_3 = 15$, $k_0 = 2$, $\lambda = 2$. Moreover, the initial conditions are given by $[x_1(0), x_2(0), x_3(0)]^T = [0.1, 0.4, 0.2]^T$, and $\hat{\theta}(0) = 0$. Simulation results indicate that the proposed controller works well and guarantees the boundedness of all the signals in the closed-loop system. The details are shown in Figs. 4, 5, 6.

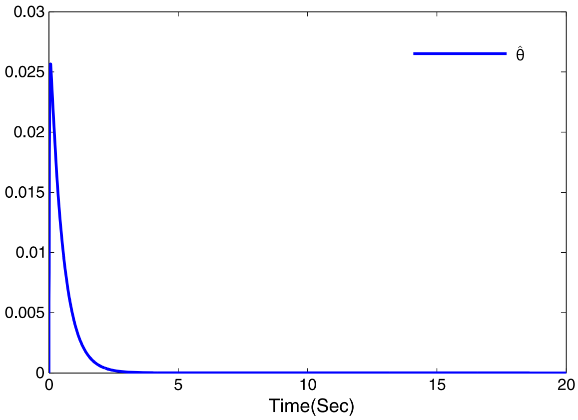

Fig. 6 Adaptive laws $\hat{\theta}$

5 Conclusion

Based on the backstepping technique, a direct adaptive control approach has been proposed for a class of stochastic nonlinear systems. The main contribution of this article is that only one adaptive parameter is needed to be estimated online no matter how many neural networks nodes are used and how large the order of systems is. The stability analysis in this note guarantees that all the signals in the closed-loop systems are bounded in probability. Two simulation examples are given to illustrate the effectiveness of the proposed approach.

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