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ORIGINAL PAPER

Adaptive-impulsive projective synchronization of drive-response delayed complex dynamical networks with time-varying coupling

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Abstract This paper investigates the adaptive-impulsive projective synchronization of drive-response delayed complex dynamical networks with time-varying coupling, in which the weights of links between two connected nodes are time varying. By the stability analysis of the impulsive functional differential equation, the sufficient conditions for achieving projective synchronization are obtained, and a hybrid controller, that is, an adaptive feedback controller with impulsive control effects is designed. The numerical examples are presented to illustrate the effectiveness and advantage of the proposed synchronization criteria.

Keywords Drive-response delayed networks · Adaptive-impulsive control · Projective synchronization

1 Introduction

Complex dynamical network has attracted significant attention because it can be used to describe many nature and artificial systems, such as the World Wide Web, various wireless communication networks, metabolic networks, biological neural works, scientific citation webs, epidemic network, traffic network, and so on. In a complex dynamical network, each node represents a dynamical system according to the different situation of the concerned problem, while the edges represent the connections between nodes. Since the discovery of small-world property [1] and scale-free feature [2] of complex network, complex network has been developed particular rapidly and gradually become a focal subject.

Synchronization, one of the typical collective behaviors of complex dynamical networks has been extensively investigated in different fields of engineering and sociology [3–19]. During the last decades, many kinds of synchronization have been proposed, such as complete synchronization, phase synchronization, lag synchronization, cluster synchronization, generalized synchronization, as well as projective synchronization. Correspondingly, various control schemes including pinning control [5–8], adaptive control [9–12], impulsive control [13–19], etc.; have been used to study the different kinds of synchronization. Among all kinds of synchronization, projective synchronization which was first studied in two coupled partially linear systems by Mainer and Reface [20], is one of the most noticeable problems because of its proportionality between the synchronized dynamical states with a scaling factor. Later, the projective synchronization has been extremely investigated in recent years, including chaotic systems [21–23] and complex dynamical networks [12, 24–27]. Hu et al. [24] discussed the pro-

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jective synchronization of a drive-response dynamical network model via pinning control. Guo et al. [25] investigated the projective synchronization in drive-response networks via impulsive control. The impulsive projective synchronization between the drive system and response dynamical network without the time delay was investigated in [26]. Time delays are ubiquitous in natural and artificial systems. The delay increases the dimensionality and the complexity of the systems. Consequently, time delay case should be considered. Sun et al. studied the projective synchronization in drive-response dynamical networks of partially linear systems with time-varying coupling delay in [27]. However, in many practical situations, some complex networks are time-varying networks. For this kind of network, the weights of links are time varying, which results in variations of the network topology and coupling configuration over time [28–33]. To simulate more realistic networks, time-varying coupling should be taken into account. Recently, some authors presented hybrid control strategy to investigate the synchronization of chaotic systems [34] and complex networks [35–37]. Compared to the conventional control method, the hybrid control method is more effective to the networks with evolutionary properties. Very recently, in [38], Cao et al. proposed the projective synchronization of a class of delayed chaotic systems via impulsive control, where the drive-response system can be synchronized to within a desired scaling factor. As far as the authors know, the projective synchronization of a drive-response dynamical network model with time delays dynamical nodes and time-varying coupling has not been reported via adaptive-impulsive control, which motivates the current study.

Inspired by the above previous works, this paper aims to handle the problem of the projective synchronization for a drive-response delayed dynamical network model with time-varying coupling via adaptive-impulsive control. The sufficient conditions for the projective synchronization are derived analytically by the stability analysis of the impulsive functional differential equation, and a hybrid controller, which contains an adaptive controller and an impulsive controller, is designed. Analytical results show that drive-response delayed dynamical networks with time-varying coupling can realize the projective synchronization within a desired scaling factor.

Notations Throughout this paper, let $S_\rho = \{x \in R^n | \|x\| < \rho\}$, where $\|\cdot\|$ denotes the Euclidean norm on R^n. $K = \{\varphi \in C(R^+, R^+) | \varphi(t)$ is strictly increasing and $\varphi(0) = 0\}$, $\kappa = \{\varphi \in K | \varphi(t) < t,\ t > 0\}$, $\Sigma = \{\varphi \in C(R^+, R^+) | \varphi(0) = 0,\ \varphi(t) > 0,\ t > 0\}$, $PC = \{\varphi : [-\tau, 0] \to R^n,\ \varphi(t)$ is continuous everywhere except at the finite number of points $\bar{t}$, where $\varphi(\bar{t}^+), \varphi(\bar{t}^-)$, exist and $\varphi(\bar{t}^+) = \varphi(\bar{t})\}$, $PC_\delta(t) = \{\varphi \in PC : \|\varphi\| < \delta\}$. $\lambda_{\max}(A), \lambda_{\min}(A)$ denote the minimum and maximum eigenvalue of matrix A, respectively. sup denotes the upper bound. I denotes an appropriate dimensional identity matrix. A^{-1} expresses an invertible matrix of A.

The rest of this paper is organized as follows: In Sect. 2, preliminaries and the model of drive-response delayed dynamical networks with time-varying coupling are given. In Sect. 3, the projective synchronization criteria are obtained. Numerical simulations are shown in Sect. 4. The conclusion is finally drawn in Sect. 5.

2 Preliminaries and model description

Consider the following impulsive control system:

$$\begin{cases} \dot{x}(t) = f(t, x), & t \neq t_k, t \geq 0, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) & \\ \qquad = I_k x(t_k^-), & t = t_k, \\ x(t_0^+) = x(t_0), & t_0 \geq 0, k = 1, 2, \ldots, \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable, $x_t(\theta) = x(t+\theta)$ for $\theta \in [-\tau, 0]$. $f : R^n \times R^+ \to R^n$ is a right continuous function. $x(t_k^-) = \lim_{t \to t_k^-} x(t)$ and $x(t_k^+) = x(t_k)$. The instant sequence $\{t_k\}$ satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots, t_k \to \infty$ as $k \to \infty$. $I_k \in C[R^n, R^n]$ denotes the incremental change of the state at time t_k with $I_k(0) = 0$.

Before proceeding, we give some necessary definitions and lemmas to derive the main results of this paper.

Definition 1 Let $V : R^n \times R^+ \to R^+$, then V is said to belong to class V_0 if

(i) V is continuous in each of the sets $R^n \times [t_{k-1}, t_k)$, and for each $x \in R^n$, $k = 1, 2, \ldots$, $\lim_{(y,t) \to (x, t_k^-)} V(y, t) = V(x, t_k^-)$ exists,
(ii) V is locally Lipschitzian in $x \in R^n$.

Definition 2 For $(x, t) \in R^n \times [t_{k-1}, t_k)$, the right and upper Dini's derivative of $V(x, t) \in V_0$ is defined as follows:

$$D^+V(x,t) \triangleq \lim_{h\to 0}\sup\frac{1}{h}\big[V\big(x+hf(x,t),t+h\big)-V(x,t)\big].$$

Lemma 1 *The matrix inequality* $2x^Ty \le x^TQx + y^TQ^{-1}y$ *holds, for any vectors and a positive-definite matrix* $Q \in R^{n\times n}$.

Lemma 2 [39] *Assume that there exist* $V \in \upsilon_0$, ω_1, $\omega_2 \in K$, $\varphi \in \kappa$ *and* $H \in \Sigma$ *such that*

(i) $\omega_1(\|x\|) \le V(t,x) \le \omega_2(\|x\|)$ *for* $(t,x) \in [t_0,+\infty] \times S_\rho$;
(ii) *For all* $x \in S_\rho, 0 < \rho_1 \le \rho$, $V(t_k, I_k(x)) \le \varphi(V(t_k^-, x))$ *for all* k;
(iii) *For any solution* $x(t)$ *of* (1), $V(t+s, x(t+s)) \le \varphi^{-1}(V(t,x(t))), -\tau \le s \le 0$, *implies that* $D^+V(t,x(t)) \le g(t)H(V(t,x(t)))$, *where* $g: [t_0,+\infty] \to R^+$ *is locally integrable,* φ^{-1} *is the inverse function of* φ;
(iv) H *is nondecreasing and there exist constants* $l_2 \ge l_1 > 0$ *and* $r > 0$ *such that for any* $\mu > 0$, $l_1 \le t_k - t_{k-1} \le l_2$ *and* $\int_{\varphi(\mu)}^{\mu}\frac{ds}{H(s)} - \int_{t_{k-1}}^{t_k} g(s)\,ds \ge r$, $k = 1,2,\dots$.

Then the zero solution of (1) *is uniformly asymptotically stable.*

Inspired by [38], the drive-response delayed network model with time-varying coupling, in which time delays dynamical nodes are partially linear time-delayed chaotic systems, is introduced by the following equation:

$$\begin{cases} \dot{u}_d = M(z)\cdot u_d(t) + \gamma\,\Gamma(u_d(t-\tau)-u_d(t)),\\ \dot{z}(t) = f(u_d(t), u_d(t-\tau), z(t), z(t-\tau)),\\ \dot{u}_{ri} = M(z)\cdot u_{ri}(t) + \gamma\,\Gamma(u_{ri}(t-\tau)-u_{ri}(t))\\ \qquad + c\sum_{j=1}^N c_{ij}(t)A(t)u_{rj}(t), \end{cases} \tag{2}$$

where the drive system and the response network systems are linked through the variable $z(t) \in R^1$. $u_d(t) = (u_d^1(t), u_d^2(t), \dots, u_d^n(t)) \in R^n$, $u_{ri}(t) = (u_{ri}^1(t), u_{ri}^2(t), \dots, u_{ri}^n(t))^T \in R^n$, the d and r stand for the drive system and response system, respectively. The constant $c > 0$ is the coupling strength to be adjusted, $\tau \ge 0$ is the time delay. $M(z) \in R^{n\times n}$ is a matrix which depends on the variable $z(t)$. $A(t) \in R^{n\times n}$ is the time-varying inner-coupling link matrix at time t. $C(t) = (c_{ij}(t))_{N\times N}$ is the outer-coupling configuration matrix, in which $c_{ij}(t) \ne 0$ if there is a link from node i to node j $(i \ne j)$, and $c_{ij}(t) = 0$ $(i \ne j)$ otherwise, the diagonal elements of matrix $C(t)$ are given by $c_{ii}(t) = -\sum_{j=1, j\ne i}^N c_{ij}(t)$, $i = 1,2,\dots,N$. Here, let the coupling matrices $A(t)$ and $C(t)$ be bounded and continuous.

If there exists a constant $\alpha(\alpha \ne 0)$ such that $\lim_{t\to\infty} = \|e_i(t)\| = \|u_{ri}(t) - \alpha u_d(t)\| = 0$ for all $i = 1,2,\dots,N$, then the projective synchronization of the drive-response delayed network (2) is achieved, where α is a desired scaling factor. In [24–27], the authors investigated the projective synchronization of the drive-response dynamical network model, but the time-varying coupling was not considered. The aim of this paper is to discuss the adaptive-impulsive control of projective synchronization in the drive-response delayed complex dynamical networks with time-varying coupling. And we choose proper the adaptive feedback controller $U_i, i = 1,2,\dots,N$, the impulsive controller B_{i_k} which is a $n\times n$ constant matrix, and the impulsive distances $\Delta_k = t_k - t_{k-1}$, $k = 1,2,\dots,$ such that the projective synchronization of system (2), that is, $\lim_{t\to\infty}\|e_i(t)\| = 0$.

Therefore, based on adaptive-impulsive control method, the drive-response delayed network (2) can be rewritten as the following impulsive differential equation:

$$\begin{cases} \dot{u}_d = M(z)\cdot u_d(t) + \gamma\,\Gamma(u_d(t-\tau)-u_d(t)),\\ \dot{z}(t) = f(u_d(t), u_d(t-\tau), z(t), z(t-\tau)),\\ \dot{u}_{ri} = M(z)\cdot u_{ri}(t) + \gamma\,\Gamma(u_{ri}(t-\tau)-u_{ri}(t))\\ \qquad + c\sum_{j=1}^N c_{ij}(t)A(t)u_{rj}(t) + U_i, \quad t \ne t_k,\\ \Delta u_{ri} = u_{ri}(t_k^+) - u_{ri}(t_k^-) = B_{i_k}[u_{ri} - \alpha u_d],\\ \quad t = t_k,\ k = 1,2,\dots, \end{cases} \tag{3}$$

where $B_{i_k} \in R^{n\times n}$ is a gain matrix, $U_i \in R^n$ is the control inputs, $u_{ri}(t_k^+) = \lim_{t\to t_k^+} u_{ri}(t)$, $u_{ri}(t_k^-) = \lim_{t\to t_k^-} u_{ri}(t)$, $k = 1,2,\dots$. Moreover, any solution of (3) is right continuous at each t_k, that is, $u_{ri}(t_k^+) = u_{ri}(t_k)$.

Here, the adaptive controller U_i and updating laws are designed as follows:

$$U_i = -d_i e_i(t), \quad i = 1,2,\dots,N, \tag{4}$$

$$\dot{d}_i = k_i e_i^T(t)e_i(t), \quad k_i > 0. \tag{5}$$

3 Main results

In this section, the criteria for the adaptive-impulsive projective synchronization of the drive-response delayed dynamical networks with time-varying coupling will be established.

Let the projective synchronization error $e_i(t) = u_{ri}(t) - \alpha u_d(t)$ $(i = 1, 2, \dots, N)$, we can derive the error dynamical network:

$$\begin{cases} \dot{e}_i(t) = M(z)e_i(t) + \gamma \Gamma(e_i(t-\tau) - e_i(t)) \\ \qquad + c\sum_{j=1}^{N} c_{ij}(t)A(t)e_j(t) + U_i, \\ \quad t \neq t_k, \\ \dot{z}(t) = f(u_d(t), u_d(t-\tau), z(t), z(t-\tau)), \\ \Delta e_i = B_{i_k} e_i(t_k^-), \quad t = t_k, k = 1, 2, \dots. \end{cases} \tag{6}$$

Then, one has the following results.

Theorem 1 *If there exist positive definite matrices P_i and positive definite diagonal matrices R_i, such that the following conditions hold*:

(i) $\max_k(\|I + B_{i_k}\|^2) = \rho_k$, $\rho_k\|P_i\|\lambda_{\max}(P_i^{-1}) \le \beta_k < 1$,
(ii) $\sup[\lambda_{\max}(P_i^{-1}\Omega_i)] > 0$,
(iii) $\sup[\lambda_{\max}(P_i^{-1}\Omega_i)](t_k - t_{k-1}) + \ln\beta_k < 0$,

where $\Omega_i = M^T(z)P_i + P_iM(z) + c\sum_{j=1}^{N} c_{ij}^2(t) \times \|P_iA(t)\|^2 I + cNI + \gamma P_i\Gamma R_i\Gamma^T P_i - 2\gamma P_i\Gamma + \frac{\gamma\lambda_{\max}(P_i^{-1}R_i^{-1})}{\beta_k}P_i - 2d^*P_i$, *$d^*$ is the minimum value of the initial feedback strength d_{i0}, $i = 1, 2, \dots, N$, then the trivial solution of the error system (6) is globally asymptotically stable, which implies the drive-response delayed network with time-varying coupling (2) achieves the projective synchronization under the adaptive-impulsive control, and the scaling factor α is the desired value in advance.*

Proof Since $\dot{d}_i = k_i e_i^T(t)e_i(t)$, $k_i > 0$, then, we have $d_i \ge d_{i0}$, d_{i0} is the initial feedback strength d_i.

Considering the following Lyapunov function:

$$V(t, e_i(t)) = \sum_{i=1}^{N} e_i^T(t)P_ie_i(t).$$

Then, we have

$$\min_{1\le i\le N}(\lambda_{\min}(P_i))\sum_{i=1}^{N} e_i^T(t)_i e_i(t) \le V(t, e_i(t)) \le \max_{1\le i\le N}(\lambda_{\max}(P_i))\sum_{i=1}^{N} e_i^T(t)e_i(t).$$

For all $e_i \in S(\rho_1)$, $0 < \rho_1 \le \rho$,

$$\begin{aligned} V(t_k, e_i(t_k)) &= \frac{1}{2}\sum_{i=1}^{N} e_i^T(t_k^-)(I + B_{i_k})^T P_i(I + B_{i_k})e_i(t_k^-) \\ &\le \rho_k\|P_i\|\lambda_{\max}(P_i^{-1})\sum_{i=1}^{N} e_i^T(t_k^-)P_ie_i(t_k^-) \\ &\le \beta_k V(t_k^-, e_i(t_k^-)), \quad k = 1, 2, \dots. \end{aligned}$$

Let $\varphi(s) = \beta_k s$, then $\varphi \in \kappa$. For any solution of (6), if

$$V(t+s, e_i(t+s)) \le \varphi^{-1}(V(t, e_i(t))), \quad \forall s \in [-\tau, 0],$$

that is,

$$\sum_{i=1}^{N} e_i^T(t+s)P_ie_i(t+s) \le \frac{1}{\beta_k}\sum_{i=1}^{N} e_i^T(t)P_ie_i(t), \quad \forall s \in [-\tau, 0].$$

Especially, for $s = -\tau$, one has

$$\sum_{i=1}^{N} e_i^T(t-\tau)P_ie_i(t-\tau) \le \frac{1}{\beta_k}\sum_{i=1}^{N} e_i^T(t)P_ie_i(t).$$

The Dini derivative of $V(t, e_i(t))$ along the trajectories of (6) is

$$\begin{aligned} D^+V(t, e_i(t)) &= \sum_{i=1}^{N} \dot{e}_i^T(t)P_ie_i(t) + \sum_{i=1}^{N} e_i^T(t)P_i\dot{e}_i(t) \\ &= \sum_{i=1}^{N}\Big[M(z)e_i(t) + \gamma\Gamma(e_i(t-\tau) - e_i(t)) \\ &\quad + c\sum_{j=1}^{N} c_{ij}(t)A(t)e_j(t) - d_ie_i(t)\Big]^T P_ie_i(t) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{N} e_i^T(t)P_i\Big[M(z)e_i(t)\\
&+\gamma\Gamma\big(e_i(t-\tau)-e_i(t)\big)\\
&+c\sum_{j=1}^{N} c_{ij}(t)A(t)e_j(t)-d_ie_i(t)\Big]\\
&=\sum_{i=1}^{N} e_i^T(t)\big[M^T(z)P_i+P_iM(z)\\
&-2\gamma P_i\Gamma-2d_iP_i\big]e_i(t)\\
&+2c\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}(t)e_i^T(t)P_iA(t)e_j(t)\\
&\le\sum_{i=1}^{N} e_i^T(t)\big[M^T(z)P_i+P_iM(z)\\
&-2\gamma P_i\Gamma-2d_{i0}P_i\big]e_i(t)\\
&+2c\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)c_{ij}(t)P_iA(t)e_j(t)\\
&+2\gamma\sum_{i=1}^{N} e_i^T(t)P_i\Gamma e_i(t-\tau).
\end{aligned}
$$

From Lemma 1, one has

$$
\begin{aligned}
&2c\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}(t)e_i^T(t)P_iA(t)e_j(t)\\
&\le c\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}^2(t)e_i^T(t)P_iA(t)A^T(t)P_ie_i(t)\\
&+c\sum_{i=1}^{N}\sum_{j=1}^{N} e_j^T(t)e_j(t)\\
&\le c\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}^2(t)\|P_iA(t)\|^2e_i^T(t)e_i(t)\\
&+cN\sum_{i=1}^{N} e_i^T(t)e_i(t),
\end{aligned}
$$

$$
\begin{aligned}
&2\gamma\sum_{i=1}^{N} e_i^T(t)P_i\Gamma e_i(t-\tau)\\
&\le\gamma\sum_{i=1}^{N} e_i^T(t)P_i\Gamma R_i\Gamma^TP_ie_i(t)\\
&+\gamma\sum_{i=1}^{N} e_i^T(t-\tau)R_i^{-1}{}_ie_i(t-\tau)\\
&\le\gamma\sum_{i=1}^{N} e_i^T(t)P_i\Gamma R_i\Gamma^TP_ie_i(t)\\
&+\frac{\gamma\lambda_{\max}(P_i^{-1}R_i^{-1})}{\beta_k}\sum_{i=1}^{N} e_i^T(t)P_ie_i(t).
\end{aligned}
$$

Then we have

$$
\begin{aligned}
&D^+V\big(t,e(t)\big)\\
&\le\sum_{i=1}^{N} e_i^T(t)\Big[M^T(z)P_i+P_iM(z)\\
&+c\sum_{j=1}^{N} c_{ij}^2(t)\|P_iA(t)\|^2I+cNI\\
&+\gamma P_i\Gamma R_i\Gamma^TP_i+\frac{\gamma\lambda_{\max}(P_i^{-1}R_i^{-1})}{\beta_k}P_i\\
&-2\gamma P_i\Gamma-2d^*P_i\Big]e_i(t)\\
&=\sum_{i=1}^{N} e_i^T(t)\Omega_ie_i(t)\\
&\le\sup\big[\lambda_{\max}\big(P_i^{-1}\Omega_i\big)\big]\sum_{i=1}^{N} e_i^T(t)P_ie_i(t).
\end{aligned}
$$

Let $g(t)=1$, $H(s)=\sup[\lambda_{\max}(P_i^{-1}\Omega_i)]s$, then

$$
\begin{aligned}
&\int_{\varphi(\mu)}^{\mu}\frac{ds}{H(s)}-\int_{t_{k-1}}^{t_k} g(s)\,ds\\
&=-\frac{\ln\beta_k}{\sup[\lambda_{\max}(P_i^{-1}\Omega_i)]}-(t_k-t_{k-1})>0.
\end{aligned}
$$

This implies that the error system (6) is globally asymptotically stable about zero. Therefore, the adaptive-impulsive projective synchronization of the drive-response delayed dynamical networks (2) is achieved. The proof is completed. □

Let the impulses be equidistant and separated by interval $t_k-t_{k-1}=\Delta$, $P_i=R_i=\Gamma=I$, $B_{i_k}=bI$ in Theorem 1, the following Corollary 1 holds.

Corollary 1 *Let* $\omega = \sup[\lambda_{\max}(M^T(z) + M(z))]$. *If there exist a constant* b *such that the following conditions hold*:

(i) $-2 < b < 0$, $\beta_k = (1+b)^2$,
(ii) $\Omega > 0$,
(iii) $\Omega\Delta + \ln(1+b)^2 < 0$,

where $\Omega = \omega + \sup[c\sum_{j=1}^{N} c_{ij}^2(t)\|P_i A(t)\|^2] + cN - \gamma - 2d^* + \frac{\gamma}{(1+b)^2}$, *then the drive-response delayed networks with time-varying coupling can realize the projective synchronization with the desired scaling factor* α.

When the time delay $\tau = 0$, the following Corollary 2 is easily obtained.

Corollary 2 *Let* $\omega = \sup[\lambda_{\max}(M^T(z) + M(z))]$. *If there exist a constant* b *such that the following conditions hold*:

(i) $-2 < b < 0$, $\beta_k = (1+b)^2$,
(ii) $\Omega' > 0$,
(iii) $\Omega'\Delta + \ln(1+b)^2 < 0$,

where $\Omega' = \omega + \sup(c\sum_{j=1}^{N} c_{ij}^2(t)\|P_i A(t)\|^2 + cN - 2d^*)$, *then the drive-response networks with time-varying coupling can realize the projective synchronization with the desired scaling factor* α.

The proof of Corollary 2 is more or less similar with the proof of Theorem 1, thus we omit it.

Remark 1 The conditions given by Theorem 1 and corollaries do not assume that the coupling matrix $C(t)$ is symmetric and irreducible and its off-diagonal elements are nonnegative, which can be applied to more real-world dynamical networks.

Remark 2 In [26], the authors discussed the impulsive control of the projective synchronization of the drive-response dynamical network model without time delay, where the network is static and undirected. However, this paper investigates the projective synchronization of a drive-response delayed dynamical network with time-varying coupling by employing the adaptive-impulsive control. Moreover, it is clear that if $\tau = 0$, $A(t) = I_n$ and $C(t) = C$ is a constant symmetric irreducible matrix with nonnegative off-diagonal elements, then the time-varying coupled network become the static coupled network. Therefore, we can regard Theorem 3 in [26] as the special case of Theorem 1.

4 Numerical simulation

In this section, we give three examples to illustrate the theoretical results obtained in the previous section, that is, the drive-response time-varying coupling networks can be synchronized to within a desired scaling factor α via adaptive-impulsive control. To verify and demonstrate the effectiveness of the proposed methods, we consider the time-delay Lorenz chaotic system as the drive system.

The Lorenz system with a time-delay is described by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \sigma(y-x) \\ (a-z)x - y \\ xy - ez \end{pmatrix} + \gamma \begin{pmatrix} x(t-\tau) - x \\ y(t-\tau) - y \\ z(t-\tau) - z \end{pmatrix}, \tag{7}$$

where $\gamma = 10$, $\tau = 5$, $\sigma = 16$, $a = 40$, $e = 4$.

Firstly, the drive-response delayed network systems with time-varying coupling are described as follows:

$$\begin{cases} \dot{u}_d(t) = M(z)\cdot u_d(t) + 10(u_d(t-5) - u_d(t)), \\ \dot{z}(t) = x(t)y(t) - ez(t) + 10(z(t-5) - z(t)), \\ \dot{u}_{ri}(t) = M(z)\cdot u_{ri}(t) + 10(u_{ri}(t-5) - u_{ri}(t)) \\ \qquad + c\sum_{j=1}^{5} c_{ij}(t)A(t)u_{rj}(t) - d_i e_i, \\ \quad t \neq t_k, \\ \Delta u_{ri} = u_{ri}(t_k^+) - u_{ri}(t_k^-) = B_{i_k}[u_{ri} - \alpha u_d], \\ \quad t = t_k, k = 1, 2, \dots, \end{cases}$$

$$i = 1, 2, \dots, 5, \tag{8}$$

where

$$M(z) = \begin{pmatrix} -\sigma & \sigma \\ a - z & -1 \end{pmatrix}.$$

Choosing the time-varying coupling configuration matrices:

$$C(t) = \begin{pmatrix} -3\sin t & -1 & 0 & 3\sin t & 1 \\ \sin t & -\sin t - 2\cos t & 0 & 0 & 2\cos t \\ 0 & \sin t & 0 & -\sin t & 0 \\ 0 & \sin t\cos t & 1 & -\sin t\cos t & -1 \\ -2 & 2\sin t & 0 & -1 & 3 - 2\sin t \end{pmatrix}$$

and

$$A(t) = \begin{pmatrix} 1 + e^{-t} & 0 \\ 0 & -0.5\cos t \end{pmatrix}.$$

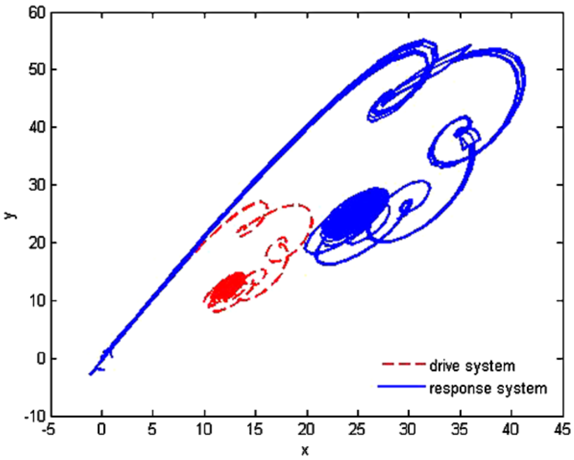

Fig. 1 The trajectories of projective synchronization in the x–y plane with $\alpha = 2$, $\tau = 5$. *The dash line* and *the solid line* represent phase graph of drive system and response system, respectively

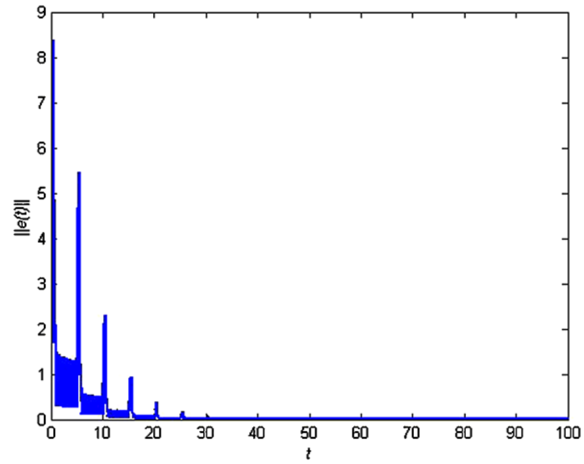

Fig. 2 Projective synchronization error $\alpha = 2$, $\tau = 5$

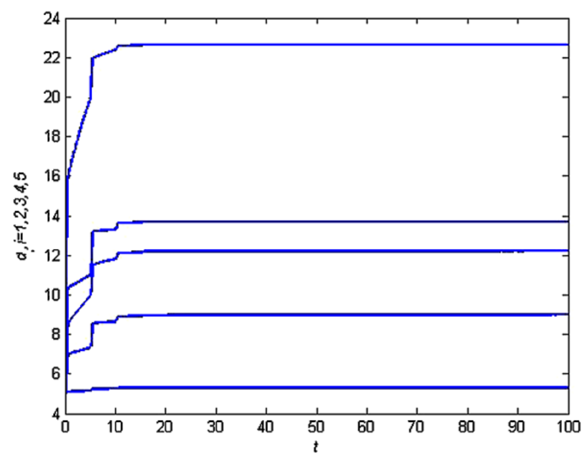

Fig. 3 The evolution of the feedback strength d_i, $i = 1, 2, \ldots, 5$

For simplicity, in the numerical simulations, we assume $c = 0.1$, $k_i = 1$, $P_i = R_i = I_2$, $B_{i_k} = \mathrm{diag}\{-0.79, -0.79\}$, $\rho_k = (1+b)^2 = 0.0441 > 0$, $d_{i0} = 5$. After calculations, getting $\lambda_{\max}(M^T(z) + M(z)) = -(\sigma+1) + \sqrt{(\sigma-1)^2 + (\sigma+a-z)^2} < 41$, $\omega = 41$, $\Omega = \omega + \sup(c\sum_{j=1}^{N} c_{ij}^2(t)\|A(t)\|^2) + cN - \gamma - 2d^* + \frac{\gamma}{(1+b)^2} = 250.9574$, then one has $0 < \Delta < -\frac{\ln(1+b)^2}{\Omega} < -\frac{\ln 0.0441}{250.9574} = 0.0124$. Taking the impulsive interval $\Delta = t_{k+1} - t_k = 0.01$, then it is easy to verify that all conditions in Corollary 1 are satisfied. Thus, the drive system and the response networks synchronized to within a desired scaling factor α. Figure 1 displays the projective synchronization trajectory of the drive-response dynamical networks with desired scaling factor $\alpha = 2$. Figure 2 shows the error between projection trajectories $\|e(t)\| = \sqrt{(x_{i1} - \alpha x_1)^2 + (y_{i2} - \alpha y_2)^2}$, $i = 1, 2, \ldots, N$. The evolution of the feedback strength d_i is shown in Fig. 3. Figure 4 displays the trajectories of state variables. The numerical results show that the adaptive-impulsive controlling scheme for the drive-response delayed complex dynamical network model with time-varying coupling is effective.

Remark 3 Furthermore, if we only adopt impulsive control strategy, the other conditions are chosen as the mentioned above, the synchronization error $\|e(t)\|$ is shown in Fig. 5. Clearly, from the Figs. 2 and 5, it is easy to find that the impulsive control effect is not as well as the adaptive-impulsive control method.

Particularly, if $\tau = 0$, the delayed system (7) becomes the Lorenz system. By Corollary 2, the projective synchronization can be achieved via the adaptive-impulsive control. The other numerical conditions are the same as above, one has $\Omega' = 34.2$, we choose impulsive interval $0 < \Delta < -\frac{\ln(1+b)^2}{\Omega'} = 0.0913$, the conditions in Corollary 2 are satisfied, the projective synchronization can be obtained with the desired scaling factor $\alpha = -2$, as shown in Figs. 6, 7, and 8.

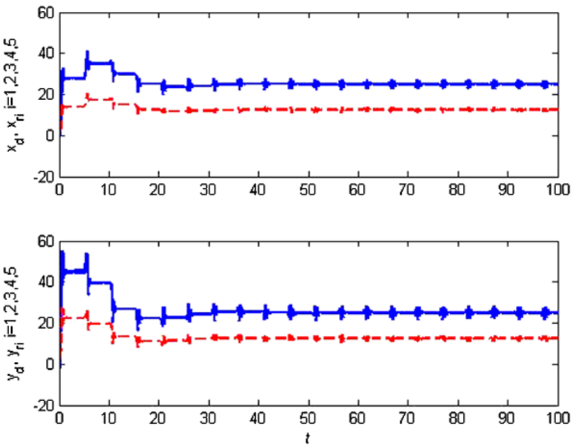

Fig. 4 State trajectories of drive system (*the dash line*) and response system (*the solid line*) $\alpha = 2$, $\tau = 5$

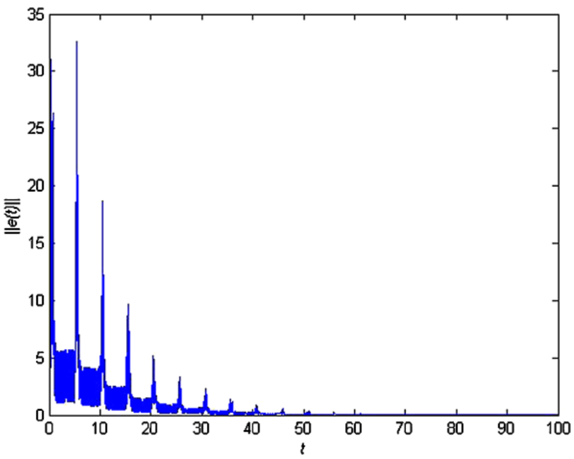

Fig. 5 Projective synchronization error under the impulsive control

Then we consider a small-world drive-response dynamical network with time delays dynamical nodes and time-varying coupling

$$\begin{cases} \dot{u}_d(t) = M(z) \cdot u_d(t) + 10(u_d(t-5) - u_d(t)), \\ \dot{z}(t) = x(t)y(t) - ez(t) + 10(z(t-5) - z(t)), \\ \dot{u}_{ri}(t) = M(z) \cdot u_{ri}(t) + 10(u_{ri}(t-5) - u_{ri}(t)) \\ \qquad + c\sum_{j=1}^{100} c_{ij} A(t) u_{rj}(t) - d_i e_i, \\ \quad t \neq t_k, \\ \Delta u_{ri} = u_{ri}(t_k^+) - u_{ri}(t_k^-) = B_{i_k}[u_{ri} - \alpha u_d], \\ \quad t = t_k, k = 1, 2, \ldots, \end{cases}$$

$$i = 1, 2, \ldots, 100. \tag{9}$$

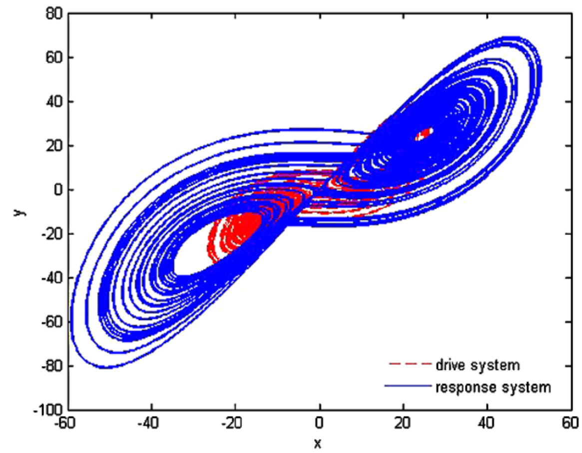

Fig. 6 The trajectories of projective synchronization in the x–y plane with $\alpha = -2$, $\tau = 0$. *The dash line* and *the solid line* represent phase graph of drive system and response system, respectively

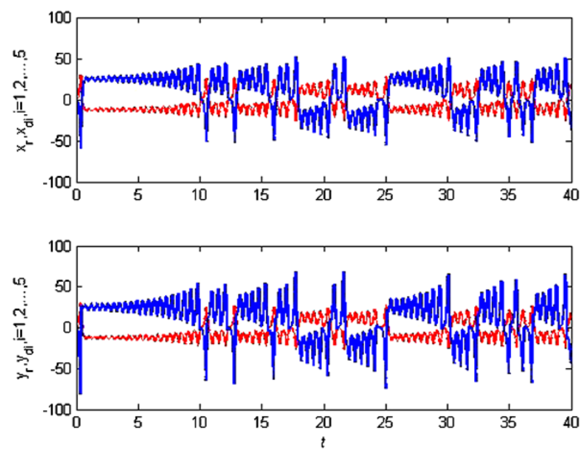

Fig. 7 State trajectories of drive system (*the dash line*) and response system (*the solid line*) $\alpha = -2$, $\tau = 0$

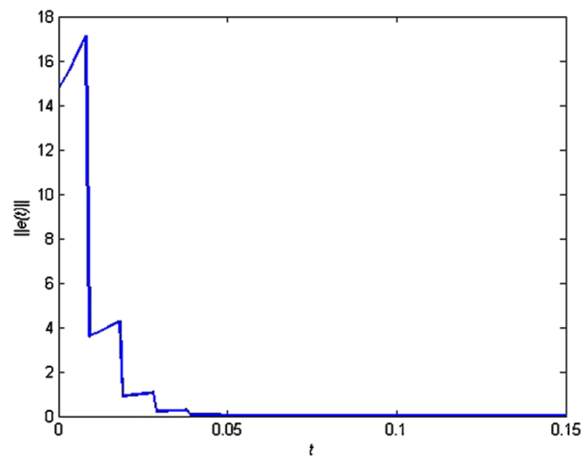

Fig. 8 Projective synchronization error $\alpha = -2$, $\tau = 0$

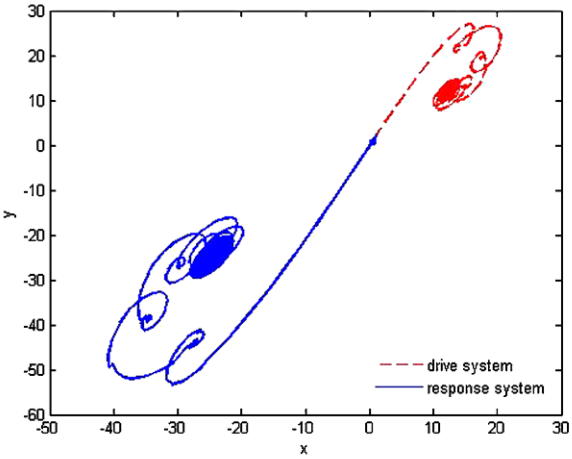

Fig. 9 The trajectories of projective synchronization in the x–y plane with $\alpha = -2$, $\tau = 5$. *The dash line* and *the solid line* represent phase graph of drive system and response system, respectively

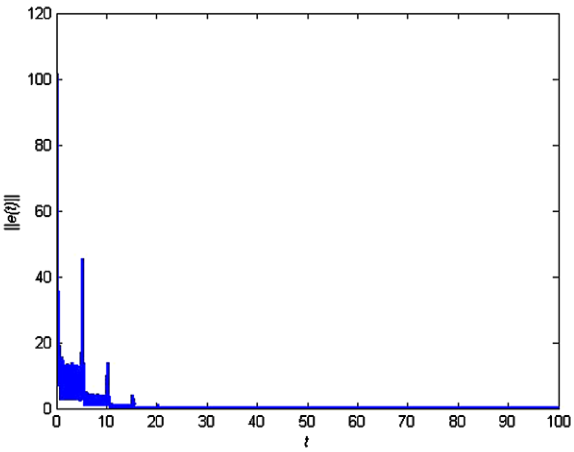

Fig. 10 Projective synchronization error $\alpha = -2$, $\tau = 5$

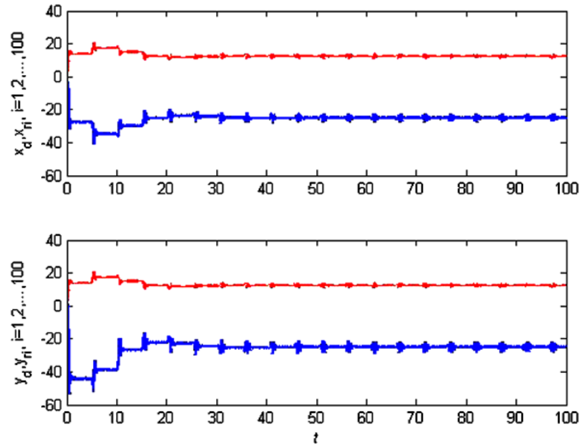

Fig. 11 State trajectories of drive system (*the dash line*) and response system (*the solid line*) $\alpha = -2$, $\tau = 5$

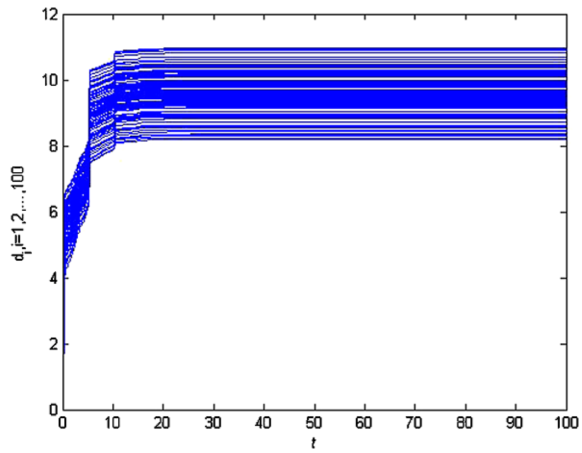

Fig. 12 The evolution of the feedback strength d_i, $i = 1, 2, \ldots, 100$

Here, let the parameters $N = 100$, $K = 2$, and $p = 0.1$, and network (9) with small-world connections can be randomly generated according to the rule [40]. Then the coupling matrix C is given as follows:

$$C = \begin{pmatrix} -98 & 0 & 1 & \cdots & 1 \\ 1 & -98 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 1 & \cdots & \cdots & -98 & 0 \\ 0 & \cdots & 1 & 1 & -98 \end{pmatrix}_{100\times 100}. \quad (10)$$

The projective synchronization can be obtained with the desired scaling factor $\alpha = -2$ (see Figs. 9, 10, 11, and 12).

5 Conclusion

In this paper, the projective synchronization of the drive-response dynamical network model with time delays dynamical nodes and time-varying coupling has been investigated by using the adaptive-impulsive control. Based on the stability analysis of impulsive system, some sufficient conditions for realizing the projective synchronization with the desired scaling factor α are established. Moreover, numerical simulations have also been given to show the effectiveness and the correctness of the theoretical analysis finally. From the simulation results, it is easy to observe that the only impulsive control effective is not as well as the adaptive-impulsive control strategy.

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