Nonlinear Dyn (2012) 67:1729–1736
DOI 10.1007/s11071-011-0100-9

ORIGINAL PAPER

Adaptive projective synchronization of dynamical networks with distributed time delays

Pengchun Rao · Zhaoyan Wu · Meng Liu

Received: 22 April 2011 / Accepted: 19 May 2011 / Published online: 11 June 2011

Abstract In this paper, the adaptive projective synchronization of dynamical network with distributed time delays is investigated. Network with unknown topology and network with both unknown topology and system parameters of node dynamics are considered respectively. Based on Lyapunov stability theory and LaSalle's invariance principle, the sufficient conditions for achieving projective synchronization are obtained. Numerical examples are provided to show the effectiveness of the proposed method.

Keywords Projective synchronization · Distributed time delays · Dynamical network

1 Introduction

In virtue of the wide applications in many fields of complex networks, such as physics, World Wide Web, communication networks, social networks, neural networks, epidemic network, traffic networks, etc., it has draw increasing attention by researchers. Since the small-world property [1] and scale-free property [2] of complex network were proposed, complex network has been developed particular rapidly and gradually become a newborn subject.

Synchronization as a kind of collective dynamical behavior of complex network has been extensively investigated in various fields of engineering and sociology [3–12]. To some extent, synchronization is a phenomenon that a drive system and a response system via coupling signals synchronize with each other. During the last decades, many kinds of synchronization have been proposed, such as complete synchronization [13], generalized synchronization [14], projective synchronization [15–19], etc., and many network models have been introduced to describe the real world more realistic [20–23]. In [22], Zheng et al. considered projective synchronization of complex networks with time-varying coupling delay. In [23], Li and Yang investigated the network with distributed time delay and proposed a sufficient condition for the occurrence of global exponential synchronization.

Due to the complexity of the real world, not all the system parameters are well known beforehand, i.e., there exist some unknown or uncertain parameters, such as the topology of network, the coupling delays and the parameter in the individual node system. Then, how to synchronize the networks with unknown parameters and identify the unknown parameters effectively becomes important and valuable task. In [16], Jia et al. studied generalized projective synchronization of a class of chaotic systems with uncertain pa-

P. Rao · Z. Wu (✉)
College of Mathematics and Information Science, Jiangxi Normal University, Nanchang, China
e-mail: zhywu7981@gmail.com

M. Liu
Jiaxing Nanyang Vocational and Technical College, Jiaxing, China

rameters. In [24], Liu et al. studied projectively lag synchronization of a new hyper-chaotic system with uncertain parameters.

The complex network with distributed time delays is usually used to describe the biological neural network, which has a distribution of transmission delays due to the presence of parallel pathways with a variety of axon sizes and lengths and cannot be modeled by complex networks with discrete time delays. Motivated by the above discussions, in this paper we consider adaptive projective synchronization of the dynamical networks with distributed time delays and unknown parameters. Through designing proper controllers and adaptive updating laws, the projective synchronization can be realized and the unknown parameters can be identified quickly and effectively under several assumptions.

The rest of this paper is organized as follows. In Sect. 2, the complex dynamical network model with distributed delays and some preliminaries are presented. In Sect. 3, the projective synchronization of the presented network with unknown parameters is considered. Firstly, only the topology of network is assumed to be unknown, based on LaSalle's invariance principle [25] and Lyapunov function method [26], the sufficient conditions for projective synchronization are obtained by designing proper controllers and the parametric update laws. Secondly, both the topology of network and the system parameter of node dynamics are assumed to be unknown, the sufficient conditions for projective synchronization are obtained as well. In Sect. 4, several numerical examples are given to verify the effectiveness of the derived result. Some conclusions are drawn in Sect. 5.

Notation Throughout this paper, for symmetric matrix P, the notation $P > 0$ ($P < 0$) means that the matrix P is positive definite (negative definite).

2 Model and preliminaries

In this section, a general dynamical network model is introduced and some preliminaries are presented. Consider the networks with distributed time delays consisting of N identical coupled nodes; the drive network can be described by

$$\dot{x}_i(t) = f\big(x_i(t)\big) + \sum_{j=1}^{N} a_{ij}\Gamma \int_0^{\tau} k(u)x_j(t-u)\,du, \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$, $i = 1, 2, \dots, N$, denotes the state vector of the ith node; $f : R^n \to R^n$ is a vector function and continuously differentiable; $\Gamma = diag\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ is the $n \times n$ real inner-coupling matrix with $\gamma_i \geq 0$ for any $i = 1, 2, \dots, n$; $\tau > 0$ is time delay, and the continuous density function $k : [0, \tau] \to [0, +\infty]$ is prescribed which satisfies $\int_0^{\tau} k(u)\,du = 1$. $A = (a_{ij})_{N\times N} \in R^{N\times N}$ is the coupling matrix, in which $a_{ij} \neq 0$ if there is a connection from node i to j ($i \neq j$); otherwise, $a_{ij} = 0$ ($i \neq j$) and the diagonal elements of the matrix A satisfy $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, $i = 1, 2, \dots, N$.

In order to obtain the sufficient conditions for achieving projective synchronization, the following lemmas are needed.

Lemma 1 (LaSalle's invariance principle [25]) *Consider the general nonlinear autonomous system as follows*:

$$\dot{x} = f(x),$$

where $x \in R^n$ and $f(x) : R^n \to R^n$ is a locally Lipschitz mapping. Suppose that $\Omega \subset R^n$ is a invariant compact set of the above equation, and there is a continuously differentiable (time-independent) function $V(x) : R^n \to R$ whose orbital derivative $\dot{V}(x)$ is negative semidefinite. Let E be the union of the union of all complete orbits contained in

$$\big\{x \in R^n \mid \dot{V}(x) = 0\big\}$$

and M the largest invariant set included in E. Then, it is valid that the orbit $X(t)$ converges asymptotically to the set M as t approaches infinity for every $X(t_0) \in \Omega$.

Lemma 2 [23] *Let Q be a symmetric $N \times N$ real matrix and $Q > 0$. Then*

$$\begin{aligned}&\int_0^{\tau} k(u)\tilde{e}_j^T(t-u)Q\tilde{e}_j(t-u)\,du\\ &\quad \geq \left(\int_0^{\tau} k(u)\tilde{e}_j(t-u)\,du\right)^T\\ &\qquad \times Q\left(\int_0^{\tau} k(u)\tilde{e}_j(t-u)\,du\right).\end{aligned}$$

Lemma 3 (Schur complement [27]) *For a given symmetric matrix S with the form $S = [S_{ij}]$, $S_{11} \in R^{r\times r}$,*

$S_{12} \in R^{r\times(n-r)}$, $S_{22} \in R^{(n-r)\times(n-r)}$, *it follows that* $S < 0$ *if and only if* $S_{11} < 0$, $S_{22} - S_{21}S_{11}^{-1}S_{12} < 0$ *or* $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{21} < 0$.

3 Main results

In this section, the adaptive projective synchronization of dynamical network with distributed time delays and unknown parameters are investigated.

Firstly, we assume that only the topology of the network is unknown, i.e., the coupling matrix $A = (a_{ij})$ is unknown. Then the response network is chosen as

$$\dot{y}_i(t) = f\big(y_i(t)\big) + \sum_{j=1}^{N} b_{ij}(t)\Gamma\int_0^{\tau} k(u)y_j(t-u)\,du + u_i\big(x_i(t), y_i(t)\big), \tag{2}$$

where $b_{ij}(t)$, $i = 1, 2, \ldots, N$, are the estimations of a_{ij}, and $u_i(x_i(t), y_i(t))$ are the adaptive controllers.

Our objective here is synchronizing the response network (2) to the drive network (1) projectively by choosing proper controllers and parametric update laws. The following assumption is required.

Assumption 1 *It is assumed that* $y_1(t-u)$, $y_2(t-u)$, $\ldots, y_N(t-u)$ *are linearly independent. That is to say, there does not exist nonzero constants* m_j $(j = 1, 2, \ldots, N)$ *such that*

$$m_1y_1(t-u) + m_2y_2(t-u) + \cdots + m_Ny_N(t-u) = 0.$$

Let $e_i(t) = y_i(t) - \alpha x_i(t)$, *then one can obtain the following error network*:

$$\begin{aligned}
\dot{e}_i(t) &= \dot{y}_i(t) - \alpha\dot{x}_i(t)\\
&= f\big(y_i(t)\big) + \sum_{j=1}^{N}\Gamma\int_0^{\tau} k(u)y_j(t-u)\,du\\
&\quad - \alpha f\big(x_i(t)\big)\\
&\quad - \alpha\sum_{j=1}^{N} a_{ij}\Gamma\int_0^{\tau} k(u)e_j(t-u)\,du\\
&\quad + u_i\big(x_i(t), y_i(t)\big)\\
&= f\big(y_i(t)\big)\\
&\quad + \sum_{j=1}^{N}\big(b_{ij}(t) - a_{ij}\big)\Gamma\int_0^{\tau} k(u)y_j(t-u)\,du\\
&\quad + \sum_{j=1}^{N} a_{ij}\Gamma\int_0^{\tau} k(u)e_j(t-u)\,du - \alpha f\big(x_i(t)\big)\\
&\quad + u_i\big(x_i(t), y_i(t)\big).
\end{aligned}$$

Theorem 1 *Suppose that Assumption 1 holds. Choose the following controller and updating laws*:

$$\begin{aligned}
&u_i\big(x_i(t), y_i(t)\big) = -d_i(t)e_i + \alpha f\big(x_i(t)\big) - f\big(y_i(t)\big),\\
&\dot{d}_i(t) = k_i\|e_i\|^2,\\
&\dot{b}_{ij}(t) = -\gamma_{ij}e_i^T(t)\Gamma\int_0^{\tau} k(u)y_j(t-u)\,du,
\end{aligned} \tag{3}$$

where k_i *and* γ_{ij}, $i, j = 1, 2, \ldots, N$, *are arbitrary positive constants. Then the response system* (2) *and the drive system* (1) *can achieve the generalized projective synchronization and the unknown topology structure can be identified as well.*

Proof Substitute the controllers $u_i(x_i(t), y_i(t))$ into the error network

$$\begin{aligned}
\dot{e}_i(t) &= \sum_{j=1}^{N}\big(b_{ij}(t) - a_{ij}\big)\Gamma\int_0^{\tau} k(u)y_j(t-u)\,du\\
&\quad + \sum_{j=1}^{N} a_{ij}\Gamma\int_0^{\tau} k(u)e_j(t-u)\,du - d_ie_i(t).
\end{aligned} \tag{4}$$

Lyapunov function is constructed as

$$\begin{aligned}
V(t) &= \sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\gamma_{ij}}\big(b_{ij}(t) - a_{ij}\big)^2\\
&\quad + \sum_{j=1}^{n}\int_0^{\tau} k(u)\bigg(\int_{t-u}^{t}\tilde{e}_j^T(\theta)\tilde{e}_j(\theta)\,d\theta\bigg)du\\
&\quad + \sum_{i=1}^{N}\frac{1}{k_i}(d_i - d^*)^2,
\end{aligned}$$

where $\tilde{e}_j = (e_{1j}, e_{2j}, \ldots, e_{Nj})^T$, d^* is sufficiently large positive constant which is to be determined. Obviously, the Lyapunov function is non-negative. Then, the derivative of $V(t)$ with respect to t along the trajectories of (4) is

$$\begin{aligned}\dot{V}(t) &= 2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T a_{ij}\Gamma \int_0^\tau k(u)e_j(t-u)\,du \\ &\quad - \sum_{j=1}^{n}\int_0^\tau k(u)\tilde{e}_j^T(t-u)\tilde{e}_j(t-u)\,du \\ &\quad + (1-2d^*)\sum_{j=1}^{n}\tilde{e}_j^T\tilde{e}_j \\ &= 2\sum_{j=1}^{n}\tilde{e}_j^T\gamma_j A\int_0^\tau k(u)\tilde{e}_j(t-u)\,du \\ &\quad - \sum_{j=1}^{n}\int_0^\tau k(u)\tilde{e}_j^T(t-u)\tilde{e}_j(t-u)\,du \\ &\quad + (1-2d^*)\sum_{j=1}^{n}\tilde{e}_j^T\tilde{e}_j.\end{aligned}$$

According to Lemma 2, and choosing $Q = I_N$, one can obtain

$$\begin{aligned}\dot{V}(t) &\le 2\sum_{j=1}^{n}\tilde{e}_j^T\gamma_j A\int_0^\tau k(u)\tilde{e}_j(t-u)\,du \\ &\quad - \sum_{j=1}^{n}\left(\int_0^\tau k(u)\tilde{e}_j(t-u)\,du\right)^T \\ &\quad \times \left(\int_0^\tau k(u)\tilde{e}_j(t-u)\,du\right) \\ &\quad + (1-2d^*)\sum_{j=1}^{n}\tilde{e}_j^T\tilde{e}_j \\ &= \sum_{j=1}^{n} H^T\begin{bmatrix}(1-2d^*)I_N & \gamma_j A \\ \gamma_j A^T & -I_N\end{bmatrix}H,\end{aligned}$$

where

$$\mathbf{H} = \begin{bmatrix}\tilde{e}_j \\ \int_0^\tau k(u)\tilde{e}_j(t-u)\,du\end{bmatrix}. \tag{5}$$

According to Lemma 3 and Lemma 1, it is obvious that there exists a sufficiently large positive constant d^* such that $\dot{V}(t) < 0$ and the largest invariant set is contained in set $E = \{e_i(t) = 0,\ i = 1, 2, \ldots, N\}$. Therefore, set M could be described as

$$\begin{aligned}M = \Bigg\{e_i = 0, \sum_{j=1}^{N}\big(b_{ij}(t) - a_{ij}\big) \\ \times \Gamma\int_0^\tau k(u)y_j(t-u)\,du = 0\Bigg\}.\end{aligned}$$

The trajectory asymptotically converges to the largest invariant M. That is to say, systems (1) and (2) are asymptotically projectively synchronous starting with arbitrary initial values.

According to Assumption 1, one can also obtain $b_{ij}(t) \to a_{ij}$ for any $i, j = 1, 2, \ldots, N$, as $t \to \infty$. The largest set M can be described as $M = \{e_i = 0, b_{ij}(t) = a_{ij}\}$. The proof is completed. □

Secondly, we assume that both the topology of the network and the system parameters of node dynamics are unknown. Then rewrite the drive network as

$$\begin{aligned}\dot{x}_i(t) &= Gx_i(t) + g\big(x_i(t)\big) + D\big(x_i(t)\big)\Theta \\ &\quad + \sum_{j=1}^{N} a_{ij}\Gamma\int_0^\tau k(u)x_j(t-u)\,du,\end{aligned} \tag{6}$$

where for $i = 1, 2, \ldots, N$, $G \in R^{n\times n}$ is coefficient matrix, $g : R^n \to R^n$ is nonlinear function, $D : R^n \to R^{n\times p}$; $\Theta \in R^p$ is the unknown system parameter vector. Let $D(x_i(t)) = (d^1(x_i(t)), d^2(x_i(t)), \ldots, d^p(x_i(t))$, where $d^l(x(t)) \in R^{n\times 1}$, $l = 1, 2, \ldots, p$.

Then, the response network is chosen as

$$\begin{aligned}\dot{y}_i(t) &= Gy_i(t) + g\big(y_i(t)\big) + D\big(y_i(t)\big)\hat{\Theta} \\ &\quad + \sum_{j=1}^{N} b_{ij}(t)\Gamma\int_0^\tau k(u)y_j(t-u)\,du \\ &\quad + u_i'\big(x_i(t), y_i(t)\big),\end{aligned} \tag{7}$$

where for $i = 1, 2, \ldots, N$, $\hat{\Theta}$ and $b_{ij}(t)$ are the estimation of Θ and a_{ij} respectively, and $u_i'(x_i(t), y_i(t))$ are controllers to be designed.

Our objective here is synchronizing the response network (7) to the drive network (6) projectively by choosing proper controllers and parametric update laws. The following assumption is required.

Assumption 2 *Assume that $d^1(x_i(t)), d^2(x_i(t)), \ldots, d^p(x_i(t)), y_1(t-u), y_2(t-u), \ldots, y_N(t-u)$ are lin-*

early independent, i.e., there does not exist nonzero constants m_j $(j=1,2,\dots,p+N)$ such that

$$m_1 d^1\big(x_i(t)\big)+\cdots+m_P d^P\big(x_i(t)\big)+m_{(P+1)}y_1(t-u)$$
$$+\cdots+m_{(p+N)}y_N(t-u)=0,$$
$$i=1,2,\dots,N.$$

Theorem 2 *Let $e_i(t)=y_i(t)-\alpha x_i(t)$. If the controller vector in (7) is designed by*

$$\begin{aligned}u_i'\big(x_i(t),y_i(t)\big)&=-d_i(t)e_i(t)+G\big[\alpha x_i(t)-y_i(t)\big]\\&\quad+\big[\alpha D\big(x_i(t)\big)-D\big(y_i(t)\big)\big]\hat{\Theta}\\&\quad+\alpha g\big(x_i(t)\big)-g\big(y_i(t)\big),\end{aligned}$$

where $i=1,2,\dots,N$; the updating laws of $d_i(t)$ and $b_{ij}(t)$ are the same in (3); $\hat{\Theta}$ is updated according to the following laws:

$$\dot{\hat{\Theta}}=-\alpha\sum_{i=1}^{N}D^T\big(x_i(t)\big)e_i(t);$$

then, the networks (6) and (7) can globally asymptotically achieve the projective synchronization. $b_{ij}(t)$ and $\hat{\Theta}$ can dynamically estimate a_{ij} and Θ respectively for any $i,j=1,2,\dots,N$.

Proof The error network can be expressed as

$$\begin{aligned}\dot{e}_i(t)&=\dot{y}_i(t)-\alpha\dot{x}_i(t)\\&=-d_i e_i(t)+\sum_{j=1}^{N}a_{ij}\Gamma\int_0^{\tau}k(u)e_j(t-u)\,du\\&\quad+\sum_{j=1}^{N}\big(b_{ij}(t)-a_{ij}\big)\Gamma\int_0^{\tau}k(u)y_j(t-u)\,du\\&\quad+\alpha D\big(x_i(t)\big)\big(\hat{\Theta}-\Theta\big).\end{aligned}\tag{8}$$

Choose the following Lyapunov function:

$$\begin{aligned}V(t)&=\sum_{i=1}^{N}e_i^T(t)e_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\gamma_{ij}}\big(b_{ij}(t)-a_{ij}\big)^2\\&\quad+\sum_{j=1}^{n}\int_0^{\tau}k(u)\bigg(\int_{t-u}^{t}\tilde{e}_j^T(\theta)\tilde{e}_j(\theta)\,d\theta\bigg)du\\&\quad+\sum_{i=1}^{N}\frac{1}{k_i}(d_i-d^*)^2+\big\|\hat{\Theta}-\Theta\big\|^2,\end{aligned}\tag{9}$$

where $\tilde{e}_j$ and d^* denote the same as the Lyapunov function in the proof of Theorem 1. Similarly to the proof of Theorem 1, we can obtain the derivative of $V(t)$ with respect to t along the trajectories of (8):

$$\dot{V}(t)\le\sum_{j=1}^{n}H^T\begin{bmatrix}(1-2d^*)I_N & \gamma_j A\\ \gamma_j A^T & -I_N\end{bmatrix}H,$$

where H denotes the same as in (5). And the largest invariant set is

$$\begin{aligned}M=\bigg\{&e_i=0,\ \alpha D\big(x_i(t)\big)\big(\hat{\Theta}-\Theta\big)\\&+\sum_{j=1}^{N}\big(b_{ij}(t)-a_{ij}\big)\\&\times\Gamma\int_0^{\tau}k(u)y_j(t-u)\,du=0\bigg\}.\end{aligned}$$

According to Lemma 3 and Lemma 1, systems (6) and (7) are globally asymptotically generalized projective synchronous. Under Assumption 2, we can obtain $b_{ij}(t)\to a_{ij}$ for any $i,j=1,2,\dots,N$ and $\hat{\Theta}\to\Theta$, as $t\to\infty$. The proof is completed. □

4 Numerical examples

In this section, two numerical examples are provided to verify the effectiveness of the derived results. Consider a complex network with distributed time delays consisting of N identical nodes in Lü chaotic system [28], which can be described by

$$\begin{cases}\dot{x}_1=a(x_2-x_1),\\ \dot{x}_2=-x_1x_3+cx_2,\\ \dot{x}_3=x_1x_2-bx_3.\end{cases}$$

Example 1 Consider the drive network consisting of 5 nodes, which can be described by

$$\dot{x}_i(t)=f\big(x_i(t)\big)+\sum_{j=1}^{5}a_{ij}\Gamma\int_0^{\tau}k(u)x_j(t-u)\,du,$$

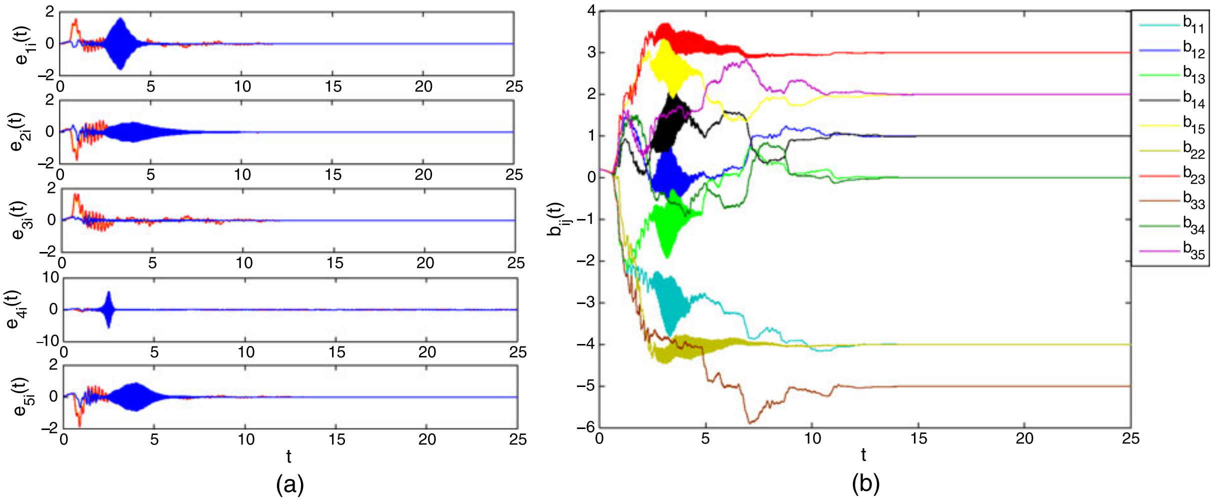

Fig. 1 (**a**) Synchronization errors e_i, (**b**) identification of network topology structure

where $a = 36$, $b = 3$, $c = 20$. Then the controlled response network is

$$\begin{aligned}\dot{y}_i(t) &= f\big(y_i(t)\big) + \sum_{j=1}^{5} b_{ij}(t)\Gamma \int_0^\tau k(u)y_j(t-u)\,du \\ &\quad + u_i\big(x_i(t), y_i(t)\big), \\ u_i\big(x_i(t), y_i(t)\big) &= -d_i(t)e_i + \alpha f\big(x_i(t)\big) - f\big(y_i(t)\big), \\ \dot{d}_i(t) &= k_i\|e_i\|^2, \\ \dot{b}_{ij}(t) &= -\gamma_{ij} e_i^T \Gamma \int_0^\tau k(u)y_j(t-u)\,du.\end{aligned}$$

For simplicity, we choose $\Gamma = diag(1,1,1)$, $k(u) = \frac{u}{2}$ and $\tau = 2$. And the unknown coupling matrix A is assumed to be

$$\mathbf{A} = \begin{bmatrix} -4 & 1 & 0 & 1 & 2 \\ 1 & -4 & 3 & 0 & 0 \\ 0 & 3 & -5 & 0 & 2 \\ 1 & 0 & 0 & -1 & 0 \\ 2 & 0 & 2 & 0 & -4 \end{bmatrix}.$$

In the numerical simulation, we choose scaling factor $\alpha = -3$, $k_i = 1$ and $\gamma_{ij} = 1$ for $i, j = 1, 2, \ldots, 5$. The initial values $b_{ij}(0) = 0.2$ and $d_i(0) = 0.5$ for all $i, j = 1, 2, \ldots, 5$. Figure 1(a) shows the synchronization errors $e_i(t) = y_i(t) - \alpha x_i(t)$ $(1 \le i \le 5)$ and Fig. 1(b) shows the identification of network topology structure.

Example 2 Rewrite the drive network consisting of 5 nodes in the form (6), and we have

$$\begin{aligned}\dot{x}_i(t) &= g\big(x_i(t)\big) + D\big(x_i(t)\big)\Theta \\ &\quad + \sum_{j=1}^{5} a_{ij}\Gamma \int_0^\tau k(u)x_j(t-u)\,du,\end{aligned}$$

where $g(x_i(t)) = (0, -x_{i1}(t)x_{i3}(t), x_{i1}(t)x_{i2}(t))^T$,

$$D\big(x_i(t)\big) = \begin{bmatrix} x_{i2}(t) - x_{i1}(t) & 0 & 0 \\ 0 & 0 & x_{i2}(t) \\ 0 & -x_{i3}(t) & 0 \end{bmatrix}.$$

$\Theta = (a, b, c)^T$ is unknown system parameter vector to be estimated. Then, the corresponding controlled response network is

$$\begin{aligned}\dot{y}_i(t) &= g\big(y_i(t)\big) + D\big(y_i(t)\big)\hat{\Theta}(t) + u_i'\big(x_i(t), y_i(t)\big) \\ &\quad + \sum_{j=1}^{N} b_{ij}(t)\Gamma \int_0^\tau k(u)y_j(t-u)\,du, \\ u_i'(x_i, y_i) &= \alpha g\big(x_i(t)\big) - g\big(y_i(t)\big) \\ &\quad + \big[\alpha D\big(x_i(t)\big) - D\big(y_i(t)\big)\big]\hat{\Theta} - d_i e_i(t), \\ \dot{d}_i(t) &= k_i\|e_i\|^2, \\ \dot{b}_{ij}(t) &= -\gamma_{ij} e_i^T \Gamma \int_0^\tau k(u)y_j(t-u)\,du, \\ \dot{\hat{\Theta}}(t) &= -\alpha \sum_{i=1}^{N} D^T\big(x_i(t)\big)e_i(t).\end{aligned}$$

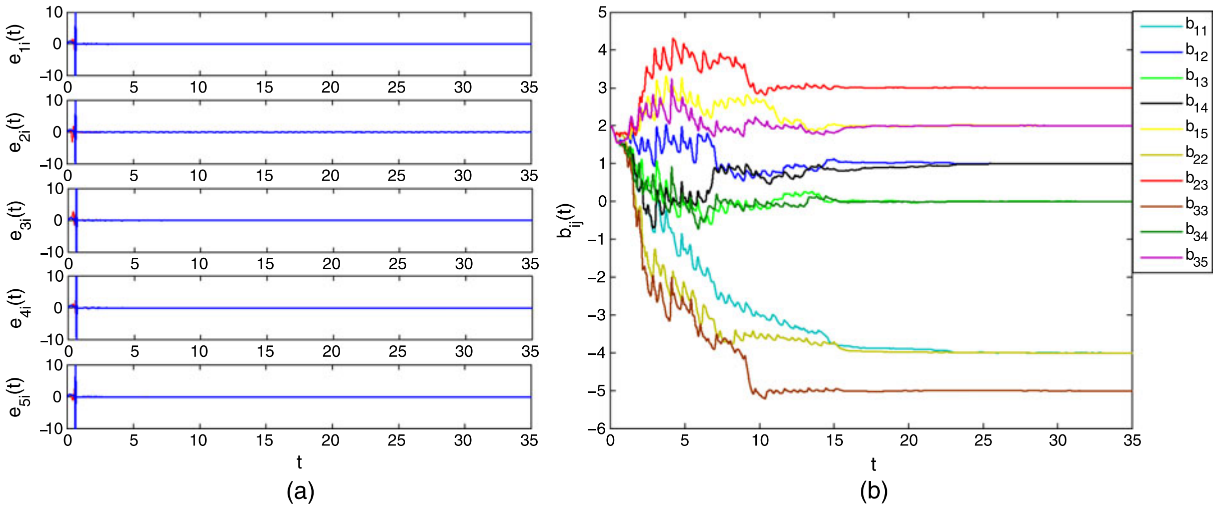

Fig. 2 (**a**) Synchronization errors e_i, (**b**) identification of network topology structure

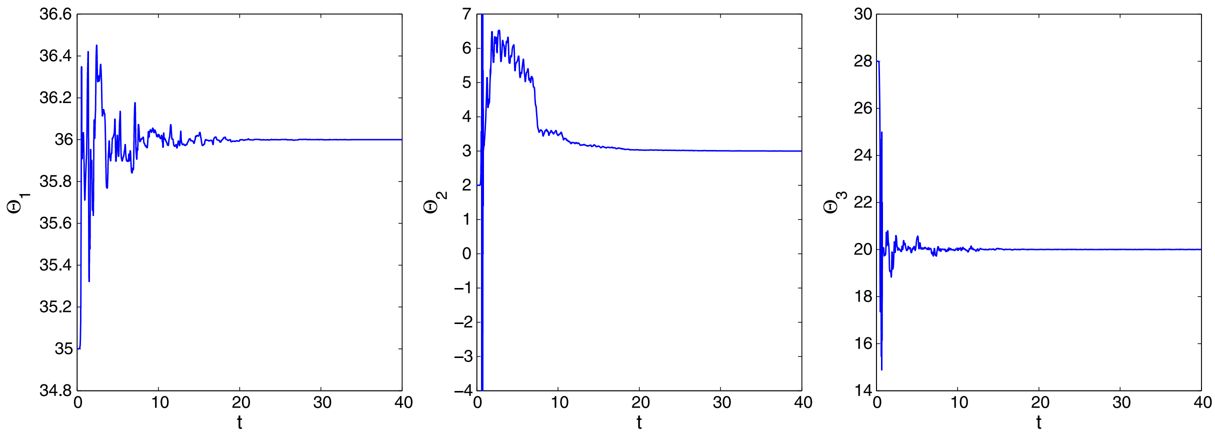

Fig. 3 Identification of system parameters $\hat{\Theta} = (\Theta_1, \Theta_2, \Theta_3)^T$

For simplicity, we choose $\Gamma = diag(1,1,1)$, $k(u) = \frac{u}{2}$ and $\tau = 2$. The unknown system parameter vector is assumed to be $\Theta = (36, 3, 20)^T$ and the unknown coupling matrix A is assumed to be

$$\mathbf{A} = \begin{bmatrix} -4 & 1 & 0 & 1 & 2 \\ 1 & -4 & 3 & 0 & 0 \\ 0 & 3 & -5 & 0 & 2 \\ 1 & 0 & 0 & -1 & 0 \\ 2 & 0 & 2 & 0 & -4 \end{bmatrix}.$$

In the numerical simulation, we choose scaling factor $\alpha = 5$, $k_i = 1$ and $\gamma_{ij} = 1$ for $i, j = 1, 2, \ldots, 5$. The initial values $\hat{\Theta}(0) = (30, 2, 25)^T$, $b_{ij}(0) = 2$ and $d_i(0) = 0.5$ for all $i, j = 1, 2, \ldots, 5$. Figure 2(a) shows the synchronization errors and Fig. 2(b) shows the identification of network topology structure. Figure 3 shows the identification of the unknown system parameters Θ.

5 Conclusions

In this paper, adaptive projective synchronization of dynamical networks with distributed time delays and unknown parameters has been investigated. Base on the Lyapunov stability theory and LaSalle's invariance principle, the sufficient conditions for projective synchronization between a drive network and a response network have been obtained. Meanwhile, the unknown network topology structure and system parameters can be identified effectively. Two typical numerical simulations are given to verify the theoretical analysis.

Acknowledgements This work is supported jointly by the Startup Fund for Ph.D. of Jiangxi Normal University (3087) and the Innovation Foundation for Graduate of Jiangxi Province.

References

1. Watts, D.J., Strogatz, S.H.: Collective dynamics of 'small-world' networks. Nature **393**, 440–442 (1998)
2. Barabasi, A.-L., Albert, R.: Emergence of scaling in random networks. Science **286**, 509–512 (1999)
3. Li, C., Sun, W., Kurths, J.: Synchronization between two coupled complete networks. Phys. Rev. E **76**, 046204 (2007)
4. Zhou, J., Lu, J.A., Lü, J.: Pinning adaptive synchronization of a general complex dynamical network. Automatica **44**, 996–1003 (2008)
5. Tang, H., Chen, L., Lu, J.A., Tse, C.: Adaptive synchronization between two complex networks with nonidentical topological structures. Physica A **387**, 5623–5630 (2008)
6. He, G., Yang, J.: Adaptive synchronization in nonlinearly coupled dynamical networks. Chaos Solitons Fractals **38**, 1254–1259 (2008)
7. Lu, J., Cao, J.: Adaptive synchronization in tree-like dynamical networks. Nonlinear Anal., Real World Appl. **8**, 1252–1260 (2007)
8. Zhou, J., Xiang, L., Liu, Z.: Global synchronization of generalized complex networks with mixed coupling delays. Physica A **385**, 729–742 (2007)
9. Sharma, B.B., Kar, I.N.: Observer-based synchronization scheme for a class of chaotic systems using contraction theory. Nonlinear Dyn. **63**, 429–445 (2011)
10. Amritkar, R.E., Hu, C.K.: Synchronized state of coupled dynamics on time-varying networks. Chaos **16**, 015117 (2006)
11. Li, X., Chen, G.: Synchronization and desynchronization of complex dynamical networks. IEEE Trans. Circuits Syst. I **50**, 1381–1390 (2003)
12. Wang, X., Chen, G.: Synchronization in small-world dynamical networks. Int. J. Bifurc. Chaos Appl. Sci. Eng. **12**, 187–192 (2002)
13. Mahmoud, G.M., Mahmoud, E.E.: Complete synchronization of chaotic complex nonlinear systems with uncertain parameters. Nonlinear Dyn. **62**, 875–882 (2010)
14. Lü, L., Li, C.: Generalized synchronization of spatiotemporal chaos in a weighted complex network. Nonlinear Dyn. **63**, 699–710 (2011)
15. Mainieri, R., Rehacek, J.: Projective synchronization in three-dimensional chaotic systems. Phys. Rev. Lett. **82**, 3042–3045 (1999)
16. Jia, Z., Lu, J.A., Deng, G., Zhang, Q.: Generalized projective synchronization of a class of chaotic (hyperchaotic) systems with uncertain parameters. Chin. Phys. B **16**, 1246–1251 (2007)
17. Li, R.H., Xu, W., Li, S.: Adaptive generalized projective synchronization in different chaotic systems based on parameter identification. Phys. Lett. A **367**, 199–206 (2007)
18. Feng, C.F.: Projective synchronization between two different time-delayed chaotic systems using active control approach. Nonlinear Dyn. **62**, 453–459 (2010)
19. Li, C.D., Liao, X.F., Yang, X.F., Huang, T.W.: New algebraic conditions for global exponential stability of delayed recurrent neural networks. Chaos **15**, 043103 (2005)
20. Chavez, M., Hwang, D.-U., Amann, A., Hentschel, H.G.E., Boccaletti, S.: Synchronization is enhanced in weighted complex networks. Phys. Rev. Lett. **94**, 218701 (2005)
21. Zhou, J., Chen, T.: Synchronization in general complex delayed dynamical networks. IEEE Trans. Circuits Syst. I **53**, 733–744 (2006)
22. Zheng, S., Bi, Q., Cai, G.: Adaptive projective synchronization in complex networks with time-varying coupling delay. Phys. Lett. A **373**, 1553–1559 (2009)
23. Li, C.H., Yang, S.Y.: Synchronization in linearly coupled dynamical networks with distributed time delays. Int. J. Bifurc. Chaos Appl. Sci. Eng. **18**, 2039–2047 (2008)
24. Liu, H., Lu, J., Zhang, Q.: Projectively lag synchronization and uncertain parameters identification of a new hyperchaotic system. Nonlinear Dyn. **62**, 427–435 (2010)
25. Hassan, K.K.: Nonlinear Systems. Prentice Hall, New York (2002)
26. Robinson, R.C.: An Introduction to Dynamical Systems: Continuous and Discrete. Pearson Education, Upper Saddle River (2004)
27. Jia, Y.: Robust H_∞ Control. Science Press, Beijing (2007)
28. Lü, J., Chen, G.: A new chaotic attractor coined. Int. J. Bifurc. Chaos Appl. Sci. Eng. **12**, 659–661 (2002)