

Identifying topology of synchronous networks by analyzing their transient processes

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Received: 3 November 2010 / Accepted: 10 May 2011 / Published online: 2 July 2011
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Abstract We explore a process for identifying the topology of networks. We find that it is possible to estimate the accurate topological structure of synchronous networks by analyzing their transient processes. Some novel conditions are given to ensure the uncertain connection topology approach to the true value. Our examples further illustrate the feasibility of these proposed methods.

Keywords Topology identification · Synchronization · Adaptive control · Complex networks

1 Introduction

The study of complex networks pervades all of science and our lives, ranging from physics to computer sci-

ence, from biology to meteorology, from transportation to the Internet, to mention a few [1, 2]. Until now, the research on complex networks has focused on dynamical analysis, control, and modeling [3–14]. The prominent issue of a network is structural, because structure always affects function. From this perspective, the researches [9–13] on identifying the topological structure of a network have attracted increasing attention in recent years. Indeed, it is crucial to solving many problems in real-world phenomena, such as the monitoring of neural networks and social networks [14, 17, 18].

So far, various studies on identifying either a single dynamical system [19–26, 31–33, 35–37] or a network topology mainly employ the adaptive control method [9–40, 44–47]. Therein, previously, a persistent excitation (PE) condition was widely applied to ensure parametric convergence. And then, recently, a linear independence (LI) condition [26, 31] is addressed for estimating parameters. In [32], based on the Gram-matrix theory, the special relationship between the persistent excitation condition and the linear independence condition was presented. Naturally, these conditions can be extended to guaranteeing the topology of networks converge to the true values.

In this paper, we explore the influence of the transient process in synchronization phenomenon on the result of network topology identification. As long as some conditions are satisfied, we find that it is possible to estimate the precise connection topology during the transient process. This issue has not been discussed in

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prior works and cannot be explained by the traditional linearly independent condition or the persistent excitation condition. We specifically study this process, propose some new conditions, and further give our explanation in detail. Two premises should be ensured as follows: (i) some special conditions should be satisfied and (ii) the persistence time (i.e., the time range which the special conditions are satisfied) is much longer than the transient time which the estimated network runs from the initial state to the stable state. Then the uncertain connection topology could approach to the true value. Experiments will further illustrate the validity of our proposed methods.

The outline of this paper is as follows. In Sect. 2, we present a scheme for identifying the topology of a network. Section 3 analyzes the feasibility of successful identification during the transient process in synchronization phenomenon. Some examples are shown for further verification in Sect. 4. Some discussions and conclusions are summarized in Sect. 5.

2 A Scheme of Network Identification

Consider a dynamical network consisting of N coupled oscillators, with each node being an n -dimensional dynamical system given by

$$\dot{\mathbf{x}}_i = \mathbf{F}_i(\mathbf{x}_i) + \sum_{j=1}^N c_{ij} \mathbf{H}(\mathbf{x}_j), \quad i = 1, 2, \dots, N, \quad (1)$$

where $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{in})^T$ is the state vector of the i th oscillator, the function \mathbf{F}_i is the corresponding nonlinear vector field, and $\mathbf{H}(\cdot)$ is the linear or nonlinear output function of the individual oscillator. The topology of the network is determined by the coupling matrix $C = (c_{ij})_{N \times N}$, in which $c_{ij} \neq 0$ if there is a coupling from i to j ($j \neq i$), and $c_{ij} = 0$ otherwise. Here, we do not concern whether C is symmetric, irreducible, or diffusive.

To estimate the elements of C , the network in (1) is taken as the drive system. If the vector function \mathbf{F}_i and $\mathbf{H}(\cdot)$ satisfy the Lipschitz condition, i.e., there exist positive constants α_i and β such that

$$\begin{aligned} \|\mathbf{F}_i(\mathbf{x}) - \mathbf{F}_i(\mathbf{y})\| &\leq \alpha_i \|\mathbf{x} - \mathbf{y}\|, \\ \|\mathbf{H}(\mathbf{x}) - \mathbf{H}(\mathbf{y})\| &\leq \beta \|\mathbf{x} - \mathbf{y}\|, \end{aligned}$$

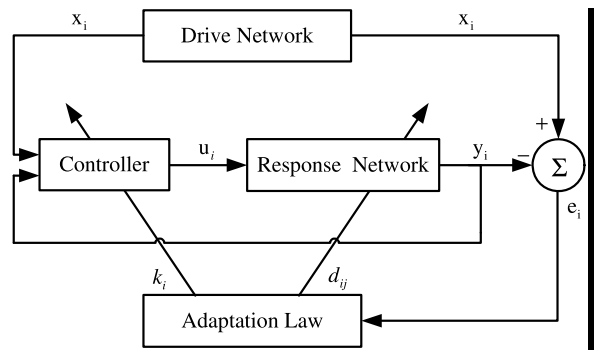


Fig. 1 The block diagram of the system

where $\|\cdot\|$ denotes the Euclidean vector norm, the response network with adaptive-feedback law can be designed as

$$\dot{\mathbf{y}}_i = \mathbf{F}_i(\mathbf{y}_i) + \sum_{j=1}^N d_{ij} \mathbf{H}(\mathbf{y}_j) + \mathbf{u}_i, \quad (2)$$

$$\mathbf{u}_i = k_i \mathbf{e}_i, \quad \dot{k}_i = a_i \|\mathbf{e}_i\|^2, \quad (3)$$

$$\dot{d}_{ij} = \mathbf{e}_i^T \mathbf{H}(\mathbf{y}_j), \quad (4)$$

where $\mathbf{y}_i = (y_{i1}, y_{i2}, \dots, y_{in})^T$, \mathbf{u}_i is the controller for the oscillator i , k_i is an adaptive parameter, a_i is a positive constant, d_{ij} represents the estimate of c_{ij} , and $\mathbf{e}_i = \mathbf{x}_i - \mathbf{y}_i$ denotes the synchronous error. The block diagram of the system is depicted in Fig. 1.

Note 1: If do not have some restrictions on the function \mathbf{F}_i and $\mathbf{H}(\cdot)$, the controller \mathbf{u}_i can be designed as other forms, such as

$$\begin{aligned} \mathbf{u}_i &= -\mathbf{F}_i(\mathbf{y}_i) + \mathbf{F}_i(\mathbf{x}_i) \\ &\quad + \sum_{j=1}^N d_{ij} (\mathbf{H}(\mathbf{x}_j) - \mathbf{H}(\mathbf{y}_j)) + \mathbf{e}_i. \end{aligned} \quad (5)$$

This type of controller has the similar effect of the controller in (3).

In the following, we will show how the unknown c_{ij} may dynamically be identified from d_{ij} in the response system in detail.

The synchronous error system between system (1) and (2) can be expressed as follows:

$$\begin{aligned} \dot{\mathbf{e}}_i &= \mathbf{F}_i(\mathbf{x}_i) - \mathbf{F}_i(\mathbf{y}_i) \\ &\quad + \sum_{j=1}^N (c_{ij} \mathbf{H}(\mathbf{x}_j) - d_{ij} \mathbf{H}(\mathbf{y}_j)) - k_i \mathbf{e}_i. \end{aligned} \quad (6)$$

Construct a Lyapunov function in the form of

$$V = \frac{1}{2} \sum_{i=1}^N \mathbf{e}_i^T \mathbf{e}_i + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N (c_{ij} - d_{ij})^2 + \frac{1}{2} \sum_{i=1}^N \frac{1}{a_i} (k_i - k^*)^2, \tag{7}$$

where k^* is a large enough constant. Furthermore, the time derivative of V along the trajectories is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N \mathbf{e}_i^T \left[\mathbf{F}_i(\mathbf{x}_i) - \mathbf{F}_i(\mathbf{y}_i) + \sum_{j=1}^N (c_{ij} \mathbf{H}(\mathbf{x}_j) - d_{ij} \mathbf{H}(\mathbf{y}_j)) \right] \\ &\quad - \sum_{i=1}^N \mathbf{e}_i^T k_i \mathbf{e}_i - \sum_{i=1}^N \sum_{j=1}^N (c_{ij} - d_{ij}) \mathbf{e}_i^T \mathbf{H}(\mathbf{y}_j) \\ &\quad + \sum_{i=1}^N (k_i - k^*) \|\mathbf{e}_i\|^2 \\ &\leq \sum_{i=1}^N (\alpha_i - k^*) \|\mathbf{e}_i\|^2 + \beta \sum_{i=1}^N \sum_{j=1}^N c_{ij} \|\mathbf{e}_i\| \cdot \|\mathbf{e}_j\| \\ &= E^T P E, \end{aligned} \tag{8}$$

where $E = (\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N)^T$, $P = \text{diag}((\alpha_1 - k^*), (\alpha_2 - k^*), \dots, (\alpha_N - k^*)) + \beta C$. As long as k^* is large enough, the matrix P can be negative definite, namely, $\dot{V} = E^T P E \leq 0$. It is obvious that $\dot{V} = 0$ if and only if $\mathbf{e}_i = 0, i = 1, 2, \dots, N$. From Barbalat’s lemma, we can obtain $\mathbf{e}_i \rightarrow 0$ as $t \rightarrow \infty, i = 1, 2, \dots, N$. The largest invariant set [12, 41] M can be described by $M = \{\mathbf{e}_i, \sum_{j=1}^N (c_{ij} - d_{ij}), (k_i - k^*) | \mathbf{e}_i = 0, k_i = k^*, \dot{d}_{ij} = 0, \sum_{j=1}^N (c_{ij} - d_{ij}) \mathbf{H}(\mathbf{x}_j) = 0, i, j = 1, 2, \dots, N\}$. In such circumstance, the following equation can be satisfied:

$$\sum_{j=1}^N (c_{ij} - d_{ij}) \mathbf{H}(\mathbf{x}_j) = 0, \quad i = 1, 2, \dots, N. \tag{9}$$

Let $c_{ij} - d_{ij} = p_j$ and then (9) can be written as

$$\sum_{j=1}^N p_j \mathbf{H}(\mathbf{x}_j) = 0, \quad i = 1, 2, \dots, N. \tag{10}$$

Denote $\mathbf{H}(\mathbf{x}_j)$ as $(h_1(\mathbf{x}_j), h_2(\mathbf{x}_j), \dots, h_n(\mathbf{x}_j))^T$ ($j = 1, 2, \dots, N$), where h_i is a subfunction of $\mathbf{H}(\mathbf{x}_j)$. Then take $g_{ij} = \int_t^{t+\tau} \mathbf{H}(\mathbf{x}_i(s))^T \mathbf{H}(\mathbf{x}_j(s)) ds$ ($i, j = 1, 2, \dots, N$) and thereby $G = (g_{ij})_{N \times N}$ which is called the Gram matrix [32, 42] of $\mathbf{H}(\mathbf{x}_1(s)), \mathbf{H}(\mathbf{x}_2(s)), \dots, \mathbf{H}(\mathbf{x}_N(s))$.

Premultiply (10) by $\mathbf{H}(\mathbf{x}_i)^T$ for both sides and integrate the equation for a period of time τ , such that

$$\int_t^{t+\tau} \mathbf{H}(\mathbf{x}_i)^T \left[\sum_{j=1}^N p_j \mathbf{H}(\mathbf{x}_j) \right] ds = 0, \tag{11}$$

$i = 1, 2, \dots, N.$

If $\mathbf{H}(\mathbf{x}_i)$ satisfies persistent excitation condition [43] or the linearly independent condition, i.e., G is full rank for any $t \geq 0$ [32], and (10) admits an unique zero solution. Then $d_{ij} = c_{ij}$, i.e., the topology of the network of (1) is successfully identified.

To summarize the above analysis, the following theorem is thus proved.

Theorem 1 *For the drive system (1) and the response system (2), provided that $\mathbf{H}(\mathbf{x}_i)$ satisfies the persistent excitation condition or the linearly independent condition, (10) ensures that $d_{ij} = c_{ij}$. Then the accurate topological identification is achieved.*

Note 2: We have proved that the persistent excitation condition is equivalent to the linearly independent condition in [32].

Note 3: If we take (5) as the controller, we merely need to design another form of Lyapunov function V and the derivation process is similar to the process when consider (3) as the controller.

3 Discussions of Successful Identification During the Transient Process of Synchronization

Some recent works showed that the topology identification would fail if the network is in a synchronous situation [9–12]. They considered that when all function $\mathbf{H}(\mathbf{x}_i)$ were linearly independent, then (10) existed unique zero solution p_j , i.e., $d_{ij} = c_{ij}$. If the drive network synchronized (take complete synchronization for example), namely, $\mathbf{H}(\mathbf{x}_i) = \mathbf{H}(\mathbf{x}_j)$, the linearly independent condition would not be tenable and $d_{ij} \neq c_{ij}$ and, therefore, the synchroniza-

tion inevitably made the network topology unidentifiable.

Hereinafter, we will investigate the successful topology identification in the process which the drive network reaches the synchronism. As stated previously, utilizing the existing linearly independent condition or persistent excitation condition can not explain this phenomenon. Consequently, we seek whether there are some new conditions which can guarantee the precise estimation as well as achieve the synchronization. Since the unknown system needs certain excitation to converge to the true value, these new conditions must at first accomplish the parameter convergence and then realize the synchronous effect. Hence, we make those traditional conditions are satisfied in a finite period time in order to estimate the accurate topological structure and after that let the drive network gradually synchronize. In this case, some new conditions are defined.

Definition 1 (Finite-time Persistent Excitation)

A continuous function $F: R_{\geq 0} \rightarrow R^{m \times n}$ is called finite-time persistently exciting if there exist two strictly positive number μ and τ during $[t_1, t_2]$ for any t such that

$$\int_t^{t+\tau} F^T(x(s))F(x(s)) ds \geq \mu I, \tag{12}$$

where $0 \leq t_1 \leq t \leq t_2$ and $t_p = t_2 - t_1$ is named as the persistence time.

Note 4: The above definition reveals that the left side of the above formula is supposed to be a positive definite matrix and must be full rank [32, 43]. Thus, the problem of judging the finite-time persistent excitation condition is reduced to calculating the rank of the matrix. The left side of the above formula can be denoted as the Gram matrix of F . And then if the Gram matrix is full rank during a period of time, the finite-time persistent excitation condition is satisfied [32].

Due to the equivalence relationship between the persistent excitation condition and the linearly independent condition, we can also define another form of *Finite-time Linearly Independent* condition [48].

Definition 2 (Finite-time Linearly Independent) The function $F_i(x(t))$ is finite-time linearly independent, if there does not exist nonzero constants k_i ($i =$

$1, 2, \dots, N$), such that $k_1 F_1(x(t)) + k_2 F_2(x(t)) + \dots + k_N F_N(x(t)) = 0$ in the time range $[t_1, t_2]$ for any t , where $0 \leq t_1 \leq t \leq t_2$ and $t_p = t_2 - t_1$ is the persistence time.

Result 1 For the network of (1), if $\mathbf{H}(\mathbf{x}_i)$ satisfies the *Finite-time Persistent Excitation* condition (or the *Finite-time Linearly Independent* condition), and the time difference ($t_p - t_f$) between the persistence time t_p and the transient time t_f is long enough, then \mathbf{e}_i and $(c_{ij} - d_{ij})$ converge to zero as $t \rightarrow \infty$ for any arbitrary initial conditions.

According to the Result 1, we will discuss the impact of the transient process in synchronization on the result of the topology identification. Running from the initial state to the stable state is a process which takes a transient time t_f for estimated error to decay to zero. As a result, the transient process is particularly important for the identification. When the persistence time t_p of finite-time persistent excitation is much longer than the transient time t_f , that is to say, there exists enough time to drive the estimated system, so that the uncertain connection topology could converge to the true value. From the perspective of energy, if the persistence time of finite-time persistent excitation is long enough, the estimated system may have adequate energy to approach to the topological structure of the drive network. The following examples will vigorously illustrate the feasibility of the above discussions.

4 Illustrative Examples

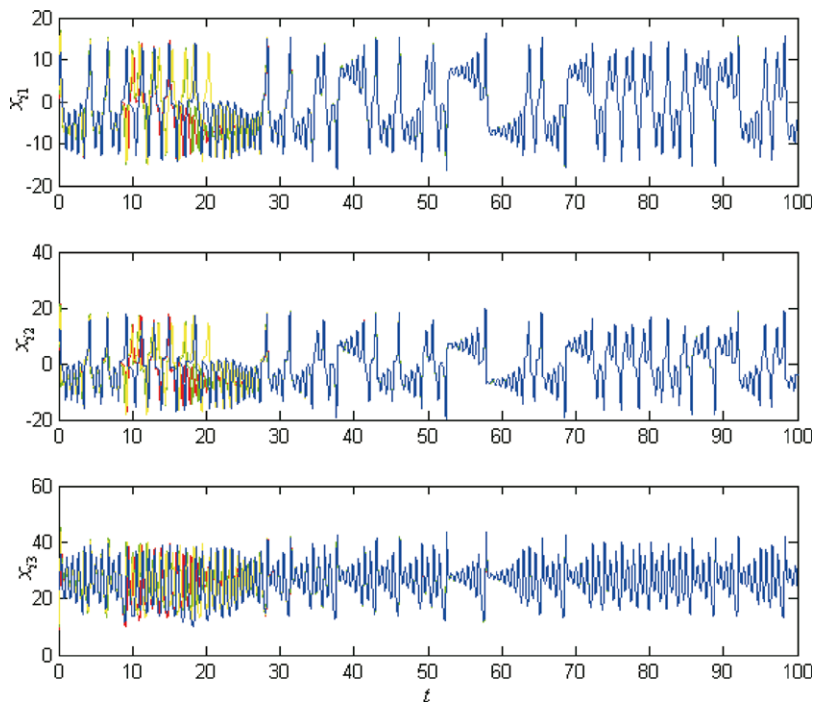
In the following, some examples are used to show the effectiveness of our proposed methods.

As an application of our strategy, we first consider the following network, which takes the chaotic Lorenz systems as the dynamics of every node in the network: $\dot{\mathbf{x}}_i = \mathbf{F}_i(\mathbf{x}_i) = (\sigma(x_{i2} - x_{i1}); \gamma x_{i1} - x_{i1}x_{i3} - x_{i2}; x_{i1}x_{i2} - bx_{i3})$, for $i = 1, 2, 3, 4$ with $\sigma = 10, \gamma = 28$, and $b = \frac{8}{3}$. The drive network is written as

$$\dot{\mathbf{x}}_i = \mathbf{F}_i(\mathbf{x}_i) + L \sum_{j=1}^N c_{ij} \mathbf{H}(\mathbf{x}_j), \tag{13}$$

with $N = 4$ and $\mathbf{H}(\mathbf{x}_j) = \mathbf{x}_j$, where the coefficient L is a constant which can control the length of t_p .

Fig. 2 (Color online)
Orbits of x_{i1}, x_{i2}, x_{i3}
($i = 1, 2, 3, 4$)



Equation (2) describes the response network. In addition, the topology is set by the coupling matrix C , where $c_{1,2} = c_{2,1} = 1, c_{1,3} = c_{3,1} = 0, c_{1,4} = c_{4,1} = 1, c_{2,3} = c_{3,2} = 1, c_{2,4} = c_{4,2} = 0, c_{3,4} = c_{4,3} = 1,$ and $c_{ii} = 0$.

In such case, we aim at adjusting the coefficient L and thereby extending the persistence time t_p , so as to make the uncertain connection topology gain enough time to tend to the true value. For this purpose, by numerical simulations, we set $L = 0.48$. Orbits of x_{i1}, x_{i2}, x_{i3} are presented in Fig. 2. Figure 3 depicts the synchronized behavior of the drive network, which takes a long time to run from the initial state to the synchronized state in the time range $[0, 30]$. And after $t = 30$, the synchronous errors of the drive network decay to zero. In what follows, we analyze whether the finite-time persistent excitation condition is satisfied or not whereby to calculate the rank of the Gram matrix.

Through precisely calculation by simulation, the result shows that the Gram matrix is full rank during the period $[0, 30]$, namely, the finite-time persistent excitation condition is satisfied and the persistence time $t_p = 30$. Figure 4 shows the corresponding process of identifying the network topology. It is approximate that after a transient time $t_f = 20, d_{ij} \rightarrow c_{ij}$, i.e., the

uncertain parameters of the adjacency matrix converge to the true values. The persistence time t_p is obviously longer than the transient time t_f . Subsequently, even if the synchronized errors of the drive network approach to zero, there is no influence on the result of the identification.

In order to further demonstrate the feasibility, another example is given which takes the chaotic Lü systems as the dynamics of every node in the network: $\dot{\mathbf{x}}_i = \mathbf{F}_i(\mathbf{x}_i) = (\sigma(x_{i2} - x_{i1}); \gamma x_{i2} - x_{i1}x_{i3}; x_{i1}x_{i2} - bx_{i3})$, for $i = 1, 2, 3, 4$ with $\sigma = 36, \gamma = 20,$ and $b = 3$. Consider (13) as the drive network and (2) as the response network, with $N = 4$ and $\mathbf{H}(\mathbf{x}_j) = \mathbf{x}_j$, where the coefficient L is a constant and the topology C is described by $c_{1,2} = 1, c_{2,1} = 0, c_{1,3} = 0, c_{3,1} = 1, c_{1,4} = c_{4,1} = 1, c_{2,3} = c_{3,2} = 1, c_{2,4} = c_{4,2} = 0, c_{3,4} = c_{4,3} = 1,$ and $c_{ii} = -2$. By numerical simulations, we set $L = 0.72$. Figure 5 depicts the synchronized behavior of the drive network and after $t = 110$, the synchronous errors of the drive network decay to zero. Calculating the rank of the Gram matrix during these time range, it shows that the finite-time persistent excitation condition is satisfied and the persistence time $t_p = 110$. Figure 6 shows the corresponding process of identifying the network topology, where the transient time $t_f = 20$. Since the persistence

Fig. 3 (Color online) Time evolution of synchronized errors e_{i1}, e_{i2}, e_{i3} ($i = 1, 2, 3, 4$) of the drive network

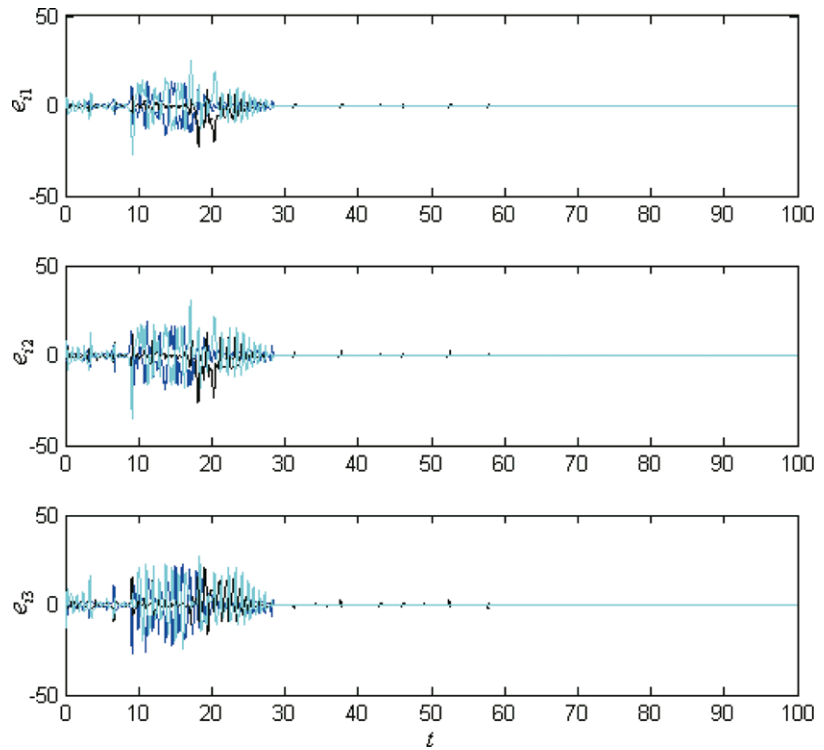
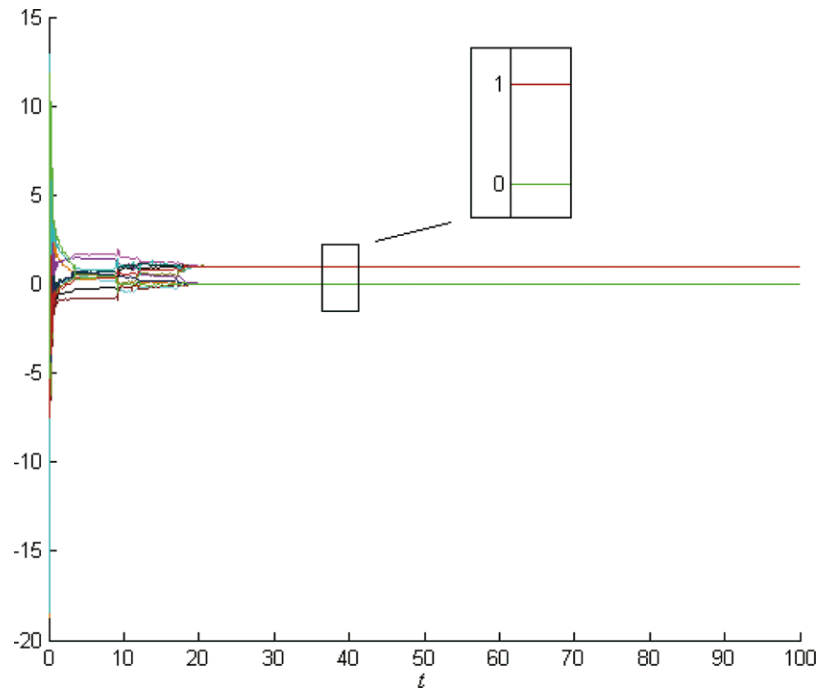


Fig. 4 (Color online) Time evolution of adaptive parameters d_{ij} in the estimated adjacency matrix



time is long enough and is longer than the transient time, the topology identification succeeds as Fig. 6 depicted.

It follows that a counterexample is given to verify that when the finite-time persistent excitation condition is not satisfied, the topology identification will

Fig. 5 (Color online) Time evolution of synchronized errors e_{i1}, e_{i2}, e_{i3} ($i = 1, 2, 3, 4$) of the drive network

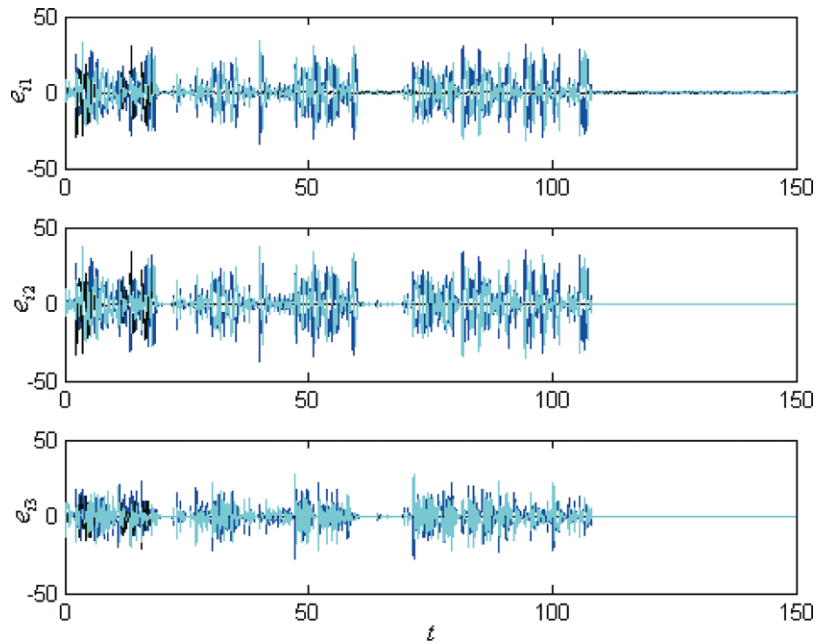
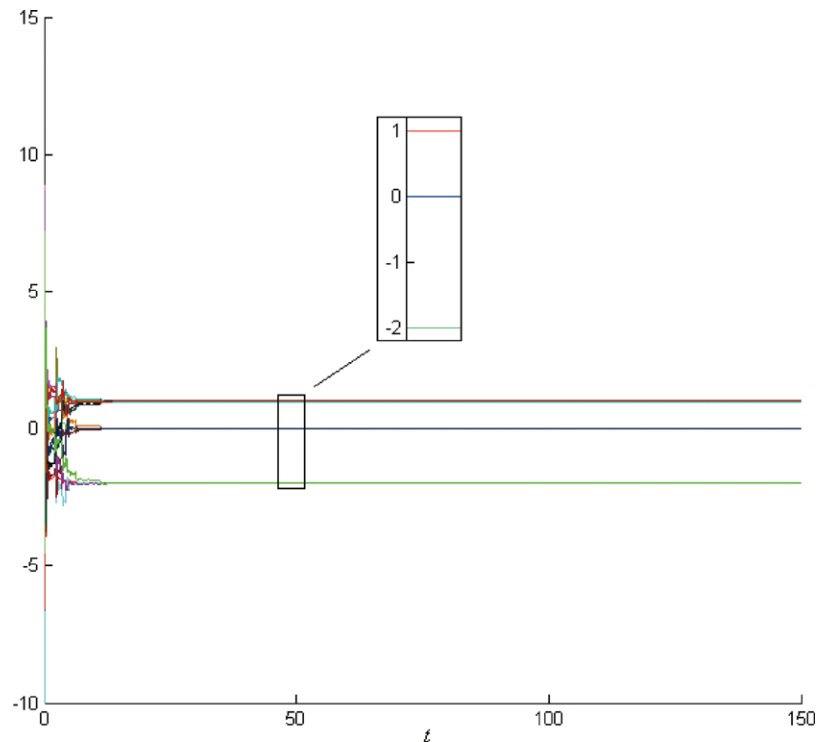


Fig. 6 (Color online) Time evolution of adaptive parameters d_{ij} in the estimated adjacency matrix



fail. Consider the same drive network and the same response network as the above examples, with the chaotic Lorenz systems as the dynamics of every node in the network: $\dot{\mathbf{x}}_i = \mathbf{F}_i(\mathbf{x}_i) = (\sigma(x_{i2} - x_{i1}); \gamma x_{i1} -$

$x_{i1}x_{i3} - x_{i2}; x_{i1}x_{i2} - bx_{i3})$, for $i = 1, 2, 3, 4$ with $\sigma = 10, \gamma = 28$, and $b = \frac{8}{3}$. The topology C is set by $c_{1,2} = 1, c_{2,1} = 0, c_{1,3} = 0, c_{3,1} = 1, c_{1,4} = c_{4,1} = 1,$
 $c_{2,3} = c_{3,2} = 1, c_{2,4} = c_{4,2} = 0, c_{3,4} = c_{4,3} = 1$, and

Fig. 7 (Color online) Time evolution of synchronized errors e_{i1} , e_{i2} , e_{i3} ($i = 1, 2, 3, 4$) of the drive network

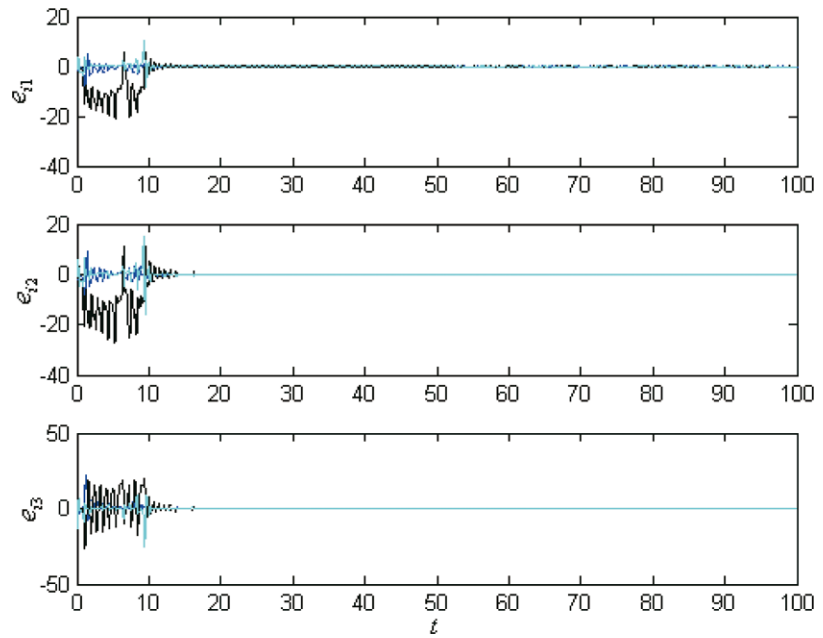
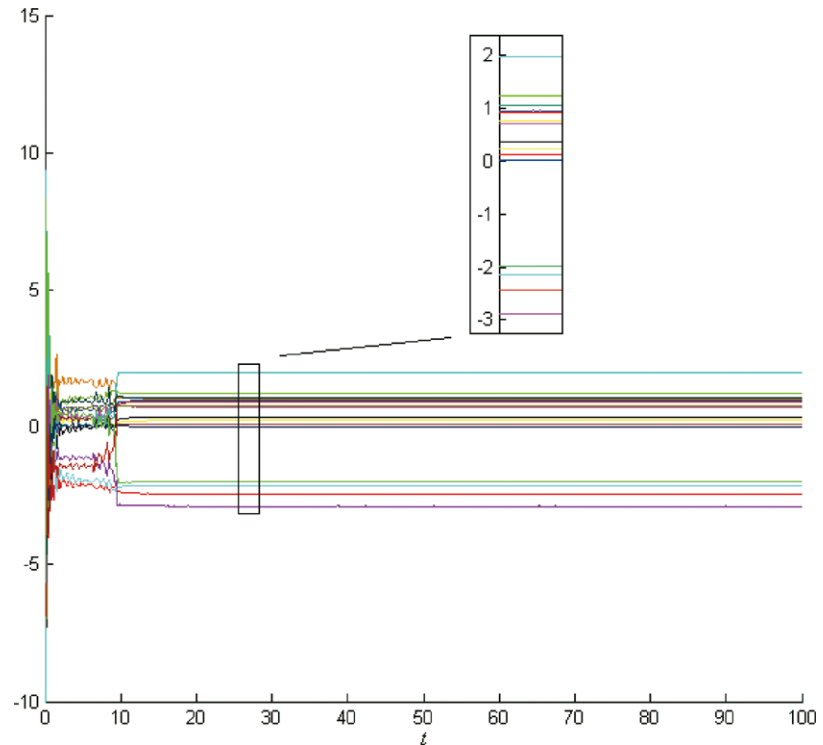


Fig. 8 (Color online) Time evolution of adaptive parameters d_{ij} in the estimated adjacency matrix



$c_{ii} = -2$. By numerical simulations, we set $L = 0.58$. Figure 7 depicts the synchronized behavior of the drive network and after $t = 15$, the synchronous errors of the drive network decay to zero. Calculating the rank

of the Gram matrix during these time range, it shows that the finite-time persistent excitation condition is not satisfied. Figure 8 shows the corresponding process of identifying the network topology, where the

transient time $t_f = 10$. Since the finite-time persistent excitation condition cannot be satisfied, the uncertain topological connection cannot converge to the true values as Fig. 8 depicted, namely, the identification fails.

The proposed methods are highly applicable in the real physical systems, such as neural network, social network and so on. Taken the biological neural network for example, firstly, our scheme can clearly describe a general form of the neural network, where each node represents each neuron. And the classical neuron model can be easily transferred to our scheme, such as the Hindmarsh–Rose (HR) model. In addition, due to the nature of the neural network, which is usually nonlinear and complex, it is suitable to identify and monitor its topology by the adaptive control approach [14, 17, 18]. Therefore, our methods can be readily employed to estimate its topological structure. The concrete procedure follows the aforementioned steps in Sect. 2.

5 Discussion and Conclusion

In this paper, we explore the network topology identification during the transient process of synchronization. We find that during the transient process, as long as the Finite-time Persistent Excitation (or the Finite-time Linearly Independent) is satisfied, and the time difference ($t_p - t_f$) between the persistence time t_p and the transient time t_f is long enough, the accurate topology identification can be achieved. Compared with the conventional PE and LI condition, the Finite-time Persistent Excitation (or the Finite-time Linearly Independent) emphasizes that it is satisfied in the time range $[t_1, t_2]$ for any t . However, the traditional PE and LI condition do not describe the restriction on the time domain which define the conditions are tenable for any $t > 0$. Therefore, the proposed conditions can clearly explain it is possible to identify the topological structure of synchronous networks by analyzing their transient processes, which cannot be illustrated by the traditional PE and LI condition. Recently, in [5], the authors presented a new method to precisely identify links among nodes based on the noise-induced relationship between dynamical correlation and topology [5]. In [49], the author studied the steady-state control based topology identification method. Since our obtained conditions are based on the adaptive control method, the above two methods are different from ours.

In conclusion, we specifically analyze the effect of the transient process in synchronization phenomenon on the topology identification. Some novel conditions are given to guarantee the successful identification of a synchronous network during the transient process. Intuitively, our approach can be generalized to estimate the topology of networks with time-delay and the weights of a weighted network in future research.

Acknowledgements This work is supported by the National Basic Research Program of China (973 Program) (Grant No. 2007CB310704), the National Natural Science Foundation of China (Grants Nos. 61070209, U0835001), the Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20100005110002) and the Fundamental Research Funds for the Central Universities (Grant No. BUPT2011RC0211).

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