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Pinning synchronization of weighted complex networks with variable delays and adaptive coupling weights

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Abstract In this paper, the synchronization for time-delayed complex networks with adaptive coupling weights is studied. A pinning strategy and a local adaptive scheme to determine coupling weights and feedback gains are proposed. It is noted that our control strategies only rely on some local information other than the global information of the whole network. Finally, the developed techniques are applied to two complex networks which are respectively synchronized to an unstable equilibrium point and a chaotic attractor.

Keywords Weighted complex network · Adaptive coupling weight · Pinning control · Synchronization

1 Introduction

Currently, complex dynamical networks are being studied across many fields of science and engineering. In general, a complex network is a large set of interconnected nodes by edges, in which each node is a fundamental unit with detailed contents. In fact, any large-scale and complicated system in nature and societies can be modeled by a complex network, where vertices are the elements of the system, and edges represent the interactions between them. Examples of complex networks include the Internet, metabolic networks, neural networks, food webs, electrical power grids, social networks, and many others [1, 2].

Over the past few decades, as a typical kind of dynamics, synchronization in complex networks attracts lots of interests in various fields of science and engineering due to the fact that it not only can well explain many natural phenomena observed, but also has many promising potential applications in image processing, secure communication, etc. From mathematical point of view, synchronization can be defined as a process wherein two (or many) dynamical systems adjust a given property of their motion to a common behavior as time goes to infinity, due to coupling or forcing [3]. Up to now, many different regimes of synchronization have been investigated, including cluster synchronization [3–7], phase synchronization [8], complete synchronization [9–12], and generalized synchronization [13–15]. Meanwhile, many effective control approaches have been developed to synchronize complex networks such as impulsive control [16–18], intermittent control [19, 20], adaptive feedback control [21–25], and so on.

On the other hand, a complex network in the real world normally has a large number of nodes, and it is usually hard and even unfeasible to control all nodes so that each follows a desired synchronous trajectory. Recalling the distributed nature of complex networks, it is feasible and reasonable to control them by acting locally on certain nodes, and then through coupling

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between nodes, achieving synchronization of the entire network, which is known as pinning control. So far, pinning control has been extensively proposed to provide an insight into regulatory mechanisms for controlling networks of coupled dynamical systems. In [26], the local and global synchronization of weighted complex dynamical networks were both considered by applying adaptive control to a fraction of network nodes and the construction of a master stability function and a Lyapunov function. Several new stability criteria of controlling a complex network with digraph topology to a homogeneous trajectory of the uncoupled system were derived in [27] by a local pinning control algorithm. In [28], a general criterion for ensuring network synchronization has been derived by using pinning control, adaptive techniques, and the authors pointed out that nodes with low degrees should be first pinned when the coupling strength is very small. In [29], low-dimensional pinning criteria for general complex dynamical networks were obtained, and it was shown that the nodes whose out-degrees are bigger than their in-degrees of a directed network should be chosen as pinned candidates. Meanwhile, some similar and useful criteria were also derived by different authors such as [30–33].

A common feature of the research works in [26–33] is that the synchronization conditions require calculating eigenvalues of the coupling weight matrix of the network; in other words, the global information of the whole networks is known beforehand, which can be obtained for the networks with small size. However, if the size of the network is very large, the prior knowledge of the network and the calculation of eigenvalues of the coupling weight matrix are both difficult. Therefore, it is natural to raise the following problem: can we pin the coupled complex network to a decoupled state with the prior partial information other than the global knowledge of the network? In fact, in many real-world networks, the coupling weights are not some constant values and cannot be known in advance, but are automatically adjusted and vary in time according to different environmental conditions. A typical example is wireless sensor networks that gather and communicate data to a central base station. Adaptation is also necessary to control networks of robots when the operating conditions change unexpectedly [23]. Motivated by these applications, the adaptive coupling weights in complex networks are more realistic and reasonable.

Nowadays, the synchronization results of complex weighted networks by using adaptive coupling weights are seriously lacking. In [34], a simple complex network was studied:

$$\dot{x}_i(t) = f\big(x_i(t), t\big) + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij}\big[x_j(t) - x_i(t)\big],$$
$$i = 1, 2, \ldots, N,$$

where $x_i(t) = (x_i^1(t), \ldots, x_i^n(t))^T$ is the state vector of the ith node, $f : R^n \times R^+$ is a continuous map, c_{ij} denotes the coupling strength between node i and node j, and $A = (a_{ij})_{N\times N}$ represents the configuration matrix. A decentralized adaptive pinning control scheme for synchronization of this kind of networks was designed by adjusting coupling strengths and feedback gains, under which the whole network was pinned to a stationary state.

In [23, 35], the authors considered the following complex network:

$$\frac{dx_i}{dt} = f(x_i) - \sum_{j\in\varepsilon_i} \sigma_{ij}(t)\big[h(x_j) - h(x_i)\big],$$
$$i = 1, 2, \ldots, N,$$

where x_i represents the state vector of the ith oscillator, $f : R^n \to R^n$ is a nonlinear vector function describing the dynamics of isolated node, $h : R^n \to R^n$ is the output function through which the systems in the network are coupled, ε_i denotes all nodes directly connected with the ith node, $\sigma_{ij}(t)$ is the coupling strength. Some novel decentralized control approaches were respectively derived by adjusting adaptively coupling strengths to ensure the synchronization of the above network.

In [36], the following coupled network was investigated:

$$\dot{x}_i(t) = F(x_i) + \sum_{j=1}^{N} a_{ij}(t)\Gamma x_j(t),$$
$$i = 1, 2, \ldots, N,$$

where $x_i(t) = (x_{i1}(t), \ldots, x_{in}(t))^T$ is the state vector representing the state variables of node i, $F(x_i) = (F_1(x_i), \ldots, F_n(x_i))^T$ is a smooth nonlinear vector-valued function, $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_{k_0}, 0, \ldots, 0)$ is the inner coupling matrix with $\gamma_i > 0$ for $i = 1, 2, \ldots, k_0$, and $a_{ij}(t)$ is the coupling strength. The asymptotically stability led by partial state variables was realized for the above network by setting adaptive update law of $a_{ij}(t)$.

Evidently, the research models are simple, and time delays are not considered in those works. However, it is well known that communication delays are ubiquitous in networks due to the finite speeds of transmission and spreading as well as traffic congestions. Therefore, it is essential to investigate the synchronization of complex networks model with time delays. To the best of our knowledge, there are few even no results concerning the asymptotical synchronization of time-delayed complex networks by adaptive undate law for coupling weights among nodes and local pinning control. In addition, the adaptive parameter for each controlled node in [3, 7, 24] contains the state information of all nodes. Just as the above discussion, it is tough to access all information of the large-scale networks. However, for a given node, the information of nodes directly connected with it can be obtained. Hence, a more satisfactory and reasonable feedback scheme for a controlled node is to incorporate the state information of the directly interacted nodes other than all nodes.

Motivated by the above discussions, our aim in this paper is to deal with the globally asymptotical synchronization of weighted complex networks with variable delays and adaptive coupling weights by virtue of pinning control and local adaptive feedback strategy without requiring global information of the networks. The main contribution of this paper lies in the following aspects. First, a pinning scheme and a local adaptive technique for coupling weights and feedback gains are designed. And then, by constructing a Lyapunov functional and applying the LaSalle's invariance principle, it is proven that the synchronization of the addressed network can be achieved under those control strategies. Particularly, some sufficient synchronization criteria for a class of coupled neural network are also derived. It is noted that the adaptive update law of coupling weights in this paper is only dependent on dynamical behaviors of partial connected directly nodes. In numerical simulation, a ring complex network and a star network are listed, which are respectively synchronized to an unstable equilibrium point of the decoupled system and a chaotic attractor, and some numerical portraits are also provided.

This paper is organized as follows. In Sect. 2, some model descriptions and useful preliminaries are given. Some control schemes are designed in Sect. 3 to ensure the synchronization of the addressed networks. In Sect. 4, two numerical examples are given to verify our theoretical results. Conclusions are drawn in Sect. 5.

2 Preliminaries

Consider a weighted complex dynamical network with time delays consisting of N identical coupled nodes, in which each node is an n-dimensional dynamical system; the entire network is described by

$$\dot{x}_i(t) = f\big(x_i(t), x_i\big(t-\tau(t)\big)\big) + \sum_{j=1, j\neq i}^{N} c_{ij}\omega_{ij}(t)\Gamma\big[x_j(t) - x_i(t)\big], \qquad (1)$$

where $i \in \mathcal{I} = \{1, 2, \dots, N\}$, $x_i(t) = (x_{i1}(t), \dots, x_{in}(t))^T$ denotes the n-dimensional state variable of the ith node, $f : R^n \times R^n \to R^n$ is a vector-valued continuous function governing the evolution of each individual node, the time delay $\tau(t)$ is bounded and satisfies $0 \le \tau(t) \le \tau$, $\Gamma \in R^{n\times n}$ is the inner connecting matrix, $\omega_{ij}(t) = \omega_{ji}(t)$ represent the time-varying coupling weights or strengths between node i and node j, and $C = (c_{ij})_{N\times N}$ is the configuration matrix representing the underlying topology structure of the network, in which c_{ij} is defined as follows: if there is a connection between node i and node j $(i \neq j)$, then $c_{ij} = c_{ji} = 1$; otherwise, $c_{ij} = c_{ji} = 0$ $(i \neq j)$, and the diagonal elements of matrix C are defined by $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$. The matrix of the weighted coupling configuration of the network denoted by $B = (b_{ij})_{N\times N}$ is defined as follows: $b_{ij} = \omega_{ij}c_{ij}$ for $i \neq j$ and $b_{ii} = \omega_{ii}c_{ii} = -\sum_{j=1, j\neq i}^{N} \omega_{ij}c_{ij}$. Suppose that the network is connected in the sense of having no isolated clusters; then the matrix C is an irreducible real symmetric matrix.

System (1) is supplemented with initial values given by

$$x_i(s) = \phi_i(s), \quad s \in [-\tau, 0], \ i \in \mathcal{I}, \qquad (2)$$

where $\phi_i(s) = (\phi_{i1}(s), \dots, \phi_{in}(s))^T \in C([-\tau, 0], R^n)$, which denotes the Banach space of all continuous functions mapping $[-\tau, 0]$ into R^n with 2-norm defined by

$$\|\psi\| = \left[\sup_{s\in[-\tau,0]} \sum_{i=1}^{n} \big|\psi_i(s)\big|^2\right]^{\frac{1}{2}}$$

for $\psi \in C([-\tau, 0], R^n)$.

Evidently, according to system (1), the dynamic behavior of each isolated node of the network (1) can be described by

$$\dot{s}(t) = f\big(s(t), s\big(t-\tau(t)\big)\big). \tag{3}$$

The main aim in this paper is to synchronize the weighted network (1) onto a desired evolution satisfying (3) by adjusting the coupling weights $\omega_{ij}(t)$ and imposing an adaptive pinning controller on system (1).

In order to obtain the main results, the following assumptions and definitions are necessary.

Assumption 1 For the vector-valued function $f(x, \bar{x})$, there exist a constant θ and a positive constant γ such that

$$\begin{aligned}&(x-y)^T\big(f(x,\bar{x})-f(y,\bar{y})\big)\\&\quad\le \theta(x-y)^T(x-y)+\gamma(\bar{x}-\bar{y})^T(\bar{x}-\bar{y})\end{aligned}$$

for any $x, \bar{x}, y, \bar{y} \in R^n$.

Assumption 2 $\tau(t)$ is a differential function with $0 \le \dot{\tau}(t) < \varepsilon < 1$.

Assumption 3 The inner coupling matrix Γ is a positive definite matrix.

Definition 1 Suppose that $A = (a_{ij}) \in R^{p\times p}$. If $a_{ij} = a_{ji} \ge 0$ for $i \ne j$ and $a_{ii} = -\sum_{i=1, j\ne i}^{p} a_{ij}$ for $i = 1, 2, \dots, p$, and A is irreducible, then we say that $A \in \mathbf{A_1}$.

Lemma 1 (see [37]) *If $A \in \mathbf{A_1}$, then all eigenvalues of the matrix $A - \Lambda$ are negative, where $\Lambda =$ $\mathrm{diag}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_p)$, ε_i are nonnegative constants satisfying $\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_p > 0$.*

Remark 1 If $f(x, \bar{x})$ is reduced to $f(x)$, Assumption 1 is degenerated to the following condition:

For any $x, y \in R^n$, there exist a constant θ such that

$$(x-y)^T\big[f(x)-f(y)\big] \le \theta(x-y)^T(x-y).$$

It can be easily proven that the above condition is equivalent to the so-called QUAD condition introduced in Refs. [3, 23, 35]. Particularly, a vector field $f: R^n \to R^n$ is QUAD iff, for any $x, y \in R^n$,

$$\begin{aligned}&(x-y)^T\big[f(x)-f(y)-\Delta(x-y)\big]\\&\quad\le -\varpi(x-y)^T(x-y),\end{aligned}$$

where Δ is an arbitrary diagonal matrix of order n, and ϖ is a nonnegative scalar. Hence, Assumption 1 can be seen as an extension of the QUAD condition to some extent.

3 Main results

Our goal is to synchronize the weighted network (1) onto a desired evolution, i.e.,

$$\lim_{t\to+\infty}\big\|x_i(t)-s(t)\big\| = 0 \tag{4}$$

for $i \in \mathcal{I}$, where the norm $\|\cdot\|$ of a vector x is defined as $\|x\| = \sqrt{x^T x}$, and $s(t)$ is an arbitrary desired state of system (3), which may be an equilibrium point, a periodic orbit, or even a chaotic orbit.

In order to achieve the aim (4), we introduce a control strategy to nodes in the network (1). The controlled network corresponding to system (1) can be described as

$$\begin{aligned}\dot{x}_i(t) &= f\big(x_i(t), x_i\big(t-\tau(t)\big)\big)\\&\quad+\sum_{j=1, j\ne i}^{N} c_{ij}\omega_{ij}(t)\Gamma\big[x_j(t)-x_i(t)\big]+u_i,\end{aligned} \tag{5}$$

where u_i is an adaptive pinning controller, which is designed as

$$u_i(t) = -\delta_i\beta_i(t)\Gamma\big(x_i(t)-s(t)\big), \tag{6}$$

here, $\delta_i = 1$ if node i is selected to control, otherwise $\delta_i = 0$, $\beta_i(t)$ is a feedback gain which is constructed by

$$\dot{\beta}_i(t) = \delta_i k_i\big(x_i(t)-s(t)\big)^T\Gamma\big(x_i(t)-s(t)\big), \tag{7}$$

where $\beta_i(0) \ge 0$ and $k_i > 0$ for $i \in \mathcal{I}$.

Based on controllers (6) and (7), we have the following results.

Theorem 1 *Under Assumptions 1–3, suppose that at least one node is controlled satisfying (6) and (7). System (5) is asymptotically synchronized to system (3) if the coupling weights $\omega_{ij}(t)$ $(i \ne j)$ satisfy the following adaptive update laws*:

$$\dot{\omega}_{ij}(t) = c_{ij}\mu_{ij}\big(x_i(t)-x_j(t)\big)^T\Gamma\big(x_i(t)-x_j(t)\big), \tag{8}$$

where $\omega_{ij}(0) \ge 0$ and $\mu_{ij} > 0$.

Proof Let the synchronic error be $e_i(t) = x_i(t) - s(t)$ for $i \in \mathcal{I}$. It follows from system (3) and (5) that

$$\dot{e}_i(t) = \tilde{f}\big(e_i(t), e_i\big(t-\tau(t)\big)\big) + \sum_{j=1, j\neq i}^{N} c_{ij}\omega_{ij}(t)\Gamma\big[x_j(t) - x_i(t)\big] + u_i, \quad (9)$$

where $\tilde{f}(e_i(t), e_i(t-\tau(t))) = f(x_i(t), x_i(t-\tau(t))) - f(s(t), s(t-\tau(t)))$.

Define the following Lyapunov functional:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{(\omega_{ij}(t)-h)^2}{\mu_{ij}} + \frac{1}{2}\sum_{i=1}^{N}\frac{(\beta_i(t)-h)^2}{k_i} + \sum_{i=1}^{N}\int_{t-\tau(t)}^{t}\frac{\gamma}{1-\varepsilon}e_i^T(s)e_i(s)\,ds,$$

where h is a positive constant to be determined later.

By virtue of Assumptions 1–3, the time derivative of $V(t)$ can be derived by

$$\begin{aligned}\frac{dV(t)}{dt} \le{}& \sum_{i=1}^{N}\left(\theta + \frac{\gamma}{1-\varepsilon}\right)e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} c_{ij}\omega_{ij}e_i^T(t)\Gamma\big(x_j(t) - x_i(t)\big)\\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}(\omega_{ij}-h)c_{ij}\big(x_i(t) - x_j(t)\big)^T\Gamma\big(x_i(t) - x_j(t)\big)\\ &- \sum_{i=1}^{N}\delta_i\beta_i e_i^T(t)\Gamma e_i(t) + \sum_{i=1}^{N}(\beta_i - h)\delta_i e_i^T(t)\Gamma e_i(t)\\ ={}& \sum_{i=1}^{N}\left(\theta + \frac{\gamma}{1-\varepsilon}\right)e_i^T(t)e_i(t) - \sum_{i=1}^{N}h\delta_i e_i^T(t)\Gamma e_i(t)\\ &+ \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} c_{ij}\omega_{ij}e_i^T(t)\Gamma\big(x_j(t) - x_i(t)\big)\\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}(\omega_{ij}-h)c_{ij}\big(x_i(t) - x_j(t)\big)^T\Gamma\big(x_i(t) - x_j(t)\big). \end{aligned} \quad (10)$$

Applying $\omega_{ij} = \omega_{ji}$ and $c_{ij} = c_{ji}$, we have

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}(\omega_{ij}-h)c_{ij}\big(x_i(t) - x_j(t)\big)^T \times \Gamma\big(x_i(t) - x_j(t)\big)\\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}(\omega_{ij}-h)c_{ij}\big(e_i(t) - e_j(t)\big)^T \times \Gamma\big(x_i(t) - x_j(t)\big)\\ &= \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}(\omega_{ij}-h)c_{ij}e_i(t)^T\Gamma\big(x_i(t) - x_j(t)\big). \end{aligned} \quad (11)$$

Substituting (11) into (10), we have

$$\begin{aligned}\frac{dV(t)}{dt} \le{}& \sum_{i=1}^{N}\left(\theta + \frac{\gamma}{1-\varepsilon}\right)e_i^T(t)e_i(t) - \sum_{i=1}^{N}h\delta_i e_i^T(t)\Gamma e_i(t)\\ &+ \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} hc_{ij}e_i(t)^T\Gamma\big(x_j(t) - x_i(t)\big)\\ ={}& \sum_{i=1}^{N}\sum_{j=1}^{N} hc_{ij}e_i(t)^T\Gamma e_j(t) - \sum_{i=1}^{N}h\delta_i e_i^T(t)\Gamma e_i(t) + \left(\theta + \frac{\gamma}{1-\varepsilon}\right)\sum_{i=1}^{N}e_i^T(t)e_i(t)\\ ={}& he^T(t)\big[(C-\Delta)\otimes\Gamma\big]e(t) + \left(\theta + \frac{\gamma}{1-\varepsilon}\right)\sum_{i=1}^{N}e_i^T(t)e_i(t), \end{aligned}$$

where $\Delta = \mathrm{diag}(\delta_1, \ldots, \delta_N)$.

It follows from Lemma 1 and Assumption 3 that $(C-\Delta)\otimes\Gamma$ is a negative matrix. Then a suitable h can be chosen such that

$$\frac{dV(t)}{dt} < -\sum_{i=1}^{N} e_i^T(t)e_i(t).$$

According to LaSalle's invariance principle [38], $e_i(t)\to 0$ as $t\to+\infty$, that is,

$$\lim_{t\to+\infty}\left\|x_i(t)-s(t)\right\|=0$$

for $i\in\mathcal{I}$. Therefore, under the adaptive controllers and updating laws (6)–(8), the controlled network (5) is asymptotically synchronized to $s(t)$. The proof of Theorem 1 is completed. □

Assume that $s^*=(s_1^*,\dots,s_n^*)$ is an equilibrium point of the decoupled system (3), which may be unstable. Evidently,

$$S^*=(\underbrace{s^*,s^*,\dots,s^*}_{N})$$

is also an equilibrium point of the network (1). From Theorem 1 the following result is directly derived.

Corollary 1 *Under Assumptions 1–3, suppose that at least one node in system (1) is controlled. The equilibrium S^* is asymptotically stable, i.e., the network (1) is asymptotically synchronized to s^* if the following adaptive update laws are satisfied*:

(1) $u_i(t)=-\delta_i\beta_i(t)\Gamma(x_i(t)-s^*)$,
(2) $\dot\beta_i(t)=\delta_i k_i(x_i(t)-s^*)^T\Gamma(x_i(t)-s^*)$,
(3) $\dot\omega_{ij}(t)=c_{ij}\mu_{ij}(x_i(t)-x_j(t))^T\Gamma(x_i(t)-x_j(t))$,

where $\beta_i(0)\ge 0$, $\omega_{ij}(0)\ge 0$, $k_i>0$, and $\mu_{ij}>0$ for $i,\ j\in\mathcal{I}$.

In the following, we consider a class of controlled coupled neural networks with variable coupling strengths described by

$$\dot x_i(t)=-Ax_i(t)+Bg\big(x_i(t)\big)+Dg\big(x_i\big(t-\tau(t)\big)\big) + \sum_{j=1,j\ne i}^{N} c_{ij}\omega_{ij}(t)\Gamma\big[x_j(t)-x_i(t)\big]+u_i, \tag{12}$$

where $i\in\mathcal{I}$, Γ, c_{ij}, and $\omega_{ij}(t)$ are defined in system (1), $x_i(t)=(x_{i1}(t),\dots,x_{in}(t))^T\in R^n$ denotes the state variable associated with the ith node, $A=\mathrm{diag}(a_1,a_2,\dots,a_n)$ is the decay constant matrix with $a_i>0$ for $i\in\mathcal{I}$, and $B=(b_{ij})_{n\times n}$ and $D=(d_{ij})_{n\times n}$ are the connection matrix and delayed connection matrix. $g(x_i(t))=(g_1(x_{i1}(t)),\dots,g_n(x_{in}(t)))^T$ is the activation function of the neurons and satisfies the following condition:

Assumption 4 For any $x,y\in R^n$, there exists a positive constant F such that

$$\big(g(x)-g(y)\big)^T\big(g(x)-g(y)\big)\le F(x-y)^T(x-y).$$

Correspondingly, the synchronization state $s(t)$ associated with system (12) is represented by

$$\dot s(t)=-As(t)+Bg\big(s(t)\big)+Dg\big(s\big(t-\tau(t)\big)\big). \tag{13}$$

The following statement is provided to ensure the asymptotical synchronization of the coupled network (12).

Corollary 2 *Suppose that Assumptions 2–4 hold and at least one node in system (12) is controlled. Under controllers (6)–(8), the network (12) is asymptotically synchronized to $s(t)$ satisfying system (13).*

Proof Evidently, only Assumption 1 should be verified. It is easy that

$$\begin{aligned} &f\big(x_i(t),x_i\big(t-\tau(t)\big)\big)\\ &\quad=-Ax_i(t)+Bg\big(x_i(t)\big)+Dg\big(x_i\big(t-\tau(t)\big)\big). \end{aligned}$$

Then,

$$\begin{aligned} &(x-y)^T\big(f(x,\bar x)-f(y,\bar y)\big)\\ &\quad=(x-y)^T\big[-A(x-y)+B\big(g(x)-g(y)\big)\\ &\qquad+D\big(g(\bar x)-g(\bar y)\big)\big]\\ &\quad\le\frac{1}{2}(x-y)^T\big(-2A+BB^T+DD^T+FI\big)(x-y)\\ &\qquad+\frac{F}{2}(\bar x-\bar y)^T(\bar x-\bar y)\\ &\quad\le\frac{\mu}{2}(x-y)^T(x-y)+\frac{F}{2}(\bar x-\bar y)^T(\bar x-\bar y), \end{aligned}$$

where μ is the largest eigenvalues of matrix $-2A+BB^T+DD^T+FI$, which shows that Assumption 1 in Theorem 1 is satisfied and it is directly obtained that the network (12) is asymptotically synchronized to the decoupled state $s(t)$. □

Remark 2 It is easy to see that the adaptive feedback gains $\beta_i(t)$ and the coupling weights $\omega_{ij}(t)$ respectively tend to some positive constants when the network (1) realizes asymptotical synchronization.

Remark 3 In Refs. [23, 34–36], the adaptive coupling strengths were well proposed to realize synchronization of complex network without time delays. In this paper, a more general complex network with time delays is investigated by automatically adjusting coupling strengths and using local pinning control scheme. Evidently, our proof techniques are simple, and the results are novel and effective. Moreover, our adaptive update law of coupling weight $\omega_{ij}(t)$ is only related to the dynamical properties of directly connected nodes i and j.

Remark 4 It follows from the control scheme (6) and the proof of Theorem 1 that only a fraction of nodes selected randomly need to control in order to realize the synchronization and other nodes are pinned to the synchronized state $s(t)$. In fact, pinning control has been extensively studied in Refs. [26–33], in which the calculation of eigenvalues for the coupling weight matrix of the network is needed; in other words, the global information of the whole networks is required. It is difficult for large-scree network to calculate the eigenvalues for the coupling weight matrix. In order to avoid this trouble, the adaptive coupling weights are proposed in this paper, and their adaptive update laws are only related to the effective information of partial interacted nodes other than the global information of the whole nodes.

4 Numerical simulations

In this section, based on the results obtained in the previous sections, two numerical examples are presented to show the effectiveness and feasibility of our results.

Example 1 Consider a coupled network composed of $N = 5$ nodes, where each node is a two-dimensional dynamical oscillator. The controlled network can be written as

$$\dot{x}_i(t) = A_1 x_i(t) + B_1 g\big(x_i\big(t-\tau(t)\big)\big) + \sum_{j=1, j\neq i}^{5} c_{ij}\omega_{ij}(t)\big[x_j(t) - x_i(t)\big] + u_i, \quad (14)$$

where $x_i = (x_{i1}, x_{i2})^T$, $g(x_i) = (\tanh(x_{i1}), \tanh(x_{i2}))^T$, $\tau(t) = \frac{e^t}{1+e^t}$, and

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}.$$

Assume that all nodes are connected as a ring, i.e., complex network (14) is a ring coupled network. The configuration matrix C is

$$C = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}.$$

Obviously, the decoupled or isolated oscillator can be described as

$$\begin{cases} \dot{s}_1(t) = -s_1(t) + 3\tanh(s_2(t-\tau(t))), \\ \dot{s}_2(t) = -s_2(t) + 3\tanh(s_1(t-\tau(t))). \end{cases} \quad (15)$$

System (15) has three equilibria

$$S_1^* = (-2.9847, -2.9847), \qquad S_2^* = (0, 0),$$
$$S_3^* = (2.9847, 2.9847).$$

It is easy to see by simulation shown in Figs. 1–2 that all solutions of system (15) with different initial values converge to either S_1^* or S_3^*, which illuminate that the equilibria S_1^* and S_3^* are both locally stable and S_2^* is unstable.

In the following, we choose $s(t) = S_2^*$ as a desired state, and the first three nodes are only controlled. Obviously, Assumptions 1–3 are satisfied. Choose control strengths $k_i = 0.5$ for $i = 1, 2, 3$ and $\beta_1(0) = 0.16$, $\beta_2(0) = 0.12$, $\beta_3(0) = 0.15$. Select $\mu_{ij} = 0.2$ in the adaptive law (8) and $\omega_{12}(0) = 0.01$, $\omega_{15}(0) = 0.02$, $\omega_{23}(0) = 0.03$, $\omega_{34}(0) = 0.02$, $\omega_{45}(0) = 0.05$. Based on those parameters and according to Corollary 1, the solution $x_i(t)$ of system (14) tends to $S_2^* = (0, 0)$, that is to say, the unstable equilibrium S_2^* is stabilized under controllers (6)–(8), which is shown in Fig. 3. From Figs. 4–5, the coupling weights ω_{ij} and the feedback gains β_i converge to some constants, respectively.

Example 2 Consider a star complex dynamic network described by

$$\dot{s}(t) = -A_2 s(t) + B_2 h\big(s(t)\big) + D_2 h\big(s\big(t-\tau(t)\big)\big), \quad (16)$$

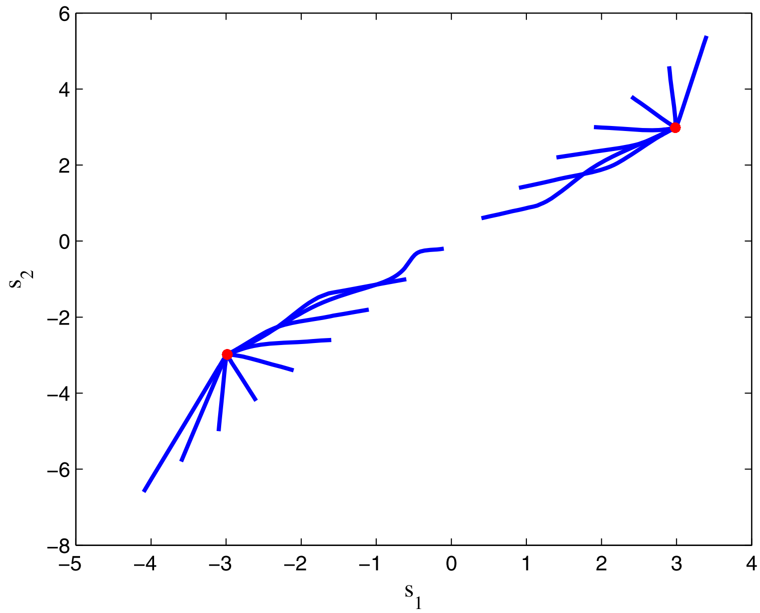

Fig. 1 The phase portrait of system (15) with different initial values

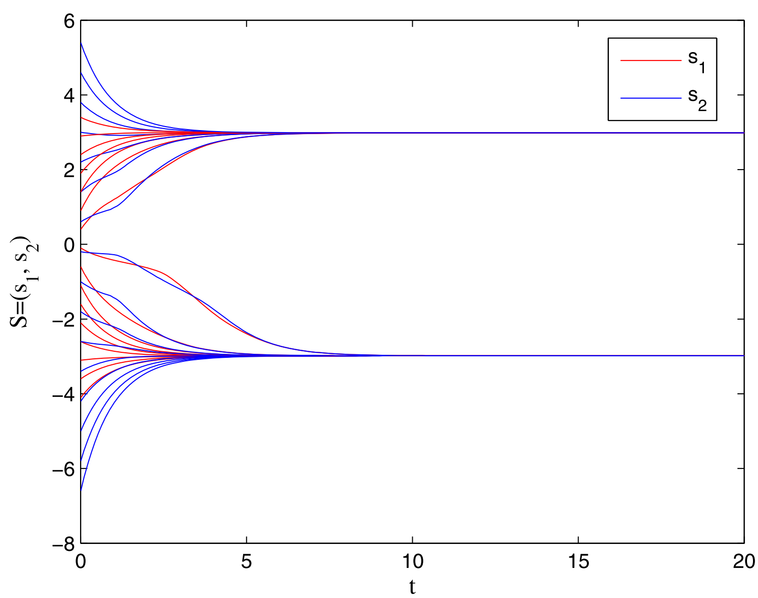

Fig. 2 Time evolution of system (15) with different initial values

where $s = (s_1, s_2)^T$, $h(s) = (0.5(|s_1 + 1| - |s_1 - 1|), 0.5(|s_2 + 1| - |s_2 - 1|))^T$, $\tau(t) = \frac{e^t}{1+e^t}$, $A_2 = \mathrm{diag}(1, 1)$, and

$$B_2 = \begin{bmatrix} 1+\frac{\pi}{4} & 20 \\ 0.1 & 1+\frac{\pi}{4} \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -\frac{1.3\sqrt{2}\pi}{4} & 0.1 \\ 0.1 & -\frac{1.3\sqrt{2}\pi}{4} \end{bmatrix}.$$

The dynamic behavior of system (16) is represented in Fig. 6, from which we can see that system (16) has a chaotic attractor.

The star complex network composed of $N = 5$ identical nodes is described by

$$\dot{x}_i(t) = -A_2 x_i(t) + B_2 h\big(x_i(t)\big) + D_2 h\big(x_i\big(t - \tau(t)\big)\big) + \sum_{j=1, j\neq i}^{5} c_{ij}\omega_{ij}(t)\big[x_j(t) - x_i(t)\big], \tag{17}$$

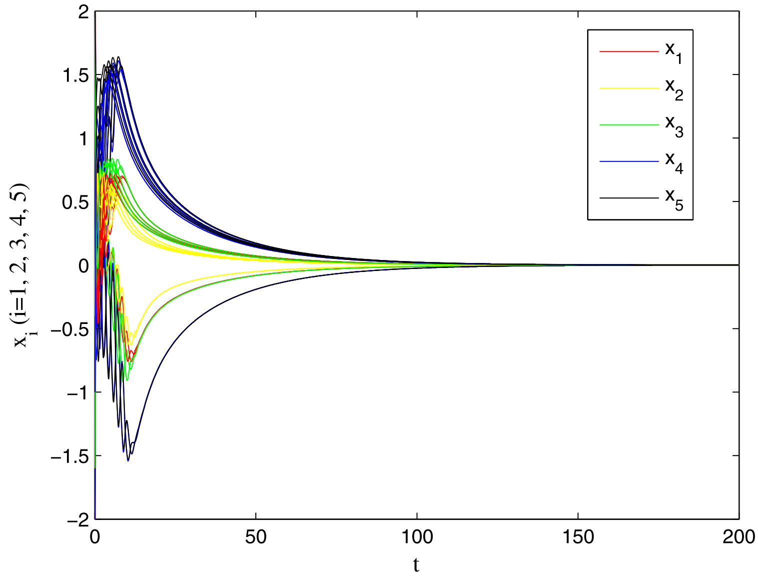

Fig. 3 Time evolution of controlled system (14) with different initial values

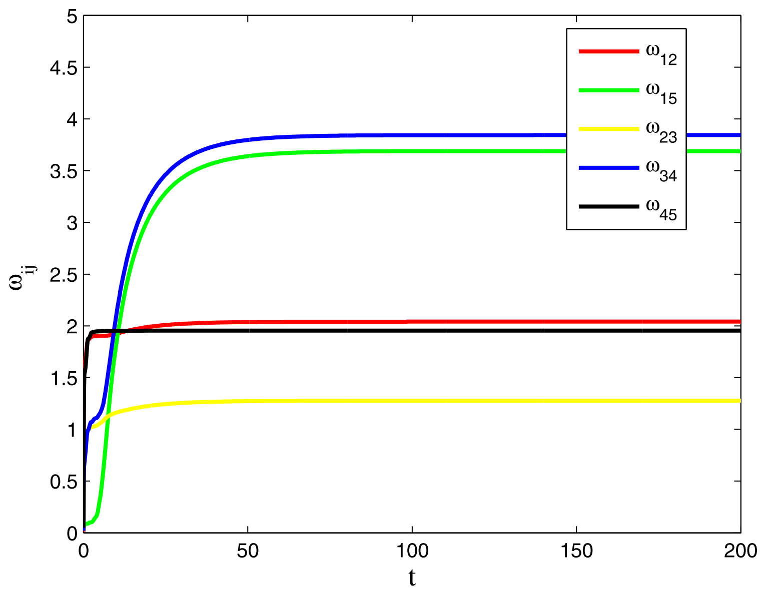

Fig. 4 Time evolution of coupling weights ω_{ij} of system (14)

for $i = 1, 2, \ldots, 5$, where A_2, B_2, D_2, $h(x_i)$, $\tau(t)$ are defined in system (16), and the configuration matrix C is given by

$$C = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

We choose chaotic attractor as the synchronized aim and only control the first node. Evidently, Assumptions 1–3 are satisfied. Take the control strength $k_1 = 0.5$ and $\beta_1(0) = 0.16$. Select $\mu_{12} = \mu_{13} = 0.01$ and $\mu_{14} = \mu_{15} = 0.02$ in the adaptive law (8) and $\omega_{12}(0) = \omega_{13}(0) = 0.05$, $\omega_{14}(0) = \omega_{15}(0) = 0.03$. It follows from Corollary 2 that system (17) is synchronized to chaotic system (16), which is verified by Fig. 7, and in this case, the coupling weights ω_{ij} and the feedback gains β_i respectively converge to some constants that are shown in Figs. 8–9.

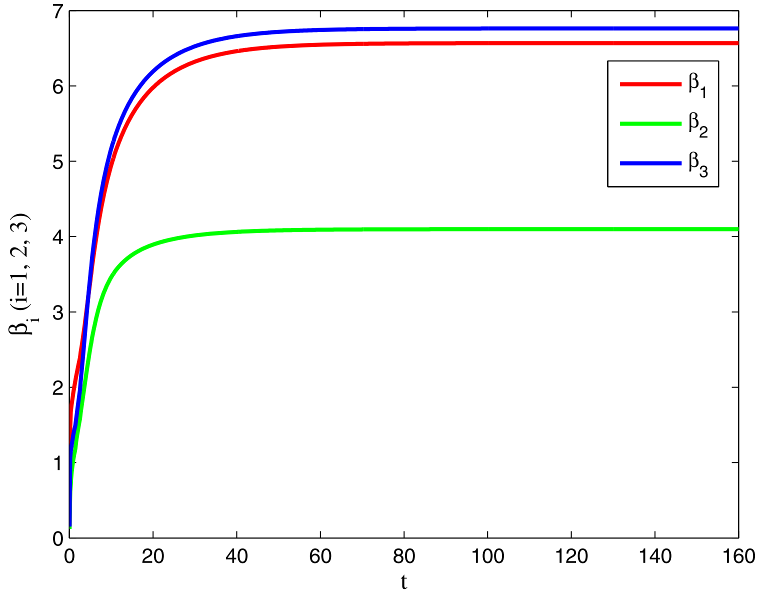

Fig. 5 Time evolution of feedback gains β_i

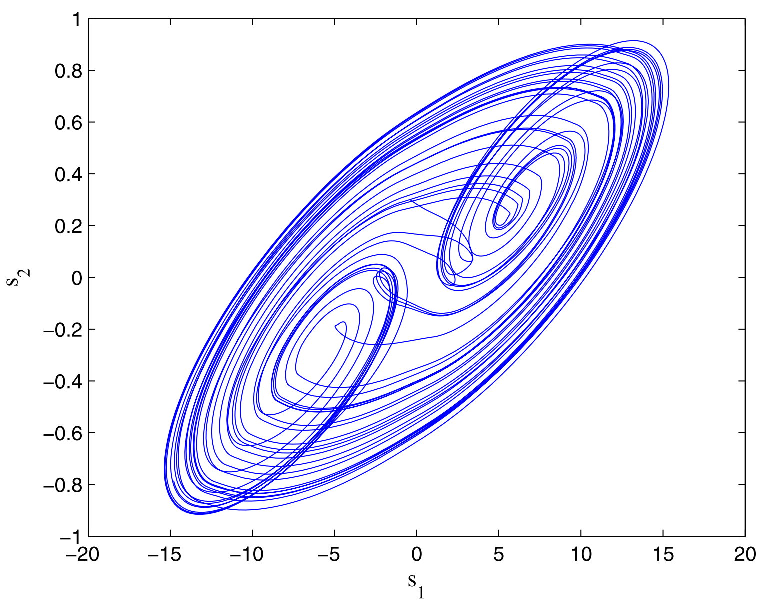

Fig. 6 Chaotic behavior of system (16) with initial values $(-0.4, 0.3)$

Remark 5 In Example 1, the unstable equilibrium point S_2^* of (15) is chosen as the desired orbit, and all nodes in ring network (14) are synchronized to S_2^* by controlling the first three nodes and adjusting the adaptive coupling weights. In Example 2, the synchronized orbit is a chaotic attractor, and all node states in star network (17) tend to this chaotic trajectory by adaptively adjusting the coupling weights and only controlling the first node based on the adaptive feedback law.

5 Conclusion

In real application, it is tough to access all information of the large-scale networks. However, for a given node, the information of nodes directly connected with it can be obtained. In light of this, our aim in this paper is to deal with the globally asymptotical synchronization of weighted complex networks with variable delays and adaptive coupling weights by virtue of controlling partial nodes and local adaptive feedback strategy without

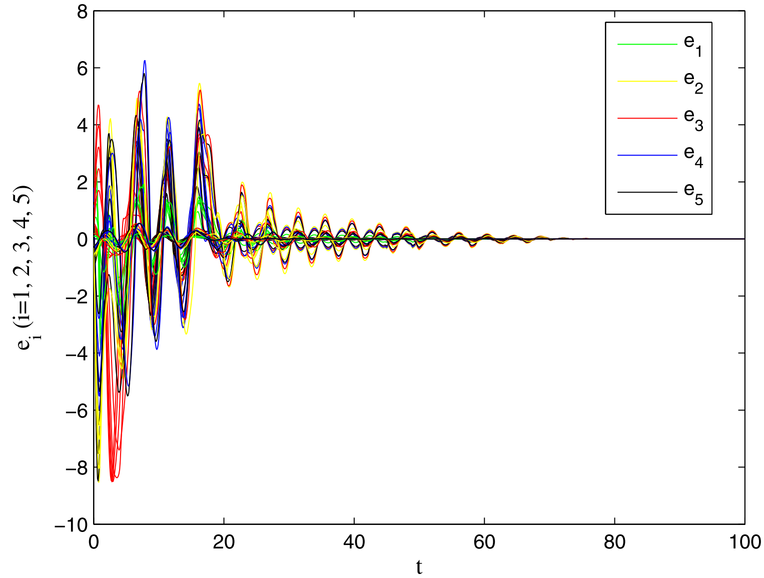

Fig. 7 Synchronization errors of the star network (17) with different initial values

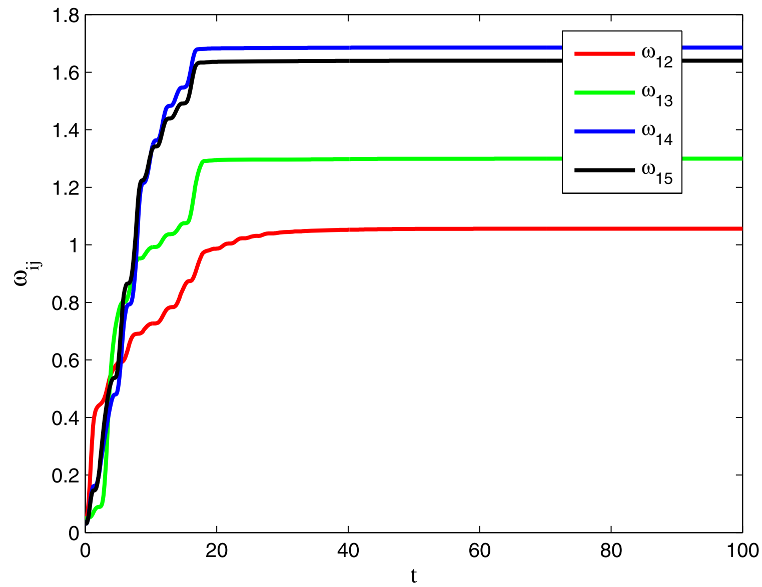

Fig. 8 Time evolution of coupling weights ω_{ij} of system (17)

requiring global information of the networks. The synchronized state is chosen as a decoupled state, which may be an equilibrium point, or even a chaotic orbit. The information of the controlled node itself and the synchronized state is only needed in our local feedback control. Moreover, our adaptive update law of coupling weight $\omega_{ij}(t)$ is only related to the dynamical behaviors of directly connected nodes i and j. Evidently, our results are derived based on local effective information other than the global information of the whole network, which is significantly predominant in application in view of the access difficulty of the global information. Finally, a ring complex network and a star network are listed, which are respectively synchronized to an unstable equilibrium point of the decoupled system and a chaotic attractor.

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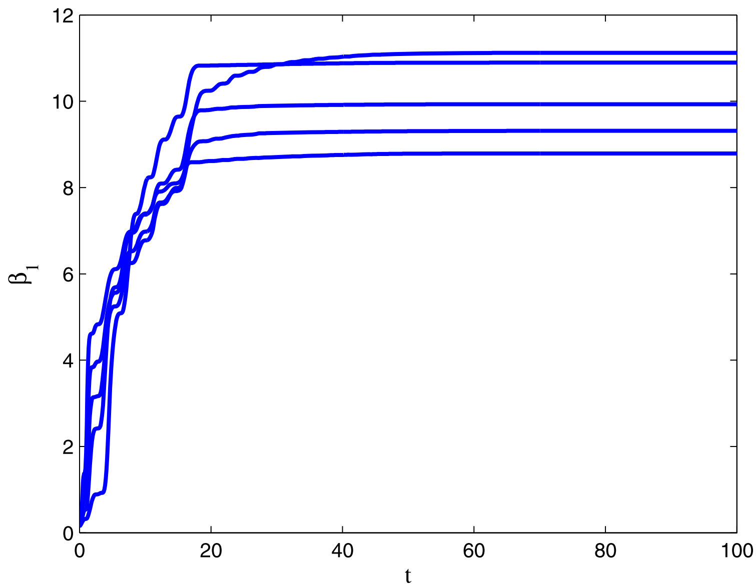

Fig. 9 Time evolution of feedback gain β_1 with different initial values

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