Nonlinear Dyn (2012) 67:829–845
DOI 10.1007/s11071-011-0029-z

ORIGINAL PAPER

# *p*th moment exponential synchronization for stochastic delayed Cohen–Grossberg neural networks with Markovian switching

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Received: 30 November 2010 / Accepted: 21 March 2011 / Published online: 13 April 2011


**Abstract** This paper is a contribution to the analysis of the $p$th moment exponential synchronization problem for a class of stochastic delayed Cohen–Grossberg neural networks with Markovian switching. The jumping parameters are determined by a continuous-time, discrete-state Markov chain, and the delays are time-varying delays.

By using the Lyapunov–Krasovskii functional, stochastic analysis theory, a generalized Halanay-type inequality as well as output coupling with delay feedback control technique, some novel sufficient conditions are derived to achieve complete $p$th moment exponential synchronization of the addressed neural networks. In particular, the traditional assumptions on the differentiability of the time varying delay and the boundedness of its derivative are removed in this paper. The results obtained in this paper generalize and improve many known results. Moreover, a numerical example and its simulation are also provided to demonstrate the effectiveness and applicability of the theoretical results.

**Keywords** $p$th moment exponential synchronization · Time-varying delay · Stochastic neural network · Cohen–Grossberg neural network · Markovian switching

This work was jointly supported by the National Natural Science Foundation of China (10801056, 60874088), the Natural Science Foundation of Ningbo (2010A610094), K.C. Wong Magna Fund in Ningbo University, and the Specialized Research Fund for the Doctoral Program of Higher Education (20070286003).

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## 1 Introduction

As is well known, control and synchronization of chaos systems have been an important research topic since Pecora and Carrol in [2, 16] originally introduced their works about the issue of chaos synchronization. The goal of chaos synchronization is that two or more chaotic systems can share a common dynamical behavior. Consequently, chaos synchronization has been widely applied to create secure communication systems, chemical and biological systems, image processing, information science, human heartbeat regulation, and harmonic oscillation generation, etc. In particular, chaos synchronization has been used to study neural networks in recent years since neural networks can exhibit chaotic behaviors when the neural networks' parameters and time delays are appropriately chosen. Therefore, it is interesting and important to study the synchronization of chaos neural networks.

Recently, a large number of interesting results on the synchronization of chaotic neural networks with or without noise disturbances have appeared in the literature; see e.g., [4, 7–12, 17, 19–21, 23, 24, 26, 29, 30, 32, 34, 37] and the references therein. By using the drive-response concept and a nonlinear feedback control law, Cui and Lou [4] investigated the state synchronization of the two identical chaotic neural networks. In [9], Li and Fu studied the exponential synchronization problem for a class of chaotic delayed neural networks with impulsive and stochastic perturbations by using the method of impulsive delay differential inequalities. With the help of complex systems theory, Posadas-Castillo et al. [17] studied the synchronization of chaotic neural networks with delays in coupled irregularly arrays in two cases: without and with chaotic master node. By using a Lyapunov–Krasovskii functional and a combination of the free-weighting matrix method, Newton–Leibniz formulation and inequality technique, Song [21] designed the controllers to achieve the asymptotical and exponential synchronization for neural networks with mixed delays. Based on the adaptive feedback control technique, Wang et al. [26] obtained some sufficient conditions to achieve adaptive synchronization for a class of recurrent neural networks. In [29], Yang and Cao proposed a unidirectional linear coupling scheme for exponential lag synchronization of a class of chaotic delayed neural networks with impulsive effects. By employing the adaptive control and linear feedback with the updated law, Zhou et al. [32] derived several sufficient conditions to guarantee adaptive synchronization for two coupled delayed neural networks. Based on the invariant principle of function differential equations and Lyapunov–Krasovskii functional as well as the adaptive control and linear feedback with update law, Zhu and Cao [34] derived some novel sufficient conditions achieving synchronization of the two coupled networks with mixed delays, which synchronously consist of constant delays, time-varying delays, and distributed delays. By using the LaSalle invariant principle of stochastic differential delay equations and the stochastic analysis theory as well as the adaptive feedback control technique, Zhu and Cao [37] studied the adaptive synchronization under almost every initial data for a class of unidirectionally coupled stochastic delayed neural networks. It is worth pointing out that all of the mentioned works do not consider the $p$th moment exponential synchronization. But on the other hand, the $p$th moment exponential stability on neural networks has been widely studied by many authors, for instance, see [5, 6, 20, 22, 25] and the references therein. Thus, it is also interesting to the $p$th moment exponential synchronization on neural networks.

On the other hand, a class of neural networks with Markovian switching called as Markovian jump neural networks has received a great deal of research attention since it can model the phenomenon of information latching, and the abrupt phenomena such as random failures or repairs of the components, sudden environmental changes, changing subsystem interconnections, and so on. When noise disturbances are considered in Markovian jump neural networks, this class of neural networks is usually called Markovian jump stochastic neural networks or stochastic neural networks with Markovian switching. It is known that a Markovian jump stochastic neural network is more complicated and comprises a general stochastic neural network as its special case. Owing to the practical importance, many journal papers have recently devoted to study the stability analysis issue for Markovian jump neural networks [1, 13, 18, 27, 28, 31, 33, 35, 36, 38–40]. However, up to now, the synchronization problem for Markovian jump neural networks has received little research attention, despite its practical importance. This situation motivates our present investigation.

Inspired by the above discussions, in this paper, we study the $p$th moment exponential synchronization problem for a class of stochastic delayed Cohen–Grossberg neural networks with Markovian switching. The jumping parameters are determined by a continuous-time, discrete-state Markov chain, and the delays are time-varying delays. To the best of the authors' knowledge, until now, the $p$th moment exponential synchronization problem for this class of generalized neural networks has not yet been solved. The main goal of this paper is to fill this gap. By using the Lyapunov–Krasovskii functional, stochastic analysis theory, a generalized Halanay-type inequality as well as output coupling with delay feedback control technique, some novel sufficient conditions are derived to achieve complete $p$th moment exponential synchronization of the addressed neural networks. In particular, the traditional assumptions on the differentiability of the time varying delays and the boundedness of its derivative are removed in this paper. The results obtained in this paper generalize and improve many known results. Finally, a numerical example and its simulation are also provided to demonstrate the effectiveness and applicability of the theoretical results.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the model and some notations as well as several necessary assumptions. By employing the Lyapunov–Krasovskii functional, a generalized Halanay-type inequality, and stochastic analysis theory as well as output coupling with delay feedback control technique, we prove that the two coupled delayed neural networks are $p$th moment exponential synchronization in Sect. 3. In Sect. 4, a numerical example and its simulation are given to illustrate the effectiveness of the obtained results. Finally, a general conclusion is drawn in Sect. 5.

## 2 Model description, notations and assumptions

*Notation* Throughout this paper, the following notations will be used. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the $n$-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\mathbb{R}^+=[0,\infty)$ and Trace $(\cdot)$ denotes the trace of the corresponding matrix. Let $\tau>0$ and $C([-\tau,0];\mathbb{R}^n)$ denote the family of continuous function $\phi$ from $[-\tau,0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\|=\sup_{-\tau\le\theta\le0}|\phi(\theta)|$. Denote by $C^p_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ the family of all $\mathcal{F}_0$ measurable, $C([-\tau,0];\mathbb{R}^n)$-valued stochastic variables $\xi=\{\xi(\theta):-\tau\le\theta\le0\}$ such that $\int_{-\tau}^0\mathbb{E}|\xi(s)|^p\,ds<\infty$, where $\mathbb{E}[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure $P$.

Let $\{r(t),\ t\ge0\}$ be a right-continuous Markov chain on a complete probability space $(\Omega,\mathcal{F},P)$ taking values in a finite state space $S=\{1,2,\dots,N\}$ with generator $Q=(q_{ij})_{N\times N}$ given by

$$P\bigl\{r(t+\Delta t)=j|r(t)=i\bigr\}$$
$$=\begin{cases}q_{ij}\Delta t+o(\Delta t) & \text{if } i\ne j,\\ 1+q_{ii}\Delta t+o(\Delta t) & \text{if } i=j\end{cases}$$

where $\Delta t>0$ and $\lim_{\Delta t\to0}\frac{o(\Delta t)}{\Delta t}=0$. Here, $q_{ij}\ge0$ is the transition rate from $i$ to $j$ if $i\ne j$ while $q_{ii}=-\sum_{j\ne i}q_{ij}$.

In this paper, we consider the following Markov jump neural networks with time-varying delays:

$$\begin{aligned}dx(t)=\Bigl\{&-\widetilde{\alpha}\bigl(x(t),r(t)\bigr)\bigl[\widetilde{\beta}\bigl(x(t),r(t)\bigr)\\&-C\bigl(r(t)\bigr)\widetilde{f}\bigl(x(t)\bigr)\\&-D\bigl(r(t)\bigr)\widetilde{g}\bigl(x\bigl(t-\tau(t)\bigr)\bigr)\bigr]+J\Bigr\}\,dt,\end{aligned}\tag{1}$$

where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^{\mathrm{T}}$ is the state vector associated with the $n$ neurons, $\widetilde{\alpha}(x(t),r(t))=\operatorname{diag}(\widetilde{\alpha}_1(x_1(t),r(t)),\widetilde{\alpha}_2(x_2(t),r(t)),\dots,\widetilde{\alpha}_n(x_n(t),r(t)))$ represents an amplification function, and $\widetilde{\beta}(x(t),r(t))=[\widetilde{\beta}_1(x_1(t),r(t)),\widetilde{\beta}_2(x_2(t),r(t)),\dots,\widetilde{\beta}_n(x_n(t),r(t))]^{\mathrm{T}}$ is the behaved function. The matrices $C(r(t))=(c_{ij}(r(t)))_{n\times n}$ and $D(r(t))=(d_{ij}(r(t)))_{n\times n}$ are the connection weight matrix and the time-varying delay connection weight matrix, respectively. $\widetilde{f}(x(t))=[\widetilde{f}_1(x_1(t)),\widetilde{f}_2(x_2(t)),\dots,\widetilde{f}_n(x_n(t))]^{\mathrm{T}}$ and $\widetilde{g}(x(t))=[\widetilde{g}_1(x_1(t)),\widetilde{g}_2(x_2(t)),\dots,\widetilde{g}_n(x_n(t))]^{\mathrm{T}}$ are the neuron activation functions, and $J=[J_1,J_2,\dots,J_n]^{\mathrm{T}}$ denotes a constant external input vector. The time-varying delay $\tau(t)$ satisfies $0\le\tau(t)\le\tau$, where $\tau$ is a positive constant.

We consider the model (1) as the drive system. The response system is

$$\begin{aligned}dy(t)=\Bigl\{&-\widetilde{\alpha}\bigl(y(t),r(t)\bigr)\bigl[\widetilde{\beta}\bigl(y(t),r(t)\bigr)\\&-C\bigl(r(t)\bigr)\widetilde{f}\bigl(y(t)\bigr)-D\bigl(r(t)\bigr)\widetilde{g}\bigl(y\bigl(t-\tau(t)\bigr)\bigr)\bigr]\\&+J+u\bigl(t,r(t)\bigr)\Bigr\}\,dt\\&+\sigma\bigl(t,r(t),y(t)-x(t),y\bigl(t-\tau(t)\bigr)\\&-x\bigl(t-\tau(t)\bigr)\bigr)\,dw(t),\end{aligned}\tag{2}$$

where $u(t,r(t))=[u_1(t,r(t)),u_2(t,r(t)),\dots,u_n(t,r(t))]^{\mathrm{T}}$ is the controller, $w(t)=(w_1,\dots,w_m)^{\mathrm{T}}$ is an $m$-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\ge0}$, and $\sigma:\mathbb{R}^+\times S\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times m}$ is the noise intensity matrix. It is known that the occurrence of external random fluctuation and other probabilistic causes often lead to this type of stochastic perturbations. We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $w(\cdot)$.

Let $e(t)=y(t)-x(t)$ be the synchronization error, the state feedback controller in the response system (2) is designed as follows:

$$\begin{aligned}u\bigl(t,r(t)\bigr)=\;&K_1\bigl(r(t)\bigr)\bigl[\widetilde{f}\bigl(y(t)\bigr)-\widetilde{f}\bigl(x(t)\bigr)\bigr]\\&+K_2\bigl(r(t)\bigr)\bigl[\widetilde{g}\bigl(y\bigl(t-\tau(t)\bigr)\bigr)\\&-\widetilde{g}\bigl(x\bigl(t-\tau(t)\bigr)\bigr)\bigr],\end{aligned}\tag{3}$$

where $K_1(r(t)), K_2(r(t))$ are the gain matrices to be scheduled. With the above control law, the dynamical system of synchronization error between system (1) and (2) can be expressed by

$$\begin{aligned} de(t) = \{&-\alpha(e(t), r(t))[\beta(e(t), r(t)) - (C(r(t)) \\ &+ K_1(r(t)))f(e(t)) \\ &- (D(r(t)) + K_2(r(t)))g(e(t-\tau(t)))]\}\,dt \\ &+ \sigma(t, r(t), e(t), e(t-\tau(t)))\,dw(t), \end{aligned} \tag{4}$$

where $\alpha(e(t), r(t)) = \widetilde{\alpha}(x(t) + e(t), r(t)) - \widetilde{\alpha}(x(t), r(t))$, $\beta(e(t), r(t)) = \widetilde{\beta}(x(t) + e(t), r(t)) - \widetilde{\beta}(x(t), r(t))$, $f(e(t)) = \widetilde{f}(x(t) + e(t)) - \widetilde{f}(x(t))$, $g(e(t)) = \widetilde{g}(x(t) + e(t)) - \widetilde{g}(x(t))$.

*Remark 1* In lots of applications, it is often to design the state-feedback controller or time-delay feedback controller as $u(t) = K_1 e(t)$ or $u(t) = K_1 e(t) + K_2 e(t - \tau(t))$. However, in some real networks, only output signals can be measured. Thus, it is necessary to consider the controller (3) in the response system. Usually, we refer to this as output coupling with delay feedback.

If taking $A(r(t)) = C(r(t)) + K_1(r(t)) = (a_{ij}(r(t)))_{n\times n}$, $B(r(t)) = D(r(t)) + K_2(r(t)) = (b_{ij}(r(t)))_{n\times n}$, then system (4) can be rewritten as

$$\begin{aligned} de(t) = \{&-\alpha(e(t), r(t))[\beta(e(t), r(t)) \\ &- A(r(t))f(e(t)) \\ &- B(r(t))g(e(t-\tau(t)))]\}\,dt \\ &+ \sigma(t, r(t), e(t), e(t-\tau(t)))\,dw(t), \end{aligned} \tag{5}$$

or equivalently,

$$\begin{aligned} de_j(t) = \Bigg\{&-\alpha_j(e_j(t), i)\Bigg[\beta_j(e_j(t), i) \\ &- \sum_{k=1}^{n} a_{ijk} f_k(e_k(t)) \\ &- \sum_{k=1}^{n} b_{ijk} g_k(e_k(t-\tau(t)))\Bigg]\Bigg\}\,dt \\ &+ \sum_{k=1}^{n} \sigma_{jk}(t, i, e_k(t), e_k(t-\tau(t)))\,dw_k(t), \end{aligned}$$

where $j = 1, 2, \dots, n$, $r(t) = i \in S$.

Throughout this paper, we always assume that $\widetilde{f}, \widetilde{g}$ and $\sigma$ satisfy the usually local Lipschitz condition and linear growth condition. It follows from [15] that for any given initial data $e(\theta) = \xi(\theta)$ on $-\tau \le \theta \le 0$ in $C^p_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, the error dynamics system (3) has a unique global solution on $t \ge 0$ and we denote the solution by $e(t; \xi)$. For simplicity, we write $e(t; \xi) = e(t)$. Let $C^2_1(\mathbb{R}^+ \times S \times \mathbb{R}^n; \mathbb{R}^+)$ denote the family of all nonnegative functions $V(t, i, e)$ on $\mathbb{R}^+ \times S \times \mathbb{R}^n$ which are continuously twice differentiable in $e$ and differentiable in $t$. If $V \in C^2_1(\mathbb{R}^+ \times S \times \mathbb{R}^n; \mathbb{R}^+)$, then along the trajectory of the system (5) we define an operator $\mathcal{L}V$ from $\mathbb{R}^+ \times S \times \mathbb{R}^n$ to $\mathbb{R}$ by

$$\begin{aligned} &\mathcal{L}V(t, i, e(t)) \\ &\quad = V_t(t, i, e(t)) + V_e(t, i, e(t)) \\ &\qquad \times \{-\alpha(e(t), i)[\beta(e(t), i) \\ &\qquad - A(i)f(e(t)) - B(i)g(e(t-\tau(t)))] + u(t)\} \\ &\qquad + \frac{1}{2}\operatorname{trace}[\sigma^{\mathrm{T}}(t, i, e(t), e(t-\tau(t))) \\ &\qquad \times V_{ee}(t, i, e(t))\sigma(t, i, e(t), e(t-\tau(t)))] \\ &\qquad + \sum_{j=1}^{N} q_{ij} V(t, j, e(t)), \end{aligned} \tag{6}$$

where

$$V_t(t, i, e(t)) = \frac{\partial V(t, i, e(t))}{\partial t},$$

$$V_e(t, i, e(t)) = \left(\frac{\partial V(t, i, e(t))}{\partial e_1}, \dots, \frac{\partial V(t, i, e(t))}{\partial e_n}\right)$$

and

$$V_{ee}(t, i, e(t)) = \left(\frac{\partial^2 V(t, i, e(t))}{\partial e_j \partial e_k}\right)_{n\times n}.$$

To prove our results, the following assumptions are necessary in this paper.

**Assumption 1** *There exist positive constants* $\alpha^0_{ij}, \alpha^1_{ij}$ $(i = 1, 2, \dots, N,\ j = 1, 2, \dots, n)$ *such that*

$$0 < \alpha^0_{ij} \le \widetilde{\alpha}_j(x_j(t), i) \le \alpha^1_{ij}$$

*for all* $x_j(t) \in \mathbb{R}$, $r(t) = i$, $i \in S$ *and* $j = 1, 2, \dots, n$.

**Assumption 2** *There exist positive constants* $\delta_{ij}$ $(i = 1, 2, \dots, N,\ j = 1, 2, \dots, n)$ *such that*

$$x_j(t)\widetilde{\beta}_j\big(x_j(t), i\big) \geq \delta_{ij} x_j^2(t)$$

*for all* $x_j(t) \in \mathbb{R}, r(t) = i, i \in S$ *and* $j = 1, 2, \dots, n$.

*Remark 2* The function $\widetilde{\beta}_j(x_j(t), i)$ in the earlier literature is required to be differentiable and its derivative is required to be over zero. However, the function $\widetilde{\beta}_j(x_j(t), i)$ in Assumption 2 is not necessarily differentiable. For example, if taking $\widetilde{\beta}_j(x_j(t), i) = 2|x_j(t)|$ $(i = 1, 2, \dots, n)$, then Assumption 2 is satisfied, but the conditions in the earlier literature do not hold. Hence, Assumption 2 of this paper is weaker than those given the earlier literature.

**Assumption 3** *There exist diagonal matrices* $U_l^- = \operatorname{diag}(u_{l1}^-, u_{l2}^-, \dots, u_{ln}^-)$, $U_l^+ = \operatorname{diag}(u_{l1}^+, u_{l2}^+, \dots, u_{ln}^+)$, $l = 1, 2$ *satisfying*

$$u_{1j}^- \leq \frac{\widetilde{f}_j(x_1) - \widetilde{f}_j(x_2)}{x_1 - x_2} \leq u_{1j}^+,$$
$$u_{2j}^- \leq \frac{\widetilde{g}_j(x_1) - \widetilde{g}_j(x_2)}{x_1 - x_2} \leq u_{2j}^+,$$

*for all* $x_1, x_2 \in \mathbb{R}, x_2 \neq x_2, j = 1, 2, \dots, n$.

*Remark 3* In Assumption 3, we do not require the boundedness of activation functions and they may be neither monotonically increasing nor continuously differentiable. Moreover, the constants $u_{lj}^-, u_{lj}^+$ $(l = 1, 2, j = 1, 2, \dots, n)$ are allowed to be *positive, negative or zero*. Hence, Assumption 3 of this paper is weaker than those given in the previous literature (see, e.g., [4, 7, 20, 23, 26, 30]).

**Assumption 4** *There exist positive constants* $\mu_{ij}, \nu_{ij}$, $i \in S$, $j = 1, 2, \dots, n$ *such that*

$$\operatorname{trace}\big[\sigma^{\mathrm{T}}(t, i, x, y)\sigma(t, i, x, y)\big] \leq \sum_{j=1}^{n}\big(\mu_{ij} x_j^2 + \nu_{ij} y_j^2\big)$$

*for all* $x, y \in \mathbb{R}^n, i \in S$ *and* $t \in \mathbb{R}^+$.

**Assumption 5** $\sigma(t, r(t), 0, 0, 0) \equiv 0$.

Under Assumption 5 and noting the facts that $\alpha(0, r(t)) = \beta(0, r(t)) = f(0) = g(0) = 0$, the system (3) admits a trivial solution $e(t; 0) \equiv 0$ corresponding to the initial data $\xi = 0$. Hence, to prove that the systems (1) and (2) achieve $p$th moment exponential synchronization, it suffices to prove that the trivial solution of the system (3) is $p$th moment exponentially stable. On the other hand, by Assumption 3, we have

$$\begin{aligned} u_{1j}^- &\leq \frac{f_j(x_1) - f_j(x_2)}{x_1 - x_2} \leq u_{1j}^+, \\ u_{2j}^- &\leq \frac{g_j(x_1) - g_j(x_2)}{x_1 - x_2} \leq u_{2j}^+, \end{aligned} \tag{7}$$

for all $x_1, x_2 \in \mathbb{R}, x_2 \neq x_2, j = 1, 2, \dots, n$.

Next, we first introduce the definition of $p$th moment exponential synchronization for the two coupled neural networks (1) and (2), and then state the notation of the upper right Dini-derivative and some preliminary lemmas, which are needed to prove our main results.

**Definition 1** The two coupled neural networks (1) and (2) are said to be $p$th $(p \geq 2)$ moment exponentially synchronized if for every $\xi \in C_{\mathcal{F}_0}^p([-\tau, 0]; \mathbb{R}^n)$, there exist positive constants $\alpha, \beta$ such that

$$\mathbf{E}\big\|e(t)\big\|^p \leq \alpha e^{-\beta t} \sup_{-\tau \leq s \leq 0} \mathbf{E}\big\|e(s)\big\|^p, \quad t \geq 0,$$

where the norm $\|e(t)\|$ satisfies $\|e(t)\| = (|e(t)|^p)^{\frac{1}{p}} = (\sum_{i=1}^n |e_i(t)|^p)^{\frac{1}{p}}$.

**Definition 2** Assume that $h$ is a continuous function. Then we can define the upper right Dini-derivative of $h$ as follows:

$$D^+\big(h(t)\big) := \limsup_{\delta \to 0^+} \frac{h(t + \delta) - h(t)}{\delta}.$$

**Lemma 1** [14, *Lemma* 4.2] *Let* $p \geq 2$ *and* $\varepsilon, a, b > 0$. *Then*

$$a^{p-1} b \leq \frac{(p-1)\varepsilon a^p}{p} + \frac{b^p}{p\varepsilon^{(p-1)}}$$

*and*

$$a^{p-2} b^2 \leq \frac{(p-2)\varepsilon a^p}{p} + \frac{2b^p}{p\varepsilon^{\frac{(p-2)}{2}}}.$$

Obviously, Letting $\varepsilon = 1$ in Lemma 1, we get the following result.

**Lemma 2** *Let $p \geq 2$ and $a, b > 0$. Then*

$$a^{p-1}b \leq \frac{(p-1)a^p}{p} + \frac{b^p}{p}$$

*and*

$$a^{p-2}b^2 \leq \frac{(p-2)a^p}{p} + \frac{2b^p}{p}.$$

## 3 Main results and proofs

In this section, the $p$th moment exponential synchronization for the two coupled neural networks (1) and (2) is investigated under Assumptions 1–5. To prove our main result, we need to introduce the following technical proposition on a generalized Halanay-type inequality.

**Proposition 1** *Assume that there are positive constant numbers $\lambda_i^{(1)}, \lambda_i^{(2)}$ $(i \in S)$ such that $\lambda_i^{(1)} > \lambda_i^{(2)} > 0$ $(i \in S)$ and $z(t,i)$ $(i \in S)$ are nonnegative continuous functions defined on $[-\tau, 0]$ and satisfies the following inequality*:

$$D^+ z(t,i) \leq -\lambda_i^{(1)} z(t,i) + \lambda_i^{(2)} \overline{z}(t,i),$$
$$t \geq 0 \text{ and } r(t) = i \in S,$$

*where $\overline{z}(t,i) = \sup_{-\tau \leq s \leq 0} z(s, r(s))$, and $\tau > 0$ is a positive constant. Then for all $t \geq 0$, we have*

$$z(t,i) \leq \overline{z}\big(0, r(0)\big) e^{-\lambda_i t},$$
$$t \geq 0 \text{ and } r(t) = i \in S,$$

*where $\lambda_i (i \in S)$ are unique positive roots of the following equation*:

$$\lambda_i = \lambda_i^{(1)} - \lambda_i^{(2)} e^{\lambda_i \tau}, \quad i \in S.$$

*Proof* The proof of Proposition 1 is very similar to that in [3, Lemma 2], and so we omit it. □

*Remark 4* Obviously, if taking $S = \{1\}$, then Proposition 1 is the same as [3, Lemma 2]. So, Proposition 1 extends and improves the correspondent result given in [3, Lemma 2].

Our main result is the following.

**Theorem 1** *Under Assumptions 1–5, the two coupled neural networks (1) and (2) can be pth moment exponentially synchronized, if there exist positive numbers $\gamma_i (i \in S), m_j$ $(j = 1, 2, \ldots, n)$ such that*

$$\lambda_i^{(1)} > \lambda_i^{(2)} > 0, \quad i \in S, \tag{8}$$

*where*

$$\begin{aligned}
\lambda_i^{(1)} = \min_{1 \leq j \leq n} \Bigg\{ & \gamma_i p \alpha_{ij}^0 \delta_{ij} - \gamma_i (p-1) \sum_{k=1}^n \alpha_{ij}^1 |a_{ijk}| u_{1k} \\
& - \gamma_i \sum_{k=1}^n \frac{m_k}{m_j} \alpha_{ik}^1 |a_{ikj}| u_{1j} \\
& - \gamma_i (p-1) \sum_{k=1}^n \alpha_{ij}^1 |b_{ijk}| u_{2k} \\
& - \frac{1}{2} \gamma_i (p-1)(p-2) \sum_{k=1}^n \mu_{ik} \\
& - \gamma_i (p-1) \sum_{k=1}^n \frac{m_k}{m_j} \mu_{ij} \\
& - \gamma_i (p-1)(p-2) \sum_{k=1}^n \nu_{ik} - \sum_{l=1}^N q_{il} \gamma_l \Bigg\},
\end{aligned}$$

$$\begin{aligned}
\lambda_i^{(2)} = \max_{1 \leq j \leq n} \Bigg\{ & \gamma_i \sum_{k=1}^n \frac{m_k}{m_j} \alpha_{ik}^1 |b_{ikj}| u_{2j} \\
& + \gamma_i (p-1) \sum_{k=1}^n \frac{m_k}{m_j} \nu_{ij} \Bigg\}.
\end{aligned}$$

*Proof* Consider the following Lyapunov–Krasovskii functional: $V(t,i,e(t)) = \gamma_i \sum_{j=1}^n m_j |e_j(t)|^p$. Noting that $V_e(t,i,e(t)) = \gamma_i p \sum_{j=1}^n m_j |e_j(t)|^{p-1}$ $\operatorname{sgn}\{e_j(t)\} = \gamma_i p \sum_{j=1}^n m_j |e_j(t)|^{p-2} e_j(t)$ and $V_{ee}(t,i,e(t)) = \gamma_i p(p-1) \sum_{j=1}^n m_j |e_j(t)|^{p-2}$ $\operatorname{sgn}\{e_j(t)\}$, it follows from Lemma 2 and (5)–(6) that

$$\begin{aligned}
&\mathcal{L}V(t,i,e(t)) \\
&\quad = -\gamma_i p \sum_{j=1}^n m_j \big|e_j(t)\big|^{p-2} e_j(t) \alpha_j\big(e_j(t), i\big) \\
&\qquad \times \Bigg[ \beta_j\big(e_j(t), i\big) - \sum_{k=1}^n a_{ijk} f_k\big(e_k(t)\big)
\end{aligned}$$

$$
\begin{aligned}
&-\sum_{k=1}^{n} b_{ijk} g_k\big(e_k\big(t-\tau(t)\big)\big)\Bigg]\\
&+\sum_{l=1}^{N} q_{il}\gamma_l \sum_{k=1}^{n} m_k \big|e_k(t)\big|^p\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n} m_j \big|e_j(t)\big|^{p-2}\mathrm{sgn}\big\{e_j(t)\big\}\\
&\times \sum_{k=1}^{n}\sigma_{jk}^2\big(t,i,e_k(t),e_k\big(t-\tau(t)\big)\big)\\
\le& -\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-2}\alpha_j\big(e_j(t),i\big)e_j(t)\\
&\times \beta_j\big(e_j(t),i\big)\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_j\big(e_j(t),i\big)\\
&\times\sum_{k=1}^{n} a_{ijk} f_k\big(e_k(t)\big)\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_j\big(e_j(t),i\big)\\
&\times\sum_{k=1}^{n} b_{ijk} g_k\big(e_k\big(t-\tau(t)\big)\big)\\
&+\sum_{l=1}^{N} q_{il}\gamma_l \sum_{k=1}^{n} m_k \big|e_k(t)\big|^p\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n} m_j \big|e_j(t)\big|^{p-2}\sum_{k=1}^{n}\big(\mu_{ik}e_k^2(t)\\
&+\nu_{ik}e_k^2\big(t-\tau(t)\big)\big)\\
\le& -\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-2}\alpha_{ij}^0\big|e_j(t)\beta_j\big(e_j(t),i\big)\big|\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_{ij}^1\sum_{k=1}^{n}|a_{ijk}|\big|f_k\big(e_k(t)\big)\big|\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_{ij}^1\\
&\times\sum_{k=1}^{n}|b_{ijk}|\big|g_k\big(e_k\big(t-\tau(t)\big)\big)\big|\\
&+\sum_{l=1}^{N} q_{il}\gamma_l \sum_{k=1}^{n} m_k \big|e_k(t)\big|^p\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n} m_j \big|e_j(t)\big|^{p-2}\\
&\times\sum_{k=1}^{n}\big(\mu_{ik}e_k^2(t)+\nu_{ik}e_k^2\big(t-\tau(t)\big)\big)\\
\le& -\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-2}\alpha_{ij}^0\delta_{ij}\big|e_j(t)\big|^2\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_{ij}^1\\
&\times\sum_{k=1}^{n}|a_{ijk}|\big|f_k\big(e_k(t)\big)-f_k(0)\big|\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_{ij}^1\\
&\times\sum_{k=1}^{n}|b_{ijk}|\big|g_k\big(e_k\big(t-\tau(t)\big)\big)-g_k(0)\big|\\
&+\sum_{l=1}^{N} q_{il}\gamma_l \sum_{k=1}^{n} m_k \big|e_k(t)\big|^p\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n} m_j \big|e_j(t)\big|^{p-2}\\
&\times\sum_{k=1}^{n}\big(\mu_{ik}e_k^2(t)+\nu_{ik}e_k^2\big(t-\tau(t)\big)\big)\\
\le& -\gamma_i p\sum_{j=1}^{n} m_j\alpha_{ij}^0\delta_{ij}\big|e_j(t)\big|^p\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_{ij}^1\sum_{k=1}^{n}|a_{ijk}|u_{1k}\big|e_k(t)\big|\\
&+\gamma_i p\sum_{j=1}^{n} m_j\big|e_j(t)\big|^{p-1}\alpha_{ij}^1\\
&\times\sum_{k=1}^{n}|b_{ijk}|u_{2k}\big|e_k\big(t-\tau(t)\big)\big|\\
&+\sum_{l=1}^{N} q_{il}\gamma_l \sum_{k=1}^{n} m_k \big|e_k(t)\big|^p
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n} m_j\left|e_j(t)\right|^{p-2}\\
&\times\sum_{k=1}^{n}\left(\mu_{ik}e_k^2(t)+\nu_{ik}e_k^2\left(t-\tau(t)\right)\right)\\
=&-\gamma_i p\sum_{j=1}^{n} m_j\alpha_{ij}^0\delta_{ij}\left|e_j(t)\right|^p\\
&+\gamma_i p\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\alpha_{ij}^1|a_{ijk}|u_{1k}\left|e_j(t)\right|^{p-1}\left|e_k(t)\right|\\
&+\gamma_i p\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\alpha_{ij}^1|b_{ijk}|u_{2k}\left|e_j(t)\right|^{p-1}\\
&\times\left|e_k\left(t-\tau(t)\right)\right|\\
&+\sum_{l=1}^{N} q_{il}\gamma_l\sum_{k=1}^{n} m_k\left|e_k(t)\right|^p\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\mu_{ik}\left|e_j(t)\right|^{p-2}e_k^2(t)\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\nu_{ik}\left|e_j(t)\right|^{p-2}\\
&\times e_k^2\left(t-\tau(t)\right)\\
\le&-\gamma_i p\sum_{j=1}^{n} m_j\alpha_{ij}^0\delta_{ij}|e_j(t)|^p\\
&+\gamma_i p\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\alpha_{ij}^1|a_{ijk}|u_{1k}\left[\frac{p-1}{p}\left|e_j(t)\right|^p\right.\\
&\left.+\frac{1}{p}\left|e_k(t)\right|^p\right]+\gamma_i p\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\alpha_{ij}^1|b_{ijk}|u_{2k}\\
&\times\left[\frac{p-1}{p}\left|e_j(t)\right|^p+\frac{1}{p}\left|e_k\left(t-\tau(t)\right)\right|^p\right]\\
&+\sum_{l=1}^{N} q_{il}\gamma_l\sum_{k=1}^{n} m_k\left|e_k(t)\right|^p+\frac{1}{2}\gamma_i p(p-1)\\
&\times\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\mu_{ik}\left[\frac{p-2}{p}\left|e_j(t)\right|^p+\frac{2}{p}\left|e_k(t)\right|^p\right]\\
&+\frac{1}{2}\gamma_i p(p-1)\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\nu_{ik}\left[\frac{p-2}{p}\left|e_j(t)\right|^p\right.\\
&\left.+\frac{2}{p}\left|e_k\left(t-\tau(t)\right)\right|^p\right]\\
=&-\gamma_i p\sum_{j=1}^{n} m_j\alpha_{ij}^0\delta_{ij}\left|e_j(t)\right|^p\\
&+\gamma_i\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\alpha_{ij}^1|a_{ijk}|u_{1k}\left[(p-1)\left|e_j(t)\right|^p\right.\\
&\left.+\left|e_k(t)\right|^p\right]\\
&+\gamma_i\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\alpha_{ij}^1|b_{ijk}|u_{2k}\left[(p-1)\left|e_j(t)\right|^p\right.\\
&\left.+\left|e_k\left(t-\tau(t)\right)\right|^p\right]+\sum_{l=1}^{N} q_{il}\gamma_l\sum_{k=1}^{n} m_k\left|e_k(t)\right|^p\\
&+\frac{1}{2}\gamma_i(p-1)\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\mu_{ik}\left[(p-2)\left|e_j(t)\right|^p\right.\\
&\left.+2\left|e_k(t)\right|^p\right]\\
&+\frac{1}{2}\gamma_i(p-1)\sum_{j=1}^{n}\sum_{k=1}^{n} m_j\nu_{ik}\left[(p-2)\left|e_j(t)\right|^p\right.\\
&\left.+2\left|e_k\left(t-\tau(t)\right)\right|^p\right]\\
=&\sum_{j=1}^{n}\left[-\gamma_i p m_j\alpha_{ij}^0\delta_{ij}\right.\\
&+\gamma_i(p-1)\sum_{k=1}^{n} m_j\alpha_{ij}^1|a_{ijk}|u_{1k}\\
&+\gamma_i\sum_{k=1}^{n} m_k\alpha_{ik}^1|a_{ikj}|u_{1j}\\
&+\gamma_i(p-1)\sum_{k=1}^{n} m_j\alpha_{ij}^1|b_{ijk}|u_{2k}\\
&+\frac{1}{2}\gamma_i(p-1)(p-2)\sum_{k=1}^{n} m_j\mu_{ik}\\
&+\gamma_i(p-1)\sum_{k=1}^{n} m_k\mu_{ij}+\frac{1}{2}\gamma_i(p-1)(p-2)\\
&\left.\times\sum_{k=1}^{n} m_j\nu_{ik}+\sum_{l=1}^{N} q_{il}\gamma_l m_j\right]\left|e_j(t)\right|^p
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\Bigg[\gamma_i\sum_{k=1}^{n}m_k\alpha_{ik}^{1}|b_{ikj}|u_{2j}+\gamma_i(p-1)\\
&\times\sum_{k=1}^{n}m_k\nu_{ij}\Bigg]\big|e_j\big(t-\tau(t)\big)\big|^p\\
&\le-\sum_{j=1}^{n}\lambda_{ij}^{(1)}m_j\big|e_j(t)\big|^p\\
&+\sum_{j=1}^{n}\lambda_{ij}^{(2)}m_j\big|e_j\big(t-\tau(t)\big)\big|^p\\
&\le-\lambda_i^{(1)}V\big(t,i,e(t)\big)+\lambda_i^{(2)}V\big(t,i,e\big(t-\tau(t)\big)\big), \quad (9)
\end{aligned}
$$

where $\lambda_i^{(1)}=\min_{1\le j\le n}\lambda_{ij}^{(1)}$, $\lambda_i^{(2)}=\max_{1\le j\le n}\lambda_{ij}^{(2)}$,

$$
\begin{aligned}
\lambda_{ij}^{(1)}&=\gamma_i p\alpha_{ij}^{0}\delta_{ij}-\gamma_i(p-1)\sum_{k=1}^{n}\alpha_{ij}^{1}|a_{ijk}|u_{1k}\\
&-\gamma_i\sum_{k=1}^{n}\frac{m_k}{m_j}\alpha_{ik}^{1}|a_{ikj}|u_{1j}\\
&-\gamma_i(p-1)\sum_{k=1}^{n}\alpha_{ij}^{1}|b_{ijk}|u_{2k}\\
&-\frac{1}{2}\gamma_i(p-1)(p-2)\sum_{k=1}^{n}\mu_{ik}\\
&-\gamma_i(p-1)\sum_{k=1}^{n}\frac{m_k}{m_j}\mu_{ij}-\gamma_i(p-1)(p-2)\\
&\times\sum_{k=1}^{n}\nu_{ik}-\sum_{l=1}^{N}q_{il}\gamma_l,\\
\lambda_{ij}^{(2)}&=\gamma_i\sum_{k=1}^{n}\frac{m_k}{m_j}\alpha_{ik}^{1}|b_{ikj}|u_{2j}\\
&+\gamma_i(p-1)\sum_{k=1}^{n}\frac{m_k}{m_j}\nu_{ij}.
\end{aligned}
$$

For any small enough $h>0$, applying the generalized Itô's formula and (9),we get

$$
\begin{aligned}
&\mathbf{E}V\big(t+h,r(t+h),e(t+h)\big)-\mathbf{E}V\big(t,r(t),e(t)\big)\\
&=\int_t^{t+h}\mathbf{E}\mathcal{L}V\big(s,r(s),e(s)\big)\,ds\\
&\le\int_t^{t+h}\big[-\lambda_{r(s)}^{(1)}\mathbf{E}V\big(s,r(s),e(s)\big)\\
&+\lambda_{r(s)}^{(2)}\mathbf{E}V\big(s,r(s),e\big(s-\tau(s)\big)\big)\big]\,ds\\
&\le\int_t^{t+h}\big[-\lambda_{r(s)}^{(1)}\mathbf{E}V\big(s,r(s),e(s)\big)\\
&+\lambda_{r(s)}^{(2)}\sup_{s-\tau\le\theta\le s}\mathbf{E}V\big(s,r(s),e(s)\big)\big]\,ds.
\end{aligned}
$$

Taking $z(t,r(t))=\mathbf{E}V(t,r(t),e(t))$, then by the above inequality we obtain

$$
D^+z(t,i)\le-\lambda_i^{(1)}z(t,i)+\lambda_i^{(2)}\overline{z}(t,i),
$$

for all $r(t)=i\in S$.

So, it follows from Proposition 1 that

$$
\begin{aligned}
&z(t,i)\le\overline{z}\big(0,r(0)\big)e^{-\lambda_i t},\\
&t\ge0 \text{ and } r(t)=i\in S,
\end{aligned}
$$

which gives

$$
\begin{aligned}
&\mathbf{E}|e(t)|^p\le\frac{\gamma_{r(0)}}{\gamma_i}\frac{\max_{1\le i\le n}m_i}{\min_{1\le i\le n}m_i}\sup_{-\tau\le s\le0}\mathbf{E}|e(s)|^p e^{-\lambda_i t},\\
&t\ge0 \text{ and } r(t)=i\in S, \qquad (10)
\end{aligned}
$$

where $\lambda_i$ is the unique positive root of the following equation:

$$
\lambda_i=\lambda_i^{(1)}-\lambda_i^{(2)}e^{\lambda_i\tau}, \quad i\in S.
$$

Taking $\alpha=\frac{\max_{i\in S}\gamma_i}{\min_{i\in S}\gamma_i}\frac{\max_{1\le i\le n}m_i}{\min_{1\le i\le n}m_i}$ and $\beta=\min_{i\in S}\lambda_i$, then by (10) we obtain

$$
\mathbf{E}\big\|e(t)\big\|^p\le\alpha e^{-\beta t}\sup_{-\tau\le s\le0}\mathbf{E}\big\|e(s)\big\|^p, \quad t\ge0.
$$

Therefore, by Definition 1, we see that the two coupled neural networks (1) and (2) are $p$th moment exponentially synchronized. □

*Remark 5* Theorem 1 is our first main result, which gives a new sufficient condition to prove that the two coupled neural networks (1) and (2) can be $p$th moment exponentially synchronized. It should be mentioned that Theorem 1 does not need the differentiability of the time-vary delay $\tau(t)$, and $\tau(t)$ is only required to be bounded, i.e., $0\le\tau(t)\le\tau$. However, all delays in [4, 7–9, 11, 12, 17, 20, 21, 23, 26, 29, 30, 32, 34] are constants or differential and their derivatives are simultaneously required to be smaller than 1, and so Theorem 1 is less conservatism than those reported in [4, 7–9, 11, 12, 17, 20, 21, 23, 26, 29, 30, 32, 34].

*Remark 6* In Theorem 1, we have discussed the $p$th moment exponential synchronization problem. However, the exponential or asymptotic or almost surely asymptotic synchronization problem is investigated in [4, 7–9, 11, 12, 17, 20, 21, 23, 26, 29, 30, 32, 34], and the $p$th moment exponential synchronization problem is not considered in [4, 7–9, 11, 12, 17, 20, 21, 23, 26, 29, 30, 32, 34]. Obviously, when $p=2$, the 2th moment exponential synchronization is the exponential synchronization, and the exponential synchronization implies the asymptotic synchronization. Therefore, Theorem 1 extends and improves those exponential or asymptotic synchronization criteria given in [4, 7–9, 11, 12, 17, 20, 21, 23, 26, 29, 30, 32, 34].

*Remark 7* In Theorem 1, Markovian switching has been employed to discuss the $p$th moment exponential synchronization problem. However, all authors in [4, 7–9, 11, 12, 17, 20, 21, 23, 26, 29, 30, 32, 34] did not consider Markovian switching. As we know, the neural network without Markovian switching can be regarded as a special case of the neural network with Markovian switching when the Markov chain $r(\cdot)$ only takes a unique value, i.e., $S=\{1\}$. Therefore, the model studied in Theorem 1 also generalizes some models discussed in the previous literature.

We now discuss some special cases of our result.

(a) When $p=2$, we have the following result based on Theorem 1.

**Theorem 2** *Under Assumptions 1–5, the two coupled neural networks (1) and (2) can be exponentially synchronized, if there exist positive numbers $\gamma_i\,(i\in S)$, $m_j$ $(j=1,2,\dots,n)$ such that*

$$\lambda_i^{(1)}>\lambda_i^{(2)}>0,\quad i\in S, \tag{11}$$

*where*

$$\begin{aligned}\lambda_i^{(1)}=\min_{1\le j\le n}\Bigg\{&2\gamma_i\alpha_{ij}^0\delta_{ij}-\gamma_i\sum_{k=1}^n\alpha_{ij}^1|a_{ijk}|u_{1k}\\&-\gamma_i\sum_{k=1}^n\frac{m_k}{m_j}\alpha_{ik}^1|a_{ikj}|u_{1j}\\&-\gamma_i\sum_{k=1}^n\alpha_{ij}^1|b_{ijk}|u_{2k}\\&-\gamma_i\sum_{k=1}^n\frac{m_k}{m_j}\mu_{ij}+\sum_{l=1}^N q_{il}\gamma_l\Bigg\},\\ \lambda_i^{(2)}=\max_{1\le j\le n}\Bigg\{&\gamma_i\sum_{k=1}^n\frac{m_k}{m_j}\alpha_{ik}^1|b_{ikj}|u_{2j}\\&+\gamma_i\sum_{k=1}^n\frac{m_k}{m_j}\nu_{ij}\Bigg\}.\end{aligned}$$

(b) When the Markov chain $r(\cdot)$ only takes a unique value, i.e., $S=\{1\}$, the two coupled neural networks (1) and (2) will become the following two coupled neural networks without Markovian switching:

$$\begin{aligned}dx(t)=\big\{&-\widetilde{\alpha}\big(x(t)\big)\big[\widetilde{\beta}\big(x(t)\big)-C\widetilde{f}\big(x(t)\big)\\&-D\widetilde{g}\big(x\big(t-\tau(t)\big)\big)\big]+J\big\}\,dt,\end{aligned}\tag{12}$$

$$\begin{aligned}dy(t)=\big\{&-\widetilde{\alpha}\big(y(t)\big)\big[\widetilde{\beta}\big(y(t)\big)-C\widetilde{f}\big(y(t)\big)\\&-D\widetilde{g}\big(y\big(t-\tau(t)\big)\big)\big]+J+u(t)\big\}\,dt\\&+\sigma\big(t,y(t)-x(t),y\big(t-\tau(t)\big)\\&-x\big(t-\tau(t)\big)\big)\,dw(t),\end{aligned}\tag{13}$$

where $C$, $D$, $\widetilde{\alpha}(x(t))$, $\widetilde{\beta}(x(t))$, $\widetilde{\alpha}(y(t))$, $\widetilde{\beta}(y(t))$, $u(t)$, $\sigma(t,y(t)-x(t),y(t-\tau(t))-x(t-\tau(t)))$ denote $C(1)$, $D(1)$, $\widetilde{\alpha}(x(t),1)$, $\widetilde{\beta}(x(t),1)$, $\widetilde{\alpha}(y(t),1)$, $\widetilde{\beta}(y(t),1)$, $u(t,1)$, $\sigma(t,1,y(t)-x(t),y(t-\tau(t))-x(t-\tau(t)))$, respectively. Accordingly, in Assumptions 1–5, we will use $\alpha_j^0,\alpha_j^1,\delta_j,\mu_j,\nu_j$, $j=1,2,\dots,n$ to denote $\alpha_{1j}^0,\alpha_{1j}^1,\delta_{1j},\mu_{1j},\nu_{1j}$, $j=1,2,\dots,n$.

Taking $V(t,e(t))=\sum_{j=1}^n m_j|e_j(t)|^p$, then by using Theorems 1 and 2, we obtain the following synchronization criteria about the two coupled neural networks (12) and (13).

**Theorem 3** *Under Assumptions 1–5, the two coupled neural networks (12) and (13) can be pth moment exponentially synchronized, if there exist positive numbers $m_j$ $(j=1,2,\dots,n)$ such that*

$$\lambda^{(1)}>\lambda^{(2)}>0, \tag{14}$$

*where*

$$\begin{aligned}\lambda^{(1)}=\min_{1\le j\le n}\Bigg\{&p\alpha_j^0\delta_j-(p-1)\sum_{k=1}^n\alpha_j^1|a_{jk}|u_{1k}\\&-\sum_{k=1}^n\frac{m_k}{m_j}\alpha_k^1|a_{kj}|u_{1j}-(p-1)\sum_{k=1}^n\alpha_j^1|b_{jk}|u_{2k}\end{aligned}$$

$$- \frac{1}{2}(p-1)(p-2)\sum_{k=1}^{n}\mu_k$$
$$- (p-1)\sum_{k=1}^{n}\frac{m_k}{m_j}\mu_k$$
$$- (p-1)(p-2)\sum_{k=1}^{n}\nu_j\Bigg\},$$
$$\lambda^{(2)} = \max_{1\le j\le n}\Bigg\{\sum_{k=1}^{n}\frac{m_k}{m_j}\alpha_k^1|b_{kj}|u_{2j}$$
$$+ (p-1)\sum_{k=1}^{n}\frac{m_k}{m_j}\nu_j\Bigg\}.$$

**Theorem 4** *Under Assumptions 1–5, the two coupled neural networks (12) and (13) can be exponentially synchronized, if there exist positive numbers $m_j$ $(j = 1, 2, \dots, n)$ such that*

$$\lambda^{(1)} > \lambda^{(2)} > 0, \tag{15}$$

*where*

$$\lambda^{(1)} = \min_{1\le j\le n}\Bigg\{2\alpha_j^0\delta_j - \sum_{k=1}^{n}\alpha_j^1|a_{jk}|u_{1k}$$
$$- \sum_{k=1}^{n}\frac{m_k}{m_j}\alpha_k^1|a_{kj}|u_{1j} - \sum_{k=1}^{n}\alpha_j^1|b_{jk}|u_{2k}$$
$$- \sum_{k=1}^{n}\frac{m_k}{m_j}\mu_j\Bigg\},$$
$$\lambda^{(2)} = \max_{1\le j\le n}\Bigg\{\sum_{k=1}^{n}\frac{m_k}{m_j}\alpha_k^1|b_{kj}|u_{2j} + \sum_{k=1}^{n}\frac{m_k}{m_j}\nu_j\Bigg\}.$$

Letting $m_i = 1$, $i = 1, 2, \dots, n$ in Theorems 1–4, then we get the following corollaries.

**Corollary 1** *Under Assumptions 1–5, the two coupled neural networks (1) and (2) can be pth moment exponentially synchronized, if there exist positive numbers $\gamma_i, i \in S$ such that*

$$\lambda_i^{(1)} > \lambda_i^{(2)} > 0, \quad i \in S, \tag{16}$$

*where*

$$\lambda_i^{(1)} = \min_{1\le j\le n}\Bigg\{\gamma_i p\alpha_{ij}^0\delta_{ij} - \gamma_i(p-1)\sum_{k=1}^{n}\alpha_{ij}^1|a_{ijk}|u_{1k}$$
$$- \gamma_i\sum_{k=1}^{n}\alpha_{ik}^1|a_{ikj}|u_{1j}$$
$$- \gamma_i(p-1)\sum_{k=1}^{n}\alpha_{ij}^1|b_{ijk}|u_{2k}$$
$$- \frac{1}{2}\gamma_i(p-1)(p-2)\sum_{k=1}^{n}\mu_{ik}$$
$$- \gamma_i(p-1)n\mu_{ij} - \gamma_i(p-1)(p-2)\sum_{k=1}^{n}\nu_{ik}$$
$$- \sum_{l=1}^{N}q_{il}\gamma_l\Bigg\},$$
$$\lambda_i^{(2)} = \max_{1\le j\le n}\Bigg\{\gamma_i\sum_{k=1}^{n}\alpha_{ik}^1|b_{ikj}|u_{2j} + \gamma_i(p-1)n\nu_{ij}\Bigg\}.$$

**Corollary 2** *Under Assumptions 1–5, the two coupled neural networks (1) and (2) can be exponentially synchronized, if there exist positive numbers $\gamma_i, i \in S$ such that*

$$\lambda_i^{(1)} > \lambda_i^{(2)} > 0, \quad i \in S, \tag{17}$$

*where*

$$\lambda_i^{(1)} = \min_{1\le j\le n}\Bigg\{2\gamma_i\alpha_{ij}^0\delta_{ij} - \gamma_i\sum_{k=1}^{n}\alpha_{ij}^1|a_{ijk}|u_{1k}$$
$$- \gamma_i\sum_{k=1}^{n}\alpha_{ik}^1|a_{ikj}|u_{1j}$$
$$- \gamma_i\sum_{k=1}^{n}\alpha_{ij}^1|b_{ijk}|u_{2k} - \gamma_i n\mu_{ij} + \sum_{l=1}^{N}q_{il}\gamma_l\Bigg\},$$
$$\lambda_i^{(2)} = \max_{1\le j\le n}\Bigg\{\gamma_i\sum_{k=1}^{n}\alpha_{ik}^1|b_{ikj}|u_{2j} + \gamma_i n\nu_{ij}\Bigg\}.$$

**Corollary 3** *Under Assumptions 1–5, the two coupled neural networks (12) and (13) can be pth moment exponentially synchronized, if the following holds:*

$$\lambda^{(1)} > \lambda^{(2)} > 0, \tag{18}$$

*where*

$$\lambda^{(1)} = \min_{1\le j\le n}\Bigg\{p\alpha_j^0\delta_j - (p-1)\sum_{k=1}^{n}\alpha_j^1|a_{jk}|u_{1k}$$

$$- \sum_{k=1}^{n} \alpha_k^1 |a_{kj}| u_{1j} - (p-1) \sum_{k=1}^{n} \alpha_j^1 |b_{jk}| u_{2k}$$
$$- \frac{1}{2}(p-1)(p-2) \sum_{k=1}^{n} \mu_k - (p-1) n \mu_j$$
$$- (p-1)(p-2) \sum_{k=1}^{n} \nu_k \Bigg\},$$
$$\lambda^{(2)} = \max_{1 \le j \le n} \left\{ \sum_{k=1}^{n} \alpha_k^1 |b_{kj}| u_{2j} + (p-1) n \nu_j \right\}.$$

**Corollary 4** *Under Assumptions 1–5, the two coupled neural networks (12) and (13) can be pth moment exponentially synchronized, if the following holds*:

$$\lambda^{(1)} > \lambda^{(2)} > 0, \tag{19}$$

*where*

$$\lambda^{(1)} = \min_{1 \le j \le n} \Bigg\{ 2\alpha_j^0 \delta_j - \sum_{k=1}^{n} \alpha_j^1 |a_{jk}| u_{1k}$$
$$- \sum_{k=1}^{n} \alpha_k^1 |a_{kj}| u_{1j} - \sum_{k=1}^{n} \alpha_j^1 |b_{jk}| u_{2k} - n \mu_j \Bigg\},$$
$$\lambda^{(2)} = \max_{1 \le j \le n} \left\{ \sum_{k=1}^{n} \alpha_k^1 |b_{kj}| u_{2j} + n \nu_j \right\}.$$

## 4 Numerical example and simulation

In this section, a numerical example and its simulation are given to demonstrate the effectiveness of the obtained results.

*Example 1* Consider the following two coupled Markovian jump neural networks with time-varying delays:

$$dx(t) = \big\{ -\widetilde{\alpha}\big(x(t), r(t)\big)\big[\widetilde{\beta}\big(x(t), r(t)\big) - C\big(r(t)\big)\widetilde{f}\big(x(t)\big) - D\big(r(t)\big)\widetilde{g}\big(x\big(t-\tau(t)\big)\big)\big] + J \big\} dt, \tag{20}$$

$$dy(t) = \big\{ -\widetilde{\alpha}\big(y(t), r(t)\big)\big[\widetilde{\beta}\big(y(t), r(t)\big) - C\big(r(t)\big)\widetilde{f}\big(y(t)\big) - D\big(r(t)\big)\widetilde{g}\big(y\big(t-\tau(t)\big)\big)\big] + J + u\big(t, r(t)\big) \big\} dt + \sigma\big(t, r(t), y(t) - x(t), y\big(t-\tau(t)\big) - x\big(t-\tau(t)\big)\big) dw(t), \tag{21}$$

with $J = (0,0)^{\mathrm{T}}$, $x(t) = (x_1(t), x_2(t))^{\mathrm{T}}$, $y(t) = (y_1(t), y_2(t))^{\mathrm{T}}$, $u(t, r(t)) = K_1(r(t))[\widetilde{f}(y(t)) - \widetilde{f}(x(t))] + K_2(r(t))[\widetilde{g}(y(t-\tau(t))) - \widetilde{g}(x(t-\tau(t)))]$, $w(t)$ is a two-dimensional Brownian motion, and $r(t)$ is a right-continuous Markov chain taking values in $S = \{1, 2\}$ with generator $Q = \left[\begin{smallmatrix} -0.06 & 0.06 \\ 0.08 & -0.08 \end{smallmatrix}\right]$. Take $\tau(t) = 0.1|\cos t| + 1$, $\widetilde{f}(x_j) = \widetilde{g}(x_j) = \tanh(x_j)$, $\alpha_j(x_j(t), i) = 0.5 + 0.3\cos(t)$, $\beta_j(x_j(t), i) = 2x_j(t)$ $(i, j = 1, 2)$,

$$\sigma\big(1, t, e(t), e\big(t-\tau(t)\big)\big) = 0.05 \begin{pmatrix} e_1(t) & e_1(t-\tau(t)) \\ (e_1(t) + e_1(t-\tau(t))) & e_2(t-\tau(t)) \end{pmatrix},$$

$$\sigma\big(2, t, e(t), e\big(t-\tau(t)\big)\big) = 0.05 \begin{pmatrix} e_1(t-\tau(t)) & e_2(t) \\ e_2(t-\tau(t)) & (e_1(t) + e_1(t-\tau(t))) \end{pmatrix}.$$

It is easy to check that Assumptions 1–4 hold. Other parameters of the two coupled neural networks (20) and (21) are given as follows:

$$C(1) = \begin{bmatrix} 4.1 & -0.3 \\ -9.9 & 6 \end{bmatrix},$$

$$D(1) = \begin{bmatrix} -3.1 & -0.2 \\ -0.5 & -4.9 \end{bmatrix},$$

$$K_1(1) = \begin{bmatrix} -4 & 0.2 \\ 10 & -6 \end{bmatrix},$$

$$K_2(1) = \begin{bmatrix} 3.2 & 0.2 \\ 0.5 & 5 \end{bmatrix},$$

$$C(2) = \begin{bmatrix} 4 & -0.2 \\ -10 & 6.2 \end{bmatrix},$$

$$D(2) = \begin{bmatrix} -3 & -0.2 \\ -0.4 & -5 \end{bmatrix},$$

$$K_1(2) = \begin{bmatrix} -4 & 0.1 \\ 10 & -6.1 \end{bmatrix},$$

$$K_2(2) = \begin{bmatrix} 3 & 0.1 \\ 0.3 & 5.1 \end{bmatrix}.$$

Figures 1 and 2 show that system (20) has a chaotic attractor. Let $p = 4$ and take $\gamma_1 = 0.5$, $\gamma_2 = 1$, then a

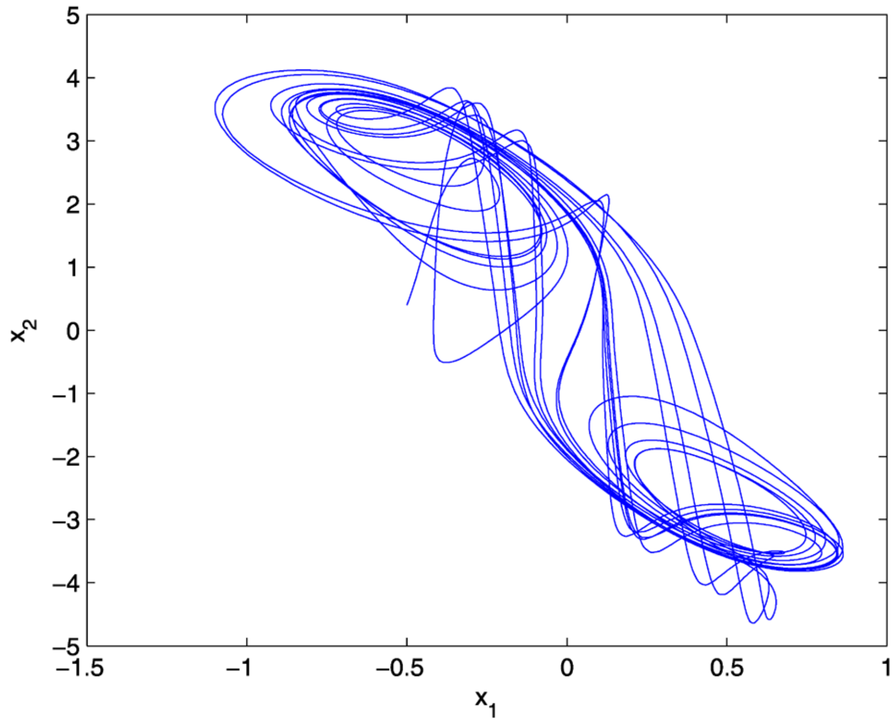


**Fig. 1** The chaotic attractor of neural network (20) in model 1

simple computation yields

$$\lambda_1^{(1)} = \min_{1\le j\le n}\Bigg\{\gamma_1 p\alpha_{1j}^0\delta_{1j} - \gamma_1(p-1)\times\sum_{k=1}^{n}\alpha_{1j}^1|a_{1jk}|u_{11k} - \gamma_1\sum_{k=1}^{n}\alpha_{1k}^1|a_{1kj}|u_{11j} - \gamma_1(p-1)\sum_{k=1}^{n}\alpha_{1j}^1|b_{1jk}|u_{12k} - \frac{1}{2}\gamma_1(p-1)(p-2)\sum_{k=1}^{n}\mu_{1k} - \gamma_1(p-1)n\mu_{1j} - \gamma_1(p-1)(p-2)\sum_{k=1}^{n}\nu_{1k} - \sum_{l=1}^{N}q_{1l}\gamma_l\Bigg\} = 1.7025,$$

$$\lambda_2^{(1)} = \min_{1\le j\le n}\Bigg\{\gamma_2 p\alpha_{2j}^0\delta_{2j} - \gamma_2(p-1)\times\sum_{k=1}^{n}\alpha_{2j}^1|a_{2jk}|u_{21k} - \gamma_2\sum_{k=1}^{n}\alpha_{2k}^1|a_{2kj}|u_{21j} - \gamma_2(p-1)\sum_{k=1}^{n}\alpha_{2j}^1|b_{2jk}|u_{22k} - \frac{1}{2}\gamma_2(p-1)\times(p-2)\sum_{k=1}^{n}\mu_{2k} - \gamma_2(p-1)n\mu_{2j} - \gamma_2(p-1)(p-2)\times\sum_{k=1}^{n}\nu_{2k} - \sum_{l=1}^{N}q_{2l}\gamma_l\Bigg\} = 3.335,$$

$$\lambda_1^{(2)} = \max_{1\le j\le n}\Bigg\{\gamma_1\sum_{k=1}^{n}\alpha_{1k}^1|b_{1kj}|u_{12j} + \gamma_1(p-1)n\nu_{1j}\Bigg\} = 1.025,$$

$$\lambda_2^{(2)} = \max_{1\le j\le n}\Bigg\{\gamma_2\sum_{k=1}^{n}\alpha_{2k}^1|b_{2kj}|u_{22j} + \gamma_2(p-1)n\nu_{2j}\Bigg\} = 2.19.$$

Obviously, $\lambda_1^{(1)} > \lambda_1^{(2)} > 0$, $\lambda_2^{(1)} > \lambda_2^{(2)} > 0$. Therefore, it follows from Corollary 1 that the two coupled neural networks (20) and (21) can be 4th moment exponentially synchronized. Moreover, Figs. 3 and 4 also show perfectly that the two coupled neural networks

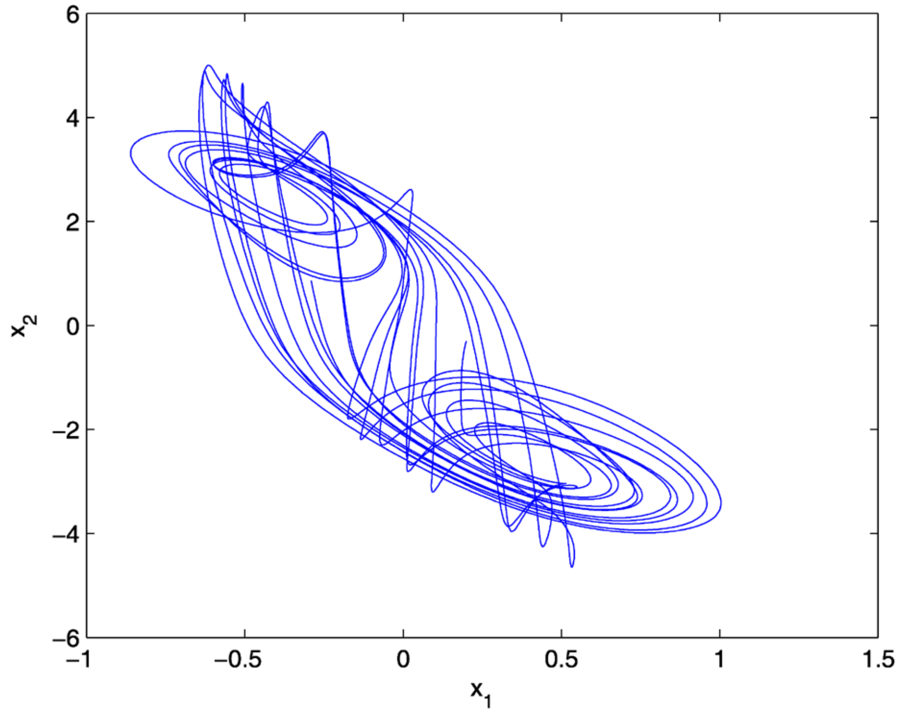


**Fig. 2** The chaotic attractor of neural network (20) in model 2

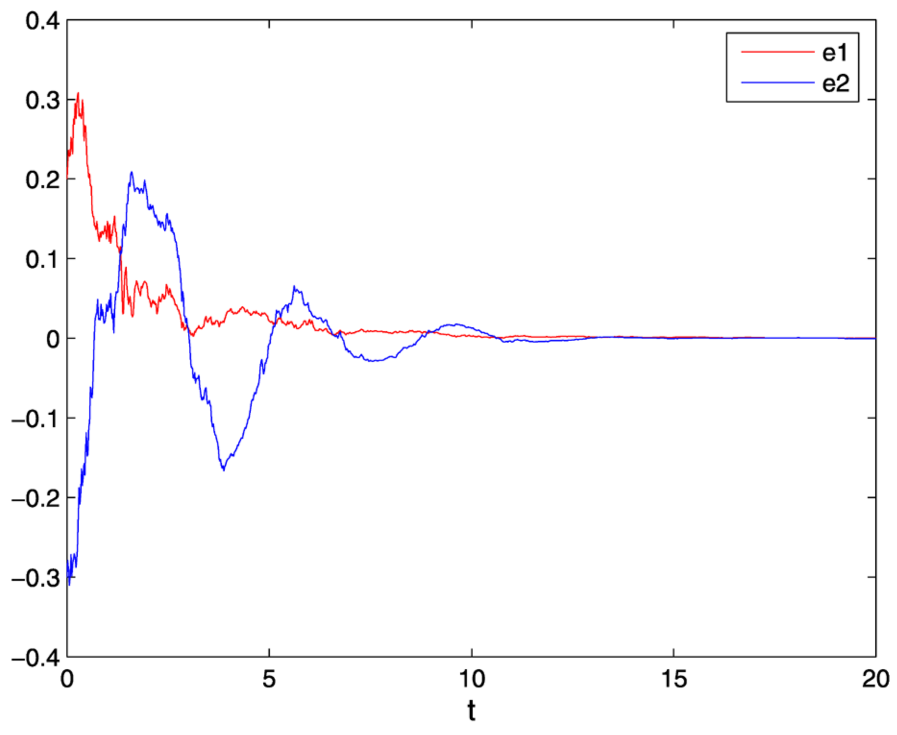


**Fig. 3** The synchronization error of the model 1 in Example 1

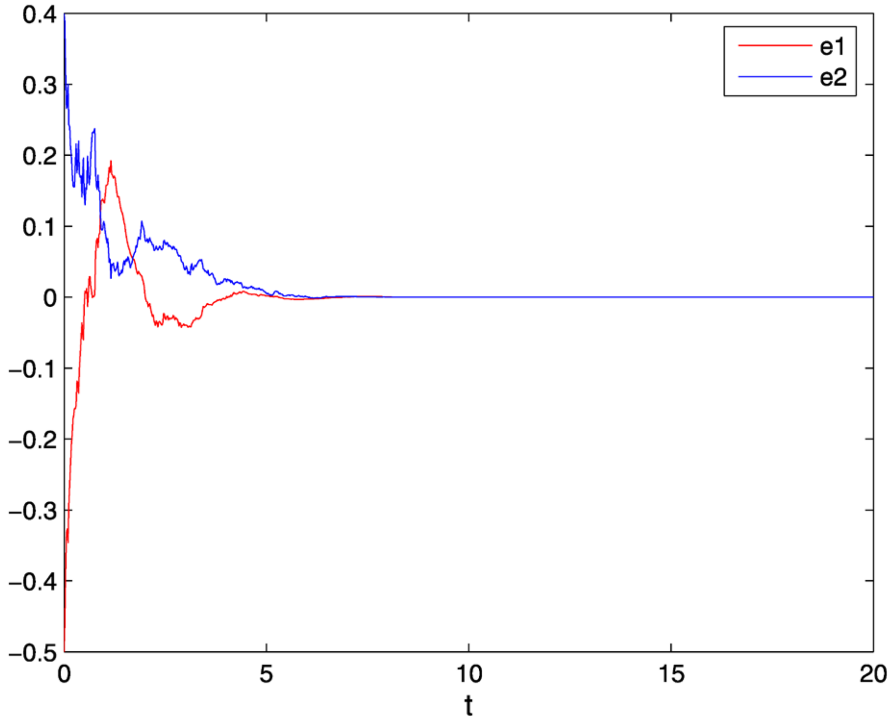


**Fig. 4** The synchronization error of the model 2 in Example 1

(20) and (21) can be 4th moment exponentially synchronized.

*Remark 8* In Example 1, we discuss a class of generalized coupled neural networks, which includes many coupled neural networks with/without Markovian switching as its special cases. In particular, the time-varying delays in Example 1 are not differential. Therefore, all of the synchronization criteria obtained in [4, 7–12, 17, 19–21, 23, 24, 26, 29, 30, 32, 34, 37] fail in our example.

## 5 Concluding remarks

In this paper, we have studied the $p$th moment exponential synchronization problem for a class of stochastic delayed Cohen–Grossberg neural networks with Markovian switching. By using the Lyapunov–Krasovskii functional, stochastic analysis theory, a generalized Halanay-type inequality as well as output coupling with delay feedback control technique, some novel sufficient conditions are derived to achieve complete $p$th moment exponential synchronization of the addressed neural networks. The results obtained in this paper generalize and improve many known results. Moreover, a numerical example and its simulation are also provided to demonstrate the effectiveness and applicability of the theoretical results. It is worth pointing out that the contribution of this paper is threefold. (1) Firstly, different from the traditional criteria such as asymptotical synchronization, almost surely asymptotical synchronization and exponential synchronization criterion, the synchronization criterion considered in this paper is $p$th moment exponential stability of the error dynamical system, which has seldom been applied to investigate the synchronization problem. (2) Secondly, the model considered in this paper is a class of stochastic delayed Cohen–Grossberg neural networks with Markovian switching, which has never been used to study the synchronization problem. (3) Thirdly, the traditional assumptions on the differentiability of the time varying delay and the boundedness of its derivative are necessary in the earlier works. However, we take off these restrictiveness in this paper. Moreover, we present a numerical example and its simulation to illustrate well the obtained results.

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