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ORIGINAL PAPER

Study on vibration reduction and mobility improvement for the flexible manipulator via redundancy resolution

Yushu Bian · Zhihui Gao · Chao Yun

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Abstract The flexible redundant manipulator, i.e., the flexible manipulator with redundant rigid degrees of freedom, possesses the same kinematic redundancy property as the rigid redundant manipulator. Some undesired effects on the flexible redundant manipulator are expected to alleviate via kinematic redundancy. Due to the presence of structural flexibility, a manipulator will inevitably vibrate when performing tasks. Therefore, how to reduce its vibration responses is a significant problem. Moreover, the manipulator's mobility, i.e., its ability to move, is another important issue, because good mobility is a desirable goal for almost all robotic manipulator systems. In this paper, how to reduce vibration and improve mobility is studied for the flexible redundant manipulator. Firstly, a method for vibration control via redundancy resolution is put forward. Secondly, the self-motions satisfying vibration reduction are analyzed, and its additional optimization ability is revealed. Based on this ability, a strategy is proposed to both reduce vibration and improve mobility for the flexible redundant manipulator. Finally, simulation results demonstrate the effectiveness of this strategy.

Y. Bian
State Key Laboratory of Robotics and System (HIT),
Harbin Institute of Technology, Harbin 150001, China

Y. Bian · Z. Gao (✉) · C. Yun
School of Mechanical Engineering and Automation,
Beihang University, Beijing 100191, China
e-mail: gaozhihui@buaa.edu.cn

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1 Introduction

The redundant manipulator, which can achieve additional performances while tracing a given end-effector trajectory, has been extensively studied [1–4]. The self-motion capability of the redundant manipulator, which allows link motion while maintaining a fixed end-effector location, has been used as a tool to resolve a lot of robotic problems such as avoidance of singularities [5], joint limits [6], and obstacles [7]. But all of the above approaches only considered rigid manipulators, which are appropriate for many industrial robots.

Compared with traditional rigid manipulators, new generation of manipulators, which are usually flexible in links or joints, have many advantages such as higher operational speed, less weight, and lower energy consumption [8, 9]. In addition, in order to perform versatile tasks and adapt to complex environment, these manipulators are usually kinematically redundant as well, i.e., the number of their rigid joint degrees of freedom is greater than the number of their end-effector degrees of freedom. In this paper, they are called "the flexible redundant manipulators." Similar to the rigid redundant manipulator, the flexible redundant manipulator also possesses the kinematic redundancy property. As a result, some undesired effects on the flexible redundant manipulator are expected to alleviate via redundancy resolution.

Due to low stiffness, a major disadvantage of the flexible manipulator is the deterioration of the end-effector accuracy. In the area of our concern, namely, analysis and control of the flexible manipulator, there have been many papers published, and different ways have been sought for controlling the flexibility [8, 9]. However, little attention appears to have been paid to the issue of how to control vibration of flexible manipulators by way of their topological characteristics. Nguyen [10] first examined the possibility of reducing vibration for the flexible manipulator with the use of the self-motions introduced by redundant degrees of freedom. In addition, some researchers [11–14] also studied vibration control for the flexible redundant manipulator by optimizing its self-motions. However, little attention has been paid to the mobility issue of the flexible redundant manipulator.

The manipulator's mobility, i.e., its ability to move, is a desirable goal for almost all robotic manipulator systems, whether or not they are redundant. The mobility of a manipulator is closely related to its kinematic dexterity. A number of mathematical measures have been proposed for quantification. Based on the singular value decomposition, several indices are put forward, like condition number, minimum singular value, and minimum condition number [15–17]. Yoshikawa defined a term of "the manipulability" as a measure for kinematic performance of a manipulator [18]. Klein compared several dexterity measures for the design and control of kinematically redundant manipulator [19]. Moreover, some extended researches have been conducted [5, 20], and some other indices are suggested, including: the global conditioning index(GCI) [21], the local conditioning index(LCI) [22], and so on.

Although some of the aforementioned indices can be used to improve mobility of the rigid redundant manipulator via the kinematic redundancy, another issue should be considered for the flexible redundant manipulator. As mentioned above, a flexible redundant manipulator will vibrate in motion due to its structural flexibility, thus resulting in the deterioration of its operation task accuracy. Therefore, as far as the flexible redundant manipulator is concerned, vibration reduction and mobility improvement should be taken into account simultaneously. However, most of the above researches on the flexible redundant manipulator mainly focused on vibration reduction, but seldom cared about kinematic performance improvement at the same time. Nguyen [10] made a tradeoff, i.e., if the flexible redundant manipulator is in regions close to singularities, choose the self-motions to maximize the manipulability measure [18] to avoid singularities; if the manipulator is in all other regions, choose the self-motions to reduce vibration. Obviously, this method cannot guarantee vibration reduction and kinematic performance improvement simultaneously. In addition, frequent switches between different methods for choosing the self-motions are prone to result in unstable motion of the flexible redundant manipulator. Until now, there are few papers concerning concurrent optimization of multiple performance indices for the flexible redundant manipulator. Therefore, it is important to seek a method for simultaneously reducing vibration and improving mobility for the flexible redundant manipulator via redundancy resolution.

In this paper, the kinematic redundancy property of the flexible manipulator is deeply researched. To deal with the adverse flexible effects, a method for vibration reduction is put forward via redundancy resolution. Then the self-motions satisfying vibration reduction are analyzed, and its additional optimization ability is revealed. It is found the self-motions satisfying vibration reduction are not unique, thus can be further used to optimize additional performance criteria. Based on this ability, a strategy is proposed to both reduce vibration and improve mobility for the flexible redundant manipulator.

2 Dynamic formulation for the flexible redundant manipulator

2.1 Kinematic analysis

In this paper, the open-chain serial flexible manipulator with redundant rigid degrees of freedom is studied. Since the nominal trajectory of a manipulator is usually planned according to its joint motion, the desired position/posture of the end-effector $\boldsymbol{x}$ is the function of joint angles $\boldsymbol{q}$, that is,

$$\boldsymbol{x} = \xi(\boldsymbol{q}) \tag{1}$$

where $\boldsymbol{x} \in \boldsymbol{R}^m$ is the nominal position/posture of the end-effector with respect to the base frame; m is the number of the end-effector degrees of freedom in the work space; $\boldsymbol{q} \in \boldsymbol{R}^{n_{\mathrm{R}}}$ is the set of joint angles; n_{R} is the number of joints.

Differentiating (1) for two times with respect to time, we obtain

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} \tag{2}$$

where $\boldsymbol{J} \in \boldsymbol{R}^{m\times n_{\mathrm{R}}}$ is the rigid Jacobian matrix, $\boldsymbol{J} = \mathrm{d}\xi/\mathrm{d}\boldsymbol{q}$; $\ddot{\boldsymbol{x}} \in \boldsymbol{R}^{m}$ is the nominal acceleration of the end-effector.

To a redundant manipulator, the number of its rigid joint degrees of freedom is greater than the number of its end-effector degrees of freedom, i.e., $n_{\mathrm{R}} > m$, hence the general solution for $\ddot{\boldsymbol{q}}$ is given by

$$\ddot{\boldsymbol{q}} = \boldsymbol{J}^{+}\left(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}\right) + \left(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J}\right)\boldsymbol{\varepsilon}_1 \tag{3}$$

where $\boldsymbol{J}^{+} \in \boldsymbol{R}^{n_{\mathrm{R}}\times m}$ is the generalized inverse matrix of $\boldsymbol{J}$; $(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\varepsilon}_1 \in \mathrm{N}(\boldsymbol{J})$; $\mathrm{N}(\boldsymbol{J})$ is the null space of $\boldsymbol{J}$; $\boldsymbol{\varepsilon}_1 \in \boldsymbol{R}^{n_{\mathrm{R}}}$ is the arbitrary vector; $\boldsymbol{I} \in \boldsymbol{R}^{n_{\mathrm{R}}\times n_{\mathrm{R}}}$ is the unit matrix.

Since (2) includes m equations with n_{R} unknown $\ddot{\boldsymbol{q}}$, there are an infinite number of solutions of $\ddot{\boldsymbol{q}}$, indicated by the arbitrary vector $\boldsymbol{\varepsilon}_1$. It means no matter how $\boldsymbol{\varepsilon}_1$ is chosen, the nominal end-effector motion of the flexible redundant manipulator cannot be affected at all, i.e., different choices of $\boldsymbol{\varepsilon}_1$ may produce different joint motion while maintaining the unchanged nominal end-effector motion. This "self-motion" capability is an attractive feature of the redundant manipulator.

2.2 Dynamic analysis

In general, the structural flexibility in a manipulator system can be mainly described as the link flexibility and the joint flexibility. In this paper, only the link flexibility is considered. Based on Kane's method and the assumed-modes method, the dynamic equations of the flexible redundant manipulator are derived (the flexible links are modeled as clamped base beams) and can be written as follows [23]:

$$\boldsymbol{M}_{\mathrm{s}}\ddot{\boldsymbol{\psi}} + \boldsymbol{C}_{\mathrm{s}}\dot{\boldsymbol{\psi}} + \boldsymbol{K}_{\mathrm{s}}\boldsymbol{\psi} = \boldsymbol{Q} \tag{4}$$

where $\boldsymbol{M}_{\mathrm{s}} \in \boldsymbol{R}^{n\times n}$ is the system mass matrix; $\boldsymbol{C}_{\mathrm{s}} \in \boldsymbol{R}^{n\times n}$ is the system damping matrix; $\boldsymbol{K}_{\mathrm{s}} \in \boldsymbol{R}^{n\times n}$ is the system stiffness matrix; $\boldsymbol{Q} \in \boldsymbol{R}^{n}$ is the sum of Coriolis, gravitational, centripetal, and control torques; $\boldsymbol{\psi} \in \boldsymbol{R}^{n}$ is the set of rigid and flexural degrees of freedom, $\boldsymbol{\psi}^{\mathrm{T}} = [\boldsymbol{q}^{\mathrm{T}}\ \boldsymbol{\varphi}^{\mathrm{T}}]^{\mathrm{T}}$; $\boldsymbol{\varphi} \in R^{n_{\mathrm{F}}}$ is the set of flexural displacement of links; n is the total number of degrees of freedom, $n = n_{\mathrm{R}} + n_{\mathrm{F}}$; n_{F} is the number of flexural degrees of freedom in the flexible links.

Equation (4) can be separated into two equations describing the dynamics of $\boldsymbol{q}$ and $\boldsymbol{\varphi}$

$$\boldsymbol{D}\ddot{\boldsymbol{q}} + \boldsymbol{U}\ddot{\boldsymbol{\varphi}} = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{R}} \tag{5}$$

$$\boldsymbol{G}\ddot{\boldsymbol{q}} + \boldsymbol{M}\ddot{\boldsymbol{\varphi}} + \boldsymbol{C}\dot{\boldsymbol{\varphi}} + \boldsymbol{K}\boldsymbol{\varphi} = \boldsymbol{f} \tag{6}$$

where $\boldsymbol{D} \in \boldsymbol{R}^{n_{\mathrm{R}}\times n_{\mathrm{R}}}$, $\boldsymbol{U} \in \boldsymbol{R}^{n_{\mathrm{R}}\times n_{\mathrm{F}}}$, $\boldsymbol{G} \in \boldsymbol{R}^{n_{\mathrm{F}}\times n_{\mathrm{R}}}$, $\boldsymbol{M} \in \boldsymbol{R}^{n_{\mathrm{F}}\times n_{\mathrm{F}}}$ are block matrices that form $\boldsymbol{M}_{\mathrm{s}}$; $\boldsymbol{\tau} \in \boldsymbol{R}^{n_{\mathrm{R}}}$ is the set of control torques applied to the joints; $\boldsymbol{\tau}_{\mathrm{R}} \in \boldsymbol{R}^{n_{\mathrm{R}}}$ is the rigid component of the nonlinear torque; $\boldsymbol{C} \in \boldsymbol{R}^{n_{\mathrm{F}}\times n_{\mathrm{F}}}$; $\boldsymbol{K} \in \boldsymbol{R}^{n_{\mathrm{F}}\times n_{\mathrm{F}}}$; $\boldsymbol{f} \in \boldsymbol{R}^{n_{\mathrm{F}}}$ is the flexible component of the nonlinear torque.

For the convenience of our study, (5) and (6) can be expressed as

$$\boldsymbol{D}\ddot{\boldsymbol{q}} + \boldsymbol{e} = \boldsymbol{\tau} \tag{7}$$

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} + \boldsymbol{C}\dot{\boldsymbol{\varphi}} + \boldsymbol{K}\boldsymbol{\varphi} = \boldsymbol{f} - \boldsymbol{G}\ddot{\boldsymbol{q}} \tag{8}$$

where $\boldsymbol{e} = \boldsymbol{U}\ddot{\boldsymbol{\varphi}} - \boldsymbol{\tau}_{\mathrm{R}}$.

Equation (8) describes the flexible vibration of the flexible redundant manipulator. When the joint motion of the flexible manipulator is known, (8) can be solved for its vibration responses, then substituting these responses into (7), the corresponding joint torques can be obtained.

Substituting (3) into (8), we obtain

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} + \boldsymbol{C}\dot{\boldsymbol{\varphi}} + \boldsymbol{K}\boldsymbol{\varphi} = \boldsymbol{u} \tag{9}$$

where $\boldsymbol{u} = \boldsymbol{f} - \boldsymbol{G}\boldsymbol{J}^{+}(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}) - \boldsymbol{G}(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\varepsilon}_1$.

Equation (9) describes vibration responses of the flexible redundant manipulator.

Substituting (3) into (7), we obtain

$$\boldsymbol{\tau} = \boldsymbol{D}\boldsymbol{J}^{+}\left(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}\right) + \boldsymbol{D}\left(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J}\right)\boldsymbol{\varepsilon}_1 + \boldsymbol{e} \tag{10}$$

Equation (10) provides the joint torques for the flexible redundant manipulator.

From (9), it is seen that the vibration equations of the flexible redundant manipulator are different from those of the flexible nonredundant manipulator. The difference is there exists in (9) the arbitrary vector $\boldsymbol{\varepsilon}_1$ corresponding to the self-motion of the redundant manipulator. The changes of the arbitrary vector $\boldsymbol{\varepsilon}_1$ may result in the changes of $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$, and $\boldsymbol{u}$, thereby change the vibration responses of the manipulator. Therefore, how to choose $\boldsymbol{\varepsilon}_1$ can directly affect the vibration responses of the flexible redundant manipulator. This effect may be favorable or adverse. Since different $\boldsymbol{\varepsilon}_1$ can result in different vibration responses of the manipulator, it is possible to damp out vibration by properly choosing $\boldsymbol{\varepsilon}_1$.

Once $\boldsymbol{\varepsilon}_1$ has been chosen for reducing vibration, the corresponding joint motion can be obtained from (3), then joint-torques for suppressing vibration can be obtained from (10). In this paper, (9) is used to reduce vibration, and (10) is used to make the flexible redundant manipulator trace a desired end-effector trajectory accurately. Therefore, (9) and (10) are the bases of our vibration reduction method for the flexible redundant manipulator.

3 Method for vibration reduction

As we know, increasing the damping and eliminating the exciting force are important ways to reduce vibration of an oscillatory system. To a flexible redundant manipulator, as mentioned above, there exist the self-motions in its joint space while maintaining a fixed nominal end-effector position/posture. This means whatever $\boldsymbol{\varepsilon}_1$ is chosen, the nominal end-effector motion cannot be affected. On the other hand, different choices of $\boldsymbol{\varepsilon}_1$ can directly affect the vibration responses of the flexible redundant manipulator. Therefore, some form of modal control force can be constructed and actively exerted to the flexible redundant manipulator system by properly choosing the arbitrary vector $\boldsymbol{\varepsilon}_1$ in the mode space, which can eliminate the exciting force and increase the damping of the manipulator.

By means of the following transformation:

$$\boldsymbol{\varphi} = \boldsymbol{P}\boldsymbol{\eta} \tag{11}$$

where $\boldsymbol{P} \in \boldsymbol{R}^{n_\mathrm{F}\times n_\mathrm{F}}$ is the mode matrix; $\boldsymbol{\eta} = \{\eta_1\ \eta_2\ \cdots\ \eta_{n_\mathrm{F}}\}^\mathrm{T}$ are the mode coordinates.

In the case of the proportional damping, (9) can be transformed into a set of uncoupled equations in the mode space, that is,

$$\ddot{\boldsymbol{\eta}} + \mathrm{diag}[2\xi_i\omega_i]\dot{\boldsymbol{\eta}} + \mathrm{diag}\big[\omega_i^2\big]\boldsymbol{\eta} = \boldsymbol{f}_\mathrm{g} \tag{12}$$

where ξ_i is the ith modal damping ratio; ω_i is the ith natural frequency; $i = 1, 2, \ldots, n_\mathrm{F}$; $\boldsymbol{f}_\mathrm{g} = \{f_{\mathrm{g}1}\ f_{\mathrm{g}2}\ \cdots\ f_{\mathrm{g}n_\mathrm{F}}\}^\mathrm{T}$ is the modal generalized force, $\boldsymbol{f}_\mathrm{g} = \boldsymbol{P}^\mathrm{T}\boldsymbol{u}$.

Equation (12) is a set of vibration equations of the flexible redundant manipulator described in the mode space.

To a flexible redundant manipulator, its modal generalized force $\boldsymbol{f}_\mathrm{g}$ can be separated as follows:

$$\boldsymbol{f}_\mathrm{g} = \boldsymbol{f}_\mathrm{v} + \boldsymbol{f}_\mathrm{c} \tag{13}$$

where

$$\begin{aligned}\boldsymbol{f}_\mathrm{v} &= \{f_{\mathrm{v}1}\quad f_{\mathrm{v}2}\quad \cdots\quad f_{\mathrm{v}n_\mathrm{F}}\}^\mathrm{T}\\ &= \boldsymbol{P}^\mathrm{T}\boldsymbol{f} - \boldsymbol{P}^\mathrm{T}\boldsymbol{G}\boldsymbol{J}^+\big(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}\big)\end{aligned} \tag{14}$$

$$\begin{aligned}\boldsymbol{f}_\mathrm{c} &= \{f_{\mathrm{c}1}\quad f_{\mathrm{c}2}\quad \cdots\quad f_{\mathrm{c}n_\mathrm{F}}\}^\mathrm{T}\\ &= -\boldsymbol{P}^\mathrm{T}\boldsymbol{G}\big(\boldsymbol{I} - \boldsymbol{J}^+\boldsymbol{J}\big)\boldsymbol{\varepsilon}_1\end{aligned} \tag{15}$$

In (14), $\boldsymbol{f}_\mathrm{v}$ is related to the joint motion of the manipulator, and mainly excites vibration, so $\boldsymbol{f}_\mathrm{v}$ is called the modal exciting force. While in (15), $\boldsymbol{f}_\mathrm{c}$ contains the arbitrary vector $\boldsymbol{\varepsilon}_1$ corresponding to the self-motion of the flexible redundant manipulator, and how to choose $\boldsymbol{\varepsilon}_1$ can directly affect its vibration responses, so $\boldsymbol{f}_\mathrm{c}$ is called the modal control force.

Equation (12) can be described as follows:

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = f_{\mathrm{v}i} + f_{\mathrm{c}i} \quad (i = 1, 2, \ldots, n_\mathrm{F}) \tag{16}$$

To suppress vibration, increasing the damping and eliminating the exciting force are expected. Since $\boldsymbol{\varepsilon}_1$ can be chosen arbitrarily, $\boldsymbol{f}_\mathrm{c}$ can be constructed by properly choosing $\boldsymbol{\varepsilon}_1$ as follows:

$$f_{\mathrm{c}i} = -2\xi_{\mathrm{c}i}\omega_i\dot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i - f_{\mathrm{v}i} \quad (i = 1, 2, \ldots, n_\mathrm{F}) \tag{17}$$

where $\xi_{\mathrm{c}i}$ is the selected modal damping ratio for suppressing vibration, and the desired damping property can be obtained by properly choosing $\xi_{\mathrm{c}i}$.

Substituting (17) into (16), we obtain

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = 0 \quad (i = 1, 2, \ldots, n_\mathrm{F}) \tag{18}$$

Equation (18) shows this form of modal control force imposed on the flexible manipulator can increase the damping and eliminate the exciting force simultaneously. As a result, vibration will be reduced effectively.

Therefore, (18) is the desired vibration reduction equation.

In order to solve the arbitrary vector $\boldsymbol{\varepsilon}_1$ satisfying vibration reduction, (17) can be described in a matrix as follows:

$$\boldsymbol{f}_{\mathrm{c}} = \boldsymbol{f}_{\mathrm{m}} - \boldsymbol{f}_{\mathrm{v}} \tag{19}$$

where $\boldsymbol{f}_{\mathrm{m}} = \{(-2\xi_{\mathrm{c}1}\omega_1\dot{\eta}_1 + 2\xi_1\omega_1\dot{\eta}_1)\cdots(-2\xi_{\mathrm{cn_F}} \times\omega_{\mathrm{n_F}}\dot{\eta}_{\mathrm{n_F}} + 2\xi_{\mathrm{n_F}}\omega_{\mathrm{n_F}}\dot{\eta}_{\mathrm{n_F}})\}^{\mathrm{T}}$.

Substituting (14) and (15) into (19), (19) can be expanded as follows:

$$\begin{aligned} &\boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\varepsilon}_1 \\ &\quad = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{f} - \boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{J}^{+}(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}) - \boldsymbol{f}_{\mathrm{m}} \end{aligned} \tag{20}$$

Equation (20) can be solved for the arbitrary vector $\boldsymbol{\varepsilon}_1$ satisfying vibration reduction.

4 Additional optimization ability

As mentioned above, a method for vibration reduction has been proposed by choosing $\boldsymbol{\varepsilon}_1$ while the desired end-effector trajectory is traced. However, in order to implement this method, some problems concerning choices of $\boldsymbol{\varepsilon}_1$ should be considered.

Equation (20) can be written as follows:

$$\boldsymbol{A}\boldsymbol{\varepsilon}_1 = \boldsymbol{b} \tag{21}$$

where $\boldsymbol{A} = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})$; $\boldsymbol{b} = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{f} - \boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{J}^{+}(\ddot{\boldsymbol{x}} - \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}) - \boldsymbol{f}_{\mathrm{m}}$.

If let rank(.) denote "the rank of", obviously $\mathrm{rank}(\boldsymbol{A}) \le \min(\mathrm{rank}(\boldsymbol{P}^{\mathrm{T}}\boldsymbol{G})\mathrm{rank}(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J}))$. In general, the matrix $\boldsymbol{G}$ has full rank except at certain special (known) manipulator configurations [10], so $\mathrm{rank}(\boldsymbol{G}) = \min(n_{\mathrm{R}}, n_{\mathrm{F}})$. Since $\boldsymbol{P}^{\mathrm{T}}$ is a full rank matrix, $\mathrm{rank}(\boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}) = \mathrm{rank}(\boldsymbol{G})$. If the manipulator is not in a singular configuration, $\mathrm{rank}(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J}) = n_{\mathrm{R}} - m$ [10]. In this paper, it is assumed $\boldsymbol{G}$ is a full rank matrix and the manipulator is not in a singular configuration.

Theoretically, a flexible redundant manipulator has infinite many vibration modes. Even though some higher modes are truncated, there are a lot of flexural degrees of freedom, but a flexible redundant manipulator usually has few redundant rigid joint degrees of freedom. It means there are more flexural degrees of freedom than the number of redundant joints available, i.e. $n_{\mathrm{R}} - m < n_{\mathrm{F}}$, which is a common case. As for $n_{\mathrm{R}} - m \ge n_{\mathrm{F}}$, it is uncommon and can be solved easily. Therefore, the first case is mainly analyzed in this paper.

If $n_{\mathrm{R}} < n_{\mathrm{F}}$, $\mathrm{rank}(\boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}) = n_{\mathrm{R}}$, then

$$\mathrm{rank}(\boldsymbol{A}) \le \min(n_{\mathrm{R}}, n_{\mathrm{R}} - m) = n_{\mathrm{R}} - m < n_R \tag{22}$$

If $n_{\mathrm{R}} \ge n_{\mathrm{F}}$, $\mathrm{rank}(\boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}) = n_{\mathrm{F}}$, then

$$\mathrm{rank}(\boldsymbol{A}) \le \min(n_{\mathrm{F}}, n_{\mathrm{R}} - m) = n_{\mathrm{R}} - m < n_{\mathrm{R}} \tag{23}$$

Either way,

$$\mathrm{rank}(\boldsymbol{A}) \le \mathrm{rank}(\tilde{\boldsymbol{A}}) \tag{24}$$

where $\tilde{A}$ is the augmentation matrix of the (21).

If $\mathrm{rank}(\boldsymbol{A}) = \mathrm{rank}(\tilde{\boldsymbol{A}})$, there are infinitely many solutions to (21) and the general solution for $\boldsymbol{\varepsilon}_1$ is given by

$$\boldsymbol{\varepsilon}_1 = \boldsymbol{A}^{+}\boldsymbol{b} + (\boldsymbol{I} - \boldsymbol{A}^{+}\boldsymbol{A})\boldsymbol{\varepsilon}_2 \tag{25}$$

where $\boldsymbol{\varepsilon}_2 \in \boldsymbol{R}^{n_{\mathrm{R}}}$ is an arbitrary vector; $\boldsymbol{A}^{+}$ is the generalized inverse matrix of $\boldsymbol{A}$.

The self-motion provided by the general solution of $\boldsymbol{\varepsilon}_1$ can suppress the end-effector vibration.

If $\mathrm{rank}(\boldsymbol{A}) < \mathrm{rank}(\tilde{\boldsymbol{A}})$, there are no solution to (21). Since $\mathrm{rank}(\boldsymbol{A}) < n_{\mathrm{R}}$, i.e. $\boldsymbol{A} \in \boldsymbol{R}^{n_{\mathrm{F}} \times n_{\mathrm{R}}}$ is not a full column rank matrix, there are infinitely many solutions of $\boldsymbol{\varepsilon}_1$ to minimize $\|\boldsymbol{A}\boldsymbol{\varepsilon}_1 - \boldsymbol{b}\|_2$, and the general solution for $\boldsymbol{\varepsilon}_1$ in the least squares sense is:

$$\boldsymbol{\varepsilon}_1 = \boldsymbol{A}^{+}\boldsymbol{b} + (\boldsymbol{I} - \boldsymbol{A}^{+}\boldsymbol{A})\boldsymbol{\varepsilon}_2 \tag{26}$$

This means that although the desired vibration control (18) cannot be achieved, but it can be approached in the least squares sense. Thus, the self-motion corresponding to the general solution for $\boldsymbol{\varepsilon}_1$ can still reduce vibration effectively. This can be verified by the following numerical simulation examples.

From (25) and (26), it can be seen that the arbitrary vector $\boldsymbol{\varepsilon}_1$ satisfying vibration reduction are not unique, and still depend on the arbitrary vector $\boldsymbol{\varepsilon}_2$. Since whatever $\boldsymbol{\varepsilon}_2$ can ensure the corresponding $\boldsymbol{\varepsilon}_1$ to satisfy vibration reduction, additional performance criteria can be optimized by properly choosing $\boldsymbol{\varepsilon}_2$ on the premise of vibration reduction.

5 Relation between mobility and joint velocity

The joint velocities required to move the end-effector with a desired speed depend on the direction of motion. The manipulator's mobility, i.e., its ability to move, is better in the directions requiring lower joint velocities [24, 25]. When the manipulator is close to a singular configuration, its joint velocities required to move in certain directions are extremely high, even the end-effector velocity is very small. In view of this, Dubey defined the velocity ratio of a manipulator as the ratio of the end-effector velocity vector norm to the joint velocity vector norm, and call it the "manipulator-velocity-ratio" (MVR) [24, 25]. If r_v is MVR, then

$$r_v = \|\dot{\boldsymbol{x}}\| / \|\dot{\boldsymbol{q}}\| \tag{27}$$

And it is proven that r_v lies in the following range [24]:

$$\sigma_m \leq r_v \leq \sigma_1 \tag{28}$$

where σ_m is the minimum singular value of $\boldsymbol{J}$, σ_1 is the maximum singular value of $\boldsymbol{J}$.

Therefore, increasing MVR along the minor-axis of MVRE (manipulator-velocity-ratio-ellipsoids) can lower the upper limit on the joint velocities required to move the end-effector in all directions [24, 25]. Since excessive joint velocities usually indicate bad kinematic performance of the manipulator, Chen [26] suggested avoiding singularity through evading joint velocity limits for the rigid redundant manipulator.

From these researches, it can be seen, as far as the rigid manipulator is concerned, the kinematic dexterity contains two meanings in the context of a manipulator's ability to move, i.e., the "magnitude" of the joint velocities and the "uniformity" of the velocity ratios. On the one hand, the joint velocities required to attain the given end-effector velocities are expected to be low. On the other hand, uniform velocity ratios in all directions of the end-effector are also expected. As for the flexible redundant manipulator, however, vibration control is considerably important issue, and thus is considered as the primary optimization objective in this paper. In our opinion, it is quite valuable to control vibration and improve kinematic performance simultaneously for the flexible redundant manipulator. Even so, however, it is difficult to simultaneously achieve vibration reduction and uniform velocity ratios in all directions of the end-effector only by using the self-motions. Therefore, it is necessary to make a compromise. Firstly, as the primary optimization objective, the joint velocities satisfying vibration reduction are obtained by redundancy resolution. As mentioned in Sect. 4, these joint velocities are not unique. Then based on the aforementioned additional optimization ability, the minimum joint velocities required to attain the given end-effector velocities are chosen from those joint velocities satisfying vibration reduction. In such a case, the manipulator's mobility in the directions of the end-effector motion can be enhanced and its vibration responses can be damped out at the same time.

6 Simultaneous optimization on vibration reduction and mobility improvement

6.1 Joint velocity resolution for vibration reduction

Substituting (25) or (26) into (3), the joint accelerations satisfying vibration reduction at time n is

$$\begin{aligned}\ddot{\boldsymbol{q}}_n &= \boldsymbol{J}_{n-1}^{+}(\ddot{\boldsymbol{x}}_{n-1} - \dot{\boldsymbol{J}}_{n-1}\dot{\boldsymbol{q}}_{n-1}) \\ &\quad + (\boldsymbol{I} - \boldsymbol{J}_{n-1}^{+}\boldsymbol{J}_{n-1})\boldsymbol{A}_{n-1}^{+}\boldsymbol{b}_{n-1} \\ &\quad + (\boldsymbol{I} - \boldsymbol{J}_{n-1}^{+}\boldsymbol{J}_{n-1})(\boldsymbol{I} - \boldsymbol{A}_{n-1}^{+}\boldsymbol{A}_{n-1})\boldsymbol{\varepsilon}_2\end{aligned} \tag{29}$$

where subscript $n-1$ and n denote the variable value at time $n-1$ and n, respectively.

From (29), it is found that the joint motions satisfying vibration reduction are not unique; they still depend on the arbitrary vector $\boldsymbol{\varepsilon}_2$. Since whatever $\boldsymbol{\varepsilon}_2$ can guarantee vibration reduction, the mobility is expected to improve by further choosing $\boldsymbol{\varepsilon}_2$.

If $\boldsymbol{\varepsilon}_2$ has been chosen for improving the mobility, $\ddot{\boldsymbol{q}}_n$ can be obtained from (29). Then $\dot{\boldsymbol{q}}_n$ and $\boldsymbol{q}_n$ can be respectively obtained by integrating $\ddot{\boldsymbol{q}}_n$ for once and twice with respect to time. If the flexible manipulator moves according to these $\dot{\boldsymbol{q}}_n$ and $\boldsymbol{q}_n$, both vibration reduction and mobility improvement can be achieved simultaneously.

According to the above analysis, vibration reduction is implemented at the acceleration level, but the mobility needs to be improved at the velocity level. Therefore, in order to achieve these two aims at the same time, the joint acceleration $\ddot{\boldsymbol{q}}_n$ should be expressed in form of velocities.

Using the Taylor Series expansion and only considering the first three items, the joint acceleration $\ddot{\boldsymbol{q}}_n$ at the time n can be expressed at the velocity level (see the Appendix):

$$\ddot{\boldsymbol{q}}_n = \frac{2}{3\Delta t}\dot{\boldsymbol{q}}_n - \frac{2}{3\Delta t^2}\boldsymbol{q}_{n-1} + \frac{2}{3\Delta t^2}\boldsymbol{q}_{n-2} \tag{30}$$

where Δt is the time step corresponding to each time subinterval of the whole process.

Let (27) = (28), we obtain:

$$\dot{\boldsymbol{q}}_n = \boldsymbol{H} + \boldsymbol{Q}\boldsymbol{\varepsilon}_2 \tag{31}$$

where $\boldsymbol{H}$ and $\boldsymbol{Q}$ are as follows:

$$\begin{aligned}\boldsymbol{H} = &\frac{1}{\Delta t}\boldsymbol{q}_{n-1} - \frac{1}{\Delta t}\boldsymbol{q}_{n-2} \\ &+ \frac{3\Delta t}{2}\big[\boldsymbol{J}_{n-1}^{+}(\ddot{\boldsymbol{x}}_{n-1} - \dot{\boldsymbol{J}}_{n-1}\dot{\boldsymbol{q}}_{n-1}) \\ &+ (\mathbf{I} - \boldsymbol{J}_{n-1}^{+}\boldsymbol{J}_{n-1})\boldsymbol{A}_{n-1}^{+}\boldsymbol{b}_{n-1}\big]\end{aligned}$$

$$\boldsymbol{Q} = \frac{3\Delta t}{2}(\boldsymbol{I} - \boldsymbol{J}_{n-1}^{+}\boldsymbol{J}_{n-1})(\boldsymbol{I} - \boldsymbol{A}_{n-1}^{+}\boldsymbol{A}_{n-1})$$

If the flexible redundant manipulator moves according to (31), it may trace the desired end-effector trajectory.

6.2 Strategy for both vibration reduction and mobility improvement

In (31), the joint velocities satisfying vibration reduction are not unique, they still depend on the arbitrary vector $\boldsymbol{\varepsilon}_2$. Since whatever $\boldsymbol{\varepsilon}_2$ can guarantee vibration reduction, the mobility can be improved by further choosing $\boldsymbol{\varepsilon}_2$ at the same time.

As mentioned in Sect. 5, if the joint velocities required to attain the given end-effector velocities can be decreased, the manipulator's mobility in the directions of the end-effector motion will be improved. Because the joint velocities in (31) can both supply the given end-effector velocities and reduce vibration, a method proposed here is to minimize these joint velocities in the least squares sense by choosing $\boldsymbol{\varepsilon}_2$, that is,

$$\text{Minimize}\left\|\dot{\boldsymbol{q}}(\boldsymbol{\varepsilon}_2)\right\|_2 \tag{32}$$

From (32), we obtain

$$\boldsymbol{\varepsilon}_2 = -\boldsymbol{Q}^{+}\boldsymbol{H} \tag{33}$$

If the flexible redundant manipulator moves according to (31) and (33), both vibration reduction and mobility improvement can be achieved at the same time.

7 Simulation and analysis

To verify the method presented above, a planar manipulator with one flexible link is used in numerical simulations, as shown in Fig. 1. The planar manipulator has one redundant rigid degree of freedom without considering its end-effector posture. Only vibration deformations in the horizontal plan are considered. The parameters are given as follows: each length of three links is 1.0 m, the first two links are made of steel, and their cross-sections are squares with side length of 0.05 m, the third link is made of aluminum with the elastic modulus of 71 GPa and the density of 2,710 kg/m^3, whose section is rectangle with the height of 0.05 m and the width of 0.005 m. The initial three joint angles are $\boldsymbol{q}(0) = \{75° \ \ -150° \ 60°\}^{\mathrm{T}}$, angular velocities are $\dot{\boldsymbol{q}}(0) = \{0° \ 0° \ 0°\}^{\mathrm{T}}$/s, and the initial end-effector velocities are $\dot{\boldsymbol{x}}(0) = \{0 \ 0\}^{\mathrm{T}}$ m/s, the desired end-effector accelerations with respect to the base frame are:

$$\ddot{\boldsymbol{x}}(t) = \begin{cases}\{0.5 \quad 0.5\}^{\mathrm{T}} \ \mathrm{m/s^2} & 0 \le t \le 1.5 \ \mathrm{s} \\ \{-0.5 \quad -0.5\}^{\mathrm{T}} \ \mathrm{m/s^2} & 1.5 \ \mathrm{s} \le t \le 3.0 \ \mathrm{s}\end{cases}$$

Our task in this example is to reduce vibration and improve mobility for the flexible redundant manipulator. Only the first two modes are considered in this example. The numerical simulations of three cases have been performed respectively as follows.

Case 1: No self-motion, i.e., $\boldsymbol{\varepsilon}_1 = 0$. The corresponding numerical simulation results are shown in Fig. 2. It is seen that the end-effector vibration cannot be reduced because of no self-motion, thereby resulting in the deterioration of the end-effector accuracy.

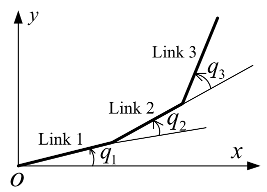

Fig. 1 Three-link planar manipulator

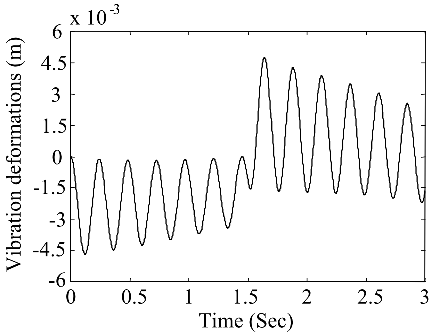

Fig. 2 Vibration deformations of the end-effector

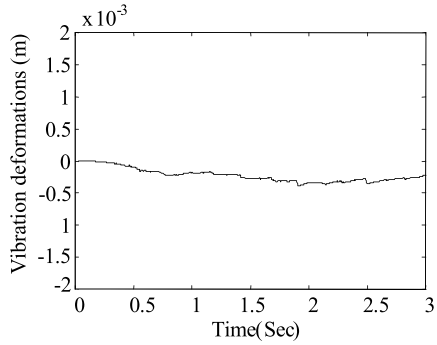

Fig. 3 Vibration deformations of the end-effector

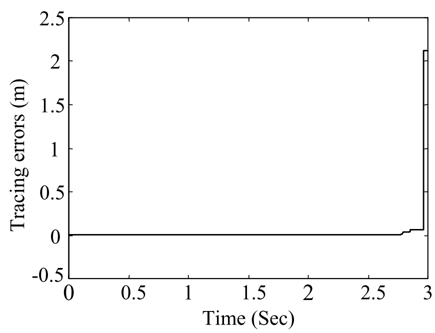

Fig. 4 Tracing errors of the end-effector

Case 2: Only considering vibration reduction, i.e., $\boldsymbol{\varepsilon}_1 \neq 0$ and $\boldsymbol{\varepsilon}_2 = 0$. In this case, the main aim is to reduce vibration but ignore joint velocities optimization. The vibration deformations of the end-effector are shown in Fig. 3. Compared with Fig. 2, it is found in Fig. 3 that the vibration deformations are eliminated effectively via redundancy resolution. However, as shown in Fig. 4, because of no simultaneous joint velocities minimization, the second joint velocity in this simulation abruptly jumps to 1003.6°/s at time $t = 2.77$ s, indicating bad mobility. As a result, the end-effector severely deviates from the prescribed trajectory, thereby causing large tracing errors. In addition, the corresponding rigid motion configurations of the manipulator are shown in Fig. 5. From this figure, it can be seen clearly that the abrupt change of the joint velocities may cause discontiguous rigid motion, resulting in the failure to trace the prescribed end-effector motion. The corresponding manipulability profile of the manipulator in this simulation is shown as line 1 in Fig. 6.

Case 3: considering both reducing vibration and improving mobility, i.e., $\boldsymbol{\varepsilon}_1 \neq 0$ and $\boldsymbol{\varepsilon}_2 \neq 0$. The vibration deformations of the end-effector are shown in Fig. 7. Compared with Fig. 2, the vibration deformations in Fig. 7 are eliminated effectively via redundancy resolution. At the same time, since the additional optimization capability is introduced in this simulation, the kinematic performance of the manipulator is improved as well. The corresponding rigid motion configurations of the manipulator are shown in Fig. 8. From this figure, it shows that the corresponding rigid motion configurations of the manipulator are smooth and continuous. The corresponding manipulability profile of the manipulator in this simulation is shown as line 2 in Fig. 6. Compared with line 1 in Fig. 6, this manipulability profile (line 2) is better. Although line 1 looks better than line 2 in the vicinity of the 3 second, since these rigid motion configurations of the manipulator during this time period in Case 2 cannot achieve the prescribed end-effector trajectory, the corresponding manipulability during this time period has no actual sense.

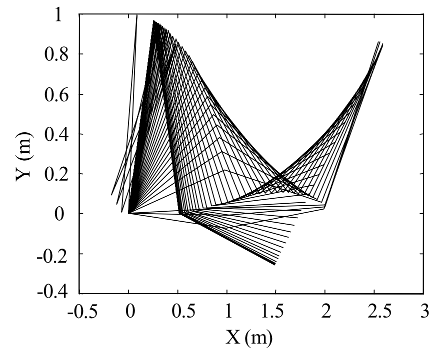

Fig. 5 Rigid motion configurations of the manipulator

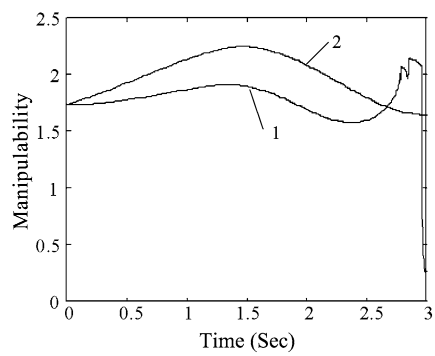

Fig. 6 Manipulability profile

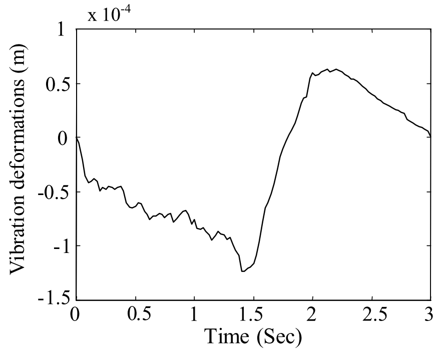

Fig. 7 Vibration deformations of the end-effector

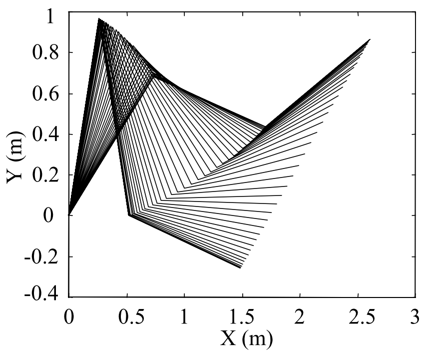

Fig. 8 Rigid motion configurations of the manipulator

All of the above simulation results show the optimization method presented in this paper is effective in reducing vibration and improving mobility.

Furthermore, it should be noted that the dynamic model of the flexible manipulator is usually highly nonlinear, thus its computation is a time-consuming task. In this study, the dynamic model of the flexible manipulator is derived based on Kane's method [27], which is an effective method for creating and solving complex flexible multibody dynamics. This method has such advantages as concision, iteration, and minor calculating works, thus is convenient for computer programming. Moreover, the modal analysis technology is employed in the vibration reduction method suggested here, thereby further reducing computation difficulty. As for the above simulation example, almost 4 seconds is spent in a computer with the Intel® Core™2 Duo T7500@2.2 GHz CPU and 3 GB RAM. On the other hand, the computation work concerning the optimization method proposed in this paper can be performed offline. In addition, the optimization method does not need to search the optimal value within specified constraints, which is usually a time-consuming work, but directly offers the final analytic expression, i.e., (31) and (33), thus can save much time. In practical applications, if a flexible redundant manipulator moves according to the optimized joint velocities obtained offline, both vibration reduction and mobility improvement can be achieved at the same time.

8 Conclusion

In this paper, we show proper choices of the self-motions are very important in both reducing undesired vibration and improving mobility for the flexible redundant manipulator. A method for vibration control is proposed via redundancy resolution. By analyzing the self-motions satisfying vibration reduction, its additional optimization ability is revealed. Based on this ability, a strategy is proposed to both reduce vibration and improve mobility for the flexible redundant manipulator. The simulation results demonstrate the effectiveness of this strategy.

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Appendix

In Sect. 6.1, the joint acceleration $\ddot{\boldsymbol{q}}_n$ at the time n is expressed at the velocity level, as shown in (30), i.e.,

$$\ddot{\boldsymbol{q}}_n = \frac{2}{3\Delta t}\dot{\boldsymbol{q}}_n - \frac{2}{3\Delta t^2}\boldsymbol{q}_{n-1} + \frac{2}{3\Delta t^2}\boldsymbol{q}_{n-2} \tag{A.1}$$

This result can be derived as follows.

Using the Taylor series expansion and only considering the first three items, the joint angles $\boldsymbol{q}(t)$ can be expanded at time n, i.e.,

$$\boldsymbol{q}(t) = \boldsymbol{q}_n + \dot{\boldsymbol{q}}_n(t-n) + \frac{\ddot{\boldsymbol{q}}_n}{2}(t-n)^2 \tag{A.2}$$

Equation (A.2) may be changed as

$$\ddot{\boldsymbol{q}}_n = \frac{2}{(t-n)^2}\left[\boldsymbol{q}(t) - \boldsymbol{q}_n - \dot{\boldsymbol{q}}_n(t-n)\right] \tag{A.3}$$

If $t = n - 1$, then

$$\ddot{\boldsymbol{q}}_n = \frac{2}{\Delta t^2}[\boldsymbol{q}_{n-1} - \boldsymbol{q}_n + \dot{\boldsymbol{q}}_n \Delta t] \tag{A.4}$$

where Δt is the time step corresponding to each time subinterval of the whole process.

If $t = n - 2$, then

$$\ddot{\boldsymbol{q}}_n = \frac{2}{4\Delta t^2}[\boldsymbol{q}_{n-2} - \boldsymbol{q}_n + 2\dot{\boldsymbol{q}}_n \Delta t] \tag{A.5}$$

Let (A.5) subtract (A.4), we obtain

$$\ddot{\boldsymbol{q}}_n = \frac{2}{3\Delta t}\dot{\boldsymbol{q}}_n - \frac{2}{3\Delta t^2}\boldsymbol{q}_{n-1} + \frac{2}{3\Delta t^2}\boldsymbol{q}_{n-2} \tag{A.6}$$

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