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ORIGINAL PAPER

Projective synchronization of nonlinear-coupled spatiotemporal chaotic systems

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Abstract The projective synchronization of one-dimensional discrete spatiotemporal chaotic systems is discussed in this paper. The coupling equation is determined by suitably separating the linear term and the nonlinear term of the dynamic function, and two coupled map lattices reach projective synchronization by the nonlinear-coupling method. Besides, this method is expanded to the projective synchronization of the complex network composed by coupled map lattices. Numerical simulations show the effectiveness of the scheme.

Keywords Coupled map lattice · Spatiotemporal chaos · Projective synchronization · Complex network

1 Introduction

Many practical systems in the real world can be described by spatiotemporal chaotic systems. Among these systems, coupled map lattice [1] is a typical discrete spatiotemporal chaotic system. It has a discretization of time and space, but its status remains continuous. Many researches [2–4] have been done based on the study of this system. As same as general chaotic systems, the synchronization of spatiotemporal chaotic systems is a focus of attention. Hu et al. presented a synchronization scheme for coupled map lattice [5]; Nekorkin et al. realized mutual synchronization of two lattices of bistable elements [6]; Xun et al. achieved synchronization of discrete spatiotemporal chaos by variable structure control [7]; Yue et al. synchronized discrete-time spatiotemporal chaos with fuzzy control method [8]; Wang et al. obtained synchronization and coherence resonance in chaotic neural networks by numerical analysis [9]. There are many other works [10–12] in this field. Generally speaking, variable coupling is a common method to synchronize two dynamic systems. In this kind of method, the coupling function can be linear or nonlinear. Linear coupling is simple and convenient, but as for chaotic systems with strong nonlinearity, nonlinear coupling is more effective, so nonlinear coupling is adopted in this paper. In order to realize projective synchronization of discrete spatiotemporal chaotic systems, the function of the response system is separated into new linear and nonlinear parts, and the nonlinear part is taken as coupling function. Two coupled map lattices are synchronized successfully by this method, and this method is expanded to projective synchronization of complex network composed by coupled map lattices. Numerical simulations show the feasibility and effectiveness of the scheme.

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2 Theory and method

Considering the following one-dimensional discrete spatiotemporal chaotic system

$$\boldsymbol{x}_{n+1}(i) = \boldsymbol{F}\big(\boldsymbol{x}_n(i)\big), \tag{1}$$

where n is the discrete time step and i $(i = 1, 2, \dots, L)$ stands for lattice coordinate, L is the number of lattice. $\boldsymbol{x}_n(i) \in \mathbb{R}^N$ is the state variable and $\boldsymbol{F} : \mathbb{R}^N \to \mathbb{R}^N$.

Separate $\boldsymbol{F}(\boldsymbol{x}_n(i))$ as follows,

$$\boldsymbol{x}_{n+1}(i) = \boldsymbol{F}\big(\boldsymbol{x}_n(i)\big) = \boldsymbol{A}\boldsymbol{x}_n(i) + \boldsymbol{H}\big(\boldsymbol{x}_n(i)\big), \tag{2}$$

where $\boldsymbol{A}$ is a matrix which satisfies $|\lambda_i| < 1$ (λ_i is eigenvalue of $\boldsymbol{A}$ and $i = 1, 2, \dots, L$), $\boldsymbol{H}(\boldsymbol{x}_n(i))$ is the rest part of $\boldsymbol{F}(\boldsymbol{x}_n(i))$.

Theorem 1 *Suppose system* (2) *is the drive system, then the response system can be described as*

$$\boldsymbol{y}_{n+1}(i) = \boldsymbol{F}\big(\boldsymbol{y}_n(i)\big) + \boldsymbol{u}\big(\boldsymbol{x}_n(i), \boldsymbol{y}_n(i)\big). \tag{3}$$

If we choose the coupling controller as

$$\boldsymbol{u}\big(\boldsymbol{x}_n(i), \boldsymbol{y}_n(i)\big) = \boldsymbol{A}\boldsymbol{y}_n(i) - \boldsymbol{F}\big(\boldsymbol{y}_n(i)\big) + \boldsymbol{H}\big(\boldsymbol{x}_n(i)\big)/\beta,$$

then system (2) *and system* (3) *will achieve projective synchronization, i.e.* $\lim_{n\to\infty} \boldsymbol{x}_n(i) = \beta \boldsymbol{y}_n(i)$, *where* $\beta = \operatorname{diag}(\beta_1, \beta_2, \dots, \beta_L)$ *stands for the proportional scale.*

Proof Suppose the projective synchronization error as $\boldsymbol{e}_n(i) = \boldsymbol{x}_n(i) - \beta \boldsymbol{y}_n(i)$, then we have

$$\begin{aligned} \boldsymbol{e}_{n+1}(i) &= \boldsymbol{x}_{n+1}(i) - \beta \boldsymbol{y}_{n+1}(i) \\ &= \big[\boldsymbol{A}\boldsymbol{x}_n(i) + \boldsymbol{H}\big(\boldsymbol{x}_n(i)\big)\big] \\ &\quad - \beta\big[\boldsymbol{A}\boldsymbol{y}_n(i) + \boldsymbol{H}\big(\boldsymbol{x}_n(i)\big)/\beta\big] \\ &= \boldsymbol{A}\boldsymbol{x}_n(i) - \beta\boldsymbol{A}\boldsymbol{y}_n(i) \\ &= \boldsymbol{A}\big[\big(\boldsymbol{x}_n(i)\big) - \beta\big(\boldsymbol{y}_n(i)\big)\big] \\ &= \boldsymbol{A}\boldsymbol{e}_n(i). \end{aligned}$$

According to the ability criterion of discrete system [13, 14], we have $\lim_{n\to\infty} \boldsymbol{e}_n(i) = 0$, i.e. with the coupling equation $\boldsymbol{H}(\boldsymbol{x}_n(i))/\beta$, system (2) and system (3) will achieve projective synchronization and the proportional scale is β. □

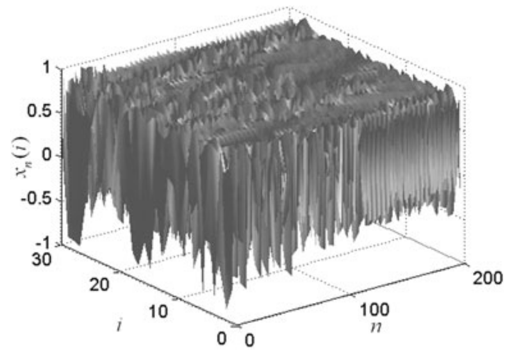


Fig. 1 With $a = 1.8$ and $\varepsilon = 0.1$, space–time evolution history of system (6)

3 Numerical simulations

Coupled map lattice (CML) [1] is adopted in numerical simulations. The dynamic function of coupled map lattice is described as

$$\begin{aligned} x_{n+1}(i) = (1-\varepsilon) f\big(x_n(i)\big) + \frac{\varepsilon}{2}\big(f\big(x_n(i-1)\big) \\ + f\big(x_n(i+1)\big)\big), \end{aligned} \tag{4}$$

where n is the discrete time step and i $(i = 1, 2, \dots, L)$ stands for lattice coordinate, L is the number of lattice, $f(x_n(i))$ is defined as

$$f\big(x_n(i)\big) = 1 - a x_n^2(i), \tag{5}$$

where a control parameter, we have

$$\begin{aligned} x_{n+1}(i) = (1-\varepsilon)\big(1 - a x_n^2(i)\big) + \frac{\varepsilon}{2}\big(\big(1 - a x_n^2(i-1)\big) \\ + \big(1 - a x_n^2(i+1)\big)\big). \end{aligned} \tag{6}$$

When $a = 1.8$, $\varepsilon = 0.1$, system (6) exhibits spatiotemporal chaos. Suppose system (6) satisfies periodic boundary, i.e. $x_n(L+1) = x_n(1)$ and the initial states of $x_n(i)$ are random from -1 to 1, then the spatiotemporal chaotic attractor of system (6) with $L = 30$ is shown in Fig. 1.

Choose system (4) as drive system, then the response system can be described as

$$\begin{aligned} y_{n+1}(i) = (1-\varepsilon) f\big(y_n(i)\big) + \frac{\varepsilon}{2}\big(f\big(y_n(i-1)\big) \\ + f\big(y_n(i+1)\big)\big) + u_n(i). \end{aligned} \tag{7}$$

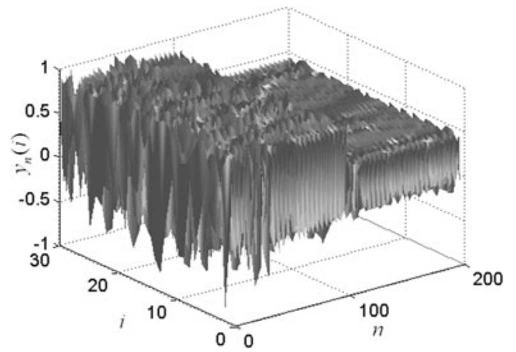


Fig. 2 Space–time evolution history of the response system (7)

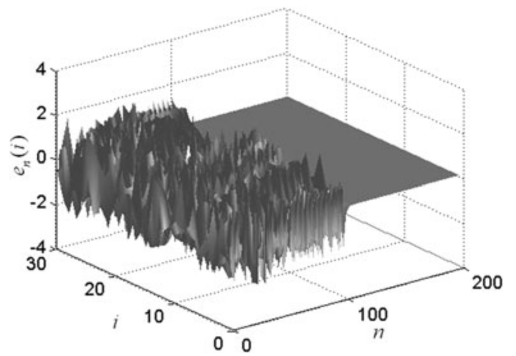


Fig. 3 Space–time evolution history of the projective synchronization error

For convenience, let $\boldsymbol{A} = \mathrm{diag}(0.5, 0.5, \ldots, 0.5)$, according to Theorem 1, it is easy to obtain

$$u_n(i) = 0.5y_n(i) - (1-\varepsilon)f\big(y_n(i)\big) + \frac{\varepsilon}{2}\big(f\big(y_n(i-1)\big) + f\big(y_n(i+1)\big)\big) + h\big(x_n(i)\big)/\beta_i, \tag{8}$$

where $h(x_n(i))/\beta_i (i = 1, 2, \ldots, L)$ is the coupling equation for every lattice, and

$$h\big(x_n(i)\big) = (1-\varepsilon)f\big(x_n(i)\big) + \frac{\varepsilon}{2}\big(f\big(x_n(i-1)\big) + f\big(x_n(i+1)\big)\big) - 0.5x_n(i). \tag{9}$$

Suppose the projective synchronization error as $e_n(i) = x_n(i) - \beta_i y_n(i)$, choose the proportional scale as $\beta = \mathrm{diag}(\beta_1, \beta_2, \ldots, \beta_L) = \mathrm{diag}(2, 2, \ldots, 2)$, start controlling at $n = 100$, let $L = 30$, then the space-time evolution history of $y_n(i)$ and $e_n(i)$ are shown in Figs. 2 and 3 respectively.

From the above simulation results, system (4) and system (7) have achieved the required projective synchronization.

4 Projective synchronization of the complex network composed by CML

Projective synchronization means that the states of drive system and response system evolve in a proportional scale. From this, we can obtain the conception of network projective synchronization. For the dynamic system in every node, if the phase difference is locked and the relevant states are in a proportional scale, then we can say that the "network projective synchronization" has been achieved. Complete synchronization and anti-synchronization may seem to be a special projective synchronization, so the study of network projective synchronization has more potential applications. In the paper, coupled map lattice (CML) will be adopted as the nodes of the target complex network.

Suppose a complex network has m nodes, and the dynamic functions of every node are one-dimensional discrete spatiotemporal chaotic systems. Suppose the states of node i as $\boldsymbol{x}_n^{(i)}$, where $\boldsymbol{x}_n^{(i)} = (x_{n1}^{(i)}, x_{n2}^{(i)}, \ldots, x_{nL}^{(i)})^{\mathrm{T}}$ and $x_{n1}^{(i)}, x_{n2}^{(i)}, \ldots, x_{nL}^{(i)} \in \mathbb{R}^n$, then the dynamic function for node i without coupling can be described as

$$\boldsymbol{x}_{n+1}^{(i)} = \boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}\big), \tag{10}$$

where $\boldsymbol{F}^{(i)} : \mathbb{R}^n \to \mathbb{R}^n$.

If considering mutual coupling, then we have

$$\boldsymbol{x}_{n+1}^{(i)} = \boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}\big) + \boldsymbol{u}^{(i)}\big(\boldsymbol{x}_n^{(1)}, \boldsymbol{x}_n^{(2)}, \ldots, \boldsymbol{x}_n^{(m)}\big), \tag{11}$$

where $\boldsymbol{u}^{(i)}(\boldsymbol{x}_n^{(1)}, \boldsymbol{x}_n^{(2)}, \ldots, \boldsymbol{x}_n^{(m)})$ stands for the coupling equation.

Suppose the proportional scale of the projective synchronization as $\beta^{(i)}$ and $\beta^{(i)} = \mathrm{diag}(\beta_1^{(i)}, \beta_2^{(i)}, \ldots, \beta_L^{(i)})$, then the projective synchronization error can be defined as

$$\boldsymbol{e}_n^{(i)} = \beta^{(i)}\boldsymbol{x}_n^{(i)} - \beta^{(i+1)}\boldsymbol{x}_n^{(i+1)} \quad (i = 1, 2, \ldots, m-1), \tag{12}$$

where $\boldsymbol{e}_n^{(i)} = (e_{n1}^{(i)}, e_{n2}^{(i)}, \ldots, e_{nL}^{(i)})^{\mathrm{T}}$. The error system can be defined as

$$\begin{aligned}\boldsymbol{e}_{n+1}^{(i)} &= \beta^{(i)}\boldsymbol{x}_{n+1}^{(i)} - \beta^{(i+1)}\boldsymbol{x}_{n+1}^{(i+1)} \\ &= \Delta\boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}, \boldsymbol{x}_n^{(i+1)}\big) + \Delta\boldsymbol{u}^{(i)},\end{aligned} \tag{13}$$

where

$$\Delta \boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}, \boldsymbol{x}_n^{(i+1)}\big) = \beta^{(i)}\boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}\big) - \beta^{(i+1)}\boldsymbol{F}^{(i+1)}\big(\boldsymbol{x}_n^{(i+1)}\big),$$

and

$$\Delta \boldsymbol{u}^{(i)} = \beta^{(i)}\boldsymbol{u}^{(i)}\big(\boldsymbol{x}_n^{(1)}, \boldsymbol{x}_n^{(2)}, \dots, \boldsymbol{x}_n^{(m)}\big) - \beta^{(i+1)}\boldsymbol{u}^{(i+1)}\big(\boldsymbol{x}_n^{(1)}, \boldsymbol{x}_n^{(2)}, \dots, \boldsymbol{x}_n^{(m)}\big).$$

If we choose

$$\Delta \boldsymbol{u}^{(i)} = -\Delta \boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}, \boldsymbol{x}_n^{(i+1)}\big) + \boldsymbol{A}\boldsymbol{e}_n^{(i)} \quad (i = 1, 2, \dots, m-1), \tag{14}$$

where $\boldsymbol{A}$ is a matrix which satisfies $|\lambda_i| < 1$ (λ_i is an eigenvalue of $\boldsymbol{A}$ and $i = 1, 2, \dots, L$), then we have

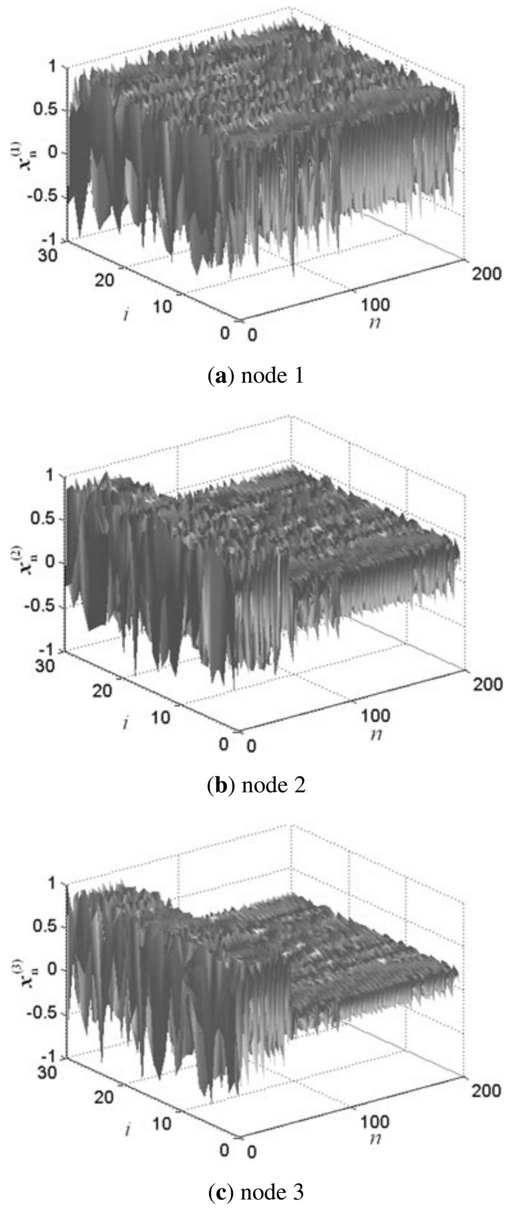
$$\begin{aligned} &\beta^{(j)}\boldsymbol{u}^{(j)} \\ &= \beta^{(1)}\boldsymbol{u}^{(1)} + \sum_{i=1}^{j-1} \Delta \boldsymbol{F}^{(i)}\big(\boldsymbol{x}_n^{(i)}, \boldsymbol{x}_n^{(i+1)}\big) - \boldsymbol{A}\sum_{i=1}^{j-1} \boldsymbol{e}_n^{(i)} \\ &= \beta^{(1)}\boldsymbol{u}^{(1)} + \beta^{(1)}\boldsymbol{F}^{(1)}\big(\boldsymbol{x}_n^{(1)}\big) - \beta^{(j)}\boldsymbol{F}^{(j)}\big(\boldsymbol{x}_n^{(j)}\big) \\ &\quad - \boldsymbol{A}\big(\beta^{(1)}\boldsymbol{x}_n^{(1)} - \beta^{(j)}\boldsymbol{x}_n^{(j)}\big) \quad (j = 2, 3, \dots, m). \end{aligned} \tag{15}$$

Choose node 1 as object system, i.e. $\boldsymbol{u}^{(1)} = 0$, then we have

$$\begin{aligned} \boldsymbol{u}^{(j)} &= \frac{\beta^{(1)}}{\beta^{(j)}}\boldsymbol{u}^{(1)} + \frac{\beta^{(1)}}{\beta^{(j)}}\boldsymbol{F}^{(1)}\big(\boldsymbol{x}_n^{(1)}\big) - \boldsymbol{F}^{(j)}\big(\boldsymbol{x}_n^{(j)}\big) \\ &\quad - \boldsymbol{A}\left(\frac{\beta^{(1)}}{\beta^{(j)}}\boldsymbol{x}_n^{(1)} - \boldsymbol{x}_n^{(j)}\right) \quad (j = 2, 3, \dots, m). \end{aligned} \tag{16}$$

It is easy to prove that (16) is equivalent to the coupling equation in (3), so according to Theorem 1, the complex network will achieve projective synchronization with the coupling described as (16). In the same way, if we choose other node as object system, we can also obtain the relevant qualified coupling equation for the rest nodes.

In the numerical simulations, suppose the network has three nodes, the dynamic function of every node is described as (6) with lattice number $L = 30$. Suppose the state variable for every node is $\boldsymbol{x}_n^{(i)}$ ($i = 1, 2, \dots, m$), choose node 1 as object node, let the proportional scale for node 2 and node 3 be diag$(2, 2, \dots, 2)$ and diag$(4, 4, \dots, 4)$, define the error between node 1 and node 2 as $\boldsymbol{e}_n^{(1,2)} = \boldsymbol{x}_n^{(1)} - 2\boldsymbol{x}_n^{(2)}$, define the error between node 1 and node 3 as $\boldsymbol{e}_n^{(1,3)} = \boldsymbol{x}_n^{(1)} - 4\boldsymbol{x}_n^{(3)}$, choose $A = \text{diag}(0.5, 0.5, \dots, 0.5)$, start controlling at $n = 50$, then the space-time evolution history of node 1, 2, 3 are shown in Fig. 4 and the error states are shown in Fig. 5. Obviously the projective synchronization of the network has been realized successfully.

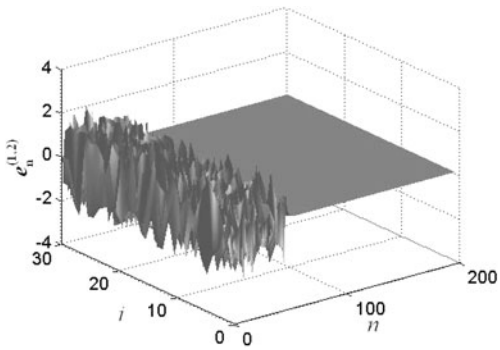


(**a**) node 1

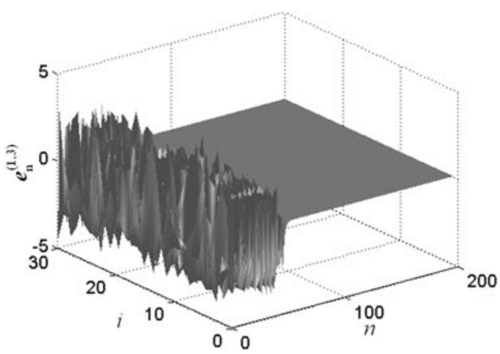
(**b**) node 2

(**c**) node 3

Fig. 4 Space–time evolution history of node 1, 2, 3



(**a**) the error between node 1 and node 2



(**b**) the error between node 1 and node 3

Fig. 5 Space–time evolution history of the error

5 Conclusions

With nonlinear coupling, the projective synchronization of one-dimensional discrete spatiotemporal chaotic systems is realized successfully. Besides, the method is expanded to the projective synchronization of the nodes of the relevant complex network. Numerical simulations show the feasibility and effectiveness of our scheme.

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