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Delay-dependent state estimation for T-S fuzzy delayed Hopfield neural networks

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Abstract This paper proposes a new delay-dependent state estimator for Takagi–Sugeno (T-S) fuzzy delayed Hopfield neural networks. By employing a suitable Lyapunov–Krasovskii functional, a delay-dependent criterion is established to estimate the neuron states through available output measurements such that the dynamics of the estimation error is asymptotically stable. It is shown that the design of the proposed state estimator for such neural networks can be achieved by solving a linear matrix inequality (LMI), which can be easily facilitated by using some standard numerical packages. An illustrative example is given to demonstrate the effectiveness of the proposed state estimator.

Keywords State estimation · Takagi–Sugeno (T-S) fuzzy Hopfield neural networks · Linear matrix inequality (LMI) · Lyapunov–Krasovskii stability theory

1 Introduction

Studying neural networks has been the central focus of intensive research activities during the last decades since neural networks have found wide applications in areas like associative memory, pattern classification, reconstruction of moving images, signal processing, and solving optimization problems to name a few. Among many neural networks, Hopfield neural networks [1] are the most popular. They have been extensively studied and successfully applied in many areas such as combinatorial optimization, signal processing, and pattern recognition [2].

Fuzzy logic theory has shown to be an appealing and an efficient approach to dealing with the analysis and synthesis problems for complex nonlinear systems. Among various kinds of fuzzy methods, Takagi–Sugeno (T-S) fuzzy models provide a successful method to describe certain complex nonlinear systems using some local linear subsystems [3, 4]. These linear subsystems are smoothly blended together through fuzzy membership functions. Recently, T-S fuzzy models are used to describe delayed Hopfield neural networks. The T-S fuzzy models can be used to represent some complex nonlinear systems by having a set of delayed Hopfield neural networks as its consequent parts. With the outstanding approximation ability of the T-S fuzzy models, T-S fuzzy delayed Hopfield neural networks [5–7] are recently recognized as an appealing and efficient tool in approximating complex nonlinear systems. Some stability problems for T-S fuzzy delayed Hopfield neural networks have been investigated in [5–7].

On the other hand, it should be noted that, for the neural networks, the neuron states are not often com-

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pletely available in the network outputs in many applications. Therefore, it becomes important to estimate the neuron states through available measurements in order to make use of the neural networks in practice. Wang et al. firstly derived a delay-independent criterion for state estimator design of delayed neural networks in [8]. The authors in [9] investigated the design of the state estimator of delayed neural networks and proposed a delay-dependent condition such that the error system is asymptotically stable. In [10], Liu et al. further dealt with the state estimation problem for a class of neural networks with discrete and distributed delays. Recently, a state estimator for a class of delayed neural networks with Markovian jumping parameters was proposed in [11]. Can we obtain a state estimator for T-S fuzzy delayed Hopfield neural networks? This paper gives an answer for this question. To the best of our knowledge, for the state estimation of T-S fuzzy delayed Hopfield neural networks, there is no result in the literature so far, which still remains open and challenging.

In this paper, we propose a new delay-dependent state estimator for T-S fuzzy delayed Hopfield neural networks. This state estimator is a new contribution to the topic of neural networks. By constructing a suitable Lyapunov–Krasovskii functional, a delay-dependent condition is developed to estimate the neuron states through available output measurements such that the estimation error system is asymptotically stable. The criterion is formulated in terms of a linear matrix inequality (LMI), which can be checked efficiently by using some standard numerical packages [12, 13].

This paper is organized as follows. In Sect. 2, we formulate the problem. In Sect. 3, an LMI problem for the delay-dependent state estimation of T-S fuzzy delayed Hopfield neural networks is proposed. In Sect. 4, a numerical example is given, and finally, conclusions are presented in Sect. 5.

2 Problem formulation

Consider the following delayed Hopfield neural network:

$$\dot{x}(t) = Ax(t) + W\phi\big(x(t-\tau)\big) + J(t), \tag{1}$$

$$y(t) = Cx(t) + Dx(t-\tau), \tag{2}$$

where $x(t) = [x_1(t)\cdots x_n(t)]^T \in R^n$ is the state vector, $y(t) = [y_1(t)\cdots y_m(t)]^T \in R^m$ is the output vector, $\tau \geq 0$ is the time-delay, $A = \mathrm{diag}\{-a_1,\dots,-a_n\} \in R^{n\times n}$ $(a_k > 0,\ k = 1,\dots,n)$ is the self-feedback matrix, $W \in R^{n\times n}$ is the delayed connection weight matrix, $\phi(x(t)) = [\phi_1(x(t))\cdots\phi_n(x(t))]^T : R^n \to R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_\phi > 0$, $C \in R^{m\times n}$ and $D \in R^{m\times n}$ are known constant matrices, and $J(t) \in R^n$ is an external input vector.

In this paper, we will consider delayed Hopfield neural networks, which are represented by T-S fuzzy models composed of a set of fuzzy implications and each implication is expressed as a linear system model. Based on the T-S fuzzy model concept, a general class of T-S fuzzy delayed Hopfield neural networks is considered here. The model of T-S fuzzy delayed Hopfield neural networks is described as follows:

Fuzzy Rule i:

IF ω_1 is μ_{i1} and $\dots\omega_s$ is μ_{is} **THEN**

$$\dot{x}(t) = A_i x(t) + W_i\phi\big(x(t-\tau)\big) + J_i(t), \tag{3}$$

$$y(t) = C_i x(t) + D_i x(t-\tau), \tag{4}$$

where ω_j $(j = 1,\dots,s)$ is the premise variable, μ_{ij} $(i = 1,\dots,r,\ j = 1,\dots,s)$ is the fuzzy set that is characterized by membership function, r is the number of the IF-THEN rules, and s is the number of the premise variables.

Using a standard fuzzy inference method (using a singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier), the system (3)–(4) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\omega)\big[A_i x(t) + W_i\phi\big(x(t-\tau)\big) + J_i(t)\big], \tag{5}$$

$$y(t) = \sum_{i=1}^{r} h_i(\omega)\big[C_i x(t) + D_i x(t-\tau)\big], \tag{6}$$

where $\omega = [\omega_1,\dots,\omega_s]$, $h_i(\omega) = w_i(\omega)/\sum_{i=1}^{r} w_j(\omega)$, $w_i : R^s \to [0,1]$ $(i = 1,\dots,r)$ is the membership function of the system with respect to the fuzzy rule i. h_i can be regarded as the normalized weight of each IF-THEN rule and it satisfies

$$h_i(\omega) \geq 0, \quad \sum_{i=1}^{r} h_i(\omega) = 1. \tag{7}$$

For the T-S fuzzy delayed Hopfield neural network (3)–(4), we design the following state estimator:

Fuzzy Rule i:

IF ω_1 is μ_{i1} and ... ω_s is μ_{is} **THEN**

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + W_i\phi(\hat{x}(t-\tau)) + J_i(t) + L(y(t) - \hat{y}(t)), \quad (8)$$

$$\hat{y}(t) = C_i\hat{x}(t) + D_i\hat{x}(t-\tau), \quad (9)$$

where $\hat{x}(t) = [\hat{x}_1(t)\cdots\hat{x}_n(t)]^T \in R^n$ is the state vector of the state estimator, $\hat{y}(t) = [\hat{y}_1(t)\cdots\hat{y}_m(t)]^T \in R^m$ is the output vector of the state estimator, and $L \in R^{n\times m}$ is the gain matrix of the state estimator to be designed. Using a standard fuzzy inference method, the state estimator (8)–(9) is inferred as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\omega)\big[A_i\hat{x}(t) + W_i\phi(\hat{x}(t-\tau)) + J_i(t) + L(y(t) - \hat{y}(t))\big], \quad (10)$$

$$\hat{y}(t) = \sum_{i=1}^{r} h_i(\omega)\big[C_i\hat{x}(t) + D_i\hat{x}(t-\tau)\big]. \quad (11)$$

Define the estimation error $e(t) = x(t) - \hat{x}(t)$. Then the estimation error system can be obtained by combining the state estimator (10)–(11) and the T-S fuzzy delayed Hopfield neural network (5)–(6), which is represented as follows:

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\omega)\big\{(A_i - LC_i)e(t) - LD_ie(t-\tau) + W_i\phi(x(t-\tau)) - W_i\phi(\hat{x}(t-\tau))\big\}. \quad (12)$$

The aim of this paper is to choose the gain matrix L so that $\hat{x}(t)$ approaches $x(t)$ asymptotically, that is to say, the estimation error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0. \quad (13)$$

3 Delay-dependent state estimation for T-S fuzzy delayed Hopfield neural networks

In this section, we design a delay-dependent state estimator for T-S fuzzy delayed Hopfield neural networks. The following theorem presents an LMI-based sufficient criterion to obtain the delay-dependent state estimator.

Theorem 1 *For a given* $S = S^T > 0$, *assume that there exist common matrices* $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $U = U^T > 0$, *and* M *such that*

$$\begin{bmatrix} [1,1] & -MD_i & U & 0 & I & PW_i \\ -D_i^TM^T & -R & -U & I & 0 & 0 \\ U & -U & -\frac{1}{\tau}Q & 0 & 0 & 0 \\ 0 & I & 0 & -\frac{1}{L_\phi^2}I & 0 & 0 \\ I & 0 & 0 & 0 & -S^{-1} & 0 \\ W_i^TP & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (14)$$

for $i = 1, \dots, r$, *where*

$$[1,1] = (PA_i - MC_i)^T + (PA_i - MC_i) + \tau Q + R.$$

Then the delay-dependent state estimation for T-S fuzzy delayed Hopfield neural networks is achieved under the following gain matrix:

$$L = P^{-1}M. \quad (15)$$

Proof Consider the following Lyapunov–Krasovskii functional

$$V(t) = e^T(t)Pe(t) + \int_{-\tau}^{0}\int_{t+\beta}^{t} e^T(\alpha)Qe(\alpha)\,d\alpha\,d\beta + \int_{-\tau}^{0} e^T(t+\sigma)Re(t+\sigma)\,d\sigma + \left[\int_{-\tau}^{0} e(t+\sigma)\,d\sigma\right]^T U\left[\int_{-\tau}^{0} e(t+\sigma)\,d\sigma\right]. \quad (16)$$

Its time derivative along the trajectory of (12) is

$$\dot{V}(t) = \dot{e}(t)^TPe(t) + e^T(t)P\dot{e}(t) + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma + e(t)^TRe(t) - e^T(t-\tau)Re(t-\tau) + \big[e(t) - e(t-\tau)\big]^T U\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T U\big[e(t) - e(t-\tau)\big]$$

$$
\begin{aligned}
&= \sum_{i=1}^{r} h_i(\omega)\big\{e^T(t)\big[(A_i - LC_i)^T P \\
&\quad + P(A_i - LC_i)\big]e(t) - e^T(t)PLD_i e(t-\tau) \\
&\quad - e^T(t-\tau)D_i^T L^T P e(t) + e^T(t) \\
&\quad \times PW_i\big(\phi\big(x(t-\tau)\big) - \phi\big(\hat{x}(t-\tau)\big)\big) \\
&\quad + \big(\phi\big(x(t-\tau)\big) - \phi\big(\hat{x}(t-\tau)\big)\big)^T W_i^T P e(t)\big\} \\
&\quad + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma \\
&\quad + e(t)^T Re(t) - e^T(t-\tau)Re(t-\tau) \\
&\quad + \big[e(t) - e(t-\tau)\big]^T U \\
&\quad \times \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T U \\
&\quad \times \big[e(t) - e(t-\tau)\big]. \qquad (17)
\end{aligned}
$$

If we use the inequality $X^T Y + Y^T X \le X^T \Lambda X + Y^T \Lambda^{-1} Y$, which is valid for any matrices $X \in R^{n\times m}$, $Y \in R^{n\times m}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in R^{n\times n}$, we have

$$
\begin{aligned}
&e^T(t)PW_i\big(\phi\big(x(t-\tau)\big) - \phi\big(\hat{x}(t-\tau)\big)\big) + \big(\phi\big(x(t-\tau)\big) \\
&\quad - \phi\big(\hat{x}(t-\tau)\big)\big)^T W_i^T P e(t) \\
&\le \big(\phi\big(x(t-\tau)\big) - \phi\big(\hat{x}(t-\tau)\big)\big)^T \big(\phi\big(x(t-\tau)\big) \\
&\quad - \phi\big(\hat{x}(t-\tau)\big)\big) + e^T(t)PW_i W_i^T P e(t) \\
&\le L_\phi^2\big(x(t-\tau) - \hat{x}(t-\tau)\big)^T \big(x(t-\tau) - \hat{x}(t-\tau)\big) \\
&\quad + e^T(t)PW_i W_i^T P e(t) \\
&= L_\phi^2 e^T(t-\tau)e(t-\tau) + e^T(t)PW_i W_i^T P e(t). \qquad (18)
\end{aligned}
$$

Using (18), we obtain

$$
\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{r} h_i(\omega)\big\{e^T(t)\big[(A_i - LC_i)^T P \\
&\quad + P(A_i - LC_i) + PW_i W_i^T P\big]e(t) \\
&\quad - e^T(t)PLD_i e(t-\tau) \\
&\quad - e^T(t-\tau)D_i^T L^T P e(t) \\
&\quad + L_\phi^2 e^T(t-\tau)e(t-\tau)\big\} \\
&\quad + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma \\
&\quad + e(t)^T Re(t) - e^T(t-\tau)Re(t-\tau) \\
&\quad + \big[e(t) - e(t-\tau)\big]^T U \\
&\quad \times \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T \\
&\quad \times U\big[e(t) - e(t-\tau)\big]. \qquad (19)
\end{aligned}
$$

Using the inequality [14],

$$
\begin{aligned}
&\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] \\
&\quad \le \tau \int_{t-\tau}^{t} e(\sigma)^T Q e(\sigma)\,d\sigma, \qquad (20)
\end{aligned}
$$

we have

$$
\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{r} h_i(\omega)\big\{e^T(t)\big[(A_i - LC_i)^T P \\
&\quad + P(A_i - LC_i) + PW_i W_i^T P\big]e(t) \\
&\quad - e^T(t)PLD_i e(t-\tau) \\
&\quad - e^T(t-\tau)D_i^T L^T P e(t) \\
&\quad + L_\phi^2 e^T(t-\tau)e(t-\tau)\big\} \\
&\quad + \tau e^T(t)Qe(t) - \frac{1}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q \\
&\quad \times \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] + e(t)^T Re(t) \\
&\quad - e^T(t-\tau)Re(t-\tau) \\
&\quad + \big[e(t) - e(t-\tau)\big]^T U\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right] \\
&\quad + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T U\big[e(t) - e(t-\tau)\big] \\
&= \sum_{i=1}^{r} h_i(\omega)\left\{\begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)\,d\sigma \end{bmatrix}^T\right. \\
&\quad \times \begin{bmatrix} (1,1) & -PLD_i & U \\ -D_i^T L^T P & (2,2) & -U \\ U & -U & -\frac{1}{\tau}Q \end{bmatrix} \\
&\quad \times \left.\begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)\,d\sigma \end{bmatrix} - e^T(t)Se(t)\right\}, \qquad (21)
\end{aligned}
$$

where

$$(1,1) = (A_i - LC_i)^T P + P(A_i - LC_i) + PW_iW_i^T P + S + \tau Q + R,$$

$$(2,2) = L_\phi^2 I - R.$$

If the following matrix inequality is satisfied,

$$\begin{bmatrix} (1,1) & -PLD_i & U \\ -D_i^T L^T P & (2,2) & -U \\ U & -U & -\frac{1}{\tau}Q \end{bmatrix} < 0, \quad (22)$$

for $i = 1, \ldots, r$, we have

$$\dot V(t) < \sum_{i=1}^{r} h_i(\omega)\{-e^T(t)Se(t)\} = -e^T(t)Se(t) \le 0. \quad (23)$$

This guarantees

$$\lim_{t\to\infty} e(t) = 0 \quad (24)$$

from Lyapunov–Krasovskii stability theory. From Schur complement, the matrix inequality (22) is equivalent to

$$\begin{bmatrix} \{1,1\} & -PLD_i & U & 0 & I & PW_i \\ -D_i^T L^T P & -R & -U & I & 0 & 0 \\ U & -U & -\frac{1}{\tau}Q & 0 & 0 & 0 \\ 0 & I & 0 & -\frac{1}{L_\phi^2}I & 0 & 0 \\ I & 0 & 0 & 0 & -S^{-1} & 0 \\ W_i^T P & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (25)$$

where

$$\{1,1\} = (A_i - LC_i)^T P + P(A_i - LC_i) + \tau Q + R.$$

If we let $M = PL$, (25) is equivalently changed into the LMI (14). Then the gain matrix of the state estimator is given by $L = P^{-1}M$. This completes the proof. □

Remark 1 The matrices P, Q, R, U, and M satisfying the LMI (14) can be obtained by interior-point algorithms. Let $F(P,Q,R,U,M)$ be the left-hand side of the LMI (14). If we consider the feasibility problem associated with $F(P,Q,R,U,M) < 0$, then one candidate barrier function is the logarithmic function

$$\phi(P,Q,R,U,M) = \begin{cases} \log\det(-F(P,Q,R,U,M)^{-1}), & F(P,Q,R,U,M) < 0, \\ \infty, & \text{otherwise.} \end{cases}$$

Under the assumption that the feasible set $\{(P,Q,R,U,M) | F(P,Q,R,U,M) < 0\}$ is bounded and nonempty, it follows that $\phi(t)$ is strictly convex, and hence it defines a barrier function for the feasibility set$\{(P,Q,R,U,M) | F(P,Q,R,U,M) < 0\}$. Therefore, there exists a unique $(P,Q,R,U,M)_{\text{opt}}$ such that $\phi((P,Q,R,U,M)_{\text{opt}})$ is the global minimum of $\phi(P,Q,R,U,M)$. The unique $(P,Q, R,U,M)_{\text{opt}}$ is usually obtained in a very efficient way from the classical Newton iteration $(P_{i+1}, Q_{i+1}, R_{i+1}, U_{i+1}, M_{i+1}) = (P_i, Q_i, R_i, U_i, M_i) - (\phi''(P_i, Q_i, R_i, U_i, M_i))^{-1}\phi'(P_i, Q_i, R_i, U_i, M_i)$, where $\phi''(P_i, Q_i, R_i, U_i, M_i)$ and $\phi'(P_i, Q_i, R_i, U_i, M_i)$ are the gradient and the Hessian of ϕ, respectively. Then the matrices P and M are obtained from $(P_i, Q_i, R_i, U_i, M_i)_{\text{opt}}$. In this paper, in order to solve $F(P,Q,R,U,M) < 0$, we utilize MATLAB LMI Control Toolbox [13], which implements state-of-the-art interior-point algorithms.

Remark 2 If we use the Newton–Leibniz formula and the following augmented Lyapunov–Krasovskii functional:

$$\begin{aligned} \bar V(t) &= e^T(t)Pe(t) \\ &+ \int_{-\tau}^{0}\int_{t+\beta}^{t} \begin{bmatrix} e(\alpha) \\ \dot e(\alpha) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \times \begin{bmatrix} e(\alpha) \\ \dot e(\alpha) \end{bmatrix} d\alpha\, d\beta \\ &+ \int_{t-\tau}^{t} \begin{bmatrix} e(\sigma) \\ \dot e(\sigma) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} e(\sigma) \\ \dot e(\sigma) \end{bmatrix} d\sigma \\ &+ \left[\int_{-\tau}^{0} e(t+\sigma)\, d\sigma\right]^T U \left[\int_{-\tau}^{0} e(t+\sigma)\, d\sigma\right], \end{aligned}$$

where

$$\begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \ge 0, \qquad \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \ge 0,$$

instead of (16), the potential conservatism of the LMI condition (14) may be reduced. The reduction of the conservatism remains as a future work.

4 Numerical example

Consider the following T-S fuzzy delayed Hopfield neural network:

Fuzzy Rule 1:

IF ω_1 is μ_{11} and $\ldots\omega_s$ is μ_{1s} **THEN**

$$\dot{x}(t) = A_1x(t) + W_1\phi\big(x(t-1)\big) + J_1(t), \tag{26}$$

$$y(t) = C_1x(t) + D_1x(t-1), \tag{27}$$

Fuzzy Rule 2:

IF ω_1 is μ_{21} and $\ldots$ ω_s is μ_{2s} **THEN**

$$\dot{x}(t) = A_2x(t) + W_2\phi\big(x(t-1)\big) + J_2(t), \tag{28}$$

$$y(t) = C_2x(t) + D_2x(t-1), \tag{29}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \qquad \phi\big(x(t)\big) = \begin{bmatrix} \frac{1}{1+e^{-x_1(t)}} \\ \frac{1}{1+e^{-x_2(t)}} \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -3.5 & 0 \\ 0 & -1.5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2.1 & 0 \\ 0 & -2.8 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.1 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 1 & -0.8 \\ 0.4 & 0.5 \end{bmatrix},$$

$$J_1(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}, \qquad J_2(t) = \begin{bmatrix} -\cos(t) \\ \sin(2t) \end{bmatrix},$$

$$C_1 = [\,1 \quad 0\,], \qquad C_2 = [\,0 \quad 1\,],$$

$$D_1 = [\,0.5 \quad 1\,], \qquad D_2 = [\,-1 \quad 0.3\,].$$

The fuzzy membership functions are taken as $h_1(\omega) = \sin^2(3x_1(t))$ and $h_2(\omega) = \cos^2(3x_1(t))$. Solving the LMI (14) by the convex optimization technique of MATLAB software gives

$$P = \begin{bmatrix} 1.2974 & 0.0060 \\ 0.0060 & 5.9058 \end{bmatrix}, \qquad M = \begin{bmatrix} 0.0139 \\ -0.0059 \end{bmatrix}.$$

When the initial conditions are given by

$$x(0) = \begin{bmatrix} -3.2 \\ 1.5 \end{bmatrix}, \qquad \hat{x}(0) = \begin{bmatrix} 2.4 \\ -2.1 \end{bmatrix},$$

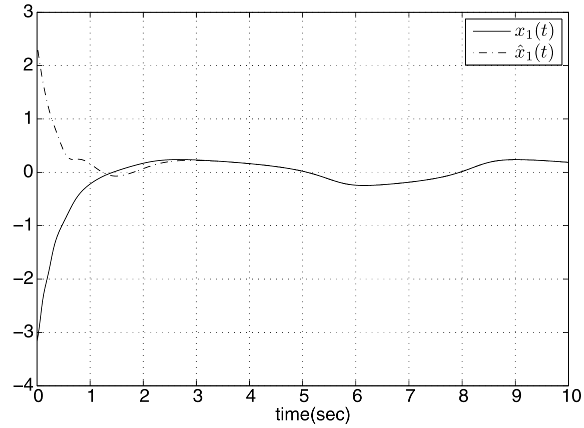

Fig. 1 Responses of the state $x_1(t)$ and its estimation $\hat{x}_1(t)$

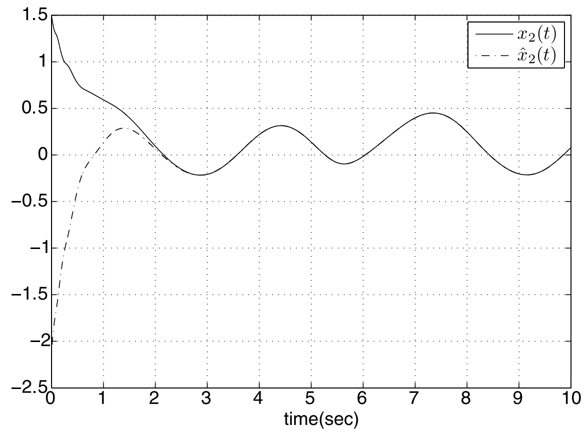

Fig. 2 Responses of the state $x_2(t)$ and its estimation $\hat{x}_2(t)$

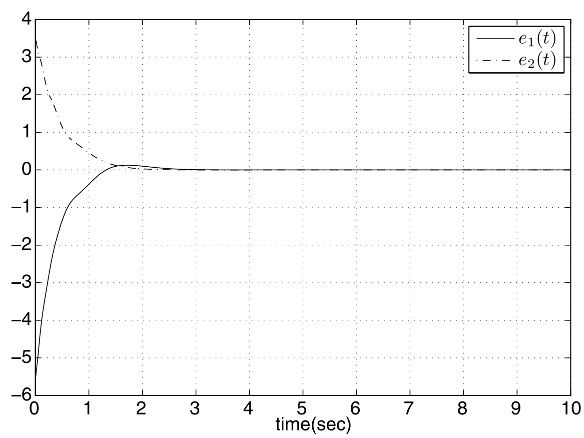

Fig. 3 Responses of the estimation error $e(t)$

the simulation results for the delay-dependent state estimator design are shown in Figs. 1–3. Figures 1 and 2 show the true states $x_1(t)$ and $x_2(t)$ and their estimations $\hat{x}_1(t)$ and $\hat{x}_2(t)$, respectively, and Fig. 3 shows the responses of the estimation error $e(t)$. The simulation results confirm that the states of the T-S fuzzy delayed Hopfield neural network (26)–(29) are tracked very well by the proposed state estimator. The effectiveness and accuracy of the proposed method is demonstrated.

5 Conclusion

For the first time, this paper proposes a delay-dependent state estimator for T-S fuzzy delayed Hopfield neural networks. By constructing an appropriatek Lyapunov–Krasovskii functional, a delay-dependent sufficient condition has been presented such that the estimation error system is asymptotically stable. It is shown that the state estimator gain matrix can be determined by solving the LMI problem. A simulation example is given to show the effectiveness of the proposed state estimator. It is expected that the results proposed in this paper can be extended to discrete-time T-S fuzzy delayed Hopfield neural networks.

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