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Output feedback $\mathcal{H}_\infty$ synchronization for delayed chaotic neural networks

Choon Ki Ahn

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Abstract In this paper, we propose a new output feedback $\mathcal{H}_\infty$ synchronization method for delayed chaotic neural networks with external disturbance. Based on Lyapunov–Krasovskii theory and linear matrix inequality (LMI) approach, the output feedback $\mathcal{H}_\infty$ synchronization controller is presented to not only guarantee stable synchronization, but also reduce the effect of external disturbance to an $\mathcal{H}_\infty$ norm constraint. The proposed controller can be obtained by solving the LMI problem. An illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords $\mathcal{H}_\infty$ synchronization · Delayed chaotic neural networks · Linear matrix inequality (LMI) · Lyapunov–Krasovskii theory · Output feedback control

1 Introduction

Neural networks have been extensively studied over the past two decades for their potential applications in modeling complex dynamics, linear and nonlinear programming, image processing, pattern recognition, associative memory, etc. [1]. Most of previous studies are predominantly concentrated on the stability analysis and periodic oscillations of neural networks [2–5]. Recently, it has been shown that neural networks can exhibit complicated dynamics and even chaotic behavior if the parameters and time delays are appropriately chosen for the neural networks [6–10]. Hence, the synchronization of chaotic neural networks has become an important area of study. In the literature, various synchronization schemes, such as Halanay inequality approach [11], adaptive control [12–14], linear coupling scheme [15], time-delay feedback control [16], impulsive control [17], and nonlinear feedback control [18], have been successfully applied to the synchronization of chaotic neural networks.

In real physical systems, one is faced with model uncertainties and a lack of statistical information on the signals. This had led in recent years to an interest in mini-max control, with the belief that $\mathcal{H}_\infty$ control is more robust and less sensitive to disturbance variances and model uncertainties [19]. In order to reduce the effect of the disturbance, Hou et al. [20] firstly adopted the $\mathcal{H}_\infty$ control concept [19] for chaotic synchronization problem of a class of chaotic systems. Recently, a dynamic controller for the $\mathcal{H}_\infty$ synchronization was proposed in [21]. To the best of our knowledge, however, for the $\mathcal{H}_\infty$ synchronization of delayed chaotic neural networks via output feedback control, there is no result in the literature so far, which still remains open and challenging.

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C.K. Ahn (✉)
Division of Electronics and Control Engineering,
Wonkwang University, Iksan, Korea
e-mail: hironaka@wonkwang.ac.kr

In this paper, a new output feedback controller for the $\mathcal{H}_\infty$ synchronization of delayed chaotic neural networks with external disturbance is proposed. By the proposed control scheme, the closed-loop error system is asymptotically synchronized and the $\mathcal{H}_\infty$ norm from the external disturbance to the synchronization error is reduced to a disturbance attenuation level. Based on the Lyapunov–Krasovskii method [24, 25] and linear matrix inequality (LMI) approach, an existence criterion for the proposed controller is represented in terms of the LMI. The LMI problem can be solved efficiently by using recently developed convex optimization algorithms [22].

This paper is organized as follows. In Sect. 2, we formulate the problem. In Sect. 3, an LMI problem for the $\mathcal{H}_\infty$ synchronization of delayed chaotic neural networks is proposed. In Sect. 4, a numerical example is given, and finally, conclusions are presented in Sect. 5.

2 Problem formulation

A class of delayed chaotic neural networks is described by the following differential equation:

$$\dot{x}(t) = Ax(t) + \bar{A}x(t-\tau) + Bf(x(t)) + \bar{B}g(x(t-\tau)) + J, \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the output vector, $\tau > 0$ is the time-delay, $A \in R^{n\times n}$ is the self-feedback matrix, $\bar{A} \in R^{n\times n}$ is the delayed self-feedback matrix, $B \in R^{n\times n}$ is the connection weight matrix, $\bar{B} \in R^{n\times n}$ is the delayed connection weight matrix, $C \in R^{m\times n}$ is a constant output matrix, and $J \in R^n$ is a constant bias vector. $f(x(t)) \in R^n$ and $g(x(t)) \in R^n$ are activation function vectors satisfying the global Lipschitz conditions with Lipschitz constants $L_f > 0$ and $L_g > 0$:

$$\|f(x_1) - f(x_2)\| \le L_f\|x_1 - x_2\|, \quad \forall x_1, x_2 \in R^n, \tag{3}$$

$$\|g(x_1) - g(x_1)\| \le L_g\|x_1 - x_2\|, \quad \forall x_1, x_2 \in R^n. \tag{4}$$

The system (1)–(2) is considered as a drive system. The synchronization problem of system (1)–(2) is considered by using the drive-response configuration. According to the drive-response concept, the controlled response system is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \bar{A}\hat{x}(t-\tau) + Bf(\hat{x}(t)) + \bar{B}g(\hat{x}(t-\tau)) + J + u(t) + Dd(t), \tag{5}$$

$$\hat{y}(t) = C\hat{x}(t), \tag{6}$$

where $\hat{x}(t) \in R^n$ is the state vector of the response system, $\hat{y}(t) \in R^m$ is the output vector of the response system, $u(t) \in R^n$ is the control input, $d(t) \in R^k$ is the external disturbance, and $D \in R^{n\times k}$ is a known constant matrix. The purpose of this paper is to design the feedback control input $u(t)$ guaranteeing the output feedback $\mathcal{H}_\infty$ synchronization. In order to design the feedback control input $u(t)$, we need information on states of drive and response systems. Thus, the control input $u(t)$ in (5) depends on states of drive and response systems. Define the synchronization error $e(t) = \hat{x}(t) - x(t)$. Then we obtain the synchronization error system

$$\dot{e}(t) = Ae(t) + \bar{A}e(t-\tau) + B(f(\hat{x}(t)) - f(x(t))) + \bar{B}(g(\hat{x}(t-\tau)) - g(x(t-\tau))) + u(t) + Dd(t). \tag{7}$$

Definition 1 (Asymptotical synchronization) The error system (7) is asymptotically synchronized if the synchronization error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0. \tag{8}$$

Definition 2 ($\mathcal{H}_\infty$ synchronization) The error system (7) is $\mathcal{H}_\infty$ synchronized if the synchronization error $e(t)$ satisfies

$$\int_0^\infty e^T(t)Se(t)\,dt < \gamma^2 \int_0^\infty d^T(t)d(t)\,dt, \tag{9}$$

for a given level $\gamma > 0$ under zero initial condition, where S is a positive symmetric matrix. The parameter γ is called the $\mathcal{H}_\infty$ norm bound or the disturbance attenuation level.

Remark 1 The $\mathcal{H}_\infty$ norm [19] is defined as

$$\|T_{ed}\|_\infty = \frac{\sqrt{\int_0^\infty e^T(t)Se(t)\,dt}}{\sqrt{\int_0^\infty d^T(t)d(t)\,dt}},$$

where T_{ed} is a transfer function matrix from $d(t)$ to $e(t)$. For a given level $\gamma > 0$, $\|T_{ed}\|_\infty < \gamma$ can be restated in the equivalent form (9). If we define

$$H(t) = \frac{\int_0^t e^T(\sigma)Se(\sigma)\,d\sigma}{\int_0^t d^T(\sigma)d(\sigma)\,d\sigma}, \tag{10}$$

the relation (9) can be represented by

$$H(\infty) < \gamma^2. \tag{11}$$

In Sect. 4, through the plot of $H(t)$ versus time, the relation (11) is verified.

The purpose of this paper is to design the output feedback controller $u(t)$ guaranteeing the $\mathcal{H}_\infty$ synchronization if there exists the external disturbance $d(t)$. In addition, this controller $u(t)$ will be shown to guarantee the asymptotical synchronization when the external disturbance $d(t)$ disappears.

3 Main results

The LMI problem for achieving the $\mathcal{H}_\infty$ synchronization for delayed chaotic neural networks is presented in the following theorem.

Theorem 1 *For given $\gamma > 0$ and $S = S^T > 0$, if there exist $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $W = W^T > 0$, and M such that*

$$\begin{bmatrix} [1,1] & P\bar{A} & W & PB & P\bar{B} & I & PD & I & 0 \\ \bar{A}^T P & -R & -W & 0 & 0 & 0 & 0 & 0 & I \\ W & -W & -\frac{1}{\tau}Q & 0 & 0 & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -\frac{1}{L_f^2}I & 0 & 0 & 0 \\ D^T P & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & -S^{-1} & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{L_g^2}I \end{bmatrix} < 0, \tag{12}$$

where (14)

$$[1,1] = A^T P + PA + MC + C^T M^T + R + \tau Q,$$

then the $\mathcal{H}_\infty$ synchronization for delayed chaotic neural networks is achieved and the output feedback controller is given by

$$u(t) = P^{-1}M(\hat{y}(t) - y(t)). \tag{13}$$

Proof The closed-loop error system with the control input $u(t) = K(\hat{y}(t) - y(t)) = KC(\hat{x}(t) - x(t))$, where $K \in R^{n\times m}$ is the gain matrix of the controller, can be written as

$$\begin{aligned}\dot{e}(t) &= (A + KC)e(t) + \bar{A}e(t-\tau) \\ &\quad + B(f(\hat{x}(t)) - f(x(t))) \\ &\quad + \bar{B}(g(\hat{x}(t-\tau)) - g(x(t-\tau))) + Dd(t).\end{aligned}$$

Consider the following Lyapunov–Krasovskii functional

$$V(e(t)) = V_1(e(t)) + V_2(e(t)) + V_3(e(t)) + V_4(e(t)), \tag{15}$$

where

$$V_1(e(t)) = e^T(t)Pe(t), \tag{16}$$

$$V_2(e(t)) \int_{-\tau}^{0}\int_{t+\beta}^{t} e^T(\alpha)Qe(\alpha)\,d\alpha\,d\beta, \tag{17}$$

$$V_3(e(t)) = \int_{t-\tau}^{t} e^T(\sigma)Re(\sigma)\,d\sigma, \tag{18}$$

$$V_4(e(t)) = \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T W\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]. \quad (19)$$

The time derivative of $V_1(e(t))$ along the trajectory of (14) is

$$\begin{aligned}
&\dot{V}_1(e(t))\\
&= \dot{e}(t)^T Pe(t) + e^T(t)P\dot{e}(t)\\
&= e^T(t)[A^T P + PA + PKC + C^T K^T P]e(t)\\
&\quad + e^T(t)P\bar{A}e(t-\tau) + e^T(t-\tau)\bar{A}^T Pe(t)\\
&\quad + e^T(t)PB(f(\hat{x}(t)) - f(x(t))) + (f(\hat{x}(t))\\
&\quad - f(x(t)))^T B^T Pe(t) + e^T(t)P\bar{B}(g(\hat{x}(t-\tau))\\
&\quad - g(x(t-\tau))) + (g(\hat{x}(t-\tau))\\
&\quad - g(x(t-\tau)))^T \bar{B}^T Pe(t)\\
&\quad + e^T(t)PDd(t) + d^T(t)D^T Pe(t).
\end{aligned}$$

If we use the inequality $X^T Y + Y^T X \le X^T \Lambda X + Y^T \Lambda^{-1} Y$, which is valid for any matrices $X \in R^{n\times m}$, $Y \in R^{n\times m}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in R^{n\times n}$, we have

$$\begin{aligned}
&e^T(t)PB(f(\hat{x}(t)) - f(x(t))) + (f(\hat{x}(t)) - f(x(t)))^T\\
&\quad \times B^T Pe(t)\\
&\le (f(\hat{x}(t)) - f(x(t)))^T (f(\hat{x}(t)) - f(x(t)))\\
&\quad + e^T(t)PB \times B^T Pe(t)\\
&\le L_f^2(\hat{x}(t) - x(t))^T(\hat{x}(t) - x(t))\\
&\quad + e^T(t)PBB^T Pe(t)\\
&= e^T(t)(L_f^2 I + PBB^T P)e(t), \qquad (20)
\end{aligned}$$

$$\begin{aligned}
&e^T(t)P\bar{B}(g(\hat{x}(t-\tau)) - g(x(t-\tau))) + (g(\hat{x}(t-\tau))\\
&\quad - g(x(t-\tau)))^T \bar{B}^T Pe(t)\\
&\le (g(\hat{x}(t-\tau)) - g(x(t-\tau)))^T (g(\hat{x}(t-\tau))\\
&\quad - g(x(t-\tau))) + e^T(t)P\bar{B}\bar{B}^T Pe(t)\\
&\le L_g^2(\hat{x}(t-\tau) - x(t-\tau))^T(\hat{x}(t-\tau) - x(t-\tau))\\
&\quad + e^T(t)P\bar{B}\bar{B}^T Pe(t)\\
&= L_g^2 e^T(t-\tau)e(t-\tau) + e^T(t)P\bar{B}\bar{B}^T Pe(t), \quad (21)
\end{aligned}$$

and

$$\begin{aligned}
&e(t)^T PDd(t) + d^T(t)D^T Pe(t)\\
&\le \gamma^2 d^T(t)d(t) + \frac{1}{\gamma^2}e(t)^T PDD^T Pe(t). \qquad (22)
\end{aligned}$$

Using (20), (21), and (22), we obtain

$$\begin{aligned}
&\dot{V}_1(e(t))\\
&\le e^T(t)\Big[A^T P + PA + PKC + C^T K^T P\\
&\quad + L_f^2 I + PBB^T P + P\bar{B}\bar{B}^T P + \frac{1}{\gamma^2}PDD^T P\Big]\\
&\quad \times e(t) + e^T(t)P\bar{A}e(t-\tau) + e^T(t-\tau)\bar{A}^T Pe(t)\\
&\quad + L_g^2 e^T(t-\tau)e(t-\tau) + \gamma^2 d^T(t)d(t).
\end{aligned}$$

The time derivative of $V_2(e(t))$ is

$$\dot{V}_2(e(t)) = \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)\,d\sigma. \quad (23)$$

Using the inequality [23]

$$\begin{aligned}
&\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]\\
&\le \tau \int_{t-\tau}^{t} e(\sigma)^T Qe(\sigma)\,d\sigma, \qquad (24)
\end{aligned}$$

we have

$$\begin{aligned}
\dot{V}_2(e(t)) &\le \tau e^T(t)Qe(t) - \frac{1}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T Q\\
&\quad \times \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]. \qquad (25)
\end{aligned}$$

The time derivative of $V_3(e(t))$ is written as

$$\dot{V}_3(e(t)) = e(t)^T Re(t) - e^T(t-\tau)Re(t-\tau). \quad (26)$$

Since $\dot{V}_4(e(t))$ yields the relation

$$\begin{aligned}
\dot{V}_4(e(t)) &= [e(t) - e(t-\tau)]^T W\left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]\\
&\quad + \left[\int_{t-\tau}^{t} e(\sigma)\,d\sigma\right]^T W[e(t) - e(t-\tau)],
\end{aligned} \quad (27)$$

we have the derivative of $V(e(t))$ as

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{V}_1(e(t)) + \dot{V}_2(e(t)) + \dot{V}_3(e(t)) + \dot{V}_4(e(t))\\
&\le \begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)d\sigma \end{bmatrix}^T
\end{aligned}$$

$$\times \begin{bmatrix} (1,1) & P\bar{A} & W \\ \bar{A}^T P & L_g^2 I - R & -W \\ W & -W & -\frac{1}{\tau}Q \end{bmatrix}$$

$$\times \begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)d\sigma \end{bmatrix} - e^T(t)Se(t)$$

$$+\gamma^2 d^T(t)d(t), \tag{28}$$

where

$$(1,1) = A^T P + PA + PKC + C^T K^T P + L_f^2 I$$
$$+ PBB^T P + P\bar{B}\bar{B}^T P + \frac{1}{\gamma^2} PDD^T P$$
$$+ \tau Q + R + S. \tag{29}$$

If the following matrix inequality is satisfied

$$\begin{bmatrix} (1,1) & P\bar{A} & W \\ \bar{A}^T P & L_g^2 I - R & -W \\ W & -W & -\frac{1}{\tau}Q \end{bmatrix} < 0, \tag{30}$$

we have

$$\dot{V}(e(t)) < -e^T(t)Se(t) + \gamma^2 d^T(t)d(t). \tag{31}$$

Integrating both sizes of (31) from 0 to ∞ gives

$$V(e(\infty)) - V(e(0)) < -\int_0^\infty e^T(t)Se(t)\,dt$$
$$+\gamma^2 \int_0^\infty d^T(t)d(t)\,dt.$$

Since $V(e(\infty)) \geq 0$ and $V(e(0)) = 0$, we have the relation (9). From Schur complement, the matrix inequality (30) is equivalent to

$$\begin{bmatrix} \{1,1\} & P\bar{A} & W & PB & P\bar{B} & I & PD & I & 0 \\ \bar{A}^T P & -R & -W & 0 & 0 & 0 & 0 & 0 & I \\ W & -W & -\frac{1}{\tau}Q & 0 & 0 & 0 & 0 & 0 & 0 \\ B^T P & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \bar{B}^T P & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -\frac{1}{L_f^2}I & 0 & 0 & 0 \\ D^T P & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & -S^{-1} & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{L_g^2}I \end{bmatrix} < 0, \tag{32}$$

where

$$\{1,1\} = A^T P + PA + PKC + C^T K^T P + R + \tau Q.$$

If we let $M = PK$, (32) is equivalently changed into the LMI (12). Then the gain matrix of the control input $u(t)$ is given by $K = P^{-1}M$. This completes the proof. □

Corollary 1 *Without the external disturbance, if we use the control input $u(t)$ proposed in Theorem 1, the asymptotical synchronization is obtained.*

Proof When $d(t) = 0$, we obtain

$$\dot{V}(e(t)) < -e^T(t)Se(t) \leqq 0 \tag{33}$$

from (31). This guarantees

$$\lim_{t\to\infty} e(t) = 0 \tag{34}$$

from Lyapunov–Krasovskii theory. This completes the proof. □

Based on Theorem 1, the optimal $\mathcal{H}_\infty$ norm bound for the $\mathcal{H}_\infty$ synchronization is obtained.

Corollary 2 *For a given $S > 0$, the optimal $\mathcal{H}_\infty$ norm bound γ is obtained by solving the following semidefinite programming problem:*

$$\min_{\gamma>0} \gamma^2 \tag{35}$$

subject to the LMI (12), $P > 0$, $Q > 0$, $R > 0$, *and* $W > 0$.

Remark 2 The LMI problem given in Theorem 1 is to determine whether the solution exists or not. It is called the feasibility problem. The LMI problem can be solved efficiently by using recently developed convex optimization algorithms [22]. In this paper, in order to solve the LMI problem, we utilize MATLAB LMI Control Toolbox [26], which implements state-of-the-art interior-point algorithms.

Remark 3 Because the LMI problem in Theorem 1 is the feasibility problem, we may find several solutions. However, we can find a unique optimal solution to the LMI problem in Corollary 2 because this problem is the convex optimization problem in terms of the LMI.

4 Numerical example

Consider the following delayed chaotic Hopfield neural network [8]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & -0.1 \\ -5 & 1.5 \end{bmatrix} \times \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix} + \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -1 \end{bmatrix} \times \begin{bmatrix} \tanh(x_1(t-1)) \\ \tanh(x_2(t-1)) \end{bmatrix}, \quad (36)$$

where $x_i(t)$ $(i = 1, 2)$ is the state variable of the neural network (36). For the numerical simulation, we use the following parameters:

$$C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1.4 & 1 \\ -1.1 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}. \quad (37)$$

For the design objective (9), let the $\mathcal{H}_\infty$ performance be specified by $\gamma = 0.2$. Applying Theorem 1 to the neural network (36) yields

$$P = \begin{bmatrix} 0.9255 & 0.1525 \\ 0.1525 & 0.7842 \end{bmatrix}, \quad M = \begin{bmatrix} -118.0857 & -126.6248 \\ 9.1317 & -108.6678 \end{bmatrix}.$$

Figure 1 shows the plot of $H(t)$ versus time when $d(t) = [10\sin(5t) \quad 10\cos(5t)]^T$. Figure 1 verifies $H(\infty) < \gamma^2 = 0.04$. This means that the $\mathcal{H}_\infty$ norm from the external disturbance $d(t)$ to the synchronization error $e(t)$ is reduced within the $\mathcal{H}_\infty$ norm bound γ. Let $d(t) = [d_1(t) \;\; d_2(t)]^T$. Phase-plane trajectories for drive and response systems are shown in Figs. 2 and 3, respectively, when the initial conditions are

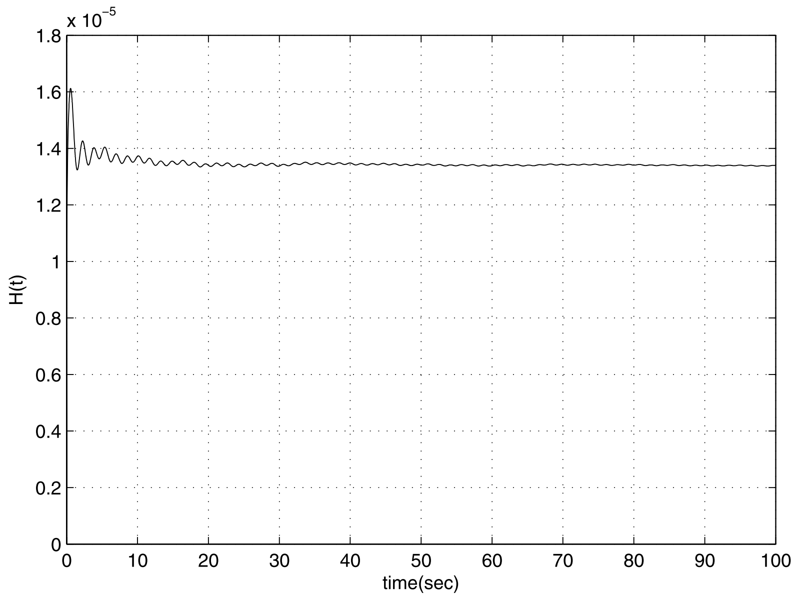


Fig. 1 The plot of $H(t)$ versus time

Fig. 2 The chaotic behavior of the drive system

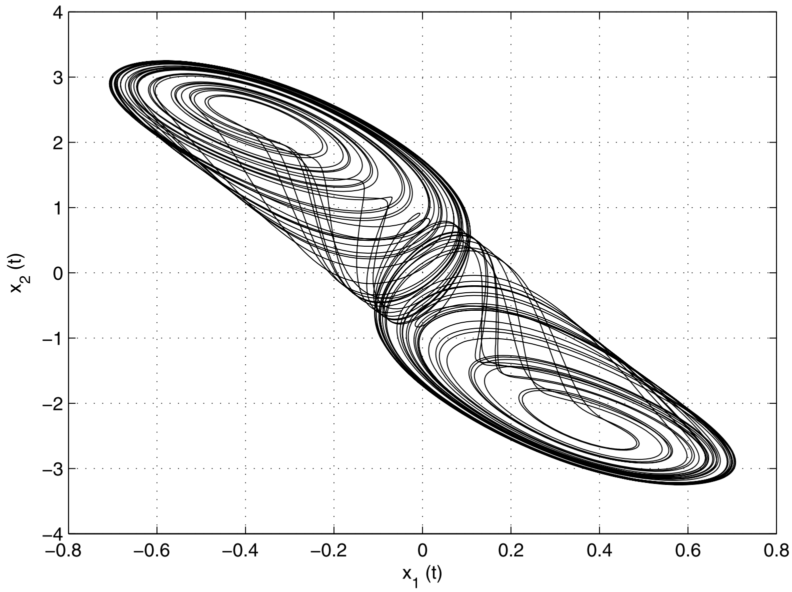
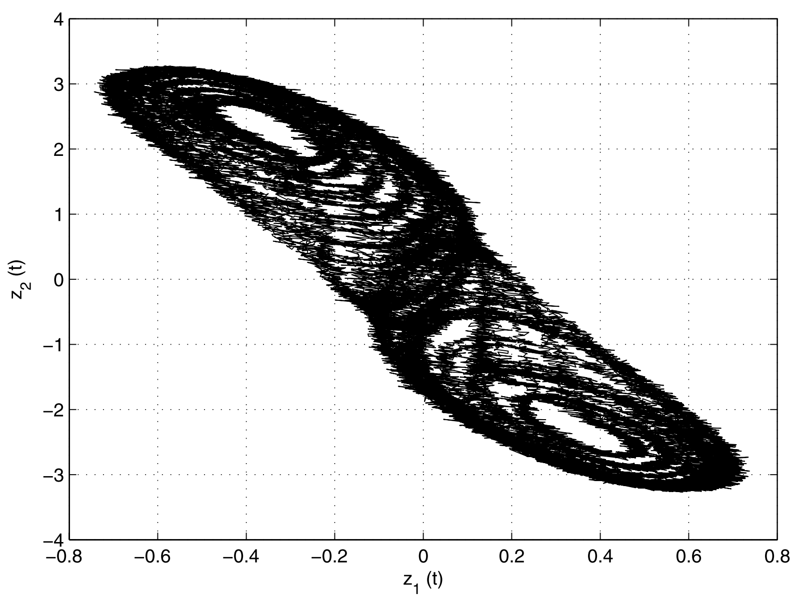


Fig. 3 The chaotic behavior of the response system



given by

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2.4 \\ 1.6 \end{bmatrix}, \quad \begin{bmatrix} \hat{x}_1(0) \\ \hat{x}_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -3.8 \end{bmatrix}, \qquad (38)$$

and the external disturbance is given by $d_i(t) = w(t)$ $(i = 1, 2)$, where $w(t)$ means a Gaussian noise with mean 0 and variance 1. Figure 4 shows state trajectories for drive and response systems when the external disturbance $d_i(t)$ $(i = 1, 2)$ is given by

$$d_i(t) = \begin{cases} w(t), & 0 \le t \le 50, \\ 0, & \text{otherwise.} \end{cases}$$

From Fig. 4, it can be seen that drive and response systems are indeed achieving chaos synchronization. Fig-

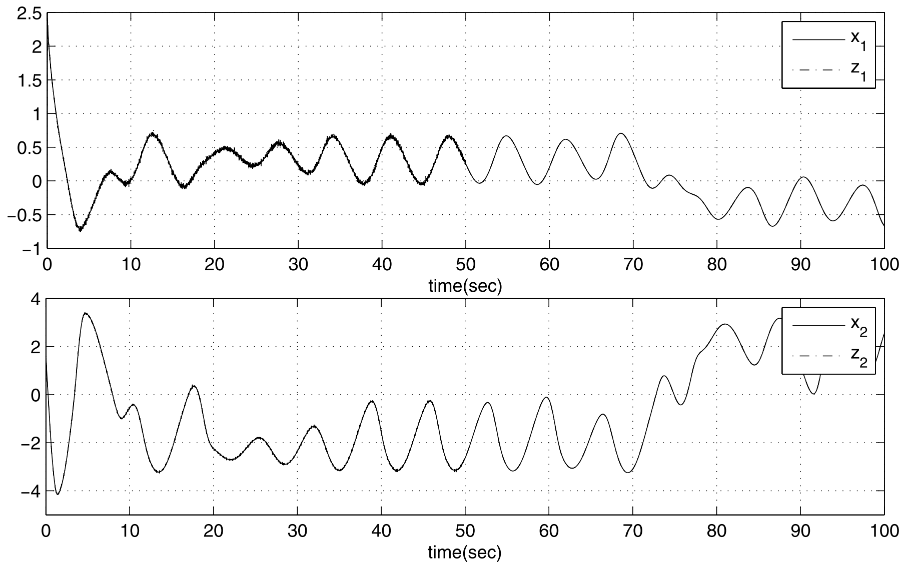


Fig. 4 State trajectories

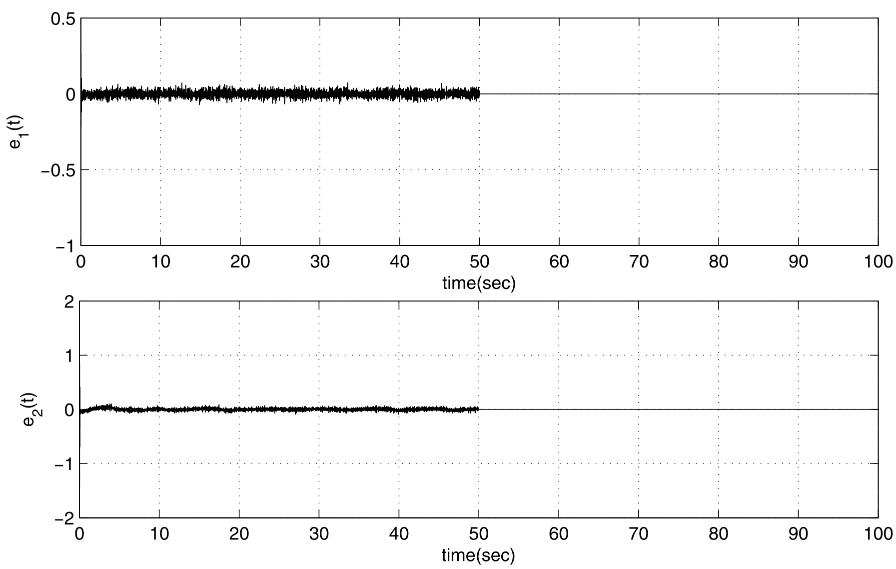


Fig. 5 Synchronization errors

ure 5 shows the proposed $\mathcal{H}_\infty$ synchronization method reduces the effect of the external disturbance $d(t)$ on the synchronization error $e(t)$. In addition, it is shown that the synchronization error $e(t)$ goes to zero after the external disturbance $d(t)$ disappears.

5 Conclusion

In this paper, a new output feedback $\mathcal{H}_\infty$ synchronization scheme for delayed chaotic neural networks with external disturbance is proposed. Based on Lyapunov–Krasovskii theory and LMI approach, the proposed method guarantees the asymptotical synchronization and reduces the $\mathcal{H}_\infty$ norm from the external disturbance to the synchronization error within a disturbance attenuation level. A simulation example is given to show the effectiveness of the proposed method. It is expected that the proposed scheme can be extended to studying $\mathcal{H}_\infty$ synchronization problems for chaotic neural networks with time-varying and distributed delays.

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