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ORIGINAL PAPER

Chaos control of a nonlinear oscillator with shape memory alloy using an optimal linear control: Part I: Ideal energy source

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Abstract In this paper, an optimal linear control is applied to control a chaotic oscillator with shape memory alloy (SMA). Asymptotic stability of the closed-loop nonlinear system is guaranteed by means of a Lyapunov function, which can clearly be seen to be the solution of the Hamilton–Jacobi–Bellman equation, thus guaranteeing both stability and optimality. This work is presented in two parts. Part I considers the so-called ideal problem. In the ideal problem, the excitation source is assumed to be an ideal harmonic excitation.

Keywords Chaos control · Shape memory alloy · Nonlinear dynamic · Linear feedback control

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1 Introduction

Shape Memory Alloys (SMA) consist of a group of metallic materials that demonstrate the ability to return to some previously defined shape when subjected to the appropriate thermal procedure. The shape memory effect occurs due to a temperature and stress dependent shift in the materials crystalline structure between two different phases called martensite and austenite. Martensite, the low temperature phase, is relatively soft whereas austenite, the high temperature phase, is relatively hard. The change that occurs within SMAs crystalline structure is not a thermodynamically reversible process and results in temperature hysteresis. SMAs have been used in a variety of actuation applications. The dynamical response of the shape memory systems is introduced in different references [1–8]. Recently, some experimental analyses confirm the presence of chaos in shape memory systems [9].

Chaotic behavior is an interesting nonlinear phenomenon, which has been intensively studied during the last three decades. Chaotic behavior is commonly detected in a wide variety of physical systems, such as electrical, mechanical, and thermal systems [10]. A fundamental characteristic of chaotic systems is its unpredictability. Controlling these complex chaotic dynamics for engineering applications has emerged as a new and attractive field and has developed many profound theories and methodologies.

In 1990, an original scheme of chaos control was put forward by [11]. The control procedure is known nowadays as the OGY method and has had a great impact on nonlinear science. The OGY method consists on stabilizing a desired unstable periodic orbit embedded in a chaotic attractor by using only a tiny perturbation on an available control parameter. This is in marked contrast with usual control methods, such as those used for periodic motion, for which tiny perturbations cause only small-size effects.

Another interesting chaos control strategy was proposed by [12]. The Pyragas method also considers the dynamical properties of a chaotic attractor to stabilize unstable periodic orbits. In this case, the method implementation requires a delayed feedback signal.

The feedback control technique has been used to suppress chaos in several nonlinear dynamical systems. To solve a general case of the optimal control design problem when a desired trajectory is a periodic or nonperiodic orbit, the linear feedback control techniques have been used by various authors, which considered the Duffing [13, 14], Rössler [13, 14], and other systems. In [15], linear feedback controllers is applied to control the chaotic Rössler system and to achieve synchronization between the hyperchaotic Rössler systems.

While design, modeling, and dynamics of SMA actuators have been studied extensively, very little work has been done in the area of control. Several researchers have used active control techniques utilizing SMA actuators. Different control strategies have been devised. Hashimoto et al. [16] applied a *PD* control scheme to the SMA wires used as actuators of a biped walking robot. Ikuta et al. [17] used active *PID* control on a segmented active endoscope made with SMA springs. Troisfontaine et al. [18] applied *PI* control on SMA actuators with an additional thermal sensor. Madill and Wang [19] used a very simple proportional control to verify the SMA system model they adapted and discuss the system stability. The control gains are tuned either online or through simulations with trial and error method. The drawback of linear *P*, *PI*, or *PID* control is that the controller may perform well in the range where the control gains are tuned, but deteriorates dramatically once outside the range.

In this paper, we proposed a linear feedback control design [13, 15] applied to a SMA oscillator system with an ideal source of power in order to stabilize the oscillations for different values of temperatures. Further, we will show that the present method can control the chaotic state of the SMA oscillator system to a limit cycle.

2 SMA constitutive modeling and problem formulation

To describe the behavior of the oscillator with shape memory, we adopt in the modeling of the problem the constitutive model proposed by [20]. This model is based on Devonshire theory and it defines a free energy of Helmholtz (ψ) in the polynomial form and it is capable to describe the shape memory and pseudoelasticity effects. The polynomial model is known more to deal with one-dimensional cases and it does not consider an explicit potential of dissipation, and no internal variable is considered. On this form, the free energy depends only on the observable state variables (temperature (T) and strain (ε)), that is, $\psi = \psi(\varepsilon, T)$, [21].

The free energy is defined in such way that, for high temperatures ($T > T_A$), the energy possesses only one point of minimum corresponding to the null strain representing the stability of the austenite phase (A); for intermediate temperatures ($T_M < T < T_A$) it presents three points of minimum corresponding to the phases austenitic (A), and two other martensitic phase (M^+ and M^-), which are induced by positive and negative stress fields, respectively; to low temperature ($T < T_M$) there are two points of minimum representing the two variants of martensite (M^+ and M^-), corresponding the null strain.

Therefore, the restrictions above are given by the following equation polynomial [21];

$$\rho\psi(\varepsilon, T) = \frac{1}{2}q(T - T_M)\varepsilon^2 - \frac{1}{4}b\varepsilon^4 + \frac{1}{6}e\varepsilon^6, \tag{1}$$

where q and b are constants of the material, T_A correspond to the temperature where the austenite phase is stable, T_M correspond to the temperature where the martensitic phase is stable and ρ is the SMA density, and the free energy has only one minimum at zero strain,

$$T_A = T_M + \frac{b^2}{4qe} \tag{2}$$

and the constant e may be expressed in terms of other constants of the material. Thus, the stress-strains relation is given by [21],

$$\sigma = q(T - T_M)\varepsilon - b\varepsilon^3 + \frac{b^2}{4q(T_A - T_M)}\varepsilon^5. \tag{3}$$

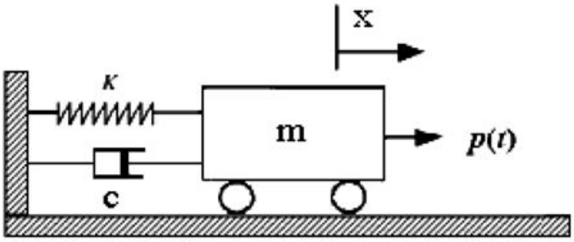

Fig. 1 Model of a SMA oscillator with ideal excitation

According to [22] the Falk's polynomial model represents in a qualitatively coherent way both martensite detwinning process and pseudoelasticity, although it does not consider twinned martensite (M). In other words, there is no stable phase for $T < T_M$ in a stress-free state, but the authors believe that this analysis is useful to the understanding of the nonlinear dynamics of shape memory systems.

The proposed model captures all of the essential features of the studied phenomenon; we consider the one degree of freedom oscillator, which consists of a mass m, connected to a rigid support through of a viscous damping with coefficient c and a shape memory element, where a periodic external force $p(t) = F\cos(\omega t)$ is applied to the system, as shown in Fig. 1.

Thus, the equation of motion that governs the vibrating system can be written as

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + K(x,T) = F\cos(\omega t). \tag{4}$$

On the other hand, the behavior of the element with shape memory can be described through the polynomial constitutive model. Therefore, the restoring force of the spring is given by

$$K = K(x,T) = \bar{q}(T - T_M)x - \bar{b}x^3 + \bar{e}x^5, \tag{5}$$

where

$$\bar{q} = \frac{qA_r}{L}; \qquad \bar{b} = \frac{bA_r}{L^3}; \qquad \bar{e} = \frac{eA_r}{L^5} \tag{6}$$

it follows that x represents the variable relative to the displacement of the element with shape memory, L is the length and A_r is the area of this element.

Then the equation of motion for the oscillator is given by:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + \bar{q}(T - T_M)x - \bar{b}x^3 + \bar{e}x^5 = F\cos(\omega t). \tag{7}$$

Next, we introduce the following dimensionless variables:

$$u = \frac{x}{L} \quad \text{and} \quad \tau = \omega_0 t. \tag{8}$$

Equation (7) can be normalized as

$$\ddot{u} + 2\mu\dot{u} + (\theta - 1)u - \alpha u^3 + \gamma u^5 = \delta\cos(\phi\tau), \tag{9}$$

where the dot represents the differentiation relatively to τ and the dimensionless variables are given by

$$\gamma = \frac{eA_r}{mL\omega_0^2}; \qquad \alpha = \frac{bA_r}{mL\omega_0^2}; \qquad \mu = \frac{c}{2m\omega_0};$$
$$\phi = \frac{\omega}{\omega_0}; \qquad \delta = \frac{F}{mL\omega_0^2}; \qquad \theta = \frac{T}{T_M}; \tag{10}$$
$$\omega_0^2 = \frac{qA_rT_M}{mL}.$$

3 Linear design for nonlinear system

We consider the nonlinear controlled system

$$\dot{v} = H(t)v + g(v) + Z, \tag{11}$$

where $v \in R^n$ is a state vector, $H(t) \in R^{n\times n}$ is a bounded matrix, $Z \in R^n$ is a control vector, and $g(v)$ is a vector whose elements are continuous functions.

We remark that the choice of $H(t)$ is not unique and this influences the performance of the resultant controller.

In several problems of engineering, physics, economy, ecology, etc., the objective is to choose a control law Z that moves the system of the disturbed regime to a desired one, either an equilibrium fixed point or a periodic or nonperiodic orbit.

Let us consider a vector function $\tilde{v}$ that characterizes the desired trajectory. The control vector consists of two parts

$$Z = \tilde{z} + z_f, \tag{12}$$

where $\tilde{z}$ is the feedforward that can be written in the following form:

$$\tilde{z} = \dot{\tilde{v}} - H\tilde{v} - g(\tilde{v}) \tag{13}$$

and the control z_f is a linear control feedback and it has the following form:

$$z_f = Bz \tag{14}$$

with $B \in R^{n\times n}$ is a constant matrix.

Defining

$$y = v - \tilde{v} \tag{15}$$

as a variation of the trajectory of the system (11) of the trajectory desired and admitting (12)–(15), arrive at the variation equation

$$\dot{y} = Hy + g(v) - g(\tilde{v}) + Bz. \tag{16}$$

The nonlinear part of the system (16) can be written as

$$g(v) - g(\tilde{v}) = G(v, \tilde{v})(v - \tilde{v}), \tag{17}$$

where $G(v, \tilde{v})$ is a limited matrix, which elements depend on v and $\tilde{v}$. If we assume (17), the dynamical system (16) has the following form:

$$\dot{y} = H(t)y + G(v, \tilde{v})y + Bz. \tag{18}$$

Then we will use the theorem done by Rafikov and Balthazar [13, 15]. If there exist matrixes $Q(t)$ and $R(t)$, positive definite, being $Q(t)$ symmetric, such that the matrix

$$\tilde{Q}(t) = Q(t) - G^T(v, \tilde{v})P(t) - P(t)G(v, \tilde{v}) \tag{19}$$

is positive definite for the limited matrix G, then the linear feedback control

$$z = -R^{-1}B^T P(t)y \tag{20}$$

is optimal, in order to transfer the nonlinear system (18) from any initial to final state

$$y(t_f) = 0 \tag{21}$$

minimizing the functional

$$J = \int_0^\infty \left(y^T \tilde{Q} y + v^T R v\right) dt, \tag{22}$$

where the symmetric matrix $P(t)$ is calculated from the algebraic nonlinear Riccati equation:

$$\dot{P} + PH + H^T P - PBR^{-1}B^T P + Q = 0. \tag{23}$$

Thus, satisfying the final condition

$$P(t_f) = 0. \tag{24}$$

According to the dynamic programming rules [23], one knows that if the minimum of functional (22) exists and if V is a smooth function of the initial conditions, then it satisfies the Hamilton–Jacobi–Bellman equation:

$$\min_v \left(\frac{dV}{dt} + y^T \tilde{Q} y + v^T R v \right) = 0. \tag{25}$$

Considering a Lyapunov function

$$V = y^T P(t) y, \tag{26}$$

where $P(t)$ is a symmetric positive definite matrix and it satisfies the Riccati differential equation (23).

Note that for positive definite matrices $\tilde{Q}$ and R, the controlled system (18) is asymptotically stable because there exists a positive definite Lyapunov function (26) which derivative is given by $\dot{V} = -y^T \tilde{Q} y - v^T R v$ that is evaluated in optimal trajectories of system (18), and it is negative definite. In addition, there exists a neighborhood of the origin such that the solution of the controlled system is locally asymptotically stable $J_{\min} = y_0^T P(0) y_0$ and the controllable system is globally asymptotically stable.

4 Linear design for SMA oscillator

In this section, we will apply the proposed method [13, 15] to reduce the instability effects of chaotic motions of the SMA oscillator leading to a more ordered system evolution. The SMA oscillator with the control law Z is described by the following differential equation:

$$\ddot{u} + 2\mu\dot{u} + (\theta - 1)u - \alpha u^3 + \gamma u^5 = \delta \cos(\phi\tau) + Z. \tag{27}$$

Let the desired trajectory be a function $\tilde{u}$. Then the desired regimen is described by following equation:

$$\ddot{\tilde{u}} = -2\mu\dot{\tilde{u}} - (\theta - 1)\tilde{u} + \alpha\tilde{u}^3 - \gamma\tilde{u}^5 + \delta\cos(\phi\tau) + \tilde{z}, \tag{28}$$

where $\tilde{z}$ is a control function which maintains the SMA oscillator in the desired trajectory. If the function $\tilde{u}$ is a solution of (27) without the control term, then $\tilde{z}=0$.

Subtracting (28) from (27) and defining

$$y=\begin{bmatrix} u-\tilde{u} \\ \dot{u}-\dot{\tilde{u}} \end{bmatrix} \tag{29}$$

we will obtain the following system

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= -2\mu y_2-(\theta-1)y_1-\alpha\tilde{u}^3+\alpha(y_1+\tilde{u})^3 \\ &\quad +\gamma\tilde{u}^5-\gamma(y_1+\tilde{u})^5+z, \end{aligned} \tag{30}$$

where $z=Z-\tilde{z}$ is feedback control.

So, (18) in this case has the following form:

$$\begin{aligned} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -(\theta-1) & -2\mu \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ \alpha(y_1+\tilde{u})^2+\alpha(y_1+\tilde{u})\tilde{u}+\alpha\tilde{u}^2-\gamma(y_1+\tilde{u})^4-\gamma(y_1+\tilde{u})^3\tilde{u}-4\gamma\tilde{u}^4-\gamma\tilde{u}^2y_1^2-3\gamma y_1\tilde{u}^3 & 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z. \end{aligned} \tag{31}$$

Using the method of multiples scales [24], we obtain an approximate analytical solution to (9) in the form [25, 26]

$$\begin{aligned} u &= a\cos(\varpi\tau-\zeta)-\hat{\varepsilon}\frac{\alpha}{32(\theta-1)}a^3\cos(3\varpi\tau-3\zeta) \\ &\quad +0(\hat{\varepsilon}^2), \end{aligned} \tag{32}$$

where $\hat{\varepsilon}$ is a small dimensionless parameter of the order of the amplitude of the solution and the amplitude a and phase ζ are governed by

$$\begin{cases} a' = -a\mu+\frac{\delta}{2\sqrt{\theta-1}}\operatorname{sen}\zeta, \\ a\zeta' = a\hat{\sigma}-\frac{10a^5\gamma}{32\sqrt{\theta-1}}+\frac{15a^5\alpha^2}{256(\sqrt{\theta-1})^3}+\frac{\delta}{2\sqrt{\theta-1}}\cos\zeta, \end{cases} \tag{33}$$

where $\hat{\sigma}$ is a detuning parameter.

When $\hat{\varepsilon}=0.001, \delta=1, \hat{\sigma}=0.1, \mu=0.1$, and $\theta=2$, the desired trajectory in the steady state is an periodic orbit, given by

$$\begin{aligned} \tilde{u} &= 0.1701\cos(\tau-0.03402) \\ &\quad -0.00002\cos(3\tau-0.10206). \end{aligned} \tag{34}$$

5 Numerical results

The numerical simulation results presented here were obtained using the Matlab-Simulink®. In all simulations, we have used the material properties presented in Table 1, and a SMA oscillator with $\hat{\varepsilon}=0.001$, $\delta=1$, $\hat{\sigma}=0.1$, and $\mu=0.1$ is used. Assuming also that the SMA element has the following values: $A_r=1.96\times10^{-5}$ m^2, $L=50\times10^{-3}$ m, and unitary mass.

As the shape memory alloys presents different properties depending on the temperature, in this article, we deal only with the study on the pseudoelastic behavior, considering a higher temperature, where austenitic phase is stable in the alloy ($\theta=2$), and consider a temperature where the martensitic phase is stable ($\theta=0.7$). For this parameter, the system without control possesses a chaotic attractor.

5.1 Analysis of the martensitic phase

The SMA material exhibits a large residual strain after the loading and unloading. This strain can be fully recovered upon heating the material.

At first, a constant temperature ($T<T_M$) is considered, where the martensitic phase is stable, therefore, here is adopted $\theta=0.7$

The chaotic behavior of the SMA oscillator is illustrated in Fig. 2. A valuable technique for the identifi-

Table 1 Material constants for a Cu-Zn-Al-Ni alloy [27]

q (MPa/K)	b (MPa)	e (MPa)	T_M (K)	T_A (K)
523.29	1.868×10^7	2.188×10^9	288	364.3

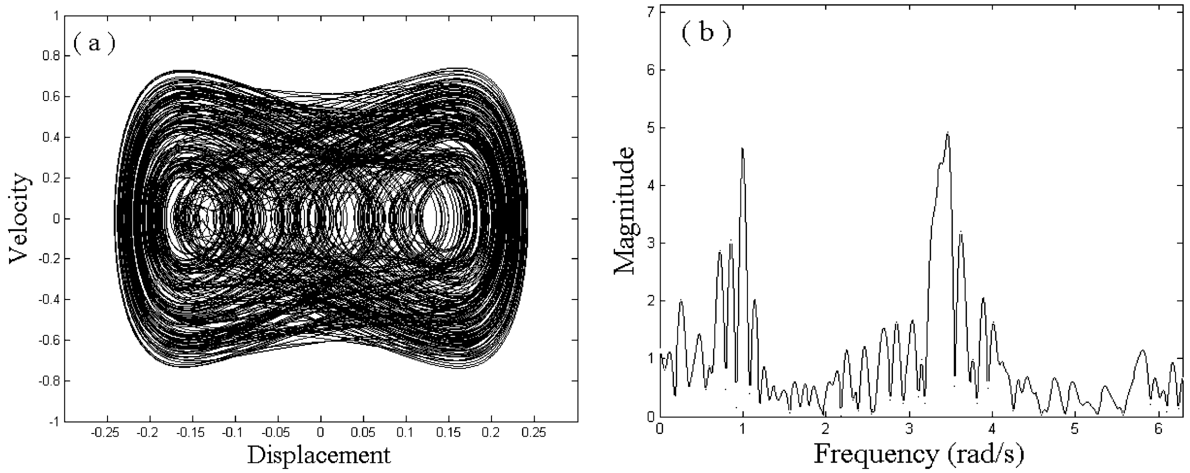

Fig. 2 Uncontrolled SMA oscillator for $\theta = 0.7$. (**a**) Chaotic behavior in the phase portrait and (**b**) power spectrum associated

cation and characterization of the system is the power spectrum. This representation is useful for dynamical analysis. The chaotic motion of the system (9) is observed by the power spectrum for the case $\theta = 0.7$.

Using the proposed feedback control [13, 15] design procedure the chaotic motion of the SMA oscillator is tracked to the desired periodic orbit (34).

For parameters $\theta = 0.7$ and $\mu = 0.1$, the matrix H has the following form:

$$H = \begin{bmatrix} 0 & 1 \\ 0.3 & -0.2 \end{bmatrix},$$

and the matrix B is considered as $B = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$.

Then taking into account that the matrix then $C = [B \ \ HB]$ has $\mathrm{Rank}(C) = 2$, the controllability conditions are verified. Choosing

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad R = [1] \tag{35}$$

then one obtains

$$P = \begin{bmatrix} 2.0111 & 1.3440 \\ 1.3440 & 1.9005 \end{bmatrix} \tag{36}$$

by solving the Riccati equation (23) using the LQR function in MATLAB®. Finally, we can conclude that the optimal function z is given by

$$z = -1.3440 y_1 - 1.9005 y_2. \tag{37}$$

The periodic motion of the controlled system is presented in Fig. 3. The phase portrait behavior is show in Fig. 3(a) and the power spectrum of the controlled SMA oscillator is given in Fig. 3(b).

The trajectory of the time histories of the system without control and controlled are shown in Fig. 4, which signalizes the effectiveness of the used techniques.

Here, we use the Poincare map to characterize the dynamic of the system. When there is no control acting on the system, a representative Poincare section shown in Fig. 5(a) shows a strange attractor behavior of the chaotic motion, but when the control is activated, the attractor chaotic is replaced by one periodic attractor represented in Fig. 5(b).

In order to characterize quantitatively the attractors involved in this study, we computed the Lyapunov exponents by using the classical method described in [28]. Lyapunov exponents can provide a qualitative and quantitative characteristic of dynamic behavior.

Essentially, Lyapunov exponents estimate the sensitive dependence to initial conditions evaluating the exponential rate of divergence or convergence of nearby orbits. Through of the signs of the Lyapunov exponents, we can distinguish among fixed point, periodic motions, quasiperiodic motions, and chaotic motions. If there is any positive Lyapunov exponent, nearby trajectories diverge, evolution is sensitive to initial conditions, and, therefore, the system presents chaotic behavior.

Table 2 contains all values of the Lyapunov exponents for the uncontrolled chaotic attractor and the controlled periodic attractor. As expected, for the chaotic attractor, one of the exponents is a positive

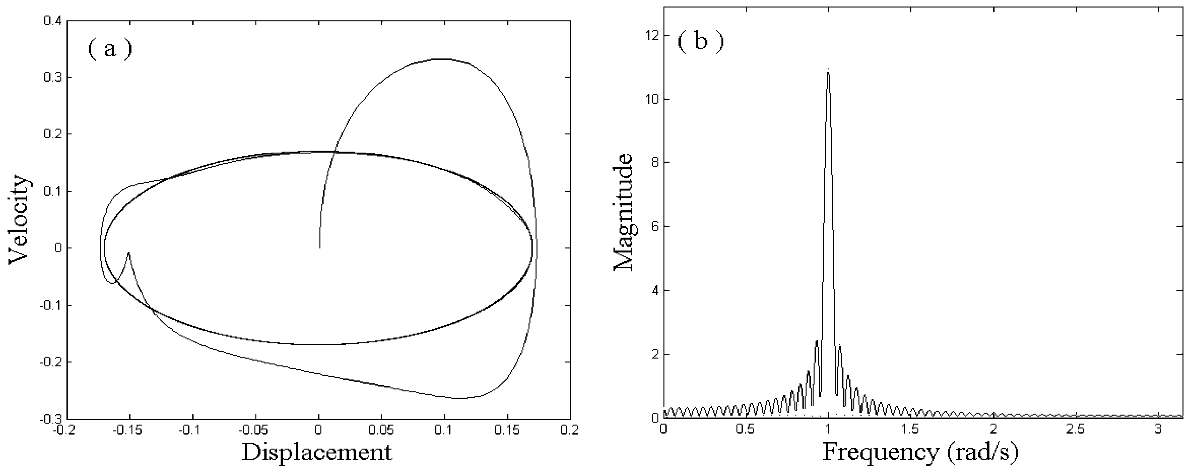

Fig. 3 Controlled SMA oscillator: (**a**) phase portrait and (**b**) power spectrum associated for $\theta = 0.7$

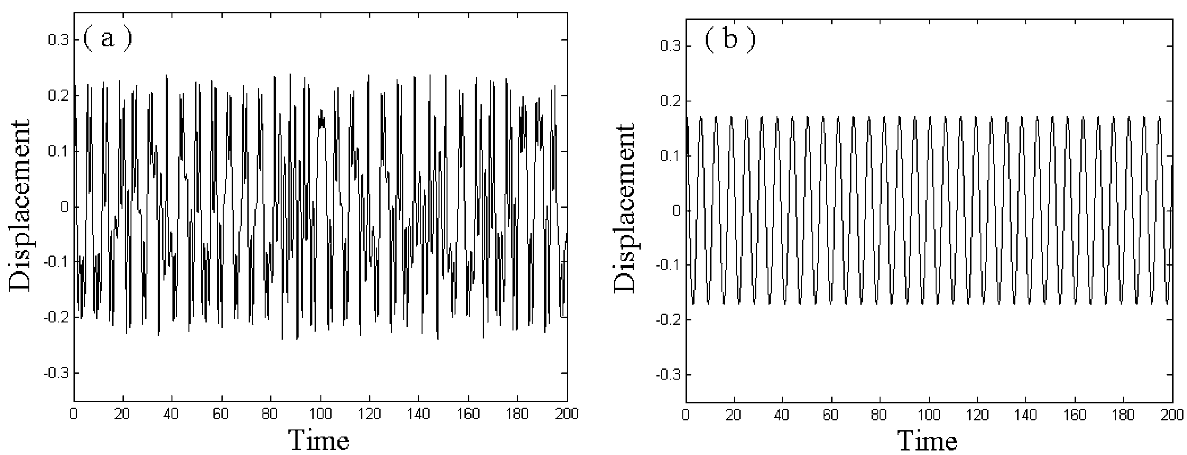

Fig. 4 Time history: (**a**) uncontrolled system and (**b**) controlled system for $\theta = 0.7$

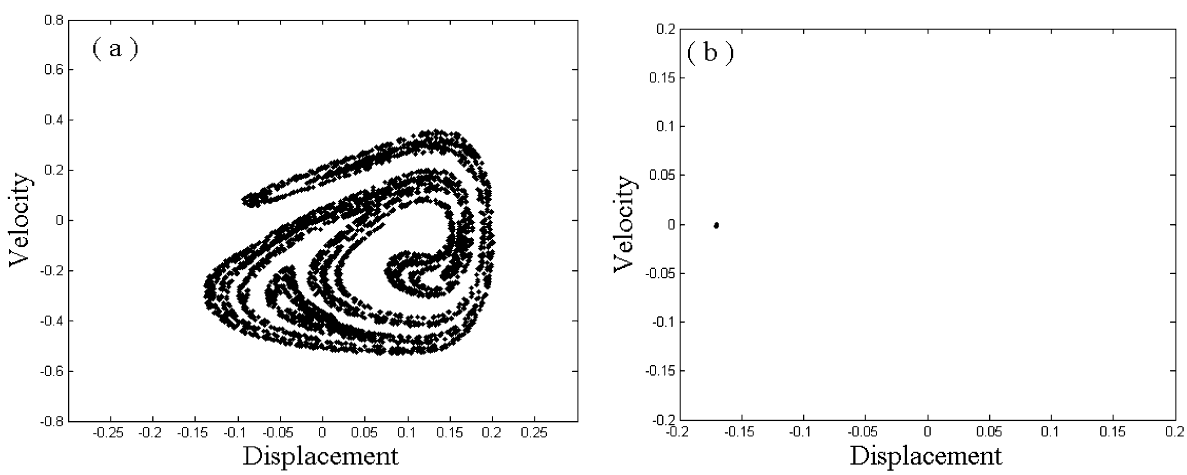

Fig. 5 Poincare map: (**a**) uncontrolled system (strange attractor) and (**b**) controlled system (periodic attractor) for $\theta = 0.7$

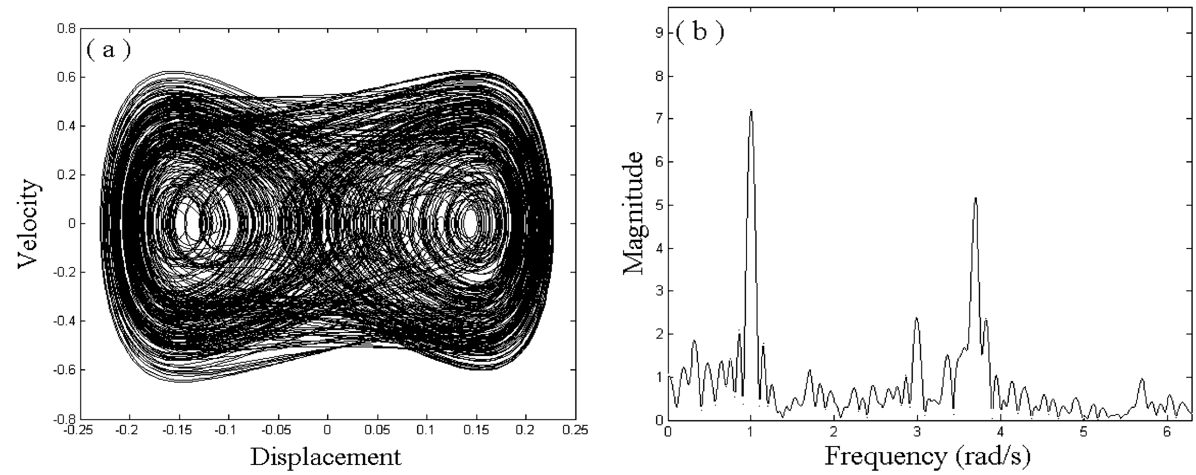

Fig. 6 Uncontrolled SMA oscillator for $\theta = 2$. (**a**) chaotic behavior in the phase portrait and (**b**) power spectrum associated

Table 2 Lyapunov's exponents

System	θ	Attractor type	λ_1	λ_2	λ_3
Uncontrolled	0.7	Chaotic	0	0.231793	−0.331791
Controlled	0.7	Periodic	0	−0.747851	−1.352649

number, whereas for the periodic attractor, there are no positive exponents.

5.2 Analysis of the pseudoelastic behavior (austenitic phase)

In the pseudoelastic effect, the SMA material achieves a very large strain upon loading that is fully recovered in a hysteresis loop upon unloading.

Now we focus on pseudoelasticity, considering a constant temperature $T > T_A$, where at a high temperature ($\theta = 2$), when the alloy is fully austenitic.

Figure 6(a) shows phase portrait for the SMA oscillator motion (velocity versus displacement of the oscillator). When there is no control acting on the system, we found for this temperature a chaotic attractor. The broadband character observed in the power spectrum (Fig. 6(b)) is a characteristic of a chaotic solution.

For parameters $\theta = 2$ and $\mu = 0.1$, the matrix H may be assumed in the following form:

$$H = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix},$$

and the matrix B is considered as $B = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$.

Then taking into account that the matrix then $C = [B\ HB]$ has $\mathrm{Rank}(C) = 2$, the controllability conditions are verified. Choosing

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad R = [1]. \tag{38}$$

Then one obtains

$$P = \begin{bmatrix} 1.8925 & 0.4142 \\ 0.4142 & 1.3323 \end{bmatrix} \tag{39}$$

by solving the Riccati equation (23) using the LQR function in MATLAB®. Finally, we can conclude that the optimal function z has the following form

$$z = -0.4142 y_1 - 1.3323 y_2. \tag{40}$$

By using the feedback control, the chaotic motion of the original system can be controlled to period-1 motion. The oscillator is controlled to the periodic orbit as shown in Fig. 7. In Fig. 7(a), we plot the phase portrait of a period-1 orbit and in Fig. 7(b) show the power spectrum of the controlling periodic motion.

Here, we show numerically that the proposed control method is effective to suppress chaos. The time history of the response of the system (9) without the feedback control is shown in Fig. 8(a), while Fig. 8(b) shows the response of the system (9) under the feedback control.

Two characteristic Poincaré maps for the uncontrolled and controlled system are presented in Fig. 9, whereas the Lyapunov exponent's values are put into Table 3.

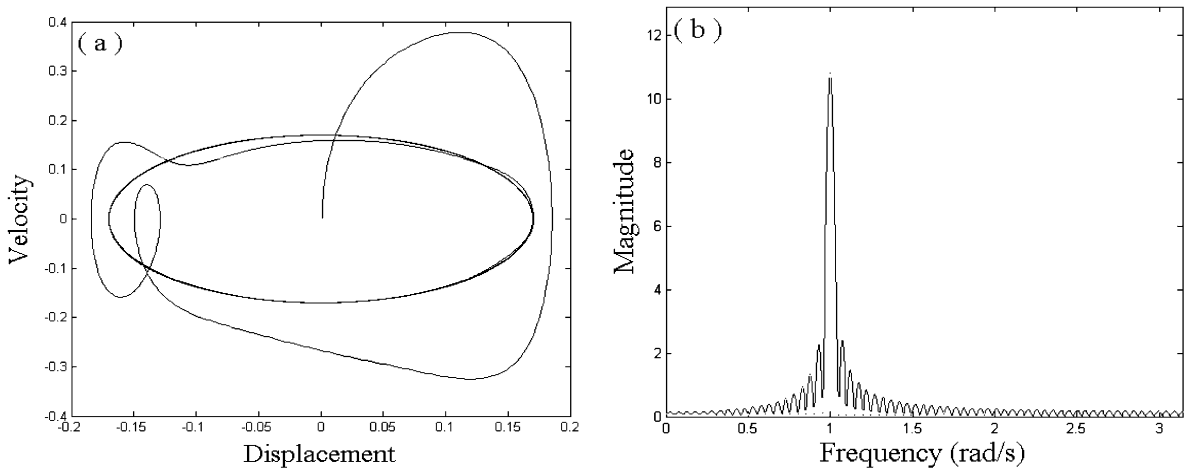

Fig. 7 Controlled SMA oscillator. (**a**) Phase portrait and (**b**) power spectrum associated for $\theta = 2$

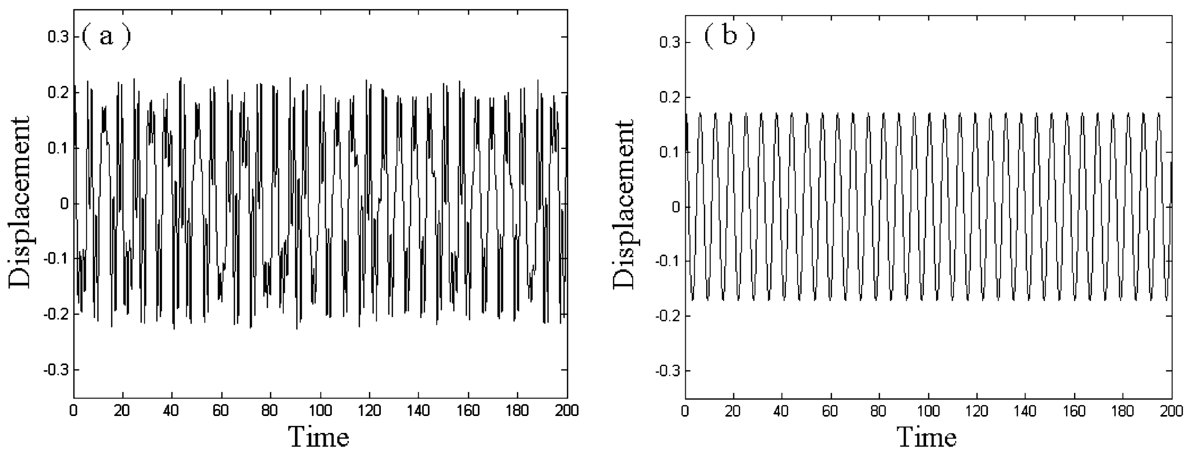

Fig. 8 Time history: (**a**) uncontrolled system and (**b**) controlled system for $\theta = 2$

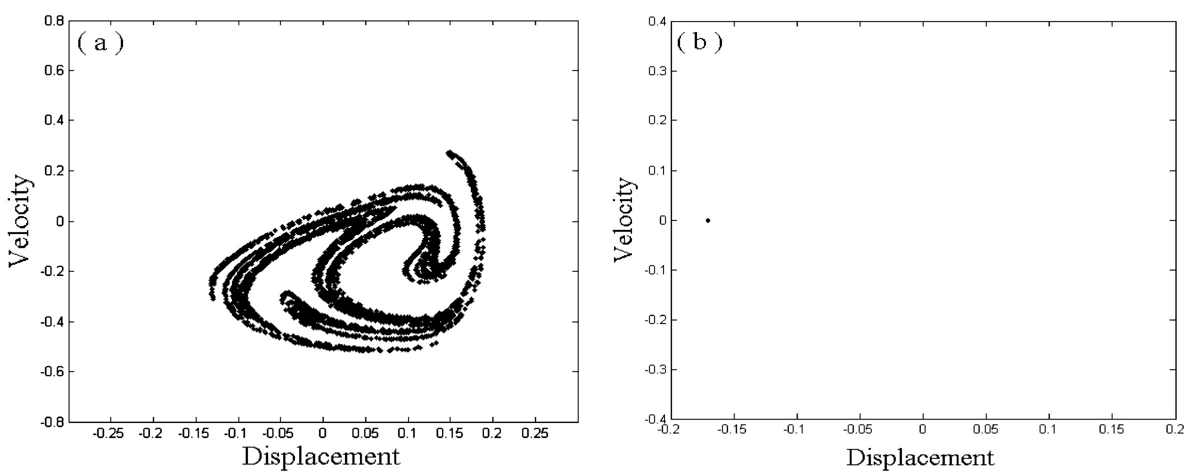

Fig. 9 Poincaré map: (**a**) uncontrolled system (strange attractor) and (**b**) controlled system (periodic attractor) for $\theta = 2$

Table 3 Lyapunov's exponents

System	θ	Attractor type	λ_1	λ_2	λ_3
Uncontrolled	2	Chaotic	0	0.083087	−0.283086
Controlled	2	Periodic	0	−0.765207	−0.767093

A strange attractor on the Poincaré map (see Fig. 9(a)) obtained for an uncontrolled system has the complicated fractal structure with features of chaotic motion. The positive sign of the maximal Lyapunov's exponents in Table 3, for $\theta = 2$, confirms that the system vibrates chaotically. The attractor in Fig. 9(b) obtained for the controlled system has different character than the former one. In this case of motion, any Lyapunov's exponents are positive (Table 3). This means that the system vibrates periodically.

6 Conclusions

In this paper, the linear feedback control problem for the SMA oscillator system has been formulated in order to obtain an optimal control. We have studied the possibility to use the active linear feedback control strategy to modify the dynamics of the SMA oscillator.

The oscillator is studied using a constitutive polynomial model describing the restitution force. We used the method of multiple scales to find analytically the periodic motion equation of the oscillator.

Asymptotic stability of the closed-loop of the nonlinear SMA oscillator system is guaranteed by means of a Lyapunov function which can clearly seen to be the solution of the Hamilton–Jacobi–Bellman equation, thus guaranteeing both stability and optimality.

The periodic and chaotic motions of the nonautonomous system are obtained by numerical methods such as power spectrum, Poincare map, and Lyapunov exponents. The chaotic motion of the uncontrolled system has been detected by using Lyapunov exponents and power spectrum. We observed that the optimal function has a distinct form depending on the temperature. The numerical simulation for the lower temperatures, where the martensitic phase is stable and for higher temperatures, where the austenite is stable shows the effectiveness of the linear feedback control.

We announce that Part II of this work is concerned about special kinds of problems called nonideal problems, that is, when the excitation is influenced by the response of the system. The ideal problems are the traditional ones. Naturally, nonideal vibrating problems have one more degree of freedom than the ideal ones, due to the action of the nonideal source (DC motor with limited power supply) and the interacting terms.

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